Solar and lunar radiometric calibration

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Solar and Lunar Radiometric Calibration

RIT/DIRS Report 01–68–163
August, 2001

Prepared for
Eastman Kodak Company
Commercial and Government Systems

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1 Introduction

1.1 Statement of the Problem

The current generation of Earth-imaging satellites is used primarily to produce photographic quality images that are corrected for inherent non-uniformity in the focal plane (flat-fielding) and non-linearity in the sensor. The next generation of satellites requires the extraction of target reflectance with 1-2% uncertainty from these images. This requires the payload to be radiometrically calibrated to 1% uncertainty, or less.

In order to achieve these levels of uncertainty, a radiometric standard is needed that is accessible from low Earth-orbit. Unfortunately, there is no such standard currently available. One common approach is to utilize man-made source standards that have calibration traceable to NIST (or other primary standards). Another method is to use detector-based standards (also traceable to NIST) and available uncalibrated sources. Unfortunately, these approaches suffer from degradation during exposure to the low Earth-orbit, which causes an unknown departure from the calibrated state (and increases the radiometric uncertainty of the standard).

1.2 Calibration Approach

The capability exists to calibrate photographic payloads for linearity and uniformity using relative methods, which do not adequately address absolute radiometric uncertainty. Current radiometric uncertainties are in the range from 5% to 10% for existing commercial systems (IKONOS, EarthWatch, and Chandra, for example).

In order to address the need for better radiometric accuracy, the photographic calibrator is being redesigned. Included in this effort are 1) reducing fixed pattern noise caused by contamination and optical surface defects, 2) characterizing the spectral throughput of the calibrator to small uncertainty, and 3) addressing the uncertainty of the illumination source (i.e., the Sun).

The solar modeling community is helping to address the uncertainty of the illumination source through some new solar models. These models have the capability to reduce the uncertainties in spectral and bolometric (integrated) solar irradiance. Included are geometric improvements that reduce the uncertainty from 3.5% to 1%, and solar activity models that further reduce the uncertainty from 1% to 0.1%.
2 General Characteristics of the Sun

The following section gives some factual information about the Sun as well as an overview of the most important features of the Sun that are usually taken into account in most solar models.

2.1 Facts about our Sun

The Sun is a self-luminous ball of gas held together by its own gravity and powered by thermonuclear fusion in its core. Our Sun, average in mass, size, and temperature, is a typical star among the various stars in the Galaxy. The distance between the Earth and Sun varies from 91,377,000 miles (147,053,000 km) at perihelion to 94,537,000 miles (152,138,000 km) at aphelion with a mean distance of 92,960,000 miles (149,591,000 km). The Sun is approximately 865,400 miles (1,392,000 km) in diameter, and its volume is about 1,300,000 times that of the Earth. Its mass is almost 700 times the total mass of all the bodies in the solar system and 332,000 times that of the Earth.

The Sun’s surface gravity is almost 28 times that of the Earth; i.e., a body on the surface of the Sun would weigh about 28 times its weight on Earth. The average density of the material composing the Sun is about one fourth that of the Earth, and about 1.41 times the density of water. At its center, the Sun has a density over 100 times that of water, a temperature of 10 to 20 million degrees Celsius, and a pressure of over 1 billion atmospheres.

Observations of sunspots and studies of the solar spectrum indicate that the Sun rotates on its axis from east to west. Because of its gaseous nature its rate of rotation varies somewhat with latitude, the speed being greatest (a period of almost 25 days) in the equatorial region and least at the poles (a period of about 35 days). The axis of the Sun is inclined at an angle of about 7° to the plane of the ecliptic.

The Sun itself is composed of many layers, the most important of which are illustrated in Figure 1. The progression of these layers, from the center of the Sun outward, is the core, radiative envelope, convective envelope, photosphere, chromosphere, and corona.

The Sun’s core is the central region where nuclear reactions consume hydrogen to form helium. The temperature in this region is about 15,000,000 °C (27,000,000 °F). These reactions release the energy that ultimately leaves the surface as visible light. The next region, the radiative envelope, extends outward from the outer edge of the core to an interface layer at the base of the convection zone. The energy generated in the core is carried by
Figure 1: Illustration showing various layers of the Sun [1].
light (photons) that bounces from particle to particle through the radiative envelope. Although the photons travel at the speed of light, they bounce so many times through this dense material that an individual photon takes about a million years to finally reach the interface layer, which is between the radiative envelope and the convection envelope. This interface layer is where the Sun’s magnetic field is thought to be generated. The convection envelope is the outer-most layer of the Sun’s interior, which extends from a depth of about 200,000 km right up to the visible surface. At the visible surface, or photosphere, the temperature is around 5,700 °K. The photosphere appears darker near the edge (limb) of the Sun’s disk because of greater absorption of light by the Sun’s atmosphere in this area. This phenomenon is called limb darkening. During an eclipse of the Sun the chromosphere and the corona (the outer layers of the Sun’s atmosphere) are observed. Also of interest is the high-speed, tenuous extension of the corona known as the solar wind.

The regions that are of interest for solar modeling are those found between the photosphere and corona. Therefore, in the next few sections, we discuss each of these layers in detail.

2.2 Photosphere

The photosphere is a luminous, apparently opaque, layer of gases that forms the visible surface of the Sun. The photosphere lies between the dense interior gases and the more attenuated gases of the chromosphere. The incandescent gases of the photosphere, estimated to be at temperatures near 6,000 °K, are so much brighter than the other layers of the Sun that they seem to form a surface (see Figure 2). These gases are in a constant state of agitation due to convection currents that reach down to 150,000 miles (241,000 km) below the photosphere. The grainy appearance of the photosphere results from differences in the density of the gases from the convective currents. Small bright patches, or granules, are several hundred miles in diameter and are constantly shifting. These granules are small cellular features (about 1000 km across) that cover the entire Sun except for those areas covered by sunspots. These features are the tops of convection cells where hot fluid rises up from the interior in the bright areas, spreads out across the surface, cools and then sinks inward along the dark lanes. Supergranules are much larger versions of granules (about 35,000 km across). These features also cover the entire Sun where the pattern is continually evolving. Other features of the photosphere, described in more detail below, are dark regions called sunspots, and bright vein-like regions often appearing near sunspots, known as faculae.
2.2.1 Sunspots

Sunspots are dark, usually irregularly shaped spots on the Sun’s surface that are actually solar magnetic storms. The temperature of the spots is lower than that of the surrounding photosphere, thus the spots are darker (see Figure 3). All but the smallest show a dark central portion (the \textit{umbra}) with a lighter outer area (the \textit{penumbra}). Studies of the spectra of sunspots show evidence of the Zeeman effect, indicating the presence of a large magnetic field. In addition, measurements of the Doppler effect in the spectral lines show that there is a vortex motion in sunspots similar to that of a tornado on Earth. The lower temperature of the gases constituting a sunspot results from the lower pressure due to the strong magnetic field. Sunspots appear usually only between latitudes from $5^\circ$ to $35^\circ$ north and south of the Sun’s equator. Sunspots are not permanent since the Sun’s surface is gaseous. Because the Sun rotates on its axis, a sunspot cannot be observed continuously for more than about two weeks.

2.2.2 Sunspot Cycle

In 1826 amateur astronomer Heinrich Schwabe began a series of solar observations (in hopes of finding planet Vulcan). By 1843 he had collected enough data to announce the existence of the sunspot cycle. An 11-year
cycle from one period of maximum activity to the next is usually observed (see Figure 4). However, a period during which most sunspots have one magnetic polarity is followed by another period during which most have the opposite magnetic polarity; thus, the cycle actually covers 22 years. During each 11-year period sunspots appear first at higher latitudes and later at latitudes closer to the solar equator as the period progresses. The spots often form in pairs or groups, with a large, long-lived leader spot matched with one or more smaller spots of opposite magnetic polarity. A number of phenomena are associated with sunspots. Sunspot activity produces various disturbances on Earth—these include magnetic storms which manifest themselves as aurorae, interference with radio reception and electric power grids, and disturbances of the magnetic compass. Periods in which an increase in sunspots is observed are called active periods.

2.2.3 Faculae

Another particularly important but less visible set of solar features are known as faculae (see Figure 5). Faculae is the plural of facula, which means “little torch” in Latin. These structures are smaller than sunspots but often more numerous, as can be seen from images of the Sun taken through a filter
which shows emission from ionized Calcium (such as the K line of Calcium at 393.3 nm). Curiously, they are brighter than their surroundings, and more so at the edge of the Sun than at the center. Faculae are associated with strong magnetic field regions, like sunspots are, but their detailed character is not well understood.

### 2.2.4 Coronal Loops

Coronal loops appear as arches that rise into the corona and back down to the Sun’s surface. They are found around sunspots and in active regions. These structures are associated with the closed magnetic field lines that connect magnetic regions on the solar surface. Many coronal loops last for days or weeks. Some loops, however, are associated with solar flares and are visible for much shorter periods. An image of such a loop can be seen in Figure 6.

### 2.3 Chromosphere

The chromosphere is a layer of rarefied, transparent gases in the solar atmosphere. It measures 6,000 miles (9,700 km) in thickness and lies between
Figure 5: Image showing bright faculae surrounding darker sunspots [4].

Figure 6: Image of coronal loop on Sun [4].
2. GENERAL CHARACTERISTICS OF THE SUN

the photosphere (the Sun’s visible surface) and the corona (its outer atmosphere). The chromosphere is an irregular layer above the photosphere where the temperature rises from 6000 °C to about 20,000 °C. At these higher temperatures hydrogen emits light that gives off a reddish color (H-alpha emission). There are numerous features of the Sun that can be seen at this wavelength such as sunspots, faculae, and plage. Plage, the French word for beach, are bright patches surrounding sunspots that are best seen in H-alpha. Plage are also associated with concentrations of magnetic fields. An H-alpha image showing sunspots, faculae, and plage can be seen in Figure 7a. The chromosphere is also visible in the light emitted by ionized calcium (CaK) in the violet part of the solar spectrum. An image of this can be been seen in Figure 7b.

2.4 Transition Region

Above the chromosphere is a very thin layer of the Sun’s atmosphere about 100 km thick over which the temperature rises drastically from 20,000 °K in the upper chromosphere to over 2,000,000 °K in the corona. This region is called the transition region. Images at various temperatures in the transition region from the Solar Ultraviolet Measurements of Radiation (SUMER) instrument onboard the Solar and Heliospheric Observatory (SOHO) can be
2.5 Corona

The last region, the corona, is a luminous envelope surrounding the Sun, outside the chromosphere. Its density is less than one billionth that of the Earth’s atmosphere. The corona is visible in total only at the time of a total eclipse of the Sun. It then appears as a halo of light with an irregular outer edge, often with streamers radiating from the Sun’s surface and contrasting with the dark lunar disk that it borders (see Figure 9). The corona consists of ionized gas at a temperature of 1,000,000 °C. By means of the coronagraph, the innermost part of the corona can be studied and photographed in full daylight. Although the visible corona extends a few solar radii above the Sun, because of its high temperature it produces a continual flow of electrically charged particles called the solar wind that moves outward through the solar system.

We have just described the majority of the layers that make up the Sun, with an emphasis on some of the external layers. The majority of the solar models used in the solar physics community only take into account the regions described in sections 2.2 and 2.3. That is, the photosphere (with associated sunspots and faculae) and chromosphere (with associated plage). This is because most of the magnetic activity that effects the Sun’s output
3 Solar Modeling

We now address the topic of actually modeling the flux output and variability from the Sun. This section gives an overview of three solar models used to predict the spectral irradiance from the Sun. The models presented are published by Lean et. al. (1998) [9], Fligge et. al. (2000) [10], and Fontenla et. al. (1999) [11]. Each model is described below in terms of the model algorithm, data source, model validation, and future outlook for the model. These models are loosely referred to by the name of the lead author in the text which follows. To thoroughly evaluate each model a series of questions were addressed to assess how the models deal with each of the areas discussed above. This line of questioning is best illustrated in Figure 10.

