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Using multispectral information to decrease the spectral artifacts in sparse-aperture imagery

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ABSTRACT

Optical sparse-aperture telescopes represent a promising new technology to increase the effective diameter of an optical system while reducing its weight and stowable size. The sub-apertures of a sparse-aperture system are phased to synthesize a telescope system that has a larger effective aperture than any of the independent sub-apertures. Sparse-apertures have mostly been modeled to date using a “gray-world” approximation where the input is a grayscale image. The gray-world model makes use of a “polychromatic” optical transfer function (OTF) where the spectral OTFs are averaged to form a single OTF. This OTF is then convolved with the grayscale image to create the resultant sparse-aperture image. The model proposed here uses a spectral image-cube as the input to create a panchromatic or multispectral result. These outputs better approximate an actual system because there is a higher spectral fidelity present than a gray-world model. Unlike its Cassegrain counterpart that has a well behaved OTF, the majority of sparse-aperture OTFs have very oscillatory and attenuated natures. When a spectral sparse-aperture model is used, spectral artifacts become apparent when the phasing errors increase beyond a certain threshold. This threshold can be based in part on the type of phasing error (*i.e.* piston, tip/tilt, and the amount present in each sub-aperture), as well as the collection conditions, including configuration, signal-to-noise ratio (SNR), and fill factor.

This research addresses whether integrating a restored multispectral sparse-aperture image into a panchromatic image will decrease the amount of spectral artifacts present. The restored panchromatic image created from integrating multispectral images is compared to a conventional panchromatic sparse-aperture image. Conclusions are drawn through image quality analysis and the change in spectral artifacts.

Keywords: sparse-aperture, multispectral, spectral artifacts, image quality, restoration, Wiener filter

1. INTRODUCTION

Sparse-aperture telescope systems were first proposed in 1970,¹ the benefits of these systems have led to their continued research to present day. They are so enticing because of the ability to synthesize a larger effective aperture from an array of smaller apertures. These sub-apertures are less costly to fabricate, thus, in theory making the sparse-aperture telescope system less expensive than its Cassegrain counterpart. Another promising aspect about these systems is the possibility of launching larger telescopes into space than current load fairing constraints permit.

Two examples of sparse-aperture system configurations are shown in Figure 1. The annulus in Figure 1 is included to show what happens as the central obscuration of the Cassegrain telescope increases, and its corresponding effect on the modulation transfer function (MTF). The MTF is the magnitude of the OTF. The tri-arm configuration is used for the simulations in this experiment.

These larger systems could either be assembled in orbit, or could unfold as planned for the James Webb Space Telescope. A larger aperture is desired because increasing the diameter permits a higher resolution to be recorded. This relationship is most easily understood by the following

$$PSF_{width} = 2.44 \frac{\lambda f}{D} = 2.44 \lambda (f\#) \quad (1)$$

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Figure 1. The Cassegrain, annulus, and tri-arm telescope configurations. The Cassegrain is the standard telescope used, the annulus is a very sparse Cassegrain, and the tri-arm is a configuration of sub-apertures. The bottom row are their corresponding MTFs.

where PSF_{width} is the width of the point spread function (PSF) as defined where it first equals zero, λ is the central wavelength of the system bandpass, f is the focal length, D is the aperture diameter, and $(f\#)$ is the system F-number. This relationship shows that as the diameter of the telescope increases, the width of the PSF decreases. This promising aspect of sparse-apertures is why so much effort is being put forth to create and model them, however, they are not without a plethora of problems. They suffer from a host of issues including poor SNR, phasing issues, highly attenuated/structured MTF, and spectral artifacts to name a few.

Many of the reasons these systems have such poor image quality is due to the optics having a smaller surface area. This means there is a smaller area to capture photons, thus, these systems have an innately lower SNR than a Cassegrain system. Intuitively, it might be thought that this would be remedied by increasing the integration time by the fill factor ($1/F$) to compensate for the loss of photons. However, this is not true due to the complex interactions of these systems, the integration time has to increase by the inverse of the fill factor cubed ($1/F^3$) instead of the inverse of the fill factor.^{2,3} Another reason for the poor image quality is the attenuated MTF characteristic of these systems. These two basic problems are exacerbated by the introduction of phasing-errors in the sub-apertures. The MTFs of these systems are highly structured when phasing-errors are introduced causing the introduction of spectral artifacts when the degraded image is restored using a filter such as the Wiener-Helstrom (Wiener) filter.^{4,5}

In theory if multiple bands are recorded in a sparse-aperture system, each band can be restored independently, reducing the spectral artifacts. The largest impact on image quality will depend on the knowledge of the phasing-errors in the system. Hopefully multiple bands can reduce the spectral artifacts to a non-noticeable point.

2. THEORY

The modeling concepts contained in this section will only be explained in a brief overview because a more in depth review of the implementation can be found in Block, et al. (2004), Introne (2004), and Introne, et al. (2005).

2.1. General Image Model

It is normally assumed when modeling telescope systems that they are linear shift invariant (LSI), while this is not usually true, it can be approximated by an isoplanatic region in the field-of-view (FOV). The noise can be

considered additive if it is uncorrelated. Using the LSI assumption, the model is normally implemented in the frequency domain

$$G(\xi, \eta) = F(\xi, \eta) \cdot H(\xi, \eta) + N(\xi, \eta) \quad (2)$$

where $G(\xi, \eta)$ is the degraded image spectrum, $F(\xi, \eta)$ is the scene spectrum, $H(\xi, \eta)$ is the transfer function (OTF), and $N(\xi, \eta)$ is the noise spectrum.

2.2. Detected Signal

Equation 2 is the general representation of an image model, the explicit equation used to calculate the detected frequency of a recorded signal without noise is⁶

$$S_{freq}^{out}(\xi, \eta, \lambda) = \frac{G_{conv}G_{elec}2^n}{S_{ADC}} \frac{\pi A_{det}T_{int}F_{fill}}{4(f\#)^2 hc} \int_0^\infty OTF(\xi, \eta, \lambda) L_{source,FT}(\xi, \eta, \lambda) \tau_{opt}(\lambda) \eta(\lambda) \lambda d\lambda \quad (3)$$

where G_{conv} is the conversion gain of the detector (volts/electron), G_{elec} is the gain from the electronic analog signal chain. n is the number of bits in the analog to digital converter (A/D), S_{ADC} is the input voltage range of the sensor (maximum output voltage), A_{det} is the area of the detector, T_{int} is the integration time of the signal, F_{fill} is the fill factor of the sparse-aperture, h is planck's constant, c is the speed of light, $L_{source,FT}$ is the spectrum of the source radiance, $\tau(\lambda)$ is the spectral transmission of the optics, $\eta(\lambda)$ is the spectral quantum efficiency of the detector, and λ is the wavelength.

The monochromatic approximation is based on two assumptions, the first being that the spatial and spectral information can be separated, the second being that the MTF is approximately equal to a real-valued system OTF.^{2,3} Again noting that the MTF is the magnitude of the OTF. If these two assumptions are used, the monochromatic model is

$$S_{freq}^{out}(\xi, \eta) = \frac{G_{conv}G_{elec}2^n}{S_{ADC}} \frac{\pi A_{det}T_{int}F_{fill}}{4(f\#)^2 hc} \frac{F_{obj,gray}(\xi, \eta)}{F_{obj,gray}(0, 0)} MTF_{poly}(\xi, \eta) \int_0^\infty L_{source}(\lambda) \tau_{opt}(\lambda) \eta(\lambda) \lambda d\lambda \quad (4)$$

where $F_{obj,gray}$ is frequency spectrum of the grayscale image, $MTF_{poly}(\xi, \eta)$ is the weighted average of the spectral OTFs, and $L_{source}(\lambda)$ is the total source radiance reaching the sensor. The assumptions that make equations 3 and 4 approximately equal work well for Cassegrain systems. Unfortunately, they do not work for highly aberrated sparse-aperture systems.^{4,5}

To model the multispectral system, equation 3 is slightly modified by changing the bounds of integration. The integration is done i times, where i is the number of bands in the multispectral image-cube.

$$S_{freq,i}^{out}(\xi, \eta, \lambda) = \frac{G_{conv}G_{elec}2^n}{S_{ADC}} \frac{\pi A_{det}T_{int}F_{fill}}{4(f\#)^2 hc} \int_{bp_i l}^{bp_i u} OTF(\xi, \eta, \lambda) L_{source,FT}(\xi, \eta, \lambda) \tau(\lambda) \eta(\lambda) \lambda d\lambda \quad (5)$$

where $S_{freq,i}^{out}$ is the system signal for channel i in bandpass bp_i . The integral goes from the lower limit of the bandpass “ $bp_i l$ ” to the upper limit “ $bp_i u$.” The total output for equation 5 is an image-cube with i number of multispectral bands. To create the panchromatic image from the multispectral cube, the multispectral image is averaged into a panchromatic image. It should also be noted that the same integration time is used to create each band in the multispectral image as the panchromatic image.

2.3. Optical Transfer Function

The OTF is calculated by taking the normalized autocorrelation of the pupil function:

$$OTF(\xi, \eta) = \frac{p(\lambda z \xi, \lambda z \eta) \star p(\lambda z \xi, \lambda z \eta)}{\int_{-\infty}^{+\infty} p(x, y) dx dy} \quad (6)$$

where the phasing-errors are already included in the pupil function p and z is the distance to the imaging plane.

2.3.1. Fill Factor

The fill factor is the ratio of the area of the sparse-aperture sub-apertures to its Cassegrain counterpart. It is generically written as

$$\text{Fill Factor} = \frac{\sum_{\text{area}} \text{sub-apertures}}{\sum_{\text{area}} \text{filled aperture}} . \quad (7)$$

As the fill factor decreases, the resulting MTF becomes more attenuated as shown in Figure 1.

2.3.2. Phasing Errors

There is a closed form solution to calculate the PSF of a sparse-aperture system, however, it is not the most useful equation because it only works for a completely unaberrated system. If any aberrations or phasing errors are introduced, then equation 6 needs to be used. The phasing errors are implemented in polar coordinates of the zernike polynomials,⁷ where the transformation from euclidean to polar geometry is

$$\begin{aligned} x &= r \cos \theta \\ y &= r \sin \theta \end{aligned} \quad (8)$$

and the optical path distance (OPD) is calculated by,

$$\begin{aligned} w[r, \theta, x_0] &= \sum_{j,m,n} W_{klm} x_0^k r^l \cos^m \theta \quad \text{with } k = 2j + m, l = 2n + m \\ &= W_{020} r^2 + W_{111} x_0 r \cos \theta + W_{200} x_0^2 \\ &\quad W_{040} r^4 + W_{131} x_0 r^3 \cos \theta + W_{222} x_0^2 r^2 \cos^2 \theta \\ &\quad W_{220} x_0^2 r^2 + W_{311} x_0^3 r \cos \theta + W_{400} x_0^4 + \text{higher order terms.} \end{aligned} \quad (9)$$

where W_{klm} are the wavefront aberration coefficients, k corresponds to the power of the x_0 term, l to the r coordinate term, and m to the trigonometric cosine term. The coefficients are defined in Table 1. The net effect

Coefficient	Form	Aberration
W_{020}	r^2	focus
W_{111}	$x_0 r \cos \theta$	tilt
W_{200}	x_0^2	piston
W_{040}	r^4	spherical
W_{131}	$x_0 r^3 \cos \theta$	coma
W_{222}	$x_0^2 r^2 \cos^2 \theta$	astigmatism
W_{220}	$x_0^2 r^2$	field curvature
W_{311}	$x_0^3 r \cos \theta$	distortion

Table 1. Wavefront aberration coefficients for first-order Gaussian and third-order Seidel aberrations.

of an aberrated pupil is an MTF that is more attenuated and structured than a sparse-aperture MTF already is. Figures 1 and 2(a) shows that all but the very low frequencies are extremely attenuated. Figure 2(b) shows that as the phasing error increases, not only does the MTF become more attenuated, but more structured as well. All of the effects create an image that is degraded much more than a Cassegrain of equivalent size.

2.4. Spectral Artifacts

It has been shown that spectral artifacts are not a noticeable problem with systems that have low amounts of phasing errors.^{4,5} Figures 2(a) through 2(d) show the spectral curves for wavelengths 0.45 μm , 0.55 μm , and 0.65 μm . These curves are a representative sample of how much the MTFs can vary with respect to wavelength. Figure 2(a) shows that the spectral MTFs for unaberrated systems have oscillations, but the magnitude stays fairly constant across the mid to high-range frequencies and the oscillations are not very large. The amount of aberrations introduced into the pupil is described as the λ rms-error, this is the mean phase error expressed as the average deviation in optical path distance (OPD) in multiples of λ ,

$$\lambda \text{ rms - error} = \sqrt{\frac{\sum_x \sum_y \sum_{\lambda} p(x, y, \lambda)^2}{d(x, y, \lambda)}} \cdot \frac{1}{\lambda_{ref}} \quad (10)$$

where $d(x, y, \lambda)$ is the number of OPD samples in the pupil function in x, y , and λ , $p(x, y, \lambda)$ is the complex pupil function that has the OPD of the sub-apertures included in it, and λ_{ref} is the reference wavelength. When phasing-error is introduced as shown in Figure 2(b), the mid and high range frequencies are no longer fairly constant valued, but now oscillate wildly, with peaks of one spectral OTF correspond to the trough of another. The effect of the aberrated MTFs on a restored image is easier to understand if the restoration filters are looked at. The simplest filter, never used because of its noise gain properties, is the inverse filter, which is the inverted transfer function. This functions in theory by undoing the effects of the transfer function from the degraded image, restoring the original scene. This works so poorly because low SNR regions become dominated by noise. However, if an inverse filter is multiplied by its transfer function, the following relationship is found,

$$H_i(\xi, \eta) \cdot \frac{1}{H_i(\xi, \eta)} = \begin{cases} 1 & \text{where } H_i(\xi, \eta) > 0 \\ 0 & \text{where } H_i(\xi, \eta) = 0 \end{cases} \quad (11)$$

where it equals one everywhere, except where it is equal to zero (after the cutoff frequency). This relationship will not hold true if the spectral transfer function of a different band ($H_j(\xi, \eta)$) is multiplied by the inverse filter because they have different shapes and cutoff frequencies as shown in Figures 2(a) and 2(b). A Wiener filter is similar to an inverse filter with a power spectrum ratio in the denominator. This ratio is included so the filter does not amplify regions with a low SNR. The following relationship with the Wiener filter will not be quite as exact as the inverse filter, but it approximates equation 11.

$$H_i(\xi, \eta) \cdot \frac{H_i^*(\xi, \eta)}{|H_i(\xi, \eta)|^2 + \frac{P_n(\xi, \eta)}{P_s(\xi, \eta)}} \approx \begin{cases} 1 & \text{where } H_i(\xi, \eta) > 0 \\ 0 & \text{where } H_i(\xi, \eta) = 0 \end{cases} \quad (12)$$

Where $P_n(\xi, \eta)$ is the noise power spectrum, and $P_s(\xi, \eta)$ is the scene power spectrum. This approximation is shown to hold true in Figure 2(c) where only the center band (*i.e.* green for this example) is used for the restoration filter. The other bands do not hold as true to this relationship. But they do not deviate from it too badly. Stated differently, when the center channel OTF is used to restore different bands, it will not do so in the above relationship. However, this relationship does not hold true when considering aberrated conditions as shown in Figure 2(d), all bands, including the band of the restoration filter (*i.e.* green for this example), oscillate greatly around the MTF value of 1. The reason this happens for the restoration band is because as the amount of phase error increases, the imaginary portion of the OTF becomes larger, meaning that the OTF is diverging from the MTF, or that the approximation becomes less valid. If the value falls below 1, the signal will be under amplified, and if the value is above 1, the signal will be over amplified. The color balance from the under/over amplification of specific frequencies from specific bands will be different from the original scene. If the color balance is shifted in an unknown way (no phasing knowledge), there will be an even worse relationship than the one shown in Figure 2(d). This shifting color balance from the original scene is why spectral artifacts become apparent. This is not a problem with filled apertures because they do not suffer from the same phasing errors as sparse-apertures, and they have a high SNR for most frequencies before the cutoff frequency.^{4,5} For the

Figure 2. (a): A sampling of the red ($0.65\mu\text{m}$), green ($0.55\mu\text{m}$), and blue ($0.45\mu\text{m}$) unaberrated spectral OTFs for the tri-arm system in Figure 1. (b): The same MTFs in (a), except aberrated with $1/4\lambda$ rms-error phasing across the aperture. (c): The unaberrated MTFs in (a) multiplied by the green channel Wiener filter. (d): The aberrated MTFs in (b) multiplied by the aberrated green channel Wiener filter.

unaberrated case in Figure 2(c), all of the channels are restored close to an MTF value of 1 except for a small range of lower frequency values. This is not true for the aberrated case where all three channels, including the channel of the restoration filter, are all oscillating to a large extent.

2.5. Normalized root-mean-square error (*nrmse*)

The quantitative metric used in this research is the *nrmse* which is defined as,

$$nrmse = \sqrt{\frac{\sum_x \sum_y |g(x, y) - f(x, y)|^2}{\sum_x \sum_y |f(x, y)|^2}}. \quad (13)$$

The normalized rms error is just the degraded image $g(x, y)$ minus the original object image $f(x, y)$, quantity squared, summed and normalized by the squared summation of the original image. Sparse-aperture image quality is distinct from Cassegrain telescopes in that they can have large amounts of spectral artifacts. Due to this reason, the *nrmse* might not have a direct correlation with image quality because it might not fully capture the effects of the artifacts. However, for the purposes of this research the *nrmse* will be used as a metric for image quality.

3. METHODOLOGY

The spectral sparse-aperture model operates on any image-cube of radiance values that has a spectral range between 0.4 to 1.0 μm . It uses the header file of the image-cube to calculate how many, and which spectral OTFs to create for the simulation. The model normally creates a panchromatic image, however it has the ability to create multispectral imagery of any number of bands not to exceed the number of bands in the original image-cube. The image-cube is a synthetically rendered image that has full spectral coverage from 0.4 to 0.9 μm created using DIRSIG (Digital Image and Remote Sensing Image Generation),⁸ a first principles model created at Rochester Institute of Technology. A spectral OTF is created for every band in the image-cube. Equation 3 is implemented with user defined parameters for all of the other variables. To create a multispectral image that is equivalent to its panchromatic counterpart, the same optics transmission is used as for the panchromatic case. In a more realistic simulation, the optics transmission from a multispectral system would have a separate transmittance for each channel rather than a single transmittance used for a panchromatic scene. The image-cube rendered by DIRSIG is a section of Rochester, NY, with 51 bands, each having a 10nm bandpass. It has a ground sampled distance (GSD) of approximately one meter, with a ground coverage of approximately 512m x 512m.

For this scenario, there will only be piston and tip/tilt phasing-errors distributed randomly across the separate sub-apertures. To compare the panchromatic and multispectral scenarios, the separate restored bands of the multispectral image are integrated to yield a panchromatic image, which is compared to a regular panchromatic image. As more phasing-error is introduced into the system, larger amounts of spectral artifacts will be introduced.

The degraded images are restored in two different ways. The first is with full phase-error knowledge (full knowledge), and the other with no phase-error knowledge (no knowledge). The full knowledge method means that the phasing errors are known and included in the OTF. The no knowledge method means that the degraded image is restored using an unaberrated OTF. The panchromatic image is restored using a “gray-world” OTF, it is formed by taking the weighted average of the spectral OTFs. The multispectral images are restored using the center OTF of each bandpass to restore its respective channel (*i.e.* H_i is used to restore channel_{*i*}).

4. RESULTS

The results of the different restorations offered insights into when using a multispectral system would give tangible benefits for increasing image quality. Figure 3(a) shows the synthetically rendered scene of Rochester, NY. The bottom row in Figures 3 - 6 is a magnified region that is highlighted in Figure 3(a). Figures 3(b) to 3(d) show that as the λ rms-error increases, the image quality of the degraded scene is negatively impacted. Visually, the no knowledge restorations do not seem as if they get any better using more bands in Figures 4(e-h) and Figures 5(e-h). However, Table 2 shows that there is a consistent decrease in the *nrmse* for the 0.15 λ rms-error case as the number of bands increase indicating the image quality is getting better, albeit, by only 0.9% when comparing the panchromatic and 17 band image. The *nrmse* decreases as well for the 0.26 λ rms-error simulations with multiple bands as well, but the trend is of a much shorter duration. The only noticeable decrease is when three bands are used instead of the panchromatic image. After three bands, there is not a noticeable increase in image quality as the number of bands are increased for 0.26 λ rms-error.

The trend of increasing image quality is more pronounced in the full knowledge restorations for both the 0.15 and 0.26 λ rms-error simulations. While there is a large increase in the 0.15 λ rms-error simulations, an increase of 2.9% *nrmse* from the panchromatic to the 3 band scenario, the 0.26 λ rms-error simulation has a larger increase in image quality for the same scenario of 15.6%. The increase in image quality is not as substantial from 3 bands to 17 bands, that increase is only 1.9% for the 0.15 λ rms-error and 3.5% for the 0.26 λ rms-error.

In previous research^{5,6} it has been shown that unaberrated sparse-aperture imagery does not exhibit significant spectral artifacts. However, this previous research only had a bandpass of 0.4 - 0.7 μm . The addition of the near infra-red region creates a larger amount of contrast especially in flora which have a high reflectivity in this

region. The scene used in this study has large amounts of flora as well as manufactured objects. The result of the larger bandpass is the introduction of spectral artifacts in restored unaberrated imagery as shown in Figure 6(a). Using 3 bands reduces spectral artifacts to an unnoticeable amount. The difference is most noticeable in the magnified region at the top right of the image by the building where two tracks cross. The track crossing is highlighted in the magnified region by a white circle in Figures 4(d), 5(d), and 6(d). If this region is looked at in Figures 6(a) and 6(b), the tracks are easier to discern in the 3 band simulation. There is some distortion in the full knowledge panchromatic image in Figure 4(a). The distortion is in the form of ringing. This ringing is most pronounced near the bright objects, especially on the roof of the building at the top of the image. The ringing is a spectral artifact from the restoration process. Figure 4(b) does not have this ringing, the spectral artifacts are reduced to an amount that is not noticeable with only 3 bands. The image quality can still be increased by using more bands as shown in Table 2.

The 0.26λ rms-error simulations obviously have a worse *nrmse* for degraded images. Less obviously, the restored images also have this trend. The panchromatic restored image in Figure 5(a) is obviously more distorted than the one in Figure 4(a), however, as the number of bands increases beyond 4, a threshold of image quality is obtained where the 0.15λ rms-error simulations *nrmse* stay approximately 1% better than the 0.26λ rms-error simulations. Visually, there is some severe distortion in Figure 5(a). The 3 band image in Figure 5(b) has some remnants of the ringing which is not evident in the 0.15λ rms-error simulation. This ringing is noticeable when the 3 band restored (full knowledge) image is compared to the 12 band restored image (Figure 5(b),(c)). Even though there is an increase in the *nrmse* by 1.2% from the 12 band to the 17 band images, there is not a visually noticeable difference in image quality (Figures 5(c) and (d)). One notable location that a difference is obvious is at the crossing tracks in the top right of the image just to the corner of the building. It is barely resolvable in the magnified region in the 3 band restoration (full knowledge), but easier to discern in the 12 band restored (full knowledge) image (Figure 5(a,b,c,d)). The correlation between the image quality and number of bands used in the restoration process is more evident for 0.26λ rms-error simulation than for 0.15λ rms-error simulations. Numerically the difference in *nrmse*'s can be seen in Table 2. The *nrmse* of the other simulations with a different number of bands is shown in Table 2.

# of bands	<i>nrmse</i>				
	0.15 λ rms-error Full knowledge	0.15 λ rms-error No knowledge	0.26 λ rms-error Full knowledge	0.26 λ rms-error No knowledge	Unaberrated
pan	0.2760	0.3156	0.3258	0.3812	0.2676
3	0.2679	0.3138	0.2749	0.3776	0.2627
4	0.2656	0.3133	0.2707	0.3770	0.2620
5	0.2642	0.3129	0.2679	0.3771	0.2616
6	0.2647	0.3127	0.2685	0.3774	0.2615
7	0.2645	0.3124	0.2685	0.3776	0.2612
8	0.2646	0.3124	0.2686	0.3775	0.2613
9	0.2651	0.3128	0.2687	0.3772	0.2616
10	0.2633	0.3124	0.2666	0.3773	0.2611
11	0.2658	0.3125	0.2698	0.3774	0.2616
12	0.2644	0.3120	0.2685	0.3776	0.2610
17	0.2627	0.3127	0.2654	0.3770	0.2613

Table 2. Table of values for the normalized root-mean-square error (*nrmse*) for the restored images for both no phase knowledge, and full phase knowledge of the pupil. The multispectral images are integrated after restoration into a panchromatic (pan) image. The number of bands are the number of bands created by the model, and restored before being integrated into a panchromatic image.

Figure 3. The top row of images are of the entire scene, the bottom row are magnified regions of the highlighted section in the panchromatic object. The images are degraded scenes with varying amounts of phase-error introduced into the pupil.

5. CONCLUSIONS

It was found that using 3 band multiband restoration to form a panchromatic image drastically reduces the spectral artifacts in an aberrated sparse-aperture system when compared to a single band approach with the same amount of phase-error. However, to visibly remove all of the spectral artifacts, more than 3 bands are needed. The actual number of bands needed to remove the artifacts is dependent on how much phase-error is present and how much phase knowledge is known. If no phase knowledge is known about the aperture, using more bands does not help considerably. It is important to note that these results are produced using a best case scenario where the dwell time for each spectral image is the same as for the single band case and perfect registration is assumed. Future work will investigate the effect of reduced dwell times on the multiband process.

It is also evident that the introduction of the near infra-red region into the bandpass increases the amount of spectral artifacts present when looking at the panchromatic simulations. This is due to the increase in the size of the bandpass, the lower cutoff frequency of the longer wavelengths, and the higher image contrast due to high reflectivity of vegetation in the near infra-red.

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Figure 4. Restored images with 0.15λ rms-error introduced into the pupil. (a)-(d) are restored using the aberrations included in the Wiener filter. (e)-(h) are restored using a Wiener filter with an unaberrated OTF. (a) and (e) are panchromatic images, (b) and (f) result from multispectral images with 3 bands, (c) and (g) result from multispectral images with 12 bands, (d) and (h) result from multispectral images with 17 bands. The circle highlights a low contrast location where two tracks cross.

Figure 5. Restored images with 0.26λ rms-error introduced into the pupil. (a)-(d) are restored using the aberrations included in the Wiener filter. (e)-(h) are restored using a Wiener filter with an unaberrated OTF. (a) and (e) are panchromatic images, (b) and (f) result from multispectral images with 3 bands, (c) and (g) result from multispectral images with 12 bands, (d) and (h) result from multispectral images with 17 bands. The circle highlights a low contrast location where two tracks cross.

Figure 6. Restored images with no phase-error introduced into the pupil. (a) is a panchromatic image, (b) is a multi-spectral image with 3 bands, (c) is a multispectral image with 12 bands, and (d) is a multispectral image with 17 bands. (b), (c), and (d) are integrated into a panchromatic image after the restoration is performed. The circle highlights a low contrast location where two tracks cross.

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