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The second moment method, a new way to test a lens

Christopher Curtis

THE SECOND MOMENT METHOD, A NEW WAY TO TEST A LENS

by

Christopher R. Curtis

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Signature of Author Christopher R. Curtis
Imaging and Photographic Science

Certified by Edward Granger
Thesis Advisor

Accepted by Name Illegible
Supervisor, Undergraduate Research

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Christopher R. Curtis

Submitted to the
Imaging and Photographic Science Division
in partial fulfillment of the requirements
for the Bachelor of Science Degree
at the Rochester Institute of Technology

ABSTRACT

The second moment method is proposed as a method of obtaining a one dimensional approximation to a two dimensional angularly averaged modulation transfer function for low spatial frequencies. For two orthogonal knife-edge scans of a point spread function, the second moments about the centroid of the corresponding line spread functions are determined. The two second moment values are then used to approximate the two dimensional angularly averaged modulation transfer function.

Two rotationally asymmetric point spread functions were

used to test the approximation, an equilateral triangle and a rectangle. The results show, for both cases, that in the 1.0 to 0.5 modulation range, there was no difference in the modulation transfer function curves determined by using the approximation and the actual angular average of the modulation transfer function. In the 0.5 to 0.1 modulation range, a difference of only 5% was calculated.

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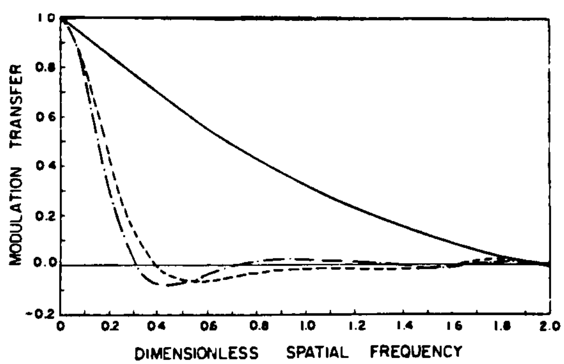
I. Introduction.

In the field of imaging, there is a need to evaluate an imaging system's ability to record detail and other information. Most imaging systems consist of a photosensitive receiving surface incorporated into an optical system[1]. Therefore, the optical component is an important part of the total imaging system, and a need exists to be able to characterize its performance. The modulation transfer function(MTF) provides this characterization and is used extensively.[2] Simply stated, the MTF is the ratio of image contrast to object contrast as a function of the spatial frequency of the object. Typically, the MTF is reported in graphical form as modulation as a function of spatial frequency. Figure I shows examples of MTF curves.



Transfer functions for perfect systems with circular aperture: (——) 0.2 wavelength defocusing, (- - -) 1.0 wavelength defocusing, (-·-) 1.2 wavelength defocusing.

Figure I: Typical Modulation Transfer Functions.[3]

Many methods exist to calculate the MTF using one of three functions; the point spread function, line spread function, or edge response function. In general, these methods can be divided into two groups; techniques using a Fourier transform and techniques not requiring the use of a Fourier transform.[4] The method chosen depends upon what information is needed from the MTF curve.

If information about the lens is needed near its frequency cut-off region, then a method that accurately calculates the MTF for high spatial frequencies should be used. Fourier transforms techniques are accurate at doing this and should be used in this case.[5]

However, there are disadvantages in using Fourier methods.[6] First, they are tedious to calculate by hand and a computer is needed. The computing times are long and storage capacities are large and may not be practicable on smaller computer systems. Second, the MTF calculations are further limited by noise; and unless many samples are taken and averaged, the calculated MTF could be incorrect. Third, most point spread functions are rotationally asymmetric; and therefore, an angularly averaged MTF is more meaningful. This requires computing more than one MTF per lens and thus compounding the computer storage problem.

At times the low spatial frequency, high modulation portion of the MTF curve is most important. In this case a method that yields an accurate MTF in this region should be used. Fourier methods would provide a good result, but in view of the disadvantages, an alternate method is more desirable.

The second moment method is such an alternative method. This is a non-Fourier techniques and thus does not suffer from the disadvantages of the Fourier methods. There are three advantages to the second moment method. First, little computing power is required. The evaluation of a real valued expontential can be performed with a programmable calculator. Second, the second moment method estimates only a single parameter, the variance of the line spread function. This implies a large amount of noise averaging and improved system performance estimates. Third, this method is a one dimensional approximation to a two dimensional MTF; and, thus no angular averaging needs to be performed. However, a disadvantage is that it is only an approximation for the low spatial frequencies and therefore limited in use.

Croteau[7] has done qualitative analysis of the second moment method by comparing it with a number of Fourier and non-Fourier techniques. He concluded that second moment method is a good predictor of the MTF in the 100 to 40% modulation range. The purpose of this research is to provide quantitative support to his conclusion.

A. History of optical testing.

The first scientific optical devices were the microscope and the telescope.[8] Not long after their invention, the need existed to quantify the quality of the instrument's optics. However, a problem was not having suitable test objects. What was often used were naturally occurring objects.[9] The microscopist used diatoms(unicellular planktonic algea). The diatoms had a periodic structure that was useful in testing. Astronomers, on the other hand, used the doulbe images of stars to compare the quality of their telescopes.

Airy and Rayleigh were the first to determine methods to characterize optical systems.[10] In 1834, Airy computed the light distribution of the image of a point of light as formed by a diffraction-limited lens. He showed that the image was a center disk containing 84% of the energy surrounded by rings of diminishing intensity. Figure II shows an example of Airy's disk.

Figure II: Airy's Disk

Rayleigh is thought to be the first person to use the term, "resolving power". He defined resolving power as the the smallest separation of two point images that could be seen. To compute this distance, he derived the formula for minimum resolvable separation(Δ),

$$\Delta=0.61\lambda f/D \qquad (1)$$

where (λ) is the wavelenght of light, (f) is the focal lenght, and (D) is the diameter of the lens. Later research, especially by Conrady,[12] showed that a more realistic coefficient was 0.50, not 0.61.

In 1896, Foucault[13] reported that lines were a better test image than points. He developed a chart, known by his name, that consisted of a large number of parallel lines that varied in spatial frequency. Using this chart, the distance between the finest pattern of lines which could just be discerned was the resolving power.

From Foucault's time until the 1940's, resolving power was used as the method of testing the quality of lenses. The reasons were:

1) It seemed reasonable that the ability to resolve fine details was an important feature of any imaging system.

2) The results were expressible in comprehensible terms.

3) Resolving power appeared to be easily measured.

However, during the post World War II years, it became increasingly obvious that the advantages of resolving power were faulty.[14] The reasons were:

1) Resolving power is not a fundumental property of a lens.

2) The criterion of resolution is uncertain. It is very dependent upon observing conditions.

3) For some lenses, the image plane of best resolving power does not correspond to the plane of maximum sharpness.

Recognizing these short comings, much effort was made to find a new method for discribing a lens' quality.

During that period, recent advances of Fourier analysis applied to the problems of information recording were extended to imaging systems.[15] From this, the modulation transfer function was derived as a way of comparing lenses.

B. Theory

1. Spread functions.

The image of a point of light is not imaged as a point, but as a distribution of light. The distribution of the energy in the image is discribed by the point spread function, P(x,y).[16] Figure III shows a three dimensional example of an image of a point of light.

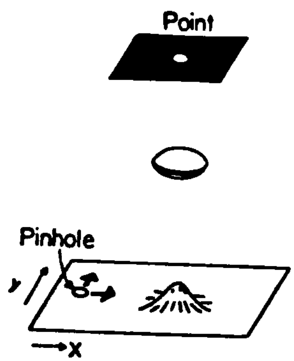


Figure III:Energy dist. of an imaged point of light.[17]

If the image is scanned with a pinhole and if the illuminance (E) through the pinhole is plotted as a function of the distance scanned, (x)and (y), the resulting graph will be that of the point spread function.

It is difficult to scan an image of a point with an even smaller pinhole, an alternate method is recommended. If a narrow, bright line of light is imaged through a lens the light will be spread like the point, but only in one direction.

This distribution of light is the line spread function, L(x).[18] Figure IV shows an example.

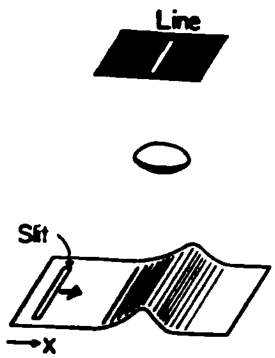


Figure IV: Energy dist. of an imaged line of light.[19]

Correspondingly, if the line spread function is scanned with a narrow slit in the (x) direction, the resulting graph is that of the line spread function.

It is obvious from Figures III and IV that the line spread function is the integral of the point spread function, integrated with respect to (y).

$$L(x)=\int_{-\infty}^{\infty}P(x,y)dy \qquad (2)$$

In addition to the point and line spread functions, there is the edge response function, E(x). If an image is made of more than one line, the total image's line spread function is the sum of the individual line spread functions.[20] In Figure V, a simple example is shown.

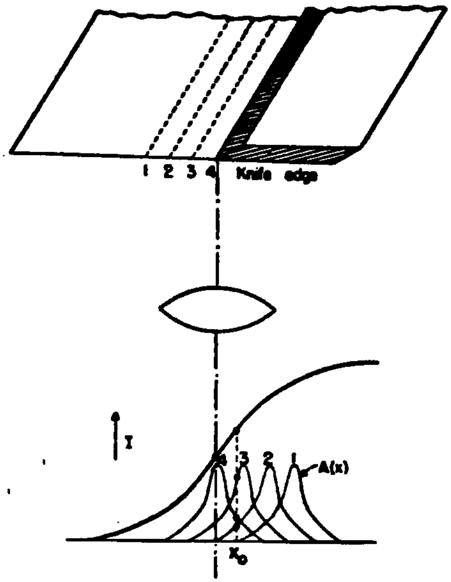


Figure V: Energy dist. of an image of an edge.[21]

A uniformly illuminated field is bounded by a straight edge. The field is made up of an infinite number of infinitely thin parallel lines(four are shown). For each line, there is a corresponding line spread function. The distribution of energy in the total image is the sum of the individual line spread functions at each point in the image, (x_o) is one such point.

If the summation of the line spread function is taken to the limit, the illuminance at the point (x_o) is the integral of the line spread function form minus infinity to (x_o), and is equal to the value of the edge response function at that point,

$$E(x_o) = \int_{-\infty}^{x_o} L(x)\,dx \qquad (3).$$

Differentiating E(x) with respect to (x) yields,

$$dE(x)/dx=L(x) \qquad (4).$$

Therefore, the derivative of the edge response function is the line spread function.[22]

The edge response function is the easiest of the three functions to determine. If either an imaged point of light or imaged line of light is scanned with a straight sharp edge, the plot of the integrated flux passing the edge as a function of the distance scanned produces the edge response function.

2. Modulation Transfer Function.

Since it is difficult to report the complex value OTF, the MTF is most often used. The MTF is equal to the modulus of the Fourier transform of the line spread function.[20]

$$MTF(f)=\left|\int_{-\infty}^{\infty} LSF(x)EXP[-i2\pi fx]dx\right| \qquad (5)$$

where (f) is spatial frequency. Therefore, the MTF can be easily plotted as a function of spatial frequency.

Most lenses have rotationally asymmetric point spread functions and thus the edge response function in one direction across the point spread function will be different than that in another direction. This will cause the calculated MTFs to be different in different directions in the frequency domain. What is done in this case is to derive a one dimensional, angular averaged MTF by averaging the MTFs determined from the scans in different directions.

3. Second Moment Method.

One way to avoid the angular averaging problem is to use the second moment method. This method involves performing two edge scans at right angle to each other, across a point spread function. From the two resulting edge response functions, the MTF is approximated using the equation,

$$MTF(f)=EXP[-\pi^2 f^2 (M2x+M2y)] \qquad (6)$$

where (f) is spatial frequency, M2x and M2y are the second moments about the centroid of the two line spread functions respectively. See Appendix A for the derivation of this equation.

C. Objectives

The objective of this research is to determine over what modulation range the second moment method is a good appproximation to the MTF. In addition, to show that the approximated MTF is independent of which two orthogonal directions are chosen to scan the point spread function.

II. Experimental.

A. General Procedure.

To test the ability of the second moment to predict the angular averaged two dimensional MTF, rotationally asymmetric point spread functions were scanned with a knife-edge scanning apparatus. See Figure VI for photograph of scanning system set-up.

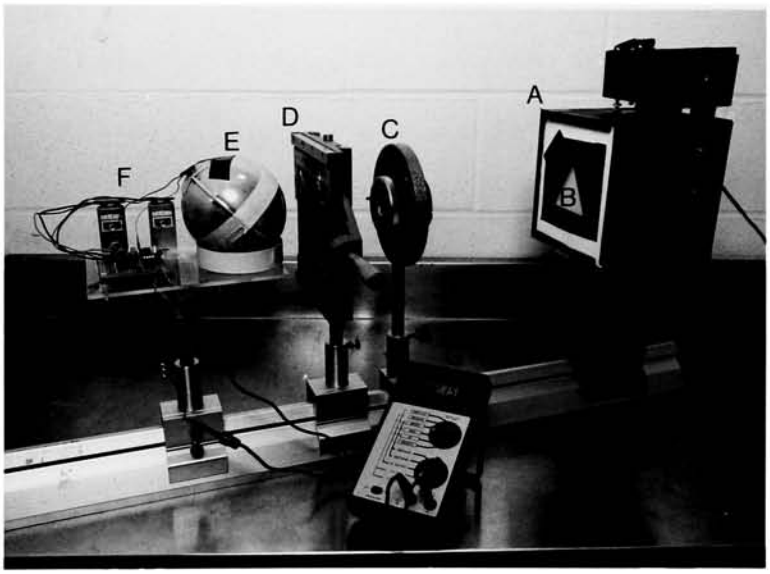


Figure VI: Scanning system set-up.

(A) is the diffuse light source, (B) is the triangle point spread function, (C) is the projection lens, (D) is scanning edge mounted on the stage, (E) is the integrating sphere, (F) is the amplifying circuit, and (G) is the volt meter.

Two point spread functions were used; an equilateral triangle and a rectangle with a side ratio of 3:1. The point spread functions were not formed by a lens but were geometric shapes photographed onto high contrast lith film. The point spread functions were illuminated by a uniform diffuse light source and projected onto the plane of the edge used for scanning. The lens used for projection was of high quality and did not degrade the image of the point spread function. The image was scanned with a straight, sharp edge that was mounted onto a stage that was moved by a micrometer. The light from the image that was not blocked by the edge was collected by a small integrating sphere. Mounted on top of the sphere was a 1 cm^2 silicon photodiode. The photodiode provided a linear voltage output as a function of the amount of light passing the knife edge. Therefore, the edge response function was in terms of voltage as a function of distance scanned on the point spread function. Two orthogonal edge response function were measured per scan. The edge response functions were then differentiated to create the line spread functions. The corresponding second moments about the centroid of the line spread function were calculated and used in Eqn. (6) to approximate the MTF.

B. Equipment used.

1. Point spread functions.

The point spread functions were made on 4x5, Kodalith ortho, 6556, Type 3 film. This allowed a clear image with sharp edges on a black background. The triangle had a side length of 6.4 cm and the longest side of the rectangle was 6.1 cm.

2. Diffuse light source.

The diffuse light source was made by a modifying an integrating box used in a color enlarger head. A flash light bulb was used as the light source and powered by a regulated 5 volt power supply. The illuminating surface was a 4x4 inch piece of white opal glass. The opal glass provided even diffusion of the light and a flat surface to mount the point spread functions.

3. Lens used for projecting the point spread functions.

The lens used for projecting the image of the point spread functions was a Canon 50 mm camera lens. The Canon lens is a high quality lens and did not degrade the image of the simulated point spread function. The reason for this is that the Canon lens has a point spread function that is approximately 100 times smaller than the simulated point spread function. The 50 mm focal length provided convienent working distances.

4. Scanning edge.

The scanning edge was made from a piece of 1/8 inch thick sheet metal. The edge was filed down to be straight and sharp. The metal was painted with flat black spray paint to reduce reflections. The edge was taped onto the micrometer stage to aid in easy adjustment for the scans in two directions. The micrometer allowed the edge to be moved in 0.50 mm increments.

5. Integrating sphere.

The integrating sphere was made from the scoops of two large kitchen ladles. The scoops are almost semi-sphereical and worked well together as an integrating sphere. The inside surfaces of the scoops were painted with flat white spray paint to provide a reflective surface. Two holes were drilled into the sphere. One hole was 3/4 inches in diameter and used as the entrance port for the light passing the scanning edge. The other hole was 1/2 inches in diameter, which the silicon photodiode was placed over. The two holes were drilled at right angles to each other.

6. Silicon photodiode and amplifying circuit.

The current output from the silicon photodiode was amplified using a National Semiconductor LM-351 operational amplifier. The LM-351 op-amp has high gain-low noise characteristics and was well suited for this application.

The voltage output from the circuit was measured with a Micronta digital multimeter. The smallest voltage that could be read was 1 millivolt. The maximum voltage output form the photodiode was approximately 7 volts. See Apendix B for the schematic of the circuit used.

C. Scanning.

For each scan performed, 50 voltage values were recorded and the distance the edge was incremented was kept constant at 0.50 mm. To demonstrate that the calculated second moments were independent of the orientation of the two orthogonal scans, scans were performed in different directions and compared. To achieve different directions, the point spread function were rotated on the light source and this caused the image on the scanning plane to rotate accordingly. For each rotation, three scans were performed and the resulting second moments were averaged. The triangle was rotated 0, 10, 20, 30, 60, and 90 degrees. The rectangle was rotated 0, 30, 45, 60, and 90 degrees.

During the scans, the voltage values from the voltmeter were enter directly into a computer and stored in data files on a diskette.

D. Second moment calculation.

A computer program was written to determine the second moment for the scans using the voltage values stored in the data files. The program normalized the edge trace, differentiated the stored values to determine the line spread function, calculated the centroid of the function and shifted the origin to the centroid. The second moment about the centroid was then calculated. See Appendix C for a listing of the program.

E. Calculating the modulation transfer function values.

To determine the MTF values for a given second moment value, a computer program was written that used equation (6). See Appendix D for a listing of the program.

F. Determination of mathematical second moment.

For both point spread functions, a mathematical second moment was determined and compared to the experimentally determined value. The shape of the point spread function was projected onto two orthogonal axes. The resulting functions were normalized to have unit area, and the second moment was determined by using the equation,

$$M_2 = \int_{-\infty}^{\infty} f(x)x^2 dx \qquad (7).$$

For each point spread function, the two resulting second moments were summed and the MTF values were determined as discussed in Section E.

G. Determination of two dimensional MTF.

For the triangle point spread function, a simple relationship was derived to compute the angular averaged MTF. For a given rotation, the triangle was projected onto the axis perpendicular to the scanning edge axis. See Figure VII for an example.

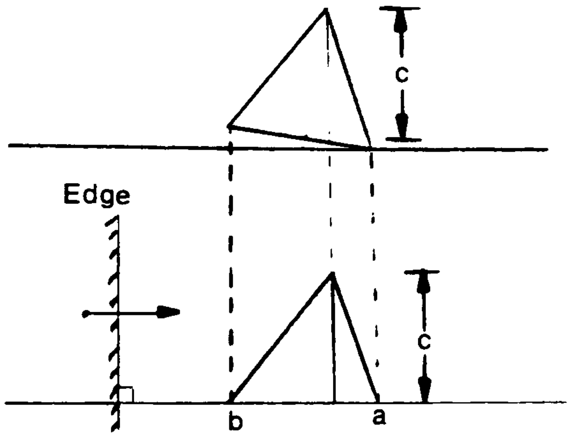


Figure VII: Projection of a rotated triangle onto an axis perpendicular to the scanning edge axis.

After normalizing the area of the projected function, the function was Fourier transformed. The result was used to angularly average the triangle MTF. The resulting equation was,

$$OTF(f)=\frac{2[\mathrm{sinc}(bf)\exp(i2\pi fb)-\mathrm{sinc}(af)\exp(-i2\pi fa)]}{i2\pi f(a+b)} \quad (8)$$

and,

$$MTF(f) = \left|OTF(f)\right| \quad (9)$$

where (a) and (b) are the absolute values of the x-intercepts from the projected function. The computer was used to implement this equation. The function was used to angularly average the triangle's MTF from 0 to 30 degrees. See Appendix D for a listing of the program.

A same procedure was used to angularly average the rectangle MTF. The two dimensional rectangle was Fourier transformed and the resulting equation was,

$$MTF(f) = \mathrm{sinc}(af_x)\mathrm{sinc}(bf_y) \qquad (10)$$

where (a) is the width of the rectangle and (b) is the length. At each frequency (f), the values for (f_x) and (f_y) were calculated using the equations,

$$f_x = f\cos(\Theta) \qquad (11)$$

$$f_y = f\sin(\Theta) \qquad (12)$$

where (Θ) was rotated from 0 to 360 degrees.

A program was written to evaluate the angular averaged MTF for the rectangle using the above equations. See Appendix E for a listing of the program.

III. Results.

The second moments obtained for several directions of scans for the triangle point spread function are given in Table I. The average second moments calculated for both directions as well the total are reported for each rotation. Table II contains the same results for the rectangle point spread function. Table III contains the mathematically calculated second moments for both point spread functions. Table IV contains the values for the one dimensional angular averaged MTFs for for both point spread function.

Figure VIII shows the angular averaged MTF for the triangle point spread function as well as the MTFs calculated for 0 and 30 degree rotations. Figure IX shows the same results for the rectangle point spread function with rotations of 0 and 90 degrees. Figure X shows the relationship between the MTF curves for the experimentally determined second moment and the mathematically determined second moment for the triangle point spread function. Figure XI shows the same results for the rectangle point spread function. Figures XII and XIII show the relationship between the one dimensional angular averaged MTF and the MTF determined from the mathematically calculated second moment for the triangle and rectangle point spread functions, respectively.

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Table I: Results for Triangle Point Spread Function.

Degree Rotation	Average M2x	Average M2y	M2x+M2y
0	6.959	6.352	13.311
10	6.683	6.873	13.556
20	7.036	6.622	13.658
30	6.890	6.373	13.263
60	5.929	6.465	12.394
90	6.336	6.810	13.167

Table II: Results for Rectangle Point Spread Function.

Degree Rotation	Average M2x	Average M2y	M2x+M2y
0	9.411	9.992	19.403
30	14.340	3.094	17.434
45	16.026	2.060	18.086
60	15.618	2.977	18.595
90	9.418	9.618	19.036

Table III: Results for Mathematical Second Moment Determination.

Point Spread Function	M2x	M2y	M2x+M2y
Rectangle	18.75	2.08	20.83
Triangle	7.04	7.03	14.07

Table IV: Results of the Angular Averaged Modulation Transfer Functions.

Frequency	Triangle	Rectangle
0	1	1
.01	.98	.98
.02	.95	.92
.03	.88	.83
.04	.80	.72
.05	.70	.59
.06	.59	.47
.07	.48	.35
.08	.38	.26
.09	.29	.18
.10	.22	.13
.11	.18	.10
.12	.15	.09

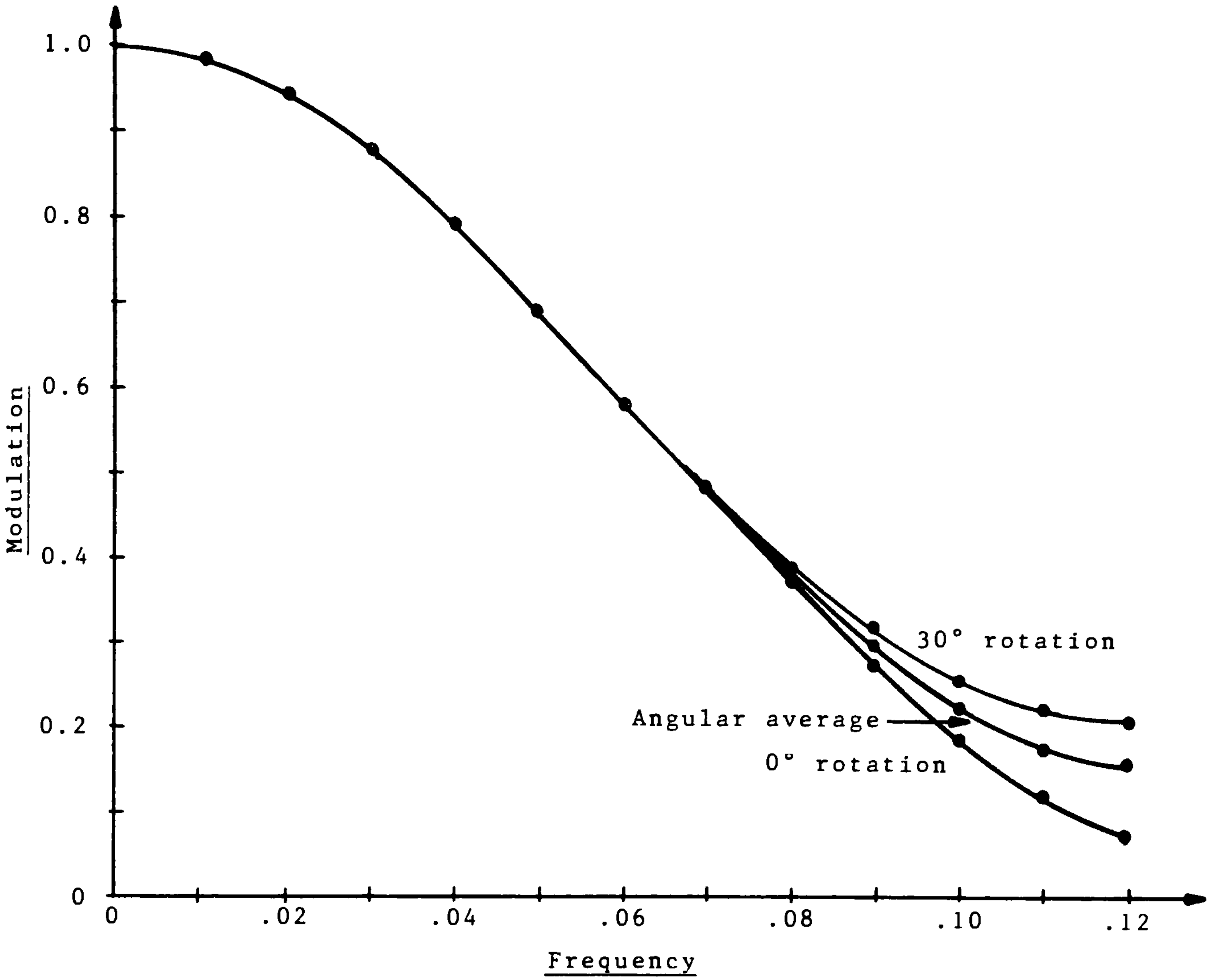

Figure VIII: Modulation transfer functions for the triangle point spread function.

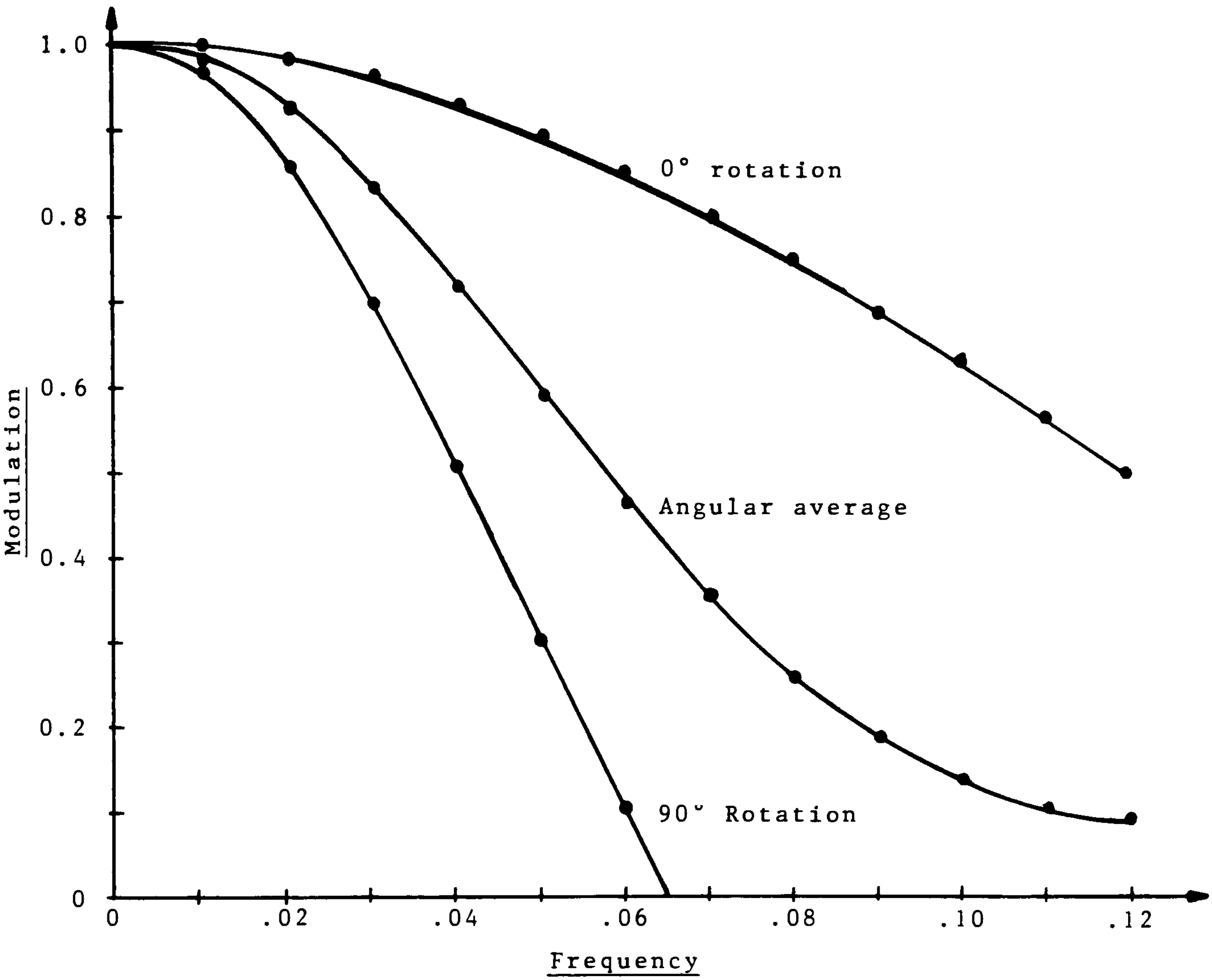

Figure IX: Modulation transfer functions for the rectangle point spread function.

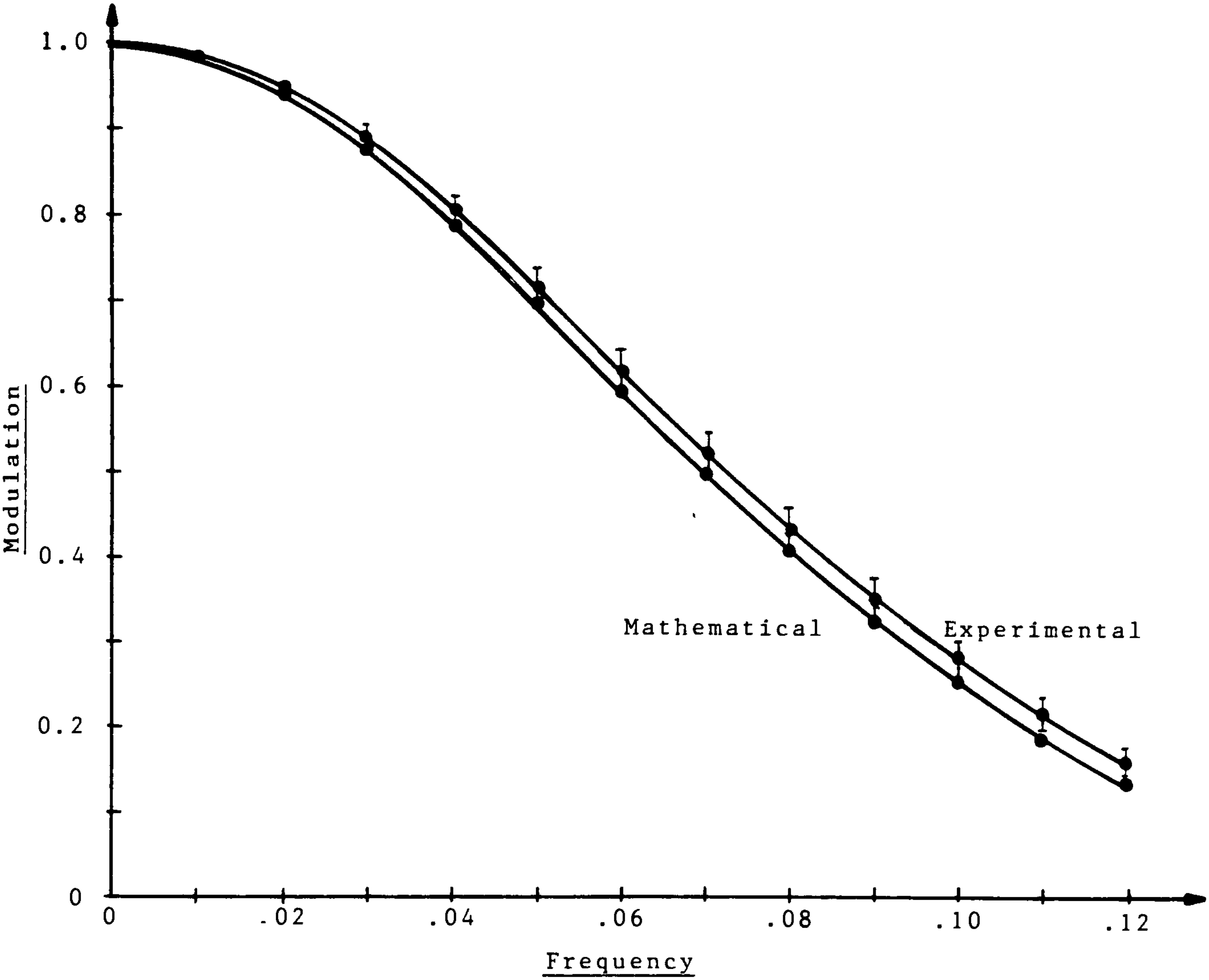

Figure X: Comparison of the MTF calculated from the mathematically derived second moment and the MTF calculated from the experimentally derived second moment, for the triangle point spread function.

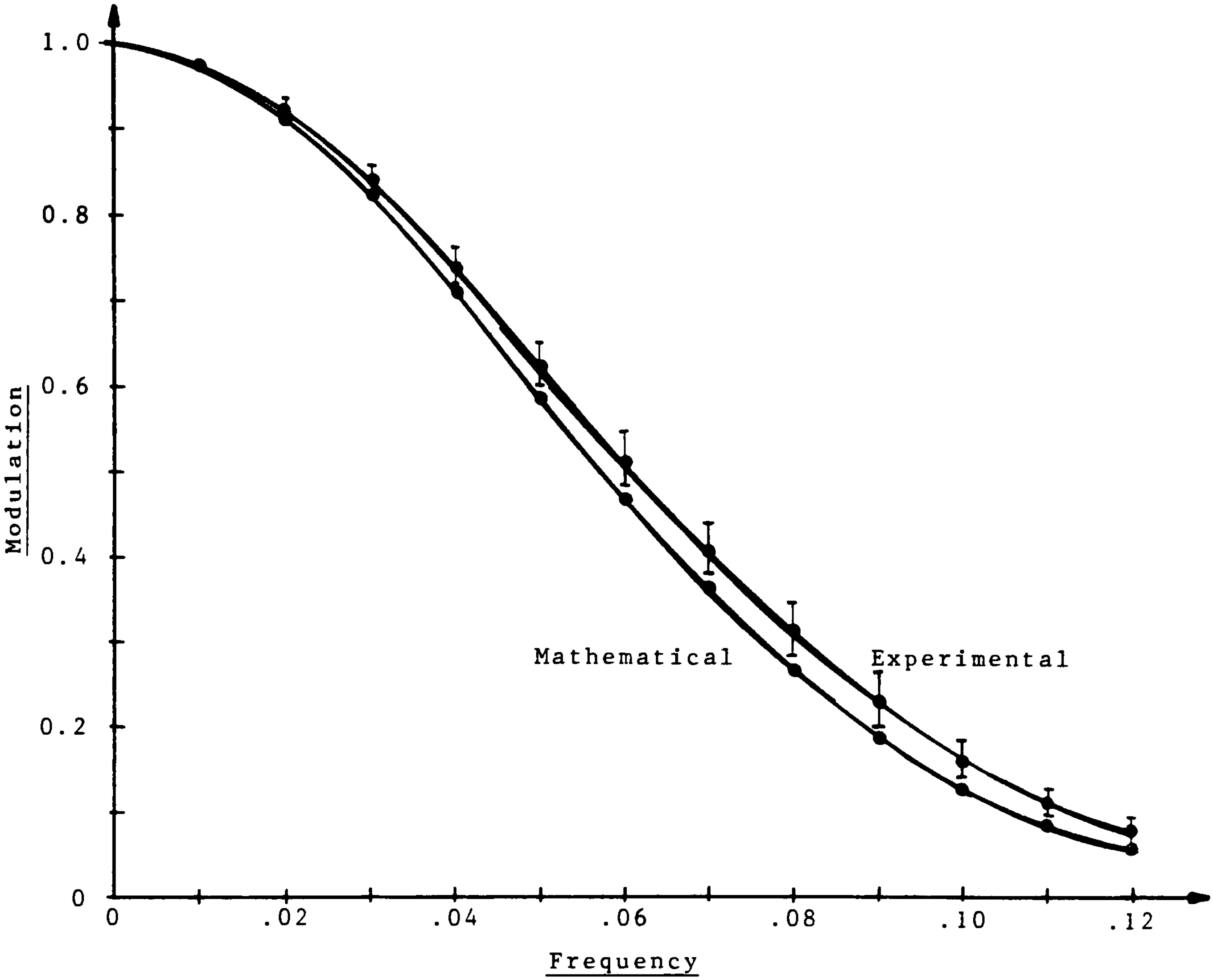

Figure XI: Comparison of the MTF calculated from the mathematically derived second moment and the MTF calculated from the experimentally derived second moment, for the rectangle point spread function.

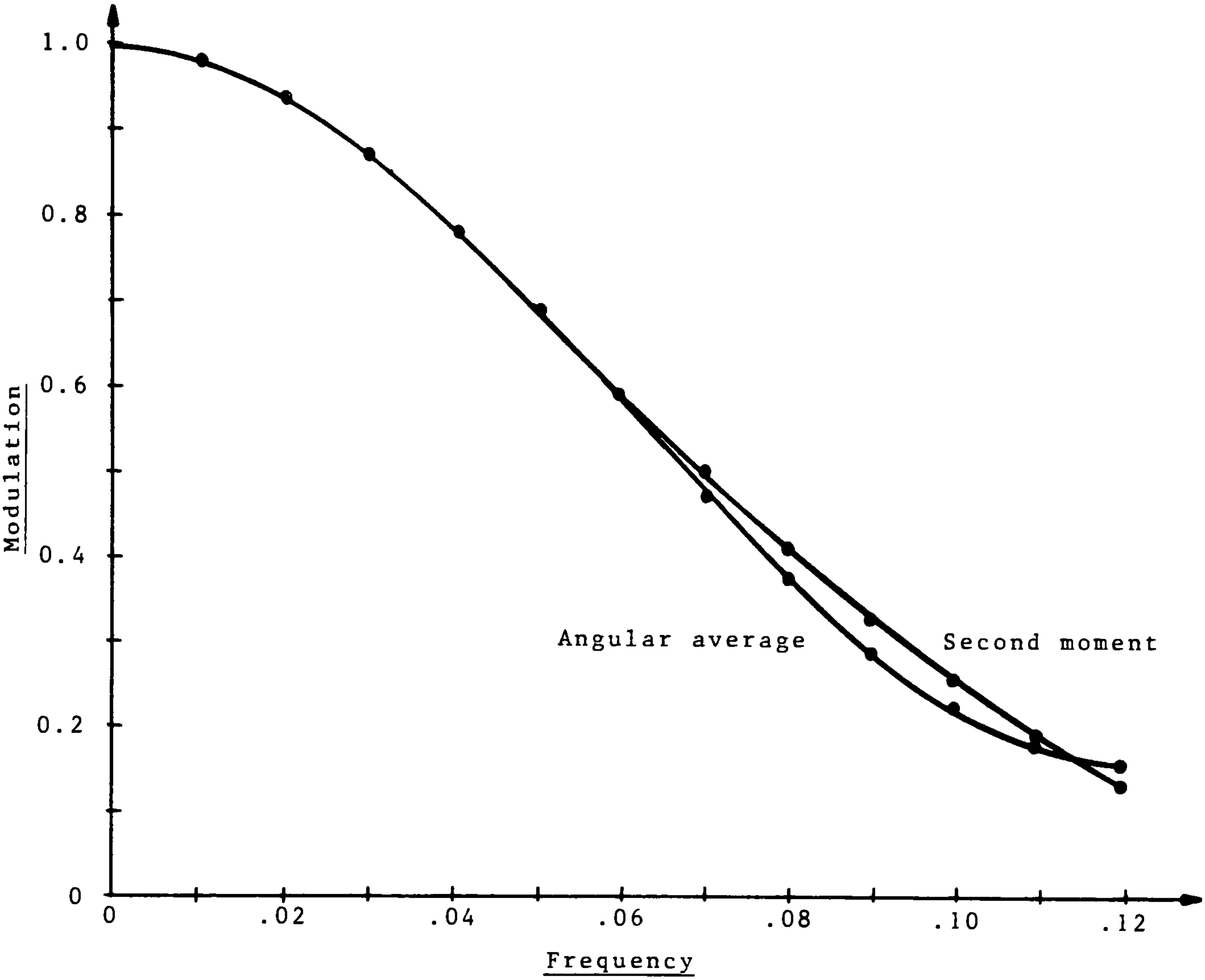

Figure XII: Comparison of the angular averaged MTF and the MTF calculated from the mathematically derived second moment, for the triangle point spread function.

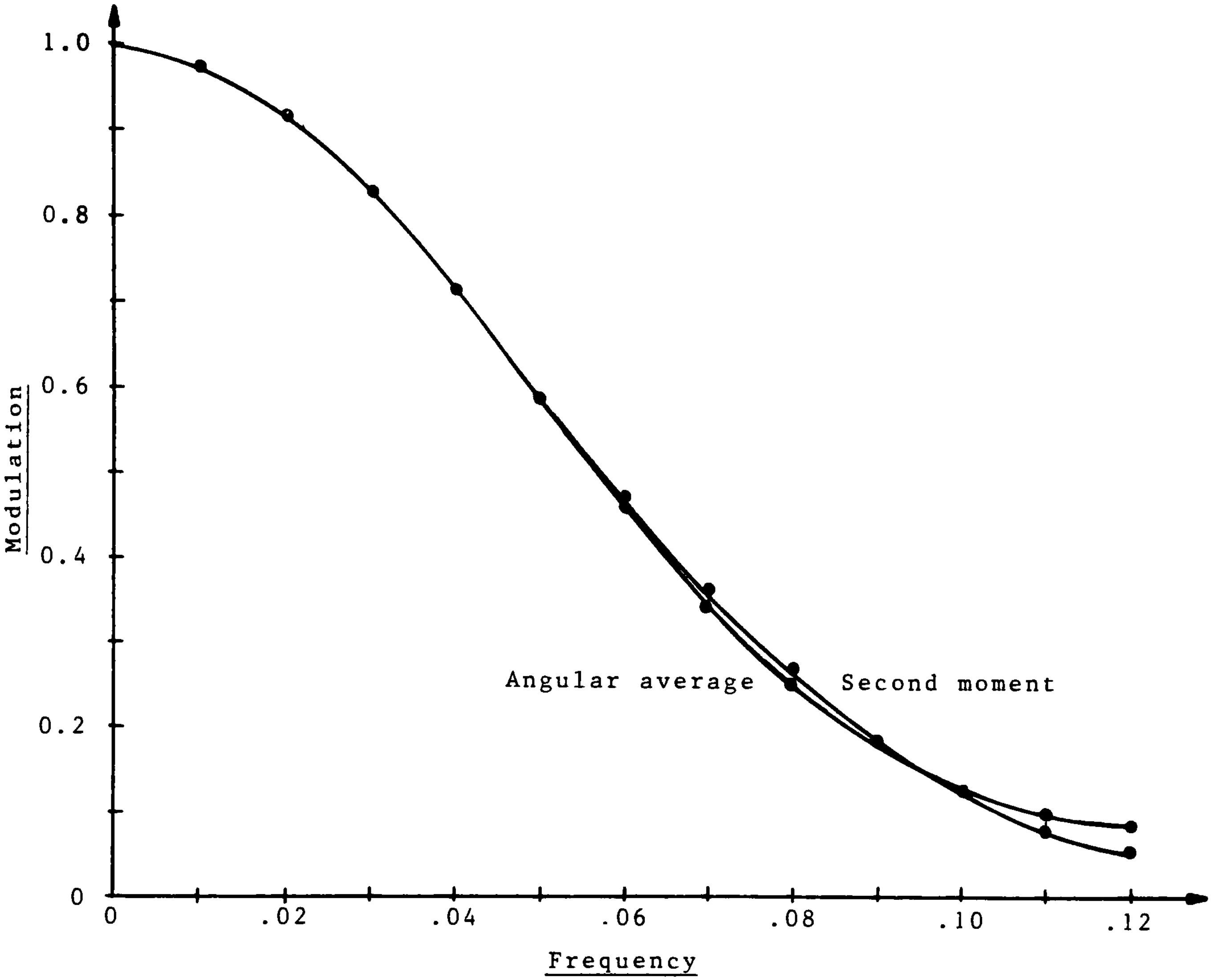

Figure XIII: Comparison of the angular averaged MTF and the MTF calculated from the mathematically derived second moment, for the rectangle point spread function.

IV. Discussion.

The results of the angularly averaged MTF for the triangle point spread function are shown in Figure VIII. For comparison, the MTFs for 0 and 30 degree rotations were plotted in addition. As expected of any average, the mean value fell within the two extremes. It is interesting to note that the three curves are virtually the same down to a modulation of 0.50 and only differ by +/-0.07 at a modulation of 0.18.

This is certainly not the case for the rectangle point spread function. Figure XI displays the angularly averaged MTF curve as well as the MTF curves for 0 and 90 degree rotations. These curves vary much more than the corresponding curves for the triangle. This is a good example of why angular averaged MTF curves are needed for rotationally asymmetric point spread functions. The MTF curve for the 90 degree rotation falls to zero modulation while the MTF curve for the 0 degree rotation drops only to a modulation of 0.85 over the save frequency interval. Accordingly, if a symmetric point spread function had been assumed, the calculated MTF would vary greatly depending upon which direction the point spread function was scanned.

To demonstrate that the scanning system calculated accurate second moments, the experimental values are compared to mathematically derived values. Figure X shows this relationship for the triangle point spread function using MTF curves. The results are excellent. The curves vary by no more than 0.03 modulation at any given frequency. Figure XI shows the same relationship for the Rectangle point spread function. The curves are not as close as in the case of the triangle, but still very good. The maximum variation is no more than 0.05 modulation.

In both cases, the experimental results plot above the mathematical results. This implies that the experimental second moment was calculated to be smaller than what is should be. This underestimation is attributed to small accumulative errors in the mathematical approximations. However, given the results, the error is not that significant.

To show that the calculated MTF is independent of the orientation of the scans across the point spread functions, 95% confidence limits were determined for the experimental MTF curves. The average second moment value for each rotation angle was calculated and the corresponding MTF values computed. From these MTF values the 95% confidence limits were calculated and plotted for specific frequencies.

Small confidence limits mean that the MTF values were close for each rotation. Figure X shows the confidence limits for the triangle point spread function. The limits are quite small, at most +/-0.02 modulation from the average.

Figure XI shows the confidence limits for the rectangle. Again, the confidence limits are small, at most +/-0.04 modulation from the average value. From this, it is easy to see that the orientation of the scan is not important in calculating the MTFs.

Figure XII displays the relationship between the MTF determined from the mathematically derived second moment and the angular averaged MTF for the triangle point spread function. Down to a modulation of 0.60 they are indistinguishable, after which they begin to spread apart. However, the greatest difference is only about 0.05 modulation in the 0.40 to 0.20 modulation range. Overall the second moment provided a good approximation. Figure XIII shows the same relationship for the rectangle point spread function. The second moment is an excellent approximation in this case. Down to a modulation of 0.10, the curves vary by no more than 0.01 modulation. As in the case for the triangle, the curves are the same down to a modulation of 0.60.

V. Conclusion.

From the data collected it is easy to conclude that the second moment method provides a very good one dimensional approximation to a two dimensional angularly averaged MTF. In the 1.0 to 0.6 modualtion range the difference is not noticable. If a small error, on the order of 0.05 modulation, can be tolerated, then the second moment method provides a good approximation down to a modulation of 0.20.

In addition it was observed that this approximation is independent of the scanning orientation.

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Appendix A.

Derivation of one dimensional second moment MTF equation for approximating a two dimensional angularly averaged MTF.

$$OTF(f)=\int_{-\infty}^{\infty} LSF(x)EXP[-i2\pi fx]dx$$

Expanding

$$EXP[-i2\pi fx]= \sum_{n=0}^{\infty}(-i2\pi fx)^n/n!$$

Can express OTF(f) in terms of the series

$$OTF(f)=\int_{-\infty}^{\infty} LSF(x) \sum_{n=0}^{\infty} (-i2\pi fx)^n /n!$$

Using the moments of LSF(x) defined by

$$m_n=\int_{-\infty}^{\infty} LSF(x)x^n dx$$

and performimg a term-by-term integration

$$OTF(f)= \sum_{n=0}^{\infty}(-i2\pi f)^n m_n/n!$$

Expanding too the first three terms

$$OTF(f)=1-i2\pi fm_1 -2\pi^2 f^2 m_2$$

Since the second term in the expansion involves the first moment of the image, it can be eliminated by shifting the coordinate system to be centered on the centroid of the image distribution. Therefore,

$$OTF(f)=1-2\pi^2 f^2 m_2$$

Note, the approximation to the MTF is gaussian.

To Find m_2;

$$m_2=\int_{-\infty}^{\infty} r^2 dm$$

The second moment about any axis is found by multiplying each element of a distribution by the square of its distance from the axis. For a light distribution,

$$m_2=\int_{-\infty}^{\infty}(x^2+y^2)PSF(x,y)dxdy$$

rewritting

$$m =\int_{-\infty}^{\infty} x^2 PSF(x,y)dx+\int_{-\infty}^{\infty} y^2 PSF(x,y)dy$$

noting that

$$LSF(x)=\int_{-\infty}^{\infty} PSF(x,y)dx$$

$$LSF(y)=\int_{-\infty}^{\infty} PSF(x,y)dy$$

Then

$$m_2=\int_{-\infty}^{\infty} x^2 LSF(x)dx+\int_{-\infty}^{\infty} y^2 LSF(y)dy$$

Recognizing that the first term is the second moment of the PSF about the x-axis and the second term is the second moment of the PSF about the y-axis.

Then

$$m_2=m(2x)+m(2y)$$

and

$$OTF(f)=EXP[-\pi^2 f^2(m(2x)+m(2y))]$$

Since

$$MTF(f)=\left|OTF(f)\right|$$

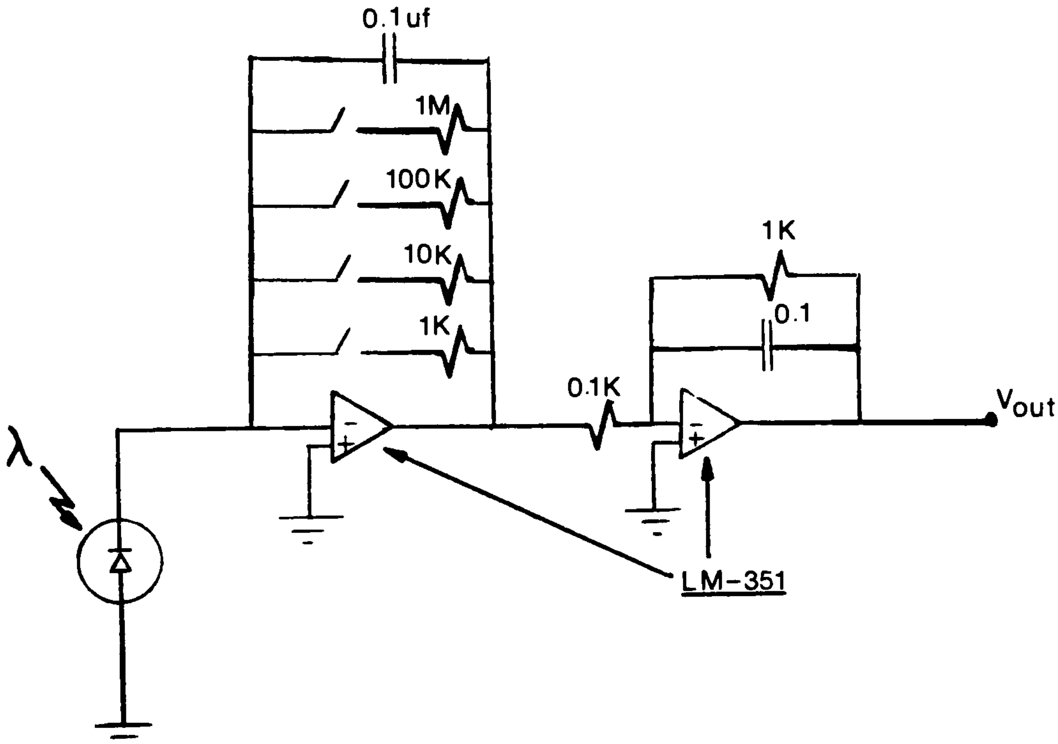
Then

$$MTF(f)=\left|EXP[-\pi^2 f^2(m(2x)+m(2y))]\right|$$

END OF DERIVATION

Appendix B.

Schematic diagram of the detector's amplifying circuit.



Appendix C.

Listing of the program that computes second moments.

```
5 REM  THIS PROGRAM CALCULATES THE SECOND MOMENT ABOUT
6 REM  THE CENTROID OF THE LINE SPEAD FUNCTION.  THE INPUT
7 REM  IS AN EDGE RESPONSE FUNCTION.
10 DIM DAT(100),DDAT(100),NDAT(100)
20 INPUT"NUMBER OF POINTS IN DATA FILE ";DP
30 PRINT""
40 INPUT"FILENAME ";F$
50 OPEN2,8,2,"0:"+F$+",S,R"
60 FOR X = 1 TO DP
70 INPUT#2,DAT(X)
80 NEXT X
90 CLOSE2
100 FOR X = 40 TO DP
110 PRINTDAT(X)
120 NEXT X
130 INPUT"LARGEST DATA VALUE ";LDV
140 FOR X = 1 TO DP
150 NDAT(X)=DAT(X)/LDV
170 NEXT X
180 FOR X = 1 TO DP-1
190 DDAT(X)=NDAT(X+1)-NDAT(X)
210 NEXT X
220 Y=0.25
230 FOR X = 1 TO DP-1
240 M0=M0+DDAT(X)*0.5
250 M1=M1+DDAT(X)*Y*0.5
260 Y=Y+0.5
270 NEXT X
280 CENTROID=M1/M0
290 Y=0.25-CENTROID
300 FOR X = 1 TO DP-1
310 M2=M2+DDAT(X)*Y^2*0.5
320 Y=Y+0.5
330 NEXT X
340 PRINTF$
345 PRINT"M2= ";M2
346 PRINT"":PRINT"":PRINT""
347 INPUT"REPEAT FOR ANOTHER FILE (Y/N) ";G$
348 IF G$="N" THEN GOTO 500
350 M0=0:M1=0:M2=0:GOTO30
500 END
```

Appendix D.

Listing of the program that computes MTF values from the second moment.

```
5 REM THIS PROGRAM CALCULATES MTF VALUES FROM AN INPUTTED M2X+M2Y
10 DIM M(50)
20 INPUT"M2X+M2Y= ";M2
30 C=^2*M2
33 OPEN4,4:CMD4
34 PRINT"M2X+M2Y= ";M2
35 PRINT""
40 FOR F=0 TO 0.2 STEP 0.005
50 M(F)=INT(1000*EXP(-C*F^2))/1000
55 PRINTF,M(F)
60 NEXT F
65 CLOSE4
140 INPUT"REPEAT ";R$
150 IF R$="Y" THEN GOTO 20
160 END
```

Appendix E.

Listing of the program that computes the angular averaged MTF for the triangle point spread function.

```
10 REM THIS PROGRAM CALCULATES THE ANGULAR AVERAGED MTF FOR THE
11 REM TRIANGLE POINT SPREAD FUNCTION
100 DIM A(1000),B(1000),R(1000),I(1000),M(1000),MOD(1000)
110 INPUT"LENGTH OF 1/2 OF ONE SIDE ";L
120 PRINT""
130 PRINT""
140 INPUT"INPUT THE DEGREE INCREMENT ";IC
145 OPEN4,4:CMD4
150 DENOM=30/IC
330 ALPHA=0
340 FOR D=0 TO DENOM-1
350 INPUT"A1= ";A1
360 INPUT"B1= ";B1
400 ALPHA=ALPHA+IC
405 F=0.0001
410 FOR C=1 TO 11
420 AARG=A1**F
430 BARG=B1**F
440 AD=(A1-B1)*A1*^2*F^2
450 BD=(A1-B1)*B1*^2*F^2
460 R(C)=(SIN(BARG))^2/BD+(SIN(AARG))^2/AD
470 I(C)=SIN(AARG)*COS(AARG)/AD-SIN(BARG)*COS(BARG)/BD
472 MOD(C)=(R(C)^2+I(C)^2)^0.5
475 F=C/100
480 NEXT C
485 FORC=1TO11:M(C)=M(C)+MOD(C)/MOD(1):NEXTC
490 NEXT D
500 PRINT""
510 PRINT"F","MODULUS"
520 FOR F=1 TO 11
540 PRINT(F-1)/100,M(F)
550 NEXT F
560 END
```

Appendix F.

Listing of the program that computes the angular averaged MTF for the rectangle point spread function.

```
5 REM  THIS PROGRAM CALCULATES THE ANGULAR AVERAGE OF THE RECTANGLE
6 REM  POINT SPREAD FUNCTION
10 DIM XS(50),YS(50)
20 INPUT"A AND B ";A,B
30 FOR F = 1 TO 10
40 FOR T = 1 TO 91 STEP 10
50 XARG=*A*F*COS(T/57.3)
60 YARG=*B*F*SIN(T/57.3)
70 XS(F)=XS(F)+SIN(XARG)/XARG
80 YS(F)=YS(F)+SIN(YARG)/YARG
90 NEXT T
100 PRINTF,XS(F)*YS(F)
110 NEXT F
120 END
```

VITA

The Author was born and raised in a small town om the Connecticut shoreline. After finishing high school on Long Island, the Author enrolled in the School of Photographic Arts and Sciences at the Rochester Institute of Technology in 1978. Plans for the future include a career in the imaging science field.