3.1 Lean and Cook Model

The Lean and Cook approach classifies the Sun into 3 separate components. These components are faculae, sunspots and the quiet Sun. The model uses processed calcium K-line (CaK) filtergrams to account for faculae on the Sun’s disk. It then uses databases of sunspot activity and measured values of the quiet Sun to produce an overall spectral irradiance as a function

Figure 9: Image of the corona surrounding the Sun (which is block out) [8].

between the UV and long wave IR is related to features found in these layers.
of heliocentric position on the Sun. Sunspots and faculae are generally considered to be the dominant contributors to solar irradiance changes on time scales of days to weeks.

3.1.1 Model Algorithm

This component model assumes that the irradiance output and variability is entirely attributable to the quiet Sun, sunspots, and faculae. The quiet Sun state describes the output of the Sun in the absence of solar activity (i.e., sunspots and faculae). This is best approximated during times of solar minimum. The quiet Sun irradiance $F_Q(\lambda)$, [$W/m^2$], as a function of wavelength, $\lambda$, reaching the Earth (at a distance from the Sun of 215 times the solar radius) can be expressed as:

$$F_Q(\lambda) = \frac{2\pi I_Q(\lambda, 1) \int R(\lambda, \mu)\,d\mu}{(215)^2}$$

(1)

Where $I_Q(\lambda, 1)$ is the quiet Sun radiance [$W/m^{-1}sr^{-1}$] at the center of the solar disk ($\mu = 1$) and $R(\lambda, \mu)$ is the center-to-limb function as a function of heliocentric position $\mu$, which ranges from 0 (at the edge) to 1 (at the
center). Furthermore, $R(\lambda, \mu)$ is defined as the ratio $I_Q(\lambda, \mu)/I_Q(\lambda, 1)$. A derivation of this expression with geometric interpretations of the variables, can be found in Appendix A.

The Sun is usually not totally free from magnetic activity, especially during times of solar maxima. To account for this additional variation, the quiet Sun is re-written as a net solar output term, $F(\lambda)$,

$$F(\lambda) = \frac{2\pi I_Q(\lambda, 1) \int C(\lambda, \mu) R(\lambda, \mu) \mu d\mu}{(215)^2}$$

where $C(\lambda, \mu)$ is called the contrast function and is defined as, $I(\lambda, \mu)/I_Q(\lambda, \mu)$, the ratio of the Sun’s radiance at heliocentric location $\mu$ relative to the surrounding quiet photosphere.

Separating radiance elements on the solar disk into those that are darker (sunspots) and brighter (faculae) than the quiet Sun, permits the rearrangement of the above expression. Therefore we have,

$$F(\lambda) = F_Q(\lambda) + \frac{2\pi I_Q(\lambda, 1)}{215^2} \int [C_S(\lambda, \mu) - 1] R(\lambda, \mu) \mu d\mu + \frac{2\pi I_Q(\lambda, 1)}{215^2} \int [C_F(\lambda, \mu) - 1] R(\lambda, \mu) \mu d\mu$$

where $C_S(\lambda, \mu) - 1$ and $C_F(\lambda, \mu) - 1$ are the residual contrasts of the darker and brighter elements.

The practical implementation of the net solar expression above is actually achieved by summing over all the elements of an image of the Sun that are darker and brighter than the background quiet Sun. Therefore, if we denote the radiative changes due to sunspots and faculae $\Delta F_S(\lambda)$ and $\Delta F_F(\lambda)$ respectively, we can write a discrete version of the net solar expression as

$$F(\lambda) = F_Q(\lambda) + \sum_{i=1}^{N_{\text{spots}}} \Delta F_S(\lambda) + \sum_{j=1}^{N_{\text{faculae}}} \Delta F_F(\lambda)$$

**Accounting for Sunspots in the Lean Model**

The spectral dependence or change in radiation of sunspots of area, $A$, at heliocentric position $\mu$ on the solar disk relative to the background (quiet) disk-integrated radiation is described as
\[
\frac{\Delta F_S(\lambda)}{F_Q(\lambda)} = \frac{5\mu A_{WDC}[C_S(\lambda) - 1]R(\lambda, \mu)}{2} \tag{5}
\]

where \( A_{WDC} = A/(2\pi R_\odot) \) is the sunspot area in units of the solar hemisphere (as given in the World Data Center records) and \( R_\odot \) is the solar radius. The above expression assumes \( C_S(\lambda) \) is the same at different positions on the solar disk. That is, it is assumed to be invariant with respect to \( \mu \).

We can additionally integrated out the spectral dependence generating a bolometric expression. Typically the bolometric form of the center-to-limb function is used, since the spectral dependency is not well understood. That is

\[
R(\mu) = \frac{3\mu + 2}{5} \tag{6}
\]

Furthermore, if we let \( S = \int F(\lambda)d\lambda \) then the sunspot equation reduces to

\[
\frac{\Delta S_S}{S_Q} = \frac{\mu A_{WDC}(C_S - 1)(3\mu + 2)}{2} \tag{7}
\]

where the \( \lambda \) dependency has been integrated out. Additional simplification allows us to use \( C_S - 1 = -0.3235 \) [12] for the sunspot contrast function. However, there also exists empirical evidence for the dependence of sunspot residual intensity contrast on sunspot area. That is, larger sunspots are darker than smaller sunspots. From this, an area dependence relationship has been developed.

\[
C_S - 1 = -[0.223 + 0.0244\log_{10}(A_{WDC})] \tag{8}
\]

However, the wavelength dependency for the contrast function is not well defined by the solar community and is the subject of ongoing research.

**Accounting for Faculae in the Lean Model**

A similar expression, to that which was developed for sunspots, can be used to account for the Sun’s variability due to faculae.

\[
\frac{\Delta F_F}{F_Q} = \frac{5\mu A_F[C_F(\lambda, \mu) - 1]R(\lambda, \mu)}{2} \tag{9}
\]
where $A_F$ is the area of the facular elements in units of the solar hemisphere. Unlike sunspots, the contrast of faculae depends strongly on $\mu$. It has been identified that faculae are brighter toward the limb than at the disk center, however, the functional form of this variation remains poorly defined and is a major source of uncertainty in the above equation. Additionally, uncertainties in the area $A_F$, and contrast term $C_F$, are significantly larger than those found in the analogous equation for sunspots. The problem is that bright magnetic elements are more dispersed on the solar disk and, at visible wavelengths, have much lower bolometric contrasts than say sunspots.

To aid in the extraction of facular features, images of the K-line of Calcium (CaK) at a wavelength of 393.3 nm are used. It is known that enhanced CaK emission occurs in the vicinity of bright faculae. Therefore, full-disk CaK images provide a methodology to quantitatively determine the influence of magnetic activity on solar irradiance. That is, each pixel of the digitized CaK image has a value that is linearly related to CaK emission from the solar disk.

For this application, the CaK images came from the Big Bear Solar Observatory (BBSO) in Big Bear City, CA. The images are first processed to remove any large-scale instrumental effects while simultaneously removing the center-to-limb variation. Histograms are then generated from these “flattened” CaK images. Even during low solar activity, the histograms (8 bit) are slightly skewed toward the higher digital counts. That is, there are usually more bright pixels than dark ones. These bright pixels show up as “tails” in the histograms. A Gaussian function is then fitted to the most abundant pixels in the histogram. There are some pixels in the high-intensity tail of the histogram, however, that are not accounted for by the Gaussian function. These pixels are the ones that are altered notably by the changing magnetic activity of the Sun and are recognized as surrogates for bright magnetic sources of solar irradiance.

One facular pixel of a CaK filtergram, with a total number of $N_{pix}$ pixels on the disk, has an area $A_F = 1/(2\mu N_{pix})$ in units of the solar hemisphere. We can then substitute this relation into the above equation for facular brightening. This yields an expression that accounts for the full disk irradiance due to this one bright pixel.

$$\frac{\Delta F_F(\lambda)}{F_Q\lambda} = \frac{5|C_F(\lambda, \mu) - 1|R(\lambda, \mu)}{4N_{pix}}$$

All such pixels in a solar image will then modify the radiative output by an amount
\[
\frac{\sum_{j=1}^{N_{\text{faculae}}} \Delta F_F(\lambda)}{F_Q(\lambda)} = \frac{5 \sum_{j=1}^{N_{\text{pix}}} [C_F(\lambda, \mu) - 1] R(\lambda, \mu)}{4N_{\text{pix}}} \tag{11}
\]

where the summation is over all pixels on the disk attributable to magnetic sources of brightness, \(N_{\text{faculae}}\), in excess of the quiet Sun.

Evaluation of the above equation requires the conversion of CaK intensity to the corresponding intensity of radiation at the wavelength of interest. According to Lean, this mapping depends on spatial resolution, since poorer resolution reduces the brightness of smaller scale features which become blended with the background quiet Sun. Lacking specific observations of the bolometric and UV contrasts corresponding to CaK emission in the BBSO filtergrams, the relationship between the intensity ratio \(C_F(\lambda, \mu)\) for radiation at wavelength \(\lambda\), from a pixel at location \(\mu\), with CaK intensity \(I_{\text{CaK}}\) is assumed to be

\[
C_F(\lambda, \mu) = (a + bI_{\text{CaK}})(c + d\mu + e^2 + f\mu^3 \ldots) \tag{12}
\]

Here, the linear relation \((a + bI_{\text{CaK}})\) converts CaK emission to an equivalent intensity ratio at the wavelength of interest [13] and the polynomial expression accounts for the \(\mu\) dependence of this intensity ratio.

### 3.1.2 Data Sources

The Lean and Cook model uses full disk CaK images, recorded with a 0.1 nm filter centered at 393.3 nm, from Big Bear Solar Observatory (BBSO) [14] and sunspot data from NOAA World Data Center (WDC) [15]. In the following section we thoroughly cover information about each data source.

Big Bear Solar Observatory

Big Bear Solar Observatory is located in the middle of Big Bear Lake (in Big Bear City, CA) to reduce the image distortion which usually occurs when the Sun heats the ground and produces convection in the air just above the ground (see Figure 11). Turbulent motions in the air near the observatory are also reduced by the smooth flow of the wind across the lake instead of the turbulent flow that occurs over mountain peaks and forests. These conditions, combined with the usually cloudless skies over Big Bear Lake and the clarity of the air at 2,000 meters (6,750 feet) elevation, make the observatory a premier site for solar observations.

The observatory was built by the California Institute of Technology in 1969. Management of the observatory, and an array of solar radio telescopes at Owens Valley Radio Observatory (OVRO) in Owens Valley, California,
was transferred to the New Jersey Institute of Technology on July 1, 1997. Funding for the operation of the observatory is from the National Aeronautics and Space Administration (NASA), the National Science Foundation (NSF), the United States Air Force, the United States Navy and other agencies. BBSO currently (year being 2001) has more than eighteen researchers in residence, including eight PhDs and eight PhD students.

The observatory’s telescopes are specially designed for solar observations. The top floor of the observatory contains a single fork mount supporting the four main telescopes: a 65 cm (26 inch) reflector, a 25 cm (10 inch), a 20 cm (8 inch), and a 15 cm (6 inch) refractor for Earthshine observations. The telescopes are equipped with highly specialized filters and cameras that isolate small portions of the visible, as well as the near infrared and ultraviolet portions, of the Sun’s spectrum. They collect full disk white-light and CaK (393.4 nm) line images, daily. They also collect H-alpha (the first atomic transition in the hydrogen Balmer series; wavelength = 656.3 nm) images, hourly, and magnetograms from the GONG [16] instrument.

