11-8-2002

Elastic Ladar Modeling for Synthetic Imaging Applications

Robin R. Burton

Rochester Institute of Technology

John R. Schott

Rochester Institute of Technology

Scott D. Brown

Rochester Institute of Technology

Follow this and additional works at: <http://scholarworks.rit.edu/article>

Recommended Citation

Robin R. Burton, John R. Schott, Scott D. Brown, "Elastic ladar modeling for synthetic imaging applications", *Proc. SPIE 4816, Imaging Spectrometry VIII*, (8 November 2002); doi: 10.1117/12.451630; <https://doi.org/10.1117/12.451630>

This Article is brought to you for free and open access by RIT Scholar Works. It has been accepted for inclusion in Articles by an authorized administrator of RIT Scholar Works. For more information, please contact ritscholarworks@rit.edu.

Elastic LADAR Modeling for Synthetic Imaging Applications

Robin R. Burton, John R. Schott, and Scott D. Brown

Chester F. Carlson Center for Imaging Science
Rochester Institute of Technology
54 Lomb Memorial Dr. Rochester, NY 14623

ABSTRACT

The Digital Imaging and Remote Sensing Image Generation (DIRSIG) model is a synthetic imagery generation model developed at the Center for Imaging Science (CIS) at the Rochester Institute of Technology (RIT). It is a quantitative first principle based model that calculates the sensor reaching radiance from the visible through to the long wave infrared on a spectral basis. DIRSIG generates a very accurate representation of what a sensor would see by modeling all the processes involved in the imaging chain. Currently, DIRSIG only models passive sources such as the sun and blackbody radiation due to the temperature of an object. Active systems have the benefit of the user being able to control the illumination source and tailor it for specific applications. Remote sensing Laser Detection and Ranging (LADAR) systems that utilize a laser as the active source have been in existence for over 30 years. Recent advances in tunable lasers and infrared detectors have allowed much more sophisticated and accurate work to be done, but a comprehensive spectral LADAR model has yet to be developed. In order to provide a tool to assist in LADAR development, this research incorporates a first principle based elastic LADAR model into DIRSIG. It calculates the irradiance onto the focal plane on a spectral basis for both the atmospheric and topographic return, based on the system characteristics and the assumed atmosphere. The geometrical form factor, a measure of the overlap between the sensor and receiver field-of-view, is carefully accounted for in both the monostatic and bistatic cases. The model includes the effect of multiple bounces from topographical targets. Currently, only direct detection systems will be modeled. Several sources of noise are extensively modeled, such as speckle from rough surfaces. Additionally, atmospheric turbulence effects including scintillation, beam effects, and image effects are accounted for. To allow for future growth, the model and coding are modular and anticipate the inclusion of advanced sensor models and inelastic scattering.

Keywords: atmospheric turbulence, elastic scattering, geometrical form factor, hyperspectral, LADAR, synthetic image generation

1. INTRODUCTION

Optical remote sensing can be divided into two categories, passive or active, based on the source of the optical illuminate. Passive systems use natural sources such as the sun or blackbody radiation due to the temperature of an object as the illuminate. Active systems use an illuminate that can be controlled and varied by the user, such as a laser. Various types of Laser Detection and Ranging (LADAR) systems exist, each based on a different scattering process and span from the ultraviolet through the infrared spectral regions. These systems have been used for a variety of commercial and military applications such as: pollution monitoring and tracing, forest fire detection, and agriculture monitoring.

The Digital Imaging and Remote Sensing Image Generation (DIRSIG) model is a synthetic imagery generation model developed at the Center for Imaging Science at the Rochester Institute of Technology.¹ It is a quantitative first principle based model that calculates the sensor reaching radiance from the visible through to the long wave infrared on a spectral basis. DIRSIG generates a very accurate representation of what a sensor would see by modeling all the processes involved in the imaging chain. Currently, DIRSIG only models passive sources. The model is an integrated collection of independent first principles based submodels that work in conjunction to produce radiance field images with high radiometric fidelity in the 0.3 – 30.0 μm region. In order to provide a tool to assist in LADAR development, this research is the first stage of a multipart effort to

