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Image quality analysis of a spectra-radiometric sparse aperture model

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ABSTRACT

Sparse aperture (SA) telescopes represent a promising technology to increase the effective diameter of an optical system while reducing overall weight and stowable size. Although conceptually explored in the literature for decades, the technology has only recently matured to the point of being reasonably considered for certain applications. In general, a sparse aperture system consists of an array of sub-apertures that are phased to synthesize a larger effective aperture. The models used to date to create predictions of sparse aperture imagery typically make use of a “gray world” assumption, where the input is a resampled black and white panchromatic image. This input is then degraded and resampled with a so-called polychromatic system optical transfer function (OTF), which is a weighted average of the OTFs over the spectral bandpass. In reality, a physical OTF is spectrally dependent, exhibiting varying structure with spatial frequency (especially in the presence of optical aberrations or sub-aperture phase errors). Given this spectral variation with spatial frequency, there is some concern the traditional gray world resampling approach may not address significant features of the image quality associated with sparse aperture systems. This research investigates the subject of how the image quality of a sparse aperture system varies with respect to a conventional telescope from a spectra-radiometric perspective, with emphasis on whether the restored sparse aperture image will be beset by spectral artifacts.

Keywords: sparse aperture, image quality, spectral artifacts, Wiener filter, restoration

1. INTRODUCTION

As technology advances, the information scientists and analysts need becomes more detailed. One piece of equipment that is used in a wide variety of fields, both civil and military, is the optical telescope. Telescopes are used in everything from trying to find the birth of the universe to treaty verification; such information is vital to understanding today’s world. In the remote sensing arena, space-based platforms utilizing telescopes have many benefits over those of ground-based or airborne systems although they also have many problems and drawbacks. At the moment it is still very expensive to launch satellites into space and it looks as if it will stay that way for quite a long time. Another problem besides cost is that the payload has to fit inside the launch vehicle. It may seem like an obvious limitation, but it is one that is a limiting factor to launching larger spaced-based imaging platforms. Here, one is generally interested in increasing the diameter (D) of the aperture to achieve a higher resolution. The diffraction-limited performance of a telescope can be characterized by the width of the point spread function (PSF_{width}) in the image plane:

$$PSF_{width} = 2.44 \frac{\lambda f}{D} = 2.44 \lambda (f\#) \quad (1)$$

where λ is the central wavelength, f is the effective focal length, and $f\#$ is the system F-number (f/D). Equation 1 shows that as the diameter of the aperture increases, the width of the point spread function decreases, implying finer detail can be resolved. An SA system involves the synthesis of a larger effective primary aperture through the combination of separate smaller optical systems (or sub-apertures) that are phased to form a common image field. This is why SA systems look so promising: they provide the ability to detect a higher resolution target while bypassing problems associated with fabricating a large monolithic optic. Potential benefits of SA systems include reduction in weight due to aperture synthesis of an array of smaller sub-apertures, less problematic launch fairing envelope constraints, decrease in the difficulty and expense of fabricating a large optic, and the ability to detect at a higher resolution. The sub-apertures can either be launched separately and constructed in space, or they can be attached to a structure that will unfold in space like the James Webb Telescope. Despite

the mission enablers, there are many problems associated with a sparse aperture system concept, including lower signal-to-noise ratio (SNR),^{1,2} phasing errors,^{3,4,5} narrow field of view (FOV),³ sensitivity to aberrations,^{2,3} increased integration time and associated image smear,^{1,2} deployment and alignment challenges,⁴ and extremely tight line-of-site requirements, to name a few. However, the potential benefits of these systems are enough to warrant continued research into developing them.

1.1. System Modeling

The theory of phasing an array of sub-apertures to synthesize a telescope with a larger effective diameter was first proposed by Meinel.⁶ Since then, there has been a plethora of research into different aspects of sparse aperture systems, including system design (*e.g.*, multiple telescope versus common secondary mirror),² sub-aperture configuration and associated diffraction-limited system performance, field of view considerations, aberrations, pupil matching and phasing errors to give a sampling of parameters under study.

One fundamental reason why these systems suffer from low SNR is that their collection area is much less than that of a monolithic system. Consequently, they detect fewer photons than their conventional counterparts (*e.g.*, Cassegrainian or Ritchey-Chretien telescopes) for a given integration period. In addition, an SA system typically has a modulation transfer function (MTF) that exhibits significant reduction in modulation in the mid and high frequency range. This physical phenomenon further contributes to a reduction in the detection SNR.¹ Since the SA system will have a lower detection SNR and significantly reduced MTF, the detected image quality will ultimately be worse than a standard Cassegrainian system. The challenge that remains is whether appropriate image restoration techniques can be invoked to recover quality without boosting noise excessively. Typical MTFs for a tri-arm SA configuration and a diffraction-limited filled aperture can be seen in Figure 1 below, where the SA system is “aberrated” as a result of random piston-tip/tilt errors represented by the phase profile on the bottom right of the diagram. From this figure, it is evident the SA’s highly modulated MTF will result in significant deterioration to the detected signal quality. Different SA configurations will have their own image quality characteristics because each has an individually unique MTF. This research endeavor is exploring multiple aperture types, including tri-arm, Golay, annular, and phased petals. For the purposes of illustration, however, this paper focuses on interim results acquired with only the general tri-arm SA configuration found in Figure 1. Meinel⁷ provides a thorough treatment of various SA configurations and their corresponding diffraction-limited MTFs for the interested reader.

To the knowledge of the authors, there has not been extensive published research on modeling the spectral effects of the detected scene and optical system characteristics on SA image quality. Much of the literature seems to indicate that SA image quality investigations performed to date have principally focused upon one of two methods. The first one develops a so-called polychromatic OTF by averaging hundreds of quasi-monochromatic OTFs over the spectral passband of interest.² This approach creates a single OTF realization that removes the spectral character at a given wavelength and therefore does not adequately account for the spectral nature of the scene. Its value lies in the fact that it captures some of the radially smearing spectral effects of the system OTF over the detection bandpass. It has been demonstrated that such an approach has been more than adequate for modeling conventional telescope performance. Unfortunately, the unique nature of SA system performance as characterized by their OTFs leads to a concern that such an approach may be inadequate for them. The second method alluded to above simply relies upon use of a quasi-monochromatic OTF at the central wavelength to approximate the overall system OTF.⁸ Obviously, this approach has even less spectral fidelity than the first method discussed above. The limitations of these approaches in capturing the true spectral character provides the impetus for this research effort.

For this investigation, there has been a concerted effort to understand the key problems facing SA systems. The possible consequences of including spectral physics into the SA model was deemed necessary to fully understand the system performance associated with these systems. With this added modeling attribute, largely uninvestigated SA image quality issues could be explored, including any spectral artifacts caused by the unique nature of the system as well as post-processing techniques that could be used to minimize them.

Figure 1. Filled and tri-arm apertures along with their corresponding MTFs. The tri-arm SA has random piston-tip/tilt phase errors associated with its sub-apertures as shown beside its MTF. The rms wavefront error across the entire tri-arm aperture is 0.24 waves.

The SA model under development uses a general image equation which makes the simplifying assumption that an isoplanatic region of the image plane can be defined where the system behaves largely as linear, shift invariant. This makes it possible to model the system through linear systems theory by convolving the object radiance field with a system PSF. The other simplifying assumption is that the noise is additive and statistically independent. With these assumptions, the general-purpose image formation equation takes the following form:

$$g(x, y) = f(x, y) * h(x, y) + n(x, y) \quad (2)$$

where $g(x, y)$ is the degraded image, $f(x, y)$ is the object radiance, $h(x, y)$ is the system impulse response or PSF, “*” is the convolution operand, and $n(x, y)$ is additive noise. These systems are usually easier to evaluate in the frequency domain because convolution becomes multiplication⁹ and the PSF transforms into the OTF. In the frequency domain, equation 2 can be recast into the following form:

$$G(\xi, \eta) = F(\xi, \eta) \cdot H(\xi, \eta) + N(\xi, \eta) \quad (3)$$

where $G(\xi, \eta)$ is the degraded image spectrum, $F(\xi, \eta)$ is the object spectrum, $H(\xi, \eta)$ is the system OTF, and $N(\xi, \eta)$ is the noise spectrum.

1.2. Optical Transfer Function

Based on the linear systems model described previously, it is apparent that characterizing either the PSF or OTF is a key component of the modeling process. For both of these system measures, the contribution from the aperture tends to be the driving factor for SA systems. To compute the diffraction-limited PSF at a given

wavelength for an SA system with circular sub-apertures, one must evaluate the following expression:

$$PSF_{sparse}(x, y) = \left(\frac{\pi D^2}{4\lambda f} \right)^2 \left(\frac{2J_1 \left[\frac{\pi r D}{\lambda f} \right]}{\left[\frac{\pi r D}{\lambda f} \right]} \right)^2 \left| \sum_{i=1}^N e^{-\frac{2\pi i}{\lambda f} (xx_i + yy_i)} \right|^2 \quad (4)$$

where (x, y) are the image plane coordinates, J_1 is the first-order Bessel function, $r = \sqrt{x^2 + y^2}$, N is the number of sub-apertures, and (x_i, y_i) defines the center location of the i^{th} sub-aperture. The first half of equation 4 is familiar as the point spread function of a clear circular aperture. The summation portion of the equation derives from the displaced centers of the sub-apertures inherent in any SA system.

Evaluation of the PSF expression above can become quite tedious for challenging aperture configurations and further development is required to address aberrations and/or phase errors. As a result, the implementation of choice tends to be the Fourier domain model that makes use of the system OTF. As mentioned previously, the principal component of the system transfer function is the aperture OTF, computed via the normalized autocorrelation of the scaled pupil function $p[x, y]$ as in the following equation:

$$OTF(\xi, \eta) = \frac{p[\lambda f \xi, \lambda f \eta] \star p[\lambda f \xi, \lambda f \eta]}{\iint_0^\infty p[x, y] dx dy} \quad (5)$$

where “ $\star$ ” represents the autocorrelation operation. From first principles, it is understood that the PSF and OTF expressions enumerated above constitute Fourier transform pairs per the following expression:

$$OTF(\xi, \eta) = \Im \{ PSF(x, y) \} \quad (6)$$

where $\Im \{ \cdot \}$ represents the Fourier transform operand. By design, the MTF is then simply the modulus of the OTF. Accordingly, either a space-domain or a frequency-domain implementation is feasible, although the Fourier domain model appears more attractive from our viewpoint in evaluating complicated, aberrated SA pupil geometries. In addition to the aperture OTF discussed above, other transfer function components can also be evaluated, including detector sampling, carrier diffusion, atmosphere, image motion, etc.

Figure 2 depicts the variation in the OTF character for an aberrated tri-arm system and diffraction-limited filled aperture at three wavelengths of interest ($0.45\mu\text{m}$, $0.55\mu\text{m}$, and $0.65\mu\text{m}$). If one compares the two system transfer functions, the spectrally-independent OTF approximations discussed previously seem problematic for an SA system considering the large amount of variation observed in the transfer function as a function of wavelength. Unlike the OTF of the filled aperture which is characterized by a monotonically decreasing slope, the OTF of the SA typically exhibits a varying structure that oscillates considerably before finally going to zero at the cutoff frequency. One will note that the effect of changing the wavelength not only affects the cutoff frequency, it also changes the locations of the peaks and troughs in the OTF. This issue is exacerbated with very sparse apertures because as the sub-aperture size decreases within a given synthesized diameter of constant size, the OTF becomes more attenuated until significant holes appear in the spatial frequency plane. These zero regions in one spectral OTF will not typically correspond to zeroes at the same frequency in other wavelengths. Such character ultimately results in blocked spatial frequency content and spatially varying modulation as a function of wavelength. This phenomenon presents issues with restoration as the “optimal” boost for one wavelength at a given spatial frequency is likely to be highly non-optimal for another wavelength. This should ultimately lead to some spectral artifacting depending on the detection SNR and the relative sparseness of the SA configuration. Clearly, the gray world OTF modeling approximations would not tend to capture such spectral effects. Since filled and conventional apertures exhibit well behaved OTFs, this spectral quality issue has not historically surfaced as a modeling problem.

Figure 2. MTF variation with wavelength for (left) a tri-arm system versus (right) a filled aperture.

1.3. Governing Signal Equation

The target scene radiance detected by the sensor is being modeled through use of first-principles atmospheric propagation and radiometry codes. The overall governing radiometric equation¹⁰ utilized in the modeling process deals with both the reflected and self-emitted regimes of the electro-magnetic spectrum. However, current research focuses on a panchromatic application (0.4-0.9 μm), so only the reflected regime is of interest. The radiance L_{source} reaching the entrance pupil of the sensor accordingly takes the following general form:

$$L_{source} [\text{Wm}^{-2}\text{sr}^{-1}\mu\text{m}^{-1}] = \left[E'_{s\lambda} \cos \sigma' \tau_1(\lambda) \frac{r(\lambda)}{\pi} + F E_{d\lambda} \frac{r_d(\lambda)}{\pi} + (1 - F) L_{b\lambda avg} r_d(\lambda) \right] \tau_2(\lambda) + L_{u\lambda} \quad (7)$$

where $E'_{s\lambda}$ is the exoatmospheric irradiance, $E_{d\lambda}$ is the downwelled irradiance, $L_{b\lambda avg}$ is the average radiance from the background, $L_{u\lambda}$ is the total upwelled radiance, τ_1 is the transmission from the sun to the target, τ_2 is the transmission along the target-sensor path, σ' is the solar declination angle to the target, $r(\lambda)$ is the total spectral reflectance of the target with no directional information, $r_d(\lambda)$ is the diffuse reflectivity of the object, and F is the fraction of the sky that can be seen by the target. This radiometry computation is performed through the joint use of the MODTRAN atmospheric propagation model and the radiance calculation engine within the Digital Image and Remote Sensing Image Generation (DIRSIG) simulation code.¹¹

Given evaluation of the spatially varying object scene radiance through methods discussed above, the detected signal in electron count is calculated through the following:

$$S_{electron} = \frac{\pi A_{det} T_{int} F_{fill}}{4 (f\#)^2 h c} \int_0^\infty L_{source}(\lambda) \tau_{opt}(\lambda) \eta(\lambda) \lambda d\lambda \quad (8)$$

where A_{det} is the area of the detector, T_{int} is the integration time, F_{fill} is the fill factor, h is Planck's constant, c is the speed of light, $f\#$ is the system F-number, τ_{opt} is the transmission of the optics in the system, and η is the spectral quantum efficiency of the sensor. With appropriate conversion factors, equation 8 provides the essence of the object image that is convolved with the system PSF in the linear systems model. Applying these

factors and addressing the degraded signal in the Fourier domain, one acquires the following equation:

$$S_{freq}^{out}(\xi, \eta, \lambda) = \frac{G_{conv}G_{elec}2^n}{S_{ADC}} \frac{\pi A_{det}T_{int}F_{fill}}{4(f\#)^2 hc} \int_0^\infty OTF(\xi, \eta, \lambda) L_{source,FT}(\xi, \eta, \lambda) \tau(\lambda) \eta(\lambda) \lambda d\lambda \quad (9)$$

where G_{conv} is the conversion gain, G_{elec} is the electronic gain, S_{ADC} is the A/D Converter input voltage range, n represents the number of binary digits associated with the A/D Converter, $OTF(\xi, \eta, \lambda)$ is the spectrally dependent optical transfer function, and $L_{source,FT}$ represents the Fourier transform of the source spectral radiance profile calculated in the overall governing radiometric equation. This expression becomes the governing equation for the source signal spectrum in the frequency domain for the implemented model.

Many of the simulation efforts performed in the past, including those addressing SA systems,² have defined a so-called “polychromatic” MTF that involves spectrally weighting individual MTF realizations at a given wavelength and then deriving an average MTF over the spectral passband of interest. The objective of defining such a transfer function is really quite simple: to make the integrand in the governing expression found in equation 9 more tractable, especially for use in resampling existing gray-scale object imagery in support of detailed sensor simulations. This is accomplished by effectively eliminating the spectral dependency of the MTF term in the expression and allowing it to be pulled outside the integral, as in the following approximation to the governing equation for the output signal spectrum:

$$S_{freq}^{out}(\xi, \eta) = \frac{G_{conv}G_{elec}2^n}{S_{ADC}} \frac{\pi A_{det}T_{int}F_{fill}}{4(f\#)^2 hc} \frac{F_{obj,gray}(\xi, \eta)}{F_{obj,gray}(0, 0)} MTF_{poly}(\xi, \eta) \int_0^\infty L_{source}(\lambda) \tau_{opt}(\lambda) \eta(\lambda) \lambda d\lambda \quad (10)$$

where $MTF_{poly}(x, y)$ is the averaged polychromatic MTF, $F_{obj,gray}(\xi, \eta)$ is the object source radiance spectrum, and $F_{obj,gray}(0, 0)$ is the source spectrum evaluated at zero spatial frequency. In effect, the above expression accounts for the spatial variability resident within the source spectral radiance profile by introducing the normalized Fourier transform of the object imagery source into the spectrum calculation. The gray world approximation found in equation 10 has worked quite effectively for conventional aperture simulations in past studies. Unfortunately, given the unique OTF character of certain systems, this approximation may not be good enough for the SA case, requiring a more direct implementation of equation 9.

1.4. System Noise

Given the previously derived expressions for object radiance and system OTF, the final physical effect that must be addressed for the linear model described in equation 2 is image noise. If individual noise components are modeled as statistically independent, they will add in quadrature to form an expression for the total noise σ_{tot} according to the following general formulation:

$$\sigma_{tot} = \left(\sum_{i=1}^N \sigma_i^2 \right)^{\frac{1}{2}} = (\sigma_{photon}^2 + \sigma_{dc}^2 + \sigma_{quan}^2 + \sigma_{elec}^2 + \sigma_r^2)^{\frac{1}{2}} \quad (11)$$

where the N independent noise sources are photon noise (σ_{photon}), dark current shot noise (σ_{dc}), quantization noise (σ_{quan}), signal chain electronic noise (σ_{elec}), and detector read noise (σ_r). For modeling purposes, this noise term is approximated in equation 2 through the following expression:

$$n(x, y) \approx \sqrt{f(x, y) * h(x, y)} n_1(x, y) + \sigma_{dc}(T_{int}) n_2(x, y) + \sigma_{read} n_3(x, y) \quad (12)$$

where $n_1(x, y)$ and $n_2(x, y)$ are demeaned random Poisson noise distributions, $n_3(x, y)$ is a zero-mean, unit-variance Gaussian noise distribution, and σ_{read} is a constant rms readout noise that captures quantization, electronic and detector read noise sources. For high SNR simulations, all of the $n_i(x, y)$ noise sources tend to converge to zero-mean, unit-variance Gaussian distributions. The sources of noise described above, in conjunction with the first-principles radiometric and Fourier optical modeling, should capture the first-order physical effects for most linear imaging systems, including the SA configurations of interest in this study.

1.5. Image Restoration

Based on the discussion in Section 1.2, it is apparent that SA systems will generally produce imagery that exhibit significant reduction in contrast or sharpness compared to equivalent conventional aperture systems due to OTF effects. In order to recover some of this lost image quality, an appropriate filtering technique is generally applied to boost the reduced modulation at certain spatial frequencies while attenuating the resulting gain in system noise. For this study, a conventional Wiener-Helstrom filter was implemented to provide the required image restoration. The classic Wiener filter $W(\xi, \eta)$ utilized in this investigation takes the following form:

$$W(\xi, \eta) = \frac{H^*(\xi, \eta)}{|H(\xi, \eta)|^2 + \frac{S_n(\xi, \eta)}{S_f(\xi, \eta)}} \quad (13)$$

where $H^*(\xi, \eta)$ is the complex conjugate of the OTF, $S_n(\xi, \eta)$ is the noise power spectrum and $S_f(\xi, \eta)$ is the object power spectrum. Since the noise and object power spectra are typically not known *a priori*, their ratio is frequently estimated via a constant or fit to an appropriate physical model.

2. METHODOLOGY

One of the overall objectives of this study is to determine whether wavelength dependency affects an SA system differently than a conventional aperture. A color Siemens resolution star chart was used as the target, where the foreground (spatially varying star) has the spectrum of red car paint and the background is of white car paint. The target object and its spectra can be seen in Figure 3. A DIRSIG-generated synthetic scene was used instead of real data to eliminate unknowns associated with material type/spectra and provide immediate flexibility in spectral/spatial resolution. In general, DIRSIG is designed to create a synthetic image with physically accurate spectral and radiometric properties from 0.3 to 14 microns.¹² The image has spectral properties attached to the materials in the scene from an external database of empirically collected radiometric data. MODTRAN is executed as a precursor to the DIRSIG run to propagate radiance through the atmosphere. The output from DIRSIG is an image cube of raw entrance pupil radiance values in as many spectral bands as desired.

Figure 3. The RGB object image and its corresponding spectra for the Siemens resolution chart.

For the purposes of illustration, a preliminary experiment using three wavelengths (0.45, 0.55, and 0.65 μm) was performed to compare a tri-arm SA configuration to a filled aperture. It should give some initial insight into

how an SA system will behave given a color object scene. To design an SA model that accurately predicts a detected signal in the presence of a polychromatic target, a direct implementation of equation 9 must be used. Appropriate image noise must subsequently be added to the degraded target signal acquired through this process. The degraded images are restored using the Wiener filter found in equation 13. In this experiment, one can restore each individual band with their corresponding OTFs (representative of a multispectral restoration) or utilize the OTF associated with the central wavelength (more representative of the restoration involved with wide bandwidth panchromatic imagery). For this case, the system OTF as well as noise and scene power spectra are known *a priori*, allowing one to “optimally” restore the detected imagery for the purposes of analysis. The results provided in this article focus on the restorations utilizing only the center wavelength OTF to demonstrate spectral issues that may surface when one attempts to restore wide bandwidth SA imagery.

The images created from the tri-arm configuration and the filled aperture were compared using the normalized root-mean-square error (*nrmse*), defined as follows:

$$nrmse = \sqrt{\frac{\sum |g(x, y) - f(x, y)|^2}{\sum |f(x, y)|^2}}. \quad (14)$$

The normalized rms error is just the degraded image $g(x, y)$ minus the original object image $f(x, y)$, quantity squared, summed and normalized by the squared summation of the original image. Preliminary conclusions can be drawn about the image predictions by using the *nrmse* along with visual interpretation of the results.

3. RESULTS

An interim output of the described model is a spectral image cube that has a depth equal to the number of wavelengths being used to approximate scene radiometry (three bands in this case). A panchromatic image can be obtained by integrating the individual spectral signatures in the image cube per equation 9. In this experiment, three wavelengths were used to give preliminary insight into the spectral nature of an SA system. The tri-arm configuration image quality is compared to that of a filled aperture in figures 4, 6, and 7.

Figure 4. The RGB image for the (a) sparse aperture and the (b) filled aperture after being restored using the center wavelength OTF in a Wiener filter using a known power spectrum ratio.

From these figures, it can be seen that the image quality of the SA system exhibits a spectral sensitivity that is not apparent with the filled aperture. For instance, the RGB images in Figure 4 indicate that the SA system tends to produce imagery that has significant color artifacts in this collection scenario, the filled aperture does not suffer from this problem. This is an especially interesting observation since the errors inherent in utilizing the center wavelength OTF for restoration of the filled aperture imagery seems to have minimal impact. On the other hand, the OTF mismatch has a significant effect on the restoration of the SA imagery. This is especially apparent in the restoration of the off-center (non-green) spectral images in Figure 7, where the SA images are significantly distorted while the filled aperture bands do not possess the same degraded effect. One will also note that the presence of zero regions in the SA system OTF generates nulls in the spatial frequency content of the produced imagery (see figures 4 and 7). These null regions are clearly located in different spatial frequency locations for different wavelengths, providing a primary source for color artifacts. From these results, it is apparent that the oscillatory character of the OTF, the severity of the aberrations present (0.24-waves rms), the detection SNR (19.5 dB), the wide detection bandwidth (0.45-0.65 μ m), and the low fill factor (0.173 based on encircled diameter) combine to contribute to significant spectral issues. Follow-on investigations will focus on trying to further understand the sensitivity of various SA configurations to these design parameters and quantify the incidence and severity of the observed spectral artifacts on overall image quality.

Table 1 shows how the *nrmse* changes as a function of wavelength for both the filled and SA system configurations pursued in this experiment. The *nrmse* provides some additional insight into how the imagery acquired from the two apertures differ. As anticipated, the degraded SA imagery has poorer *nrmse* than that of the filled aperture for all bands due to the nature of the OTFs depicted in Figure 2. In addition, the restored imagery demonstrates significant relative improvement from the degraded figures of merit. The apparent anomaly of the red band *nrmse* appears to be the result of the complex interaction between the red target spectra and the high signal background. This quantitative data suggests that the *nrmse* captures some of the nature of the image degradation that gives rise to color artifacts, although the visual effects of the restoration apparent in Figure 4 are definitely beyond what the *nrmse* can adequately explain.

		<i>nrmse</i>		
λ	Filled (degraded)	Tri-arm (degraded)	Filled (restored)	Tri-arm (restored)
0.45 μ m	0.164	0.427	0.146	0.433
0.55 μ m	0.209	0.469	0.100	0.185
0.65 μ m	0.086	0.172	0.056	0.148

Table 1. The normalized rms error of the separate spectral “images” for the tri-arm and filled configurations.

Future research will focus on evaluation of the integrated radiance at higher spectral resolution and include complex synthetic scenes that are more representative of remotely sensed scenes. Two such scenes under consideration can be seen in figure 5, where RGB spectral images were created using the DIRSIG modeling package.

4. CONCLUSIONS

The preliminary results acquired through this experiment continue to motivate further research to understand the properties of sparse aperture (SA) systems and how they interact with a radiometrically and spectrally accurate scene. These initial results show that the imagery acquired from moderately aberrated, low fill factor SA systems manifest spectral artifacts that are not characteristic of filled apertures. This observation is most notable by looking at the restored imagery predictions in figures 4 and 7, where the SA predictions demonstrate significant color effects that are not present in the filled aperture data. It is possible that some of these problems may be reduced as additional wavelength data is considered within the model, but it is still anticipated that spectral artifacting will be an issue of concern for SA telescope configurations.

Figure 5. (a) Synthetic DIRSIG scene with 18-inch GSD; (b) synthetic scene with 6-inch GSD.

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Figure 6. Comparison of the degraded RGB image planes for (1) a tri-arm sparse aperture and (2) a filled aperture at three separate wavelengths: (a) 0.65 μm , (b) 0.55 μm , (c) 0.45 μm . The spectral “images” were degraded by their respective system OTFs.

Figure 7. Comparison of the restored RGB image planes for (1) a tri-arm sparse aperture and (2) a filled aperture at three separate wavelengths: (a) $0.65\mu\text{m}$, (b) $0.55\mu\text{m}$, (c) $0.45\mu\text{m}$. The spectral “images” were restored via a Wiener filter with the center wavelength OTF and a known power spectrum ratio