The 25 cm and 65 cm telescopes observe sunspots and active regions as well as filaments, network, and tiny intra-network elements. The 20 cm telescope monitors the whole Sun. The H-alpha full disk images are
obtained with a large format digital CCD and enhanced with modern image processing techniques. Every clear day (300 on average) the 20 cm full disk telescope observes the Sun from sunrise to sunset, obtaining an image every 30 seconds. The high quality of the final full disk image is ideal for the detailed study of the evolution of fine solar structures as well as large-scale solar features. The full disk telescope is part of a network with observing stations at the Kanzelhhe Solar Observatory [17] in Austria and the Yunnan Observatory [18] in China.

BBSO has an open-data policy on the World Wide Web. The data is freely used in classrooms from primary schools to universities, and at other observatories, government agencies and private industry throughout the world.

The observatory plans to build the world’s largest solar telescope housing a 2 meter aperture. The hope is to attract new global attention to the observatory.

World Data Center
Scientific data gathering has a long history, but mechanisms for data distribution and exchange are more recent. The first large-scale international scientific enterprises were the International Polar Years of 1882-1883 which eventually led to the International Geophysical Year (IGY) of 1957-1958. Planning of the IGY was coordinated by the Comite Special de l’Annee Geophysique Internationale (CSAGI), which is the Special Committee for the IGY set up by the International Council of Scientific Unions (ICSU) [19]. CSAGI established the World Data Center system to serve the IGY, and developed data management plans for each IGY scientific discipline. Because of its success, the World Data Center (WDC) system was made permanent and used for post-IGY data.

Over the years the tally of WDCs has changed. A comprehensive set of WDCs was established in China in 1988. The WDC in the U.S.A. has expanded and the WDC in Russia is now operated by three different organizations. Some of the WDC centers in Europe and Asia have moved or have closed, but new centers have opened. World Data Centers are now referenced by the type of center without reference to the country operating the center, i.e. WDC for Glaciology. If there is more than one WDC for a discipline, the name of the city where the WDC resides is appended, i.e. WDC for Glaciology, Boulder. All centers now have computer facilities and most use electronic networks to meet requests, exchange catalog information, and transfer data.
Today the WDC system is healthy and viable. Most centers are maintaining their funding, though not without struggle. Data acquisition, storage, and distribution are expensive. WDCs cost money, but they are cost-effective in transferring data to users and their operational costs represent a tiny fraction of worldwide scientific activity.

The sunspot data for the Lean and Cook model can be found at one of the 13 WDC discipline centers located in the USA. Boulder, Colorado, houses the World Data Center for Solar Terrestrial Physics (STP) [20]. The U.S. Department of Commerce and the National Oceanic and Atmospheric Administration (NOAA) maintain the data center. The WDC for STP, Boulder, is operated by, and collocated with, the National Geophysical Data Center (NGDC).

WDC for Solar-Terrestrial Physics (STP), Boulder collects, analyzes, archives, and disseminates data that describe the space environment from the surface of the Sun to the surface of the Earth including the Sun, interplanetary space, the magnetosphere, the ionosphere, the thermosphere, geomagnetism, and cosmic rays. Solar activity data sets cover a wide range of routine solar measurements and events including sunspot numbers, solar radio emissions, solar x-rays, energetic particles, listings of event characteristics, and solar magnetic fields.

Digital and analog data are available on a variety of media. Digital data are provided for online file transfers or publication quality prints, magnetic media or CD-ROM. WDC-A for STP supports the Space Physics Interactive Data Resource (SPIDR) that allows Worldwide Web users to access, browse, display, and analyze STP data. Electronic access can be gained via the methods in Table 1.

<table>
<thead>
<tr>
<th>Access Method</th>
<th>Address/URL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gopher</td>
<td>gopher.ngdc.noaa.gov</td>
</tr>
<tr>
<td>Worldwide Web</td>
<td><a href="http://www.ngdc.noaa.gov/stp.html">http://www.ngdc.noaa.gov/stp.html</a></td>
</tr>
<tr>
<td>FTP</td>
<td>ftp.ngdc.noaa.gov</td>
</tr>
<tr>
<td>Bulletin Board via modem access</td>
<td>+1 303 497 7319, login: “online”</td>
</tr>
</tbody>
</table>

Table 1: Electronic data access for WDC-A.

Data are available on most media including CD-ROM, via Internet, and other media on request. Specifically for sunspot information, we looked at the section on Solar and Interplanetary Phenomena [21]. This lead to a page on sunspot number (dating back to 1610), position, area, and classifications which one can FTP the data taken from 12 different sites. The sites that contain the data are compiled by the National Geophysical Data Center
Another location that has a record of sunspot activity from 1874 - 2001 is the Royal Greenwich Observatory (RGO) [23]. The Royal Greenwich Observatory (RGO) compiled sunspot observations from a small network of observatories to produce a dataset of daily observations starting in May of 1874. The observatory concluded this dataset in 1976 after the US Air Force (USAF) started compiling data from its own Solar Optical Observing Network (SOON). This work was continued with the help of the US National Oceanic and Atmospheric Administration (NOAA) with much of the same information being compiled through to the present. The entire dataset is available as ASCII text files containing records for individual years. Each file consists of records with information on individual sunspot groups for each day that spots were observed.

3.1.3 Model Validation

The Lean and Cook model compares its results (for brightness components only, i.e., faculae) to bolometric data from ACRIM II onboard the Upper Atmosphere Research Satellite (UARS) and spectral data from the Solar Stellar Irradiance Comparision Experiment (SOLSTICE) also onboard UARS. Though SOLSTICE has a spectral range from 119 - 420nm, Lean only performs her analysis at $\lambda=200\text{nm}$.

Bolometric Irradiance

The bolometric brightness component from the ACRIM II data is computed by subtracting the sunspot darkening component and the bolometric quiet Sun from the total irradiance (see Lean and Cook reference for details). This was performed on a data set in 1991, from October to June. The ACRIM II data is then compared to the bolometric brightness component calculated directly from the residual CaK histograms.

The results of this comparison showed that the measured and constructed bolometric brightness components tracked each other closely during the entire descending phase of cycle 22 with a standard deviation of 0.21 $[W/m^2]$ and a correlation coefficient of 0.9. This was roughly 10% of the 2 $[W/m^2]$ decrease in the bolometric brightness that occurred during the decline of the solar cycle. The comparison was based on 779 data points.

UV Irradiance for Faculae Effects Only

At wavelengths below 200 nm, bright facular emission dominates irradiance variations. This is because the contrast of faculae increases rapidly relative
to that of the competing sunspot depletion mechanism. For this comparison, only the background quiet Sun needs to be subtracted from the overall SOLSTICE irradiance to obtain the brightness component. Additionally, the CaK images are processed to obtain estimates of the UV brightness.

The results here showed that the measured and constructed UV brightness components at 200 nm tracked each other well with a standard deviation of 0.05 \( [mW m^{-2}nm^{-1}] \) (which is less than 10% of the 0.6 \( [mW m^{-2}nm^{-1}] \) 200 nm brightness decrease for the cycle 22 decline) and a correlation coefficient of 0.94. This analysis was based on 614 data points.

As Lean points out, unlike empirical models of solar irradiance variability, in which regressions of facular proxies with the actual radiometric measurements determine the scalings needed to replicate the radiometry, the facular brightening calculations illustrated earlier are independent of the radiometry, except for specification of the absolute value of the quiet Sun irradiance, to place the reconstruction’s on the same absolute irradiance scale as the measurements.

Uncertainties do exist in the construction of the brightening terms. This is because of the lack of rigorous radiometric control of the BBSO CaK images. Uncertainties also come from the space-based radiometry itself. The ultimate interpretation of the sources of variability are limited due to inconsistencies among different total irradiance data sets. However, the close fit of the modeled data to the measured data show that this technique is fairly robust.

3.1.4 Future Outlook for this Approach of Modeling

To improve this type of modeling, more precise determination and characterization of the facular sources of irradiance need to be determined. The NSF RISE Precision Solar Photometric Telescope (PSPT) will help immensely in achieving this goal. Additionally, long term records of the 11-year irradiance cycle need to be maintained and further developed. Here, measurements by UARS and SOHO assist in this goal, but only with the continuing deployment and overlap of radiometric platforms will the uncertainties about the solar cycle be driven down.

3.2 Fligge and Solanki Model

Similar to the Lean and Cook model, the Fligge and Solanki model also partitions the Sun into faculae, sunspots, and quiet Sun, though sunspots are further broken up into umbra and penumbra. They believe the changing
surface area coverage and distribution of magnetic features, like sunspots and faculae, basically explain solar irradiance variations. The model allows the calculation of solar spectral irradiance variations from 0.160 \( \mu \text{m} \) to 160 \( \mu \text{m} \) with varying spectral resolution. It is entirely based on the changing solar surface magnetic field.

3.2.1 Model Algorithm

The Fligge and Solanki model assumes that solar irradiance changes are entirely produced by solar surface magnetic features like sunspots and faculae, though sunspots are broken up into umbral and penumbral regions to be treated separately. This algorithm, like the Lean model, calculates sunspot and faculae information as a function of wavelength and position on the solar disk. However, unlike the Lean model, which uses CaK images from BBSO to extract facular information, Fligge and Solanki use averaged data from Michelson Doppler Interferometer (MDI) magnetograms (see section 3.3.2) to obtain “filling factors” for sunspots and faculae.

The model requires two components to reconstruct solar irradiance variations. First a detailed description of the distribution of the magnetic field on the solar surface and its evolution in time is needed. This information comes from the full-disk MDI magnetograms. Next, the model requires the intensity of each individual component as a function of wavelength and position on the solar disk. This information comes from Robert Kurucz’s spectral synthesis code ATLAS9 (see section 3.3.2) which generates plane parallel model atmospheres of the quiet Sun, sunspots (umbra and penumbra), and faculae.

Similar to the Lean model, which uses a component approach, the intensity, \( I_{i,j}(\lambda, t) \), of a given element \((i, j)\) on the solar disk at time \( t \), and wavelength \( \lambda \), is given by

\[
I_{i,j} = \{ 1 - \alpha_{i,j}^u(\Phi, t) - \alpha_{i,j}^p(\Phi, t) - \alpha_{i,j}^f(\Phi, t) \} \ I^u(\mu_{i,j}, \lambda) \\
+ \alpha_{i,j}^u(\Phi, t) \ I^u(\mu_{i,j}, \lambda) \\
+ \alpha_{i,j}^p(\Phi, t) \ I^p(\mu_{i,j}, \lambda) \\
+ \alpha_{i,j}^f(\Phi, t) \ I^f(\mu_{i,j}, \lambda)
\]

(13)

where \( \alpha^u(\Phi, t) \), \( \alpha^p(\Phi, t) \) and \( \alpha^f(\Phi, t) \) are the filling factors per surface element for sunspot umbra, sunspot penumbra and faculae, respectively, which are extracted from MDI magnetogram images. The intensities are
calculated by the ATLAS9 program where $I^q(\mu, \lambda)$ is the intensity spectrum of the quiet Sun. The filling factors used here are the fractional coverage of a pixel by a particular component (i.e., umbra, penumbra, etc.). In this representation, $\Phi = \phi/\mu$ where $\phi$ is the magnetogram signal and $\mu$ is the heliocentric position. The magnetogram flux $\Phi$, is then converted into the corresponding filling factor $\alpha$.

For sunspots, umbra, and penumbra, the filling factors are set to 1. This is because the sunspots are well resolved by the MDI magnetograms. Faculae, however, are loosely packed and filling factors of less than one are expected. The conversion is based on a linear relationship between the filling factor and the flux value (see the Fligge and Solanki reference [10] for details). That is, the facular filling factor increases linearly from some threshold flux to a saturation flux, where the threshold flux is given by the noise level of the magnetogram. The irradiance $[W/m^2]$ is then calculated by integrating the intensity over the entire solar disk.

### 3.2.2 Data Sources

The Fligge and Solanki model uses two basic data types to reconstruct solar irradiance variations. This information comes from Michelson Doppler Interferometer (MDI) [24] magnetograms and a spectral synthesis code called ATLAS9 [25] by Robert Kurucz. The ATLAS9 code description is deferred to section 3.3.2 where it is additionally used in the Fontenla model.

**Michelson Doppler Interferometer Magnetograms**

The MDI instrument is onboard the Solar and Heliospheric Observatory (SOHO) [26], which was launched on December 2, 1995. MDI is a project of the Stanford-Lockheed Institute for Space Research and is a joint effort of the Solar Oscillations Investigation (SOI) in the W.W. Hansen Experimental Physics Laboratory of Stanford University and the Solar and Astrophysics Laboratory of the Lockheed-Martin Advanced Technology Center.

SOI and the MDI Team form an international collaboration to study the interior structure and dynamics of the Sun. The MDI team was responsible for the design and fabrication, and now for the operation, of the MDI instrument on-board the SOHO spacecraft.

The SOHO spacecraft functioned well after launch in December 1995. Contact with SOHO was lost on June 25, 1998 in which the recovery process began immediately. Contact was reestablished in July/August and by October SOHO was once again operational. With the loss of its last re-
maining gyro, SOHO went off line again for several weeks at the end of 1998. SOHO has since been reprogrammed and is now working again.

The center of the wavelength bandpass is centered on the Ni I photospheric absorption line at 676.78 nm. The width of the bandpass is fixed at 94 mÅ. Nominally, 15 MDI magnetograms are taken each day with a 96 minute cadence. An example of such a magnetogram can be seen in Figure 12.

A directory listing of the magnetogram data set for a particular day can be found at the MDI website [27]. There, files with the extension “.fits.Z” can be found which contain the actual magnetic data. The file sequence number indicates the 96-minute slot. However, these plots may not be available until a few weeks after data collection.

### 3.2.3 Model Validation

The Fligge and Solanki model was compared to VIRGO data on both short (days to weeks) and long term (years) time scales. The VIRGO instrument contains 3 spectral channels (402nm-blue, 500nm-green, and 862nm-red) as
well as a total integrated (TI) channel. The TI absolute accuracy was about 0.16%. The absolute specifications for the sunphotometer were unavailable.

**Short-Term Variability**
The short-term comparison was done over a period of one month and two months, respectively. During the first time frame of a month, a faculae dominated active region crossed the solar disk. The model reproduced this region fairly well in all 4 VIRGO channels (RGB and TI). A visual inspection of the plotted results shows a variation from measured data of 80ppm (0.0080%) for the red and total integrated bands. The green and blue bands showed a variation from measured data of around 180ppm (0.0180%).

The second period of two months had a sunspot dominated active region crossing the solar disk. Here again, the model successfully reproduced the VIRGO data. A closer look at the plotted results shows irradiance variations from measured data on the order of 125ppm (0.0125%).

**Long-Term Variability**
For long term variability, the model reconstructed total solar irradiance variations for approximately 700 individual days between the end of 1996 and mid 2000 using the same model atmospheres and input parameters as for the short-term reconstructions. Again, the reconstructed data fit measured VIRGO data quite well during both minimum and maximum activity periods. Upon visual inspection of plotted results, the overall variability of the Sun’s output irradiance during minimum activity was about 0.03%. The reconstructions track this variability extremely well with percent differences much lower than 0.03%.

Long-term comparisons in a spectral sense were made but with satisfactory results. This is because the long-term sensitivity of VIRGO’s sunphotometers is not stable enough to allow a comparison of the reconstructed and measured spectral irradiance records.

These close fits to measured data illustrate the precision of the model though the data itself may already be pre-biased.

### 3.2.4 Future Outlook for this Approach of Modeling

This model seems to be the first such model to make direct use of magnetic maps (together with calculated intensity spectra) to reconstruct irradiance, rather than proxies such as Ca II K or Mg II radiation which are formed in chromospheric layers. These layers are dominated by different physical processes than the photospheric layers from which most of the radiation
contributing to the total solar irradiance arises.

The success of this type of modeling further strengthens the hypothesis that the magnetic field is the dominant driver of solar irradiation variations. In order to further fine-tune the input parameters of the model, increase the statistical significance of the reconstruction’s and the basic assumptions, longer time periods have to be considered which, preferentially, should cover different levels of solar activity.

The next step in this type of modeling is to use the above technique to reconstruct a longer time series of total and spectral irradiance. What needs to be improved in this type of modeling is the connection between magnetic filling factor and spectrum in faculae. As the filling factor increases the temperature within magnetic flux tubes decreases. The model takes this effect into account but rather crudely.

3.3 Fontenla et. al. Model (Spectral Synthesis)

To address solar variability issues, the National Science Foundation (NSF) started a program called Radiative Inputs from Sun to Earth (RISE). Its mission is to measure and understand the origins of solar radiative variability including measurements and modeling. The modeling or synthesis of the solar spectrum is sponsored by the RISE program. This group addresses the calculation of the amount of energy radiated by different structures on the visible solar surface and how they combine to produce the observed irradiances.

The synthesis work focuses on determining the distribution, evolution, and the physical origins of surface features that contribute to irradiance variations. They combine this information with spectral synthesis models of lines and broad spectral bands to compute spectral and total irradiance for a particular state of the Sun.

The Smithsonian Astrophysical Observatory (SAO) has been developing computer programs for computing synthetic stellar spectra and opacity databases since 1965. These programs have been described by Kurucz et. al. [28, 29]. In performing spectral synthesis, line and atomic data from the SAO program are used in the development of a new code that allows inclusion of new models of structures seen on the solar disk and line source functions computed with various methods, to account for departures from local thermodynamic equilibrium (LTE). This work encompasses a series of approximations, semiempirical parameters, and detailed atmospheric models needed for computing solar spectral irradiances.

The task of calculating the spectrum in detail over broad wavelength
ranges with all relevant lines included is being carried out by Kurucz, [30, 31] who has compiled both measured and calculated opacity data for over 58 million atomic and molecular lines for the purpose of detailed synthesis of solar and stellar spectra. His line opacities can be divided into two broad categories: lines with accurately measured wavelengths, and lines whose wavelengths are uncertain because the corresponding energy levels have been calculated rather than measured. His complete list appears to include most of the lines in the solar spectrum and almost all of the broad-band opacity as a function of wavelength. However, many lines are not at their observed positions.

As for the Sun’s surface, separate atmospheric models are constructed for each of the different structures that appear on the solar disc. A listing of the 7 different structure models, with 3 more to be included in future work, can be see in Table 2. Each of these structures has been observed to have different spectral characteristics corresponding to different magnetic field configurations.

<table>
<thead>
<tr>
<th>Model Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Faint supergranule cell interior</td>
</tr>
<tr>
<td>C</td>
<td>Average supergranule cell interior</td>
</tr>
<tr>
<td>E</td>
<td>Average network</td>
</tr>
<tr>
<td>F</td>
<td>Bright network / faint plage</td>
</tr>
<tr>
<td>H</td>
<td>Average plage</td>
</tr>
<tr>
<td>P</td>
<td>Bright plage</td>
</tr>
<tr>
<td>S</td>
<td>Sunspot umbra</td>
</tr>
</tbody>
</table>

Table 2: Structure models used in synthesis calculations [12].

As the angle $\theta$, formed by the Earth line of sight and solar radial direction, changes there is a gradual falloff in observed intensity. This is called the center-to-limb variation (CLV). To account for this, the model computes the emergent intensity, for each structure type, at $10\mu$ ($\mu = \cos \theta$) positions using an atmospheric model representative of each solar structure. The model additionally interpolates the intensity value for intermediate values of $\mu$. Therefore this approach could be thought of as a 70 component model.

### 3.3.1 Model Algorithm and Theory

The following section describes the spectral irradiance computation, its basis in radiative transfer, the scheme for including departures from local ther-
modynamic equilibrium, and the use of the Fontenla, Avrett, and Loeser (FAL) stellar atmosphere models.

We now discuss the method for computing the intensity at each wavelength and position on the solar disk for each of the model solar atmospheres used in the different structure types (as was seen in Table 2).

As Fontenla explains, the formal integral solution to the radiative transfer equation is

$$I = \int_{0}^{\infty} S \exp(-\tau) d\tau$$  \hspace{1cm} (14)

where $I$ is the intensity $[W m^{-2} sr^{-1}]$, $S$ is the monochromatic source function, and $\tau$ is the monochromatic optical depth along the line of sight. This optical depth $\tau$, is the integral of the total opacity, $\kappa + \sigma$. That is,

$$\tau(x) = \int_{0}^{x} (\kappa + \sigma) ds$$  \hspace{1cm} (15)

where $\kappa$ is the absorption coefficient, $\sigma$ is the monochromatic scattering coefficient, and $x$ is the geometrical depth along the line of sight.

The source function $S$, is the ratio of the total emissivity to the total opacity. That is

$$S = \frac{\epsilon + \sigma J}{\kappa + \sigma}$$  \hspace{1cm} (16)

where $J$ is the local mean intensity and $\epsilon$ is the emissivity. The atmospheric parameters $\epsilon, \kappa, \text{and} \sigma$ along the line of sight determine the variation of the coefficients in equation 15 and 16. These thereby determine the values of $\tau$ and $S$ along the line of sight.

The model assumes a tangent plane-parallel model atmosphere and neglects the surface curvature of the solar surface. In this way, the line of sight optical depth in equation 15 can by replace by $\tau/\mu$, where $\tau$ is now the radial optical depth and $\mu$ is the cosine of the angle between the line of sight and the local radius. This approximation does, however, break down very close to the limb. The break down introduces small variations at the boundaries and is thus ignored. The emergent intensity is then computed from the source function $S$, and optical depth $\tau$, in the single model assigned to the feature at the solar surface.

**Calculation of Absorption Coefficient and Source Function**

This section deals with the approximations and methods used to determine the source function and optical depth scale for each of the 7 component
models. The intensity equation above (equation 14) is rather easy to integrate numerically once the source function is known as a function of radial optical depth. A principal problem in the synthesis of the solar spectrum for estimating the irradiance variability is an accurate accounting for the many absorption lines in the spectrum that reduce the total solar output to about 18% below the value for a line-free continuum. For this purpose the model makes use of the extensive lists of line opacities published by Kurucz and Bell [32].

To increase the computational efficiency in the synthetic spectra, the model computes all values on a denser wavelength grid near the lines, where the opacity and emissivity vary rapidly as a function of wavelength. The model computes intensities at 20 points spaced at 0.01Å intervals around each of the identified spectral lines. Outside the lines the spacing changes to 0.5Å. Because of the large number of lines in the Kurucz tabulation, there are almost no intervals free of lines. Even with this dynamic spacing, the number of calculations are rather large (about 60,000 for each 200Å interval in the blue). An advantage of this type of modeling is that it permits one to identify the sources of the variation easily by looking at the zoomed in spectra where all the line details are visible.

The synthesis model extends from the deepest observable photospheric layers that influence the visible and IR continuum to the lower coronal layers, including the transition region where the most energetic UV line and continuum emissions originate. Through this range the line and continuum formation changes from practically LTE (at the photosphere) to optically thin (non-LTE in the upper chromospheric layers).

**Calculation of the Line Source Function**

In the deep layers (photosphere) where the continuum optical depth at the visible wavelengths is much larger than unity, the source function is very close to the Planck function. That is, LTE approximations apply. For the Sun, the departure from LTE is negligible at the photospheric regions where the bulk of the emitted continuum radiation originates. However, the departure from LTE becomes very large in lower density and optically thin regions responsible for absorption and emission lines (i.e., the upper chromosphere). In these regions the Planck approximation for the source function can not be used. Instead a modified, more detailed, version is used which has a more detailed analysis of the absorption and scattering coefficients. A simple LTE approach would give incorrect results for lines originating in the chromosphere and above.
Atmospheric Models

The synthesis model uses seven semi-empirical atmospheric models (FAL) in order to compute the intensity for the most common solar surface structures. The focus is on accurately reproducing the daily-averaged spectra in the set of surface structures with the reliance on future advances to produce a theoretically consistent picture of all the phenomena in the solar atmosphere. Additionally, no physically consistent models that reproduces all the observed spectral features from the photosphere to the base of the corona are currently available. The available dynamic models are still sketchy and lack the opacity data and non-LTE effects that are included in the spectral synthesis approach.

The semiempirical models are constructed to reproduce observed intensities and profiles at wavelengths from the UV to radio wavelengths. The expectation is to get intensities, computed from these models, to give reasonable absolute intensities and hence, good irradiance estimates.

Atmospheric Model: The Photospheric Layer

The photospheric parts of the models are derived from the observed continua in the visible (VIS) and IR spectrum between 310 nm and 10 mm. The photospheric layers are defined here as those where the visible continuum is formed. Early studies of the solar photosphere showed that, on average, it is very close to radiative equilibrium [33]. The main opacity source in this region is H− with its free-bound and free-free contributions. Since photospheric radiation dominates the total radiative output of the Sun, continued improvements in photospheric models will improve the synthesis of the solar output.

Atmospheric Model: The Low Chromospheric Layer

The low chromospheric layers are defined here as extending from the top of the photosphere (at about 200 km height) up to the temperature minimum at about 500 km height. The low chromospheric layers are mostly transparent in the VIS and IR continuum spectra. However, they have very significant continuum opacity in the near UV. The low chromosphere significantly decreases the radiative output in the VIS because of numerous absorption lines.

Atmospheric Model: The Upper Chromospheric Layer

The upper chromosphere is far more variable than the low chromosphere
with the variability being related to magnetic fields. This region extends up to a height of 1500-2200 km with temperatures varying from 6000 to 8000 °K. The models used for this region are less certain than those of the lower chromosphere. The upper chromosphere is fairly important in the UV even though the contribution to the total solar irradiance is less than 5%. Many of the emissions formed in this layer have large departures from LTE thus making the model difficult to construct while forcing them to include non-LTE calculations.

3.3.2 Data Sources

The model relies on the use of the Fontenla, Avrett, and Loeser (FAL) stellar atmosphere models as well as synthetic stellar spectra and opacity databases such as ATLAS9 by Kurucz. Through the ATLAS9 link, one can download a complete package [34] for spectral synthesis. Several different programs are available which can be used on their own and step by step, calculating a normalized spectrum.

The Kurucz 1993 ATLAS is also available through the astronomical catalog portion of Calibration Data Base and Operations (CDBS) [35]. This catalog contains several atlases consisting of both observed and model stellar, galactic, and emission line object spectra as well as HST standard stars spectra. The Kurucz 1993 ATLAS contains about 7600 stellar atmosphere models covering a wide range of metallicities, effective temperatures, and gravities. The original atlas was created on August 22, 1993 by Kurucz (CD-ROM No. 13). For more details about the CDBS version of this atlas see the read-k93.ascii [36] file. This atlas also contains model spectra for Alpha Lyrae and the Sun. For details see K93MODELS [37]. One can also visit Kurucz’s website [38] to gain access to both ATLAS9 and ATLAS12.

The Fontenla, Avrett, and Loeser (FAL) models have been published throughout the literature [39, 40, 41].

3.3.3 Model Validation

Some initial validation was performed using the spectral synthesis model as compared to data from UARS/SOLSTICE and Neckel and Labs (which is believed to be absolute, not relative) [42]. Several bands from 121.6 nm to 2.2 μm were computed and analyzed. Quiet-Sun computed and observed irradiances were within 3% for Lyα (a chromospheric/transition region line). The departure of the calculations from the Neckel and Labs data for the photospheric bands is 3% or better in the near IR wavelengths at 865.0
nm and less than 1% in the red at 608.0 nm. However, the variation is between 9% and 19% for some wavelengths of the blue 410 nm band. Here the large variation is due to missing absorption lines in the synthetic spectra.

### 3.3.4 Future Outlook for this Approach of Modeling

A future paper will show results from including in the calculation the full non-LTE ionization. This includes diffusion in the transition region for the most abundant elements in the solar atmosphere. Currently, work is being done on calculating spectra in the 392-412 nm, 490-510 nm, and 852-872 nm bands. These are the bands being measured by the Variability of Irradiance and Gravity Oscillations (VIRGO) instrument on board SOHO. Calculations of the bands at 408-412 nm and 605-610 nm for the SunRISE Precision Solar Photometric Telescope (PSPT) are finished and will be studied in a future paper.

### 4 Solar Instrumentation

The previous sections dealt with modeling the flux output from the Sun. All of these sections used some type of instrument platform to validate their results. This next section illustrates some of these instruments. Below are tables that illustrate some of past, current, and future programs used in studying the Sun. Table 3 lists a host of Active Cavity Radiometers (ACR) while Tables 4 and 5 lists some of the spectral instruments used to study the Sun.

### 5 Solar Model Discussion

Thus far we have presented some background information about the Sun, which lead to the explanation of three solar models that attempt to predict its flux output. The next section will compare and contrast each models algorithmic approach while simultaneously illustrating highlights, both good and bad. Additionally, topics such as data origin and reliability for all the models will be addressed.

#### 5.1 Lean and Cook Model Discussion and Analysis

The first model discussed, Lean and Cook, looked at solar variability on time scales of days to weeks. This model assumed that the major contributors to solar irradiance changes were due to sunspots and faculae. The authors
therefore used a 3-component approach (including a background quiet Sun) to model the output flux from the Sun.

When compared to ACRIM II (accuracy N/A) in a bolometric brightness (faculae) sense, the modeled yielded a correlation coefficient of 0.9 and a standard deviation of 0.21 \( [W/m^2] \), which was roughly 10% of the 2 \( [W/m^2] \) decrease in the bolometric brightness that occurred during the decline of the solar cycle. The model was also compared to 200 nm data from SOLSTICE, which has a relative accuracy of 2% and absolute accuracy of 10%. For the SOLSTICE comparison, again, only the effects of faculae, which are much more difficult to model than sunspots, were analyzed. The results here showed that the measured and modeled UV brightness components at 200 nm (faculae) tracked each other very well with a correlation coefficient of 0.94 and a standard deviation of 0.05 \( [mWm^{-2}nm^{-1}] \).

The theoretical approach to the model assumes a fixed distance from the sun of 215 times the solar radius. Though this may be a reasonable approximation, a correction factor that takes into account the Earth’s elliptical orbit should be taken into account. However, it should be noted that the actual data may already have this correction implemented. In fact, this orbit correction for the Sun-Earth distance has been documented and imple-
 Instrument | Project | Operated By | Coverage | Objective | Satellite | Operation  
--- | --- | --- | --- | --- | --- | ---  
4 instruments on it [46] | | | | | | 
Relative Accuracy: NA  
Absolute Accuracy: NA  
Solar Ultraviolet Spectral Irradiance Monitor, Atmospheric Laboratory for Applications and Science | LASP-Univ. of Colorado | 115-440 nm | Uncertainty in SME data was increased by the SeaWiFs [56] community who use the Moon as an irradiance standard. This correction would be fairly trivial to implement in the Lean and Cook model. | SME | Solar Mesosphere | 4-14-89  
SUSIM-ATLAS [47] | 5 Shuttle missions | NRL-Navel research labs | 115-440 nm | Relative Accuracy: NA  
Absolute Accuracy: NA  | |  
SOLSTICE [48] | LASP-Univ. of Colorado | 119-420 nm |  | UARS | Since 9-1991  
Solar Stellar Irradiance Comparison Experiment | Relative Accuracy: 2%  
Absolute Accuracy: 10%  | | One full solar spectrum per day via 15 individual observation | | | 
SOLSPEX [49] | French-Belgian | UV | Irradiance Monitor | |  
Solar Spectrum Measurement | Relative Accuracy: NA  
Absolute Accuracy: NA  | | | | | 
Solar Ultraviolet Spectral Irradiance Monitor | Relative Accuracy: 2%  
Absolute Accuracy: 6%  | | | | | 
SPM [51] | VIRGO | ESA/NASA | 402, 500, 862 nm | | SOHO | Since 12-1995  
Triple Sunphotometer | Relative Accuracy: NA  
Absolute Accuracy: NA  | | | | | 
Table 4: Past and current spectral instruments.  

Another thing to note about the Lean and Cook model is the use of a bolometric center-to-limb function. Though the form of the bolometric center-to-limb function is widely accepted, it lacks the important spectral character needed to obtain accurate irradiance falloff as a function of wavelength. However, there are limb-darkening models that do exist such as one from the Kitt Peak Observatory. This model contains polynomial coefficients that replicate the falloff for 161 spectral bands ranging from 0.3 μm to 1.1 μm.  
The lack of spectral character is also seen in sunspots. The Lean and Cook model simplifies the spectral dependence of sunspots by integrating out the wavelength term in the contrast function. The contrast function is then set equal to a constant (called the sunspot contrast function) or is expressed in a form that has sunspot area dependence. This seems like a very crude approximation for it is known that sunspots do have a wavelength dependency [57]. However, it is unclear at this time as to what that dependency really is. This is a topic of future research.  
When incorporating the faculae dependency in the model, it is empha-
sized that faculae vary strongly with heliocentric position. However, the function form of this variation remains poorly defined and is a major source of uncertainty in the model. Additionally, the uncertainties in the area and contrast terms are significantly larger than those for sunspots. Even with these uncertainties, K-line Calcium images (CaK) were used to quantitatively determine the influence of magnetic activity due to faculae with reasonable success.

### 5.1.1 Lean and Cook: Data Sources, Reliability, and Analysis

The Lean and Cook Model uses sunspot data and CaK images to predict irradiance variability. The sunspot data for the model comes from databases such as those found at the World Data Center (WDC). The WDC, with locations across the world, has been around for some time and has its origins dating as far back at 1882. The U.S. has 13 such centers with one for Solar Terrestrial Physics (STP) located in Boulder, Colorado. The U.S. Department of Commerce and NOAA maintain this facility so it seems to have a reliable source of funding.

The CaK images that Lean and Cook used came from Big Bear Solar Observatory (BBSO) which was built in 1969. Funding for this facility comes from NASA, NSF, USAF, USN, as well as other agencies. The CaK index

<table>
<thead>
<tr>
<th>Instrument</th>
<th>Project</th>
<th>Operated By</th>
<th>Coverage</th>
<th>Objective</th>
<th>Satellite</th>
<th>Operation</th>
</tr>
</thead>
<tbody>
<tr>
<td>SDOE [53]</td>
<td>Solar EUV Experiment</td>
<td>Relative Accuracy: NA</td>
<td>0.1-200 nm</td>
<td>TIMED (4 inst)</td>
<td>Launch 7-1-2001</td>
<td></td>
</tr>
<tr>
<td>SOLSTICE [54]</td>
<td>Solar Stellar Irradiance Comparison Experiment</td>
<td>Relative Accuracy: 0.5%</td>
<td>115-300 nm (0.1-0.2 nm resolution)</td>
<td>LASP-Univ. of Colorado</td>
<td>by Orbital Sciences Corporation</td>
<td>Launch July 2002 (6yr life)</td>
</tr>
<tr>
<td>SIM [55]</td>
<td>Spectral Irradiance Monitor</td>
<td>Relative Accuracy: 0.01%</td>
<td>200-2000 nm (1-34nm res) resolution</td>
<td>LASP-Univ. of Colorado</td>
<td>Make the first precise measurements of the visible solar irradiance</td>
<td>by Orbital Sciences Corporation</td>
</tr>
<tr>
<td>SOLSTICE II [56], or EOS SOLSTICE</td>
<td>Relative Accuracy: NA</td>
<td>5-440 nm</td>
<td>MTPE Mission to Planet Earth</td>
<td>EOS Earth Observing System</td>
<td>Not yet Launched</td>
<td></td>
</tr>
</tbody>
</table>

Table 5: Past and current spectral instruments.
is a very popular proxy and is acquired by other observatories as well such as the Mees Observatory [58].

5.2 Fligge and Solanki Model Discussion and Analysis

The Fligge and Solanki model is entirely based on the changing solar surface magnetic field. Like the Lean model, they believe sunspots and faculae explain solar irradiance variations. This model was compared to the Sunphotometer (SPM) data on VIRGO on both short and long term time scales. The relative and absolute accuracy of the SPM is unknown to the author at this time. A visual inspection of plotted results for short-term variability showed a difference of 0.01% from measured data. For long-term, minimum activity, this difference was 0.03%. For long-term, maximum activity, the difference was around 0.22%.

The algorithm is slightly different than the Lean and Cook model in that it treats the umbra and penumbra of sunspots differently. Additionally, the model uses magnetograms to generate filling factors for sunspots and faculae where the intensity of each of these features comes from Kurucz’ spectral synthesis code ATLAS9. Conversely, the Lean and Cook model modulates a quiet sun intensity based on sunspots form data records and faculae from BBSO images.

The generation of the faculae filling factors is performed using a linear relationship between the filling factor and the flux value. Fligge and Solanki assume the facular filling factor increases linearly from some threshold flux to a saturation flux. It is unclear at this point if the relationship is indeed linear. Furthermore, the magnitude of error associated with this assumption is also unknown.

5.2.1 Fligge and Solanki: Data Sources, Reliability, and Analysis

The Fligge and Solanki model is based on two data sources. That is, magnetograms from MDI and Kurucz’ spectral synthesis code ATLAS9. The discussion of the Kurucz code is deferred to section 5.3.

The MDI instrument, onboard SOHO, is a project of the Stanford-Lockheed Institute for Space Research and Solar Oscillations Investigation (SOI) in the W.W. Hansen Experimental Physics Laboratory of Stanford University. In the past SOHO has gone off-line so this data source is not without its problems. It should be noted, however, that there are other observatories around the world taking similar data. Locations include the US National Observatory at Kitt Peak and Big Bear Solar observatory.
5.3 Fontenla et. al. Model Discussion and Analysis

The NSF started the RISE program, which sponsors the spectral synthesis approach, to address the issue of solar variability. Unlike the Lean/Cook and Fligge/Solanki models, the Fontenla et. al. approach sets out to determine the physical origins of surface features that contribute to irradiance variations. Using opacity databases containing line and atomic data from SAO and Kurucz, they generate spectra for 7 different structures at 10 annular positions on the Sun. This equates to a 70 component model which is significantly different than the 3 component models used by Lean/Cook and Fligge/Solanki.

Solanki and Unruh [59], who use the same model as Fligge and Solanki, also have made estimates of solar spectral irradiance variations. Here the authors use a simple approach based on height of formation, which does not take into account detailed solution of the radiative transfer equations for many lines with their departures from LTE. Even with these differences, the preliminary results using the synthesis approach agree with Solanki and Unruhs findings. Additionally, both parties agree that the changes of the solar spectral irradiance can be fully explained by the magnetic structures on the solar disk.

The problem with the synthesis approach is the inclusion of the millions of lines throughout the spectrum. Though an extensive amount of work has been done to populate the opacity databases, many lines are still missing.

The Fontenla approach attempts to model the Sun’s output from 0.160 \( \mu m \) to 160 \( \mu m \). In doing so, the model must take into account the interactions from the deepest observable photospheric layers to the lower coronal layers. Through this range the line and continua formation changes from LTE at the photosphere to optically thin which is in a non-LTE state in the upper chromosphere. However, in the wavelength range from 0.4 \( \mu m \) to 2.5 \( \mu m \), the origins for solar variability occur on the photosphere only. That is to say, the use of LTE models and approximations fully apply. By using the LTE assumption, which is a very good one at the photospheric layer, we can use rather simple source functions, such as the Planck function, without much modification. The fact that we limit ourselves to the photosphere means that we only have to take into account photospheric effects and not worry about non-LTE models which are mathematical complicated and only increase uncertainty.
5.3.1 Fontenla et. al. Model: Data Sources, Reliability, and Analysis

This approach to modeling relies heavily on the Fontenla, Avrett, and Loeser (FAL) stellar atmospheric models as well as spectra and opacity databases from Kurucz. The opacity databases have been under development since 1965 at the Smithsonian Astrophysics Laboratory. It seems, however, unlikely that this work will be discontented anytime soon given the amount of time already invested and the heavy reliance on it from others in the solar modeling community, such as Fligge and Solanik. A similar statement can be made for the FAL atmosphere models seeing how, both, Kurucz and Avrett work at the Harvard-Smithsonian Center for Astrophysics.

6 Conclusions and Recommendations for use of the Sun as a Standard

There is an on-going thrust to explore other means of calibration as more imaging satellites go on-line. One such technique is to use the Sun as a radiometric standard. In doing so, one needs to explore the workings, make-up, and origins of its variability. In the last few sections we have documented work done in the solar community that addressed these topics. Specifically, three solar models were looked at in which two of the three had similar approaches. Two of the models (Lean/Cook and Fligge/Solanki) used a three component approach that divided the Sun up into sunspots, faculae, and a quiet Sun state. The third model (Fontenla et. al.) used more of a physics based approach to understand the origins of the Sun’s variability, thus dividing the sun into 70 components.

In order to use the Sun as a standard the origins of the variability should be known with as much certainty as possible. Certainly the approach of using a 70 component model versus a 3 component model appears to be more robust. With this said, the use of the spectral synthesis model seems to be most applicable to the problem at hand. There are other factors that support this recommendation as well including the fact that the program is NOAA and NASA supported. Additionally, future experimental data, to compare to synthesis results, will come from the NASA based Solar Radiation and Climate Experiment (SORCE), which is set to launch in July, 2002. The SORCE requirements, which will cover the range from 200 to 2000 nm, are based on the National Polar-Orbiting Operational Environmental Satellite System (NPOESS). The Spectral Irradiance Monitor (SIM) on SORCE is
specified to have an absolute accuracy of 0.03% and a relative accuracy of 0.01%. This new spectral data, which was previously unavailable, will help to validate all solar models including the spectral synthesis approach.

7 Lunar Modeling

The Moon, in addition to the Sun, can also be a potential candidate as a radiometric standard. A second aspect of this project looked at the status of lunar calibration and its current and future uses. The overall goal was to gather a complete view of what data or methods for radiometric calibration are currently being used and are available. Additionally, lunar calibration errors, such as libration, phase angle, opposition effect, and reflectance were investigated while simultaneously searching for means to reduce such errors.

Lunar calibration can be used to verify the stability of absolute calibration however it is not currently available as an absolute calibration source. Hugh Kieffer has started the development of a radiometric model for absolute calibration with expected uncertainties of 1% relative and 2.5% absolute [86]. Lunar surface knowledge is limited to Apollo missions and relatively recent observations. The complete knowledge necessary to encapsulate the phase angle dependence, libration, and reflectance variations and their dependence on each other is not currently available, whereas in Kieffer's work he is essentially bi-passing these issues by taking data over the total possible range of fluctuations.

8 Lunar Background and Significance

Currently the majority of satellites use some method of on-board calibration device like a lamp or reflectance panel, for example. These devices have associated with them certain detrimental properties which can make lunar calibration an appealing alternative. This part of the project is significant because it is believed that by using an extraterrestrial target, such as the Moon, as a source of radiance, a more accurate calibration can be possible. Because the Moon is a relatively stable source, this type of lunar calibration may be easier to implement in a system. This is especially apparent when comparing to onboard methods that degrade with time and are sometimes damaged during launch.

Immediately we know that use of the Moon diminishes the necessity of relying solely on pre-flight calibrations and post flight corrections. If we could radiometrically characterize the Moon’s output, then it would be
possible to use it as a source for post flight calibration. In addition, it would be possible to re-calibrate old imagery to greater accuracy as more information became available for the calibration model.

The long-term objective of this research is to develop a model that would allow a user to calibrate a satellite system to greater accuracy than is currently possible. This could be done with any data that can decrease uncertainty in the model making it possible to improve past results by simply re-implementing the calibration. The goal is to get an overall uncertainty in the calibration of significantly less than 5%. To achieve this, it will be necessary to predict the radiance at the aperture to a high degree of accuracy.

The onboard methods for calibration that are currently used are subject to launch and space conditions, both of which will change the overall calibration and increase uncertainty. Furthermore, onboard calibrators are designed for long-term stability, though drifts of a few percent are commonly accepted in today’s standards [60]. The idea to implement absolute calibration is relatively new while the research to support this type of calibration method has already begun by various Earth Observing Systems (EOS). Many of these Earth Observing Systems use the Moon to verify their calibrations and stability while ignoring absolute calibration. Lastly, it can been seen that the reasons stated above make the possibility of using the Moon as a means of calibration very exciting and worth investigating. One of the most important features of this research is the fact that it will be applicable to basically any orbiting system.

8.1 Advantages of Lunar Calibration

A more detailed look at the advantages for using the Moon as a calibration source reveals a number of assets. First, the Moon’s surface is relatively stable. Previous research into its reflective stability has shown that its surface is photometrically stable to $10^{-8}$ per year for irradiance and $10^{-7}$ per year for radiance at common spacecraft imaging resolutions [61]. Secondly, the spectral features of lunar samples are broad so that the color temperature and structure of sunlight are reasonably well reproduced. Additionally, the Moon’s spectral radiance is within the dynamic range of most Earth-imaging instruments. The Moon, when imaged from Earth at certain phase angles, is surrounded by a black field in the reflective and thermal bands which allows for minimal uncertainty due to stray light. Due to the stability of the Moon’s surface and the fact that it is completely separate from any Earth Observing System (EOS) it will be possible to calibrate multiple systems using the same target [60].
8.2 Complexities with Lunar Calibration

In order to use the Moon as a calibration source, it is necessary to deal with some limitations. There are a few particular features, which make lunar calibration difficult. These features are:

- Lunar surface reflectance characteristics
- Calculation and inclusion of Earth-Moon and Sun-Moon distances for inverse square law distance corrections
- Phase angle effects on brightness
- Libration effects
- Solar output accuracy (not addressed specifically in this report)

The Moon is a non-lambertian surface, which lends to a complicated luminosity function. To develop formulas that describe the radiance of an element of the surface, it is necessary to use three or more independent angles; the incidence angle \( i \), the emergence angle \( E \), and the phase angle \( g \), of the viewing geometry (see Figure 13) [60].

Another factor making the Moon a difficult spectral target is the fact that the observer-Sun-Moon distance is constantly changing. The effects of libration also make using the Moon as a spectral source difficult. This
means that the Moon doesn’t show the same face to the Earth at all times. This libration pattern changes slowly over an 18.6-year period making its characterization a prolonged task [60]. Another difficulty occurs when trying to characterize the surface due to its non-uniformity. Unlike Figure 13, the lunar surface exhibits a large number of crater holes and mountainous regions. In addition to mountains there are seas which are relatively flat. The problem with this is that the seas are much darker than the non-sea areas of the lunar surface and thus would not make good targets for most calibration purposes.

9 Sources of Lunar Calibration Error

9.1 Libration

Libration effects will change the radiance seen from any EOS. Specifically there are two kinds of libration, geometric and physical. Geometric librations are apparent oscillations arising from the fact that the Moon is observed from slightly different directions at different times. Physical libration arises from slight variations in the rotation of the Moon on its axis, caused by minor distortions in its physical shape. Geometric libration occurs in both latitude and longitude, resulting from the Moon’s axis of rotation not being perpendicular to its orbital plane (allowing more of the lunar polar regions to be observed) and from the nonuniform orbital motion of the Moon respectively. This phenomenon enables 59% of the Moon’s surface to be observed from Earth over an 18-year period [63].

In order to completely characterize how this would effect a lunar calibration, it would be necessary to observe the Moon over the libration cycle, simply because it would be extremely difficult to accurately predict reflective properties for portions of the Moon which are not visible now, but perhaps would be a few months from now (perhaps some method could be derived to map soil constituents from Clementine data and used that to extrapolate to a BRDF estimate). Lunar libration is being studied, however, through the USGS by Hugh Keiffer on project ROLO (Robotic Lunar Observatory) [64]. An excellent graphic which shows in detail the libration phenomena can be seen at the following reference [65].

Currently libration effects can be seen in the Sea-viewing Wide Field-of-view Sensor (SeaWiFS) calibration effort. Libration causes the integrated radiance to vary by approximately 11% over the course of the libration cycle [66].
9 SOURCES OF LUNAR CALIBRATION ERROR

Figure 14: Integral phase curve of the Moon at three wavelengths. The small phase angles (<5°) were derived from Clementine data, whereas the larger phase angles were adopted and renormalized from Lane and Irvine (1973) [69].

9.2 Observer-Sun-Moon Distance

The Moon-Earth distance needs to be accounted for in any correction for satellite observations of the Moon. This affects the geometry of the problem and is fairly simply resolved because of the exactness with which the distance can be computed over time. In addition the Sun-Moon distance needs to be corrected for by simply taking into account the inverse square law.

9.3 Opposition Effect

The opposition effect is the narrow peak in the intensity of light scattered from a particulate medium (i.e., the lunar soil) directly back in the direction toward the source [67]. This phenomena can be observed in nearly any surface type, including the lunar regolith. In this case the brightness of any area on the lunar surface increases by nearly 40% within the 48 hours surrounding a full Moon (i.e., phase angle of zero degrees) [67]. This time period translates to approximately 11 degrees of phase [68] on either side of the full Moon and can be seen in Figure 14.

Using Figure 14 as a guide, it can be seen that for low phase angles the jump in brightness is tremendous. This will definitely come into play when using the Moon for lunar calibration. Already the SeaWiFS Calibration and Validation Team (CVT) has decided to avoid the extremely low phase angles
by observing at 7 degrees of phase for their calibration. Indeed if absolute
 calibration were to take place, the characterization of the first ten degrees
 of phase would be very difficult. If this were possible, however, it would
 result in the use of the 48-hour period surrounding zero degrees phase (0
degrees to about 10 degrees) for calibration. There are tradeoffs that exist
 between the sensitivity of the instrument and the signal level which can be
 achieved. While our accuracy would go up, because of the increased stability
 in brightness at larger phase angles, the brightness drops dramatically and
 thus the use of the Moon becomes less appealing.

10 Practicality of Lunar Calibration

In order to get a rough estimate of the apparent exoatmospheric reflectance
of the Earth, a series of MODTRAN runs were performed with the Earth
at various albedos. The Earth was treated as a uniform lambertian reflector
with reflectance values ranging from 10% to 30%. The atmospheric visibil-
ity was set to 25 km while the sensor altitude was fixed at 100 km. The
apparent reflectance as a function of wavelength $R(\lambda)$, was calculated using
the following relation,

$$R(\lambda) = \frac{L(\lambda)}{E_s(\lambda)} \cos \sigma$$

where $L(\lambda)$ is the spectral sensor radiance, $E_s(\lambda)$ is the exoatmospheric
irradiance, and $\sigma$ is the solar zenith angle. The results of these runs can
be seen in Figure 15a. It is seen that, with a typical Earth reflectance of
20% in the visible, the top-of-the-atmosphere reflectance actually takes on a
value of 25%. This 5% increase is mainly due to upwelled or path radiance
reaching the sensor. Therefore, the Earth’s dynamic range extends from 0
to 25%, for a 20% albedo.

The result above says that for optimal calibration we would like the
reflectance of the lunar surface to be one of the end points in the Earth’s
dynamic range scale. Typically though the actual lunar reflectance is around
12%, as can be seen in Figure 15b. There does exits, however, measurements
of lunar soil samples that were found to be as high as 40% in the visible.
These measurements, from the Apollo 16 mission, can be seen in Figure 16.
Though this lunar reflectance is brighter than that of the Earth, it should be
noted that these measurements are not representative of the lunar surface,
on average.
Figure 15: a) Apparent Earth reflectance at 3 albedos and b) four albedo measurements from different dates, including one diffuse measurement of sample 62231 from Apollo 16 [70].

Figure 16: Clementine filters overlay reflectance spectra of Apollo Missions (12 and 16). Additionally, 67455 is another Apollo 16 sample. Note the variability in sample reflectance’s taken from Apollo 16 site. The bright value, 67455, is likely a sample for the brightest materials taken from the moon, while the other Apollo 16 sample is probably a high average. The Apollo 12 sample is most likely a low average [71].
11 Current Available Lunar Data

11.1 Albedo Value

Lunar albedo values can be found in various places. Albedo is defined as a “photometric term for the fraction of incident light that is diffusely reflected from a surface” [72]. More specifically, NASA’s Moon Fact Sheet [73] distinguishes between two values for lunar albedo, bond albedo and visual geometric albedo. Bond albedo is the fraction of incident solar radiation reflected back into space without absorption, dimensionless. It is also called planetary albedo. Visual geometric albedo is the ratio of the body’s brightness at a phase angle of zero to the brightness of a perfectly diffusing disk with the same position and apparent size, dimensionless [74]. Values given for albedo are 0.11 for bond and 0.12 for visual geometric.

11.2 Spectral Data

In the whole history of human studies on the Moon a total of three sources have been identified for studying lunar samples on Earth. The first two sources are based on missions to the Moon and the locations from which the samples were taken are known. Both the Apollo missions (US) and the Luna missions (unmanned U.S.S.R.) have returned soil samples as seen Figure 17. The third source for lunar soil comes from the Antarctic ice cap where 10 meteorites have been recovered. Chemical studies have provided convincing evidence for their lunar origin, although it is believed that they are not from the same areas as the nine sites sampled by the Apollo and Luna missions [75]. Lunar soil samples were obtained on multiple Apollo missions to the Moon. These samples have since been used in laboratory analysis to try and characterize spectral reflectivity of the lunar surface.

The question then arises, how can these soil samples help in a lunar calibration? There are several possibilities for this. The data could be used solely to generalize what lunar surface reflectance’s are over a region of the surface. While this method is very difficult to put uncertainty values on, the data could be compiled to some degree of accuracy if used in conjunction with another lunar imager. Another method would be to try implementing this data into a model for lunar surface reflectance’s (see section 13). Perhaps the soil samples in this case are only useful for applications such as verifying what surface models or new imagers can produce.
11 CURRENT AVAILABLE LUNAR DATA

Figure 17: a) Map of lunar landing mission locations. The mare regions (light gray) are actually darker regions of the moon. Note that this accounts for the difference in spectra for Apollo missions 12 and 16 in Figure 16 [76]. b) Map for comparison with (a) again showing Apollo landing sites overlaying lunar image. Reflectances are more clearly represented here [77].

11.3 Clementine Observations

The Clementine mission to map the Moon was quite a success. It returned data from both the near and far side of the Moon ranging over the entire surface. The bandpass regions were illustrated earlier in Figure 16. Although the data from this mission may not be enough for a complete calibration, it is enough to help verify results seen from other observations which are around the bandpass regions of the Clementine instrument. The data could be used simply as a check for new data in certain locations. Perhaps, if a specific region was to be used for calibration, the fine spatial resolution (125-250 meters per pixel) [78] of the Clementine data could be helpful in characterizing the surface reflectance. Additionally and perhaps more importantly, the Clementine data would be useful for identifying an area on the surface of the Moon which is highly uniform. This kind of data would allow the selection of a region of interest for use by sideslither detectors. A sample Clementine image is shown in Figure 18.

If it were necessary to compile data for use in characterizing the surface, the Apollo 16 landing site would be the location on the Moon to use. The reasons for this can be seen in the increase in reflectance as well as the
Figure 18: Clementine mosaic of the Apollo 16 Landing site [79].
large amount of soil returned on the mission. The amount of error that is
introduced due to non-uniformity over this site is unknown, however, of the
areas on the Moon this site is clearly one of the best choices.

12 Current Lunar Calibration Projects

12.1 SeaWiFS

The SeaWiFS CVT uses monthly lunar calibrations to monitor long-term
stability of the radiometric calibration of SeaWiFS over the course of the
mission. The SeaWiFS instrument is rotated so that it points at the Moon,
and thus takes a picture of the full Moon, with space in the background. This
is done normally once a month at 7 degrees of phase (or as close to 7 degrees
as possible in order to avoid the rapid brightness changes caused by the
opposition effect described earlier). The data is corrected for a few factors,
including Sun-Moon distance, the SeaWiFS-Moon distance, the illuminated
fraction of the lunar surface as a function of phase angle, and a fourth factor
to correct for oversampling of the lunar image during the calibration (this is
a function of pitch rate and the apparent size of the Moon). Once corrected
it is compared, on a monthly basis, to previous calibrations to determine
stability changes [81]. Essentially the same method is currently employed
by a number of satellites which are on orbit or will be coming into orbit in
the near future. The SeaWiFS paradigm for calibration is shown in Figure
19.

Advantages of this method for lunar calibration include use of the full
imaging optics. The problem is, however, that this type of calibration is not
absolute. The step from this type of calibration to an absolute calibration
is tremendous, both in what has to be known and methods of finding such
information.

There also exists the possibility of transferring the SeaWiFS calibration
to another instrument. SeaWiFS was calibrated when it was put into orbit.
One possibility for transferring the calibration would be to invert SeaWiFS
original measurement from radiance to reflectance units. It then would be
possible to use this data for another instrument looking at the Moon. A
problem herein is that the resolution with which SeaWiFS images the Moon
is not extremely high. Typical observations result in an oversampling of the
Moon due to pitch rate, giving approximately 26 scan lines by about 7 pixels
diameter, as seen in Figure 20 [81].

Difficulties with this kind of calibration arise when trying to capture the
Moon at 7 degrees of phase. Although a correction can be used to counter
Figure 19: SeaWiFS calibration paradigm [80].

Figure 20: SeaWiFS lunar calibration image [82].
differences from this phase angle, data taken from within a threshold limit is invalid in this calibration and cannot be used. Another issue with this is that libration is essentially not accounted for. On a monthly basis however, the variation from this effect (again 11% variation in radiance values over the total libration cycle) is small enough that if the calibration were to be off it would be a noticeable difference.

SeaWiFS phase correction, which is currently the only known phase correction with application to lunar calibration, is discussed briefly here. Up until 1999 SeaWiFS phase angle correction had been monochromatically applied across the bands. Barnes [83] presented a new version which did allow for some spectral dependence on the phase correction. It is necessary to point out, however, that the SeaWiFS project doesn’t possess a model of the lunar surface that will allow the correction for lunar reflectances at phase angles 2 degrees or more away from 7 degrees. It is expected though that this phase angle correction factor be only a temporary measure until the model from project ROLO is available [83].

Made clear in the paradigm above is the use of the Moon for calibration, yet it is not nearly as major a component in the calibration process as are other methods. That is, this type of calibration, while it serves a purpose, is not an absolute calibration method and thus its utility is limited. The utility of this calibration has proven itself in providing changes derived from the lunar measurements. Changes to the overall system were characterized by 1% uncertainty [84]. Shown in Figure 21 is the variation in the system derived from lunar observations.

12.2 ROLO

The United States Geological Survey (USGS) operates an automated observatory called the Robotic Lunar Observatory (ROLO) which is dedicated to the radiometry of the Moon [60]. These observations have been going on since 1995 in the visible region and more recently have expanded into the short-wave infrared. The specific goal of the project is to come up with a model which will allow any satellite to be calibrated using the Moon. Project ROLO operates every night during the half of the month when the Moon is gibbous (the two weeks centered on the full Moon), with a typical run observing about 60 photometric stars and the Moon. This data is compiled into above-the-atmosphere lunar brightness images and is used to develop a photometric model of the Moon.

ROLO’s resolution will be equivalent to 15 meters [86]. Radiance models of the Moon can then be produced for the calibration of Earth-orbiting
Figure 21: Changes in the radiometric sensitivity of SeaWiFS determined from lunar measurements. These results have been normalized to the average of bands 3 and 4 for each measurement. The fitted curves shown here are to be used in the overall calibration equation. The curves are combinations of straight lines and one or more quadratic functions [84].
imaging satellites. The current system can collect wavelengths from 350 to 2450 nm [85]. The observatory does atmospheric correction by looking at about 10 stars which have known brightnesses, thus it computes extinction coefficients. The onboard calibration is done with NIST standard lamps. This model is particularly appealing and is being sought by numerous agencies because it is truly one of the best ways to characterize lunar brightness as a function of wavelength and spatial attributes. It in effect eliminates the cyclical libration effects because of the span of the observation period. It does the same for phase angle changes and although not all phase angles are having data taken for them, the model when available, will have a much better degree of accuracy than anything else available to date. The model can be continually built upon and improved with the only limiting factor here being a giant change in the lunar surface feature. Expected errors are under 1% relative and 2.5% absolute [86].

The output of project ROLO will be radiance images which are mapped to the lunar grid at 120,000 points for each filter [86]. The images will be of exo-atmospheric radiance and will be based on the current libration-phase relationship. Hugh Kieffer will be presenting an update on the model at the SPIE conference in the beginning of August 2001. Hopefully this will be to announce the upcoming release of some of the data which has been collected. The ROLO bandpass regions are shown below in Figure 22. These are of significance because they are by far the most detailed consistent look at the Moon we have had to date. They cover the range that will be necessary for absolute calibration with the sensors of today. A brief summary comparing the ROLO and SeaWiFS programs can be seen in Table 6.

<table>
<thead>
<tr>
<th>Programs</th>
<th>Type of Calibration</th>
<th>Applicability to Other Spacecraft</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>SeaWiFS</td>
<td>Monthly, to verify long term stability of calibration</td>
<td>Could apply initial SeaWiFS data by using inversion to develop reflectance image (would not correct for some phase and libration effects)</td>
<td>Detected 1% change in instrument stability</td>
</tr>
<tr>
<td>Project ROLO</td>
<td>Long term observation to generate database of radiance maps for Moon at different phase and libration angles</td>
<td>Will be useful for any EOS, specifically those that can make the attitude maneuver to image the Moon with their primary optics</td>
<td>Root sum square of total errors given as min, expected, and max respectively: 0.41, 1.05, 2.45 [86]</td>
</tr>
</tbody>
</table>

Table 6: Comparison of SeaWiFS and ROLO projects.
Figure 22: a) VNIR bandpass regions and b) SWIR bandpass regions for ROLO. Green line is the normalized solar spectrum. The black line is the typical atmospheric transmission curve for Flagstaff (airmass = 1.0). The magenta line is the detector quantum efficiencies. The blue line is the filter transmissions for stellar bandpass pairs and regions of high lunar contrast while the red line is the filter transmissions for cross-calibration with other EOS instruments [87].

13 Hapke Model

Hapke’s (1981, 1984, 1986) bidirectional reflectance equations provide the most rigorous description of planetary and specifically lunar soils photometric behavior in terms of physically meaningful parameters. According to Hapke’s 1986 version, the soil is characterized by six model parameters [88]. These parameters are:

1. The efficiency with which average particles scatter and absorb light is described by the average particle single scattering albedo, \( w \).

2. The opposition effect is characterized by two parameters. First, \( h \), relates the angular width of the opposition surge to the combined effects of regolith (lunar soil) compaction, rate of compaction with depth, and particle-size distribution.

3. The second term \( S(0) \), describes the contribution of light scattered from close to the surface of an average particle at zero phase.

4. The particle phase function is modeled by a Legendre polynomial. The first-order coefficient is \( b \).
5. The second-order coefficient is $c$.

6. The final parameter is $(\theta_{\text{bar}})$ which is a mean topographic slope angle. This parameter provides a measure of macroscopic roughness at sub-resolution scales. According to Hapke [89], “This parameter is the mean slope angle averaged over all distances on the surface between upper and lower limits that are determined by the angular resolution of the detector and the physics of the detector on the surface of the planet, which in planetary remote sensing is typically hundreds of meters to kilometers”. Hapke also states that it might be thought that the lower limit is given by the sizes of the particles making up the surface. However, it must be remembered that the radiative transfer equation for a particulate medium, on which solutions for the reflectance are based, implicitly averages the radiancve over distances that are large compared with the distances between particles. Thus, the lower limit is several times the mean particle separation. A typical size for this lower limit is of the order of a millimeter.

Hapke’s model is by far the leading model for using physical characteristics to describe surface soil reflectances. Some variations and improvements to Hapke’s model have been made since its inception in the 1960’s (Helfenstein, Veverka and Hillier, 1997). While perhaps Hapke’s model in the long run will not prove helpful in using the whole Moon for absolute calibration (as it requires surface characteristics and soil features), it may prove useful in helping to characterize a specific region of the Moon like the Apollo 16 landing site. It may be possible to use the data from lunar soils taken and implement this into Hapke’s model. While it would be essential to give the model a wavelength dependence, this could be done by using soil samples from which we have obtained complete BRDF’s. The albedo term could be replaced by these values, thus giving the spectral dependence necessary for an absolute calibration. The problems that this could present to an already highly intricate model are numerous and extremely involved. The spectral dependence of all other terms would need to be known or approximated in some way. This is, by far the most difficult question to answer. Additionally, how large an area of the Moon would we be able to say is characterized? Given perhaps limitations of current imaging instruments, this may not be feasible as the area might not be big enough. It should also be noted that a highly detailed look into the utility of the model as well as the inclusion of a spectral dependency, has not been presented in this report.
14 Lunar Conclusions and Recommendations

Lunar calibration is currently possible using the Moon as a constant standard from which to determine variation in overall calibration. By looking over a time-scale, any large variation in what is seen by an instrument can be traced to the overall calibration and perhaps corrected for. The SeaWiFS project is an example of how this type of calibration is implemented (it has detected a 1% change in absolute calibration), and is continually being improved upon. The use of the Moon for absolute calibration would be difficult because of a lack in fundamental knowledge of the Moon. While we have soil samples which provide spectrally dependent reflectance data, this is some of the only spectral data we have from the Moon.

Other factors that are not well characterized, which would effect an absolute calibration, are phase angle changes in reflectance and libration effects. Despite this lack of knowledge the hope of the remote sensing community seems to ride on the work being done in Flagstaff, Arizona, on Project ROLO. This is a USGS sponsored project being headed by Hugh Kieffer. Its intentions are to characterize the Moon well enough that it could be used for absolute calibration. It does this not by understanding the physics of the problems like phase and libration, but by simply observing over a complete cycle of phase and libration changes and then saying that as the Moon again cycles through these changes, it will be the same as the previous cycle. The hope is to have a model of radiance images accurate to within 1% relative and 2.5% absolute.

Future missions to the Moon could provide even more data which is necessary for this type of calibration. One in particular is the SIR (Smart-1 Infrared Spectrometer) instrument on SMART 1 (Small Mission for Advanced Research and Technology) which is expected to launch in December, 2002. SIR is a compact low mass spectrometer designed to measure reflectance spectra of the lunar surface from 900 to 2400 nm. The primary mission will be for mineralogical composition studies, and unlike Clementine, will measure complete spectra with much higher spectral resolution. Also of use will be the spectra obtained over a large range of viewing geometries for studying phase angle and space weathering effects [90].
15 Appendix A

15.1 Derivation of Quiet Sun Irradiance Reaching the Earth

We derive an expression that computes the top-of-atmosphere (TOA) irradiance from the Sun as seen from Earth, as a function of heliocentric position, \( \mu \), and wavelength \( \lambda \). We start by introducing a definition for radiance:

\[
L(\lambda) = \frac{dE(\lambda)}{d\Omega \cos \alpha}
\]

(18)

where the definition for the given variables are illustrated in Figures 23 and 24. We then write this differential form as irradiance, \( E \).

\[
dE(\lambda) = L(\lambda) \cos \alpha \ d\Omega
\]

(19)

upon integration of both sides over the solar disk we have

\[
E(\lambda) = \int_{\text{solar disk}} L(\lambda) \cos \alpha \ d\Omega
\]

(20)

Since the angle of the Sun, as viewed from the Earth is about 0.52 degrees, we will use a small angle approximation, \( \alpha \approx 0.26 \). Thus \( \cos \alpha \rightarrow 1 \) and the \( \alpha \) dependency is gone.

\[
E(\lambda) = \int_{\text{solar disk}} L(\lambda) d\Omega
\]

(21)

We now need to convert \( d\Omega \) into a more tractable form to perform the integration. By definition:

\[
d\Omega = \frac{dA_{\text{proj}}}{r_s^2}
\]

(22)

where

\[
dA_{\text{proj}} = dA \cos \theta
\]

(23)

by construction we have

\[
dA_{\text{proj}} = (r_s d\theta)(r_s \sin \theta d\phi) \cos \theta
\]

(24)

Therefore upon substitution we have

\[
d\Omega = \frac{r_s^2 \sin \theta \cos \theta}{r_s^2} d\theta \ d\phi
\]

(25)
Figure 23: Irradiance from the Sun’s (hemi) sphere as seen from Earth.
Figure 24: Irradiance from the Sun’s (hemi) sphere projected as a disk.

We can now substitute this expression in the relation $E(\lambda)$, with a new dependency on $\theta$ while introducing the explicit dependency of $L$ on $\theta$. $L(\theta, \lambda)$ is the radiance at $\theta$ toward the observer.

$E(\lambda) = \int \int_{\phi=0}^{\phi=0/2} \int_{\theta=0}^{\theta=\pi/2} \frac{L(\theta, \lambda) r_s^2 \sin \theta \cos \theta}{r_s^2} d\theta d\phi$  

Upon rearrangement we have

$E(\lambda) = \int \int_{\phi=0}^{\phi=0/2} \frac{L(\theta, \lambda) r_s^2 \sin \theta \cos \theta}{r_s^2} d\theta d\phi$  

We will assume that there is no azimuthal $\phi$, dependence across the disk. That is, for every zenith angle $\theta$, the energy from the precession of $\phi$, which will form continuous contour lines in Figure 24, is constant. Therefore,

$E(\lambda) = 2\pi \int_{\theta=0}^{\theta=\pi/2} \frac{L(\theta, \lambda) r_s^2 \sin \theta \cos \theta}{r_s^2} d\theta$  

We now write the expression to integrate the radiance $L(\theta = 0, \lambda)$ at the center of the disk $\theta = 0$ over the entire disk using the center-to-limb function
$R(\theta, \lambda)$ to define the ratio of $L(\theta, \lambda)/L(0, \lambda)$ at heliocentric position $\theta$ which ranges from 0 to $\pi/2$.

$$E(\lambda) = 2\pi L(0, \lambda) \int_{\theta=0}^{\pi/2} \frac{R(\theta, \lambda) r_s^2 \sin \theta \cos \theta}{r_es^2} \, d\theta$$

we now let $\cos \theta = \mu$. Therefore, $d\mu = -\sin \theta \, d\theta$. Additionally, the ratio $r_es/r_s \approx 215$. This yields

$$E(\lambda) = -2\pi L(1, \lambda) \int_1^0 \frac{R(\mu, \lambda) \mu}{(215)^2} \, d\mu$$

we can rearrange the limits of integration, which leads to

$$E(\lambda) = \frac{2\pi L(1, \lambda) \int_0^1 R(\mu, \lambda) \mu \, d\mu}{(215)^2}$$

Lastly, we bring some commonality to the nomenclature used by renaming the variables to match those found in the Lean and Cook model. We can do this if we let

$$F_Q(\lambda) = E(\lambda)$$
$$I_Q(1, \lambda) = L(1, \lambda)$$

This simple substitution yields

$$F_Q(\lambda) = \frac{2\pi I_Q(1, \lambda) \int_0^1 R(\mu, \lambda) \mu \, d\mu}{(215)^2}$$
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