Correspondence: E-mail: rrb9581@cis.rit.edu; Telephone: 585-475-7780; Fax: 585-475-2529

Figure 1. This overview illustrates all of the physical processes included in the model. These processes include atmospheric attenuation, atmospheric turbulence, speckle, and multiple bounce off of topographical targets. The atmosphere and topographical returns are modeled spatially, temporally, and spectrally.

incorporate a first principle based LADAR model into DIRSIG. The model at this point considers only elastic scattering processes. It calculates the irradiance onto the focal plane based on the system characteristics and the assumed atmosphere.

There are several pre-existing LADAR models. These models come in two types: those that calculate the return or signal-to-noise ratio for a particular system without attempting to model the return from an actual scene and those that produce a synthetic image. Models of the first type include BACKSCAT,² the System Optimization Numerics for DIAL³ (SONDIAL), Lidar-PC,⁴ and Atmospheric Lidar End-to-end Simulator⁵ (ALIENS). The only model that currently produces an actual synthetic image is the Infrared Modeling and Analysis (IRMA) code. The IRMA model was developed for the Air Force Research Laboratory Munitions Directorate by CSC/Nichols.⁶ IRMA is a fully developed synthetic imagery generation model that can model detailed sensor effects. IRMA contains a passive channel, an active millimeter channel, and a LADAR channel. Some of the advantages of IRMA are that it has both direct and coherent detection models. Additionally, IRMA keeps track of relative phase in order to model the speckle return from surfaces. IRMA currently has several limitations that restrict its use. IRMA can model a spectral system, but each wavelength must be processed separately. Additionally, the LADAR channel can not model the aerosol return and does not include atmospheric turbulence effects. Furthermore, it cannot model bistatic systems (i.e. the source and the receiver must be in the same position). Finally, it does not account for multiple bounce. The model being developed for DIRSIG addresses some of these deficiencies while giving up some of the advantages of IRMA. The phenomenology included in the DIRSIG model is covered in the next section and the implementation strategy in Section 3.

2. RELEVANT PHYSICAL PROCESSES

Figure 1 illustrates all the physical processes which the model takes into account. The pulse begins at some user defined point in the atmosphere, a certain height above the ground, and pointing in a certain direction. The pulse itself has several dimensions, not only is it spread out over space, it is spread out over time and has a spectral distribution. The pulse then propagates through the atmosphere and is attenuated by both molecular absorption and Mie and Rayleigh scattering. The atmosphere also introduces additional effects because of turbulence, including a broadening of the pulse both spatially and temporally. Additionally, the beam will be

randomly deflected from its initial propagation direction. This deflection will cause the position of the beam centroid to vary from one pulse to the next. Atmospheric turbulence also causes random spatial and temporal fluctuations in the beam intensity. These fluctuations superimpose a speckle type pattern onto the beam. Some of the scattered energy will make it back to the receiver and create an atmospheric return, referred to as the aerosol return. How much energy makes it back to the receiver depends upon the volume backscatter coefficient and the overlap of the beam and the receiver field-of-view. Part of the energy which makes it to the ground will reflect in the direction of the receiver and create the topographical return. If the scene is complicated, parts of the beam may be reflected by several surfaces before being reflected back in the direction of the receiver. Reflection by a topographical surface also imposes a speckle pattern onto the beam due to the surface roughness. As the beam propagates to the receiver, it experiences the same atmospheric effects as before. The broadening of the beam results in a blur in the focal plane and the varying deflection of the beam from pulse to pulse causes the image to move about the focal plane. The receiver, as currently implemented, uses direct detection and is only sensitive to the intensity of the beam.

2.1. LADAR Basics

This section introduces the basic theory necessary to create the core LADAR model. The basic aerosol and topographical LADAR equations, introduced in Section 2.1.1, are the fundamental equations implemented in the model. The next section, 2.1.2, introduces the geometrical form factor. The geometrical form factor is an essential quantity and is carefully included in the model for both coaxial and biaxial systems with and without a central obscuration. Finally, the model will calculate the basic equations on a spectral basis based upon a laser spectral distribution of power provided by the user and assuming no change in the laser linewidth during the scattering process.

2.1.1. Basic Equation

The foundation of the model is the basic LADAR equation as formulated by Measures⁷

$$P(\lambda_L, R) = P_L \frac{c\tau_L}{2} \frac{A_0}{R^2} \xi(\lambda_L) \xi(R) \beta(\lambda_L, R) \exp \left[-2 \int_0^R \kappa(\lambda_L, r) dr \right], \quad (1)$$

where $P(\lambda_L, R)$ is the total scattered laser power received in watts at a time corresponding to the leading edge of the laser pulse propagating to a range R , λ_L is the wavelength of the transmitted light, R is the distance of the scattering volume from the transmitter-receiver, P_L is the average power in the laser pulse, c is the speed of light, τ_L is the effective pulse duration, $c\tau_L$ is the effective laser pulse length, A_0 is the area of the objective lens or mirror (i.e., the active area of the receiver telescope), and $\xi(\lambda_L)$ is the receiver's spectral transmission factor at wavelength λ_L . The geometrical form factor, $\xi(R)$, is a function of the geometrical probability factor, $\xi(R, \mathbf{r})$. The geometrical probability factor or overlap is the probability of radiation from position r in the target plane at range R reaching the detector based on geometrical considerations. In this equation, the overlap is assumed to be unity where the field of view of the receiver optics overlaps the laser beam and zero elsewhere. $\beta(\lambda_L, R)$ is the volume backscatter coefficient obtained from MODTRAN at distance R and $\kappa(\lambda_L, R)$ is the total atmospheric extinction including molecular absorption and Mie and Rayleigh scattering. When working with a topographical reflector, Measures⁷ modifies the basic LADAR equation to the form

$$P(\lambda_L, t) = P_L \frac{A_0}{R_T^2} \xi(\lambda_L) \xi(R_T) \frac{\rho^s}{\pi} \exp \left[-2 \int_0^{R_T} \kappa(\lambda_L, r) dr \right], \quad (2)$$

where R_T is the range to the topographical target and ρ^s is the topographical target's scattering efficiency.

The aerosol return calculated from Equation 1, the upper curve in Figure 2, is equal to the summation of the return from the leading edge of the pulse at each distance convolved with the temporal shape of the pulse. Therefore, the aerosol equation already accounts for the temporal shape of the pulse. Since the topographic

Figure 2. The illustration on the left shows that the aerosol equation is the summation of the return from each distance and already accounts for the temporal shape of the pulse. The graph on the right is of the Hufnagel-Valley Model and shows the dependence of the model on the parameter A , the value of the index of refraction structure constant one meter above the ground. The upper curve has $A = 1 \cdot 10^{-14}$ and the lower curve has $A = 1 \cdot 10^{-13}$.

equation has not been convolved with the temporal shape of the pulse, the convolution must be performed before adding it to the modeled aerosol return. As the pulse reflects from a topographical surface, part of the pulse is generating a topographical return while the portion of the pulse still in the air is generating an aerosol return. Care must be taken in the model to step down the contribution from the aerosol return as the topographic return is added in because Equation 1 assumes that the entire pulse is contributing to the aerosol return.

2.1.2. Geometrical Form Factor

The geometrical form factor is defined by Measures⁷ as

$$\xi(R) = \frac{1}{\pi W^2(R)} \int_{\psi=0}^{2\pi} \int_{r=0}^{r_T} \xi(R, r, \psi) F(R, r, \psi) r dr d\psi, \quad (3)$$

where $F(R, r, \psi)$ is the distribution of laser power over the target plane and $W^2(R) = W_0^2 + \theta^2 R^2$ is the squared radius of the laser pulse in the target plane; here, W_0 is the laser output aperture radius and θ is the laser's half divergence angle. The geometrical probability factor is one where the receiver optics field-of-view overlaps the laser beam and zero everywhere else.

2.1.3. Spectral Distribution

The basic LADAR equation introduced in section 2.1.1 assumes that the laser emission is monochromatic and that it elastically scatters with no change in wavelength. The output of any laser has some spectral distribution $l(\lambda)$. The laser linewidth is often as large as the vibrational-rotational absorption bands in the infrared and monochromaticity cannot be assumed. We followed the approach by Mégie⁸ who simply multiplied the basic LADAR equation by $l(\lambda)$ and integrated over λ . This approach assumes that the spectral distribution does not change during the scattering process.

2.2. Atmospheric Propagation

In order to model the effects of turbulence, one must first model the index-of-refraction structure constant as a function of altitude. The index-of-refraction structure constant is a measure of the strength of the changes in the refractive index due to turbulence. There are several models for this constant. The models range from

no-parameter models, which represent average profiles, to parametric models, which can incorporate measured radiosonde data. Beland⁹ compiled a description of these various models, how they were developed, and their limitations. Since we need a model that extends all the way to the ground and can be calculated easily, the model incorporated into our LADAR simulation is the Hufnagel-Valley Model as formulated by Schmitt et al.³:

$$C_n^2(h) = 8.2 \times 10^{-26} v^2 h^{10} e^{-h} + 2.7 \times 10^{-16} e^{-\frac{h}{1.5}} + \frac{A}{(1000h)^{\frac{4}{3}}} e^{-10h}, \quad (4)$$

where h is the height in kilometers, A is the value of C_n^2 one meter above the ground, and v is the root-mean-square wind speed between five and twenty kilometers above the ground in m/s. Equation 4 is graphed in Figure 2 for $v = 21 \text{ m/s}$, $A = 1 \cdot 10^{-14}$ and $v = 21 \text{ m/s}$, $A = 1 \cdot 10^{-13}$.

2.2.1. Atmospheric Turbulence

Now that we have a model for the index-of-refraction structure constant, we are ready to introduce the statistics which use this quantity: scintillation, beam effects, and image effects. The spatial and temporal variation in intensity caused by atmospheric turbulence is called scintillation and resembles a speckle pattern. The statistics of scintillation can be found in several excellent texts including the treatments by Beland⁹ and Goodman.¹⁰ Unfortunately, these discussions assume propagation through a turbulent atmosphere in one direction. In many LADAR systems the beam propagates through a turbulent atmosphere, reflects off a surface, and then propagates back through effectively the same turbulent atmosphere to the receiver. This double passage causes Enhanced Backscatter (EBS) effects. These EBS effects are discussed by Andrews.¹¹

Scintillation is due to the effect of turbulence on the log amplitude of the wavefront. In contrast, image and beam effects are due to atmospheric turbulence distorting the phase of the wavefront and result in the loss of higher spatial frequencies. The equations presented in this section describing image and beam effects were taken from Beland⁹ and Nelson et al.¹² They are only valid within the inertial range, which is the spatial region where inertial forces dominate and the random fluctuations can be assumed spatially homogeneous and isotropic. The lower end of the inertial range defines the inner scale of turbulence, while the upper end defines the outer scale of turbulence. There are two types of beam effects, beam spread and centroid wander. Beam spread describes how the radius of the laser pulse in the target plane is affected by turbulence. Large scale eddies can deflect the beam while small scale eddies can scatter it. There is both a long term and a short term broadening effect. The long term beam radius, ρ_L , is related to the short term beam radius, ρ_s , and the short term beam centroid, ρ_c , by

$$\langle \rho_L^2 \rangle = \langle \rho_s^2 \rangle + \langle \rho_c^2 \rangle. \quad (5)$$

The short term beam radius is the sum of a term due to diffraction, a term due to focusing, and a term due to turbulence and has the following form:

$$\langle \rho_s^2 \rangle = \frac{4L^2}{(kD)^2} + \left(\frac{D}{2}\right)^2 \left(1 - \frac{L}{\gamma}\right)^2 + \frac{4L^2}{(k\rho_{0s})^2} \left[1 - 0.62 \left(\frac{\rho_{0s}}{D}\right)^{1/3}\right]^{6/5}, \quad (6)$$

where ρ_{0s} is the short-term transverse coherence length, L is the propagation distance, k is the wavenumber, D is the initial diameter of the beam, and γ is the radius of curvature of the beam. The short-term transverse coherence length for a gaussian wave is

$$\rho_{0s,gauss} = \rho_{0s,plane} \left[\frac{\left(1 - \frac{L}{\gamma}\right)^2 + \frac{4L^2}{k^2 D^4} \left[1 + \frac{1}{3} \left(\frac{D}{\rho_{0s,plane}}\right)^2\right]}{1 - \frac{13}{3} \left(\frac{L}{\gamma}\right) + \frac{11}{3} \left(\frac{L}{\gamma}\right)^2 + \frac{4L^2}{3k^2 D^4} \left[1 + \frac{1}{4} \left(\frac{D}{\rho_{0s,plane}}\right)^2\right]} \right]^{\frac{1}{2}}, \quad (7)$$

where the short-term transverse coherence length for a plane wave is

$$\rho_{0s,plane} = \rho_{0,plane} \left[1 + 0.37 \left(\frac{\rho_{0,plane}}{D} \right)^{\frac{1}{3}} \right]. \quad (8)$$

The long term transverse coherence length for a plane wave is

$$\rho_{0,plane} = \left[1.46 \sec(\varphi) k^2 \int_0^L C_n^2(z) dz \right]^{-\frac{3}{5}}. \quad (9)$$

Centroid wander, the motion of the beam centroid from pulse to pulse, is given by the short term beam centroid and has the following form

$$\langle \rho_c^2 \rangle = \frac{2.97 L^2}{\left(k^2 \rho_{0s}^{5/3} D^{1/3} \right)}. \quad (10)$$

The beam effects, beam spread and centroid wander, effect the beam on the way down to the ground while the image effects occur on the path back up. There are two types of image effects, image blurring and image dancing. Beland⁹ follows Fried¹³ and gives a modulation transfer function (MTF) in the focal plane due to image blurring of the form

$$MTF(\nu) = \exp \left[-3.44 \left(\frac{\lambda f \nu}{r_0} \right)^{5/3} \right], \quad (11)$$

where r_0 is Fried's coherence length, f is the focal length, and ν is the spatial frequency. Fried's coherence length is related to the long term transverse coherence length by $r_0 = 2.1\rho_0$. Spatial coherence describes the phase difference between two points on the same wave front. If the phase difference between these two points remains constant in time, then these two points are coherent. If this is true for any two points, then the wave has perfect spatial coherence. Temporal coherence describes the phase difference of the same point at two different moments in time. If the phase difference is constant over this time interval, then the wave is temporally coherent for that time interval. If this is true for any time interval, then the wave has perfect temporal coherence. The long term transverse coherence length is a measure of the spatial coherence of the beam. Propagation through turbulence will reduce the spatial coherence of any beam which has a large initial coherence to the long term transverse coherence length.

The MTF given in Equation 11 assumes long time averages. Long term image blurring is caused by small-scale eddies that move or "dance" around the focal plane. This movement is caused by the advection of large eddies that tilt the wavefront. Beland⁹ asserts that time scales less than 0.01 seconds correspond to the short term case and have a MTF of

$$MTF_s(\nu) = \exp \left\{ -3.44 \left(\frac{\lambda f \nu}{r_0} \right)^{5/3} \left[1 - b \left(\frac{\lambda f \nu}{D} \right)^{1/3} \right] \right\}, \quad (12)$$

where $b = 1$ in the near-field and $b = 0.5$ in the far field, D is the diameter of the aperture.

The wavefront tilt can be characterized by calculating the mean-square displacement in the focal plane. Beland⁹ gives the expression for the mean-square displacement in the focal plane for a plane wave

$$\langle \delta^2 \rangle = 2.91 f^2 D^{-1/3} \int_0^L C_n^2(x) dx \quad (13)$$

The equations presented in this section are used to incorporate the image and beam effects into the LADAR model. Since we are assuming that LADAR systems will have pulse lengths of less than 0.01 seconds, the short term equations will be used. Now that we have the image and beam effects the review of atmospheric turbulence is complete. In order to complete the description of the properties of the medium through which the wave is propagating, we now turn our attention to atmospheric transmission.

2.2.2. Atmospheric Transmission

Atmospheric attenuation is caused by molecular absorption and scattering. Absorption at a particular wavelength depends upon numerous conditions such as the molecules present, their relative absorption strength, temperature, and pressure. The HITRAN (high-resolution transmission molecular absorption) database contains detailed spectral-line parameters and is used by the atmospheric transmission codes MODTRAN (Moderate Resolution Transmittance Code) and FASTCODE (Fast Atmospheric Signature Code). Currently, DIRSIG uses either MODTRAN or FASTCODE, depending on the resolution needed, to calculate atmospheric transmission.¹ The MODTRAN Code calculates atmospheric transmittance and radiance for frequencies from 0 to 50,000 cm⁻¹ at 2 cm⁻¹ resolution. FASCODE is similar to MODTRAN except it has higher resolution and does not calculate scattering coefficients. Due to the resolution needed, FASTCODE will be used in the normal fashion to calculate the atmospheric absorptance. Since resolution is not an issue with scattering because the scattering functions are assumed to be spectrally flat, a modified version of MODTRAN will be used to calculate the transmission, reflectance, as well as the angular scattering coefficients. Conservation of energy requires that

$$\alpha + \tau + \rho = 1, \quad (14)$$

where α is the absorptance, τ is the transmission, and ρ is the reflectance. Since we are using both FASCODE and MODTRAN it is possible to violate this relationship if working right on an absorption line. When that occurs, the atmospheric absorptance from FASCODE is the trusted quantity and the transmission and reflectance are adjusted so that the relationship is no longer violated.

2.3. Speckle

A speckle pattern is produced in an image when coherent light scatters off a surface that is rough on the order of a wavelength. Images that exhibit speckle effects have a grainy appearance. This granular appearance is due to the wavelets that make up the wavefront travelling varying distances before reflecting off the surface. The coherent, but now dephased, wavelets then interfere to produce the speckle pattern. The statistical properties of speckle can be found in the work by J.W. Goodman in the text by Dainty.¹⁴ By developing the probability density functions, Goodman shows that the mean value of the intensity is equal to the standard deviation. Since the contrast, C , is the ratio of the standard deviation to the mean value, it is always equal to one.

In addition to the first-order statistics which describe speckle measured at a single point, we also need the second-order statistics to describe the spatial structure of the speckle pattern. The width of the autocorrelation function gives a reasonable estimate of the average width of a speckle. Goodman¹⁴ also derives the autocorrelation of the speckle intensity:

$$R_I(\Delta x, \Delta y) = \langle I \rangle^2 \left[1 + |\mu_A(\Delta x, \Delta y)|^2 \right], \quad (15)$$

where $\mu_A(\Delta x, \Delta y)$ is the complex coherence factor

$$\mu_A(\Delta x, \Delta y) = \frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} I(\xi, \eta) e^{i \frac{2\pi}{\lambda R} (\xi \Delta x + \eta \Delta y)} d\xi d\eta}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} I(\xi, \eta) d\xi d\eta}, \quad (16)$$

and $I(\xi, \eta)$ is the spatial intensity profile across the target. When working with an imaging system the equation for the autocorrelation needs to be modified. The intensity profile across the target is replaced with the intensity

transmittance of the aperture. In summary, the size of the speckle pattern before the aperture is determined by the size of the beam while the size of the speckle pattern on the focal plane is determined by the size of the aperture.

How this description of speckle is incorporated into the model is explained in section 3.2, but we will address its limiting assumptions now. The statistics presented are based on the assumption that all surfaces are rough on the order of a wavelength and that fully developed speckle results. These assumptions should be valid, even for most manmade objects, throughout the visible and long wave infrared. It is also assumed that the source is coherent. It does not take into account relationships between the speckle pattern as a function of wavelength. The DIRSIG model will apply the same speckle pattern at each wavelength regardless of the spectral line shape given to the beam. As long as the user is modeling systems which have a narrow line shape, this assumption should not introduce much error.

3. METHODOLOGY

3.1. Photon Mapping

The initial work to include a LADAR model into DIRSIG used the typical ray tracing approach. Using this technique, the biggest problem was the numerical integration of the overlap region in the GFF calculation. The GFF must be computed many times, ideally for each pixel at every range. The numerical integration was slow and possibly a source of systematic error. One solution that decreased the computational burden was to precompute a large database and use interpolation. Additional ray tracing would then be needed to compute any shadowing and/or absorption of the beam because the GFF calculation assumes that the object plane within the overlap region is uniform. When the ray tracer sends out a ray and hits object A, it is assumed that the overlap area is filled with object A and it might not be. Another question arose in how to calculate the GFF for multiply bounced photons. Multiply bounced photons can hit several targets and the return from each target must be added into the overall return at the correct time as shown in Figure 3. This motivated the search for another method and led to replacing DIRSIG's traditional raytracer with photon mapping.

A photon map is a three-dimensional data structure that stores photons that have been forward propagated from sources in the scene. Photons start at the sources and are bounced around the scene. At each bounce, the energy, direction, and time of incidence are stored in the map. The direct and indirect load at a location and time are computed by querying the map for photons that are nearby. The map can be queried for either those photons within a certain distance or for the N closest. Then the query result can be tested to find only the photons within the desired time window. The return for any time window requires the integration of all surfaces along the path. The major advantages of the photon map implementation are that it automatically incorporates the GFF and gives us a method to account for multiplied bounced photons.

3.2. Speckle

There are several methods to model speckle. IRMA includes speckle by using Fourier optics to define a transfer function and then performs a numerical complex convolution. Another method is to do a full beam propagation as outlined by Nelson¹² that accounts for both turbulence and speckle. To create a model that accounts for turbulence effects, speckle, and complex three-dimensional scenes, a full beam propagation model is not practical nor is its implementation path clear. Additionally, beam propagation programs experience difficulties with the discrete nature of the Fourier transform. Therefore, it is desirable to create a texture pattern that can be applied to the geometrical optic solution and produce an image with the correct statistics. The system we are trying to model consists of a beam reflecting from a rough surface, propagating to the aperture/lens, and then propagating to the focal plane as shown in Figure 4. Using the Fraunhofer approximation, the propagation step from one plane to the next as given by Goodman¹⁵ is

$$g(x, y) = \frac{e^{ikz}}{i\lambda z} e^{\frac{ik}{2z} \left(\left(\frac{x}{\lambda z} \right)^2 + \left(\frac{y}{\lambda z} \right)^2 \right)} B \left(\frac{x}{\lambda z}, \frac{y}{\lambda z} \right), \quad (17)$$

where x and y are the coordinates in the plane, $B \left(\frac{x}{\lambda z}, \frac{y}{\lambda z} \right)$ is the Fourier transform of the beam at the starting plane evaluated at $(\xi \rightarrow \frac{x}{\lambda z}, \eta \rightarrow \frac{y}{\lambda z})$, k is the wavenumber of the light, λ is the wavelength of the light, and

Figure 3. In a three dimensional scene photons can bounce multiple times. For example, the beam might first hit a leaf where part of the beam is reflected back to the sensor while the rest is transmitted and continues on to the ground. At the ground, part of the beam is again reflected back to the sensor while part of the beam is scattered and hits a tree before heading back to the sensor. The travel time must be accounted for in order to add the return from each surface into the overall return at the correct time.

z is the distance between the planes. We are interested in the irradiance, which is the squared magnitude of Equation 17 such that

$$|g(x, y)|^2 = \frac{1}{(\lambda z)^2} \left| B\left(\frac{x}{\lambda z}, \frac{y}{\lambda z}\right) \right|^2 = \frac{1}{(\lambda z)^2} \varepsilon(x, y), \quad (18)$$

where $\varepsilon(x, y)$ is the total energy contained in the beam at the starting plane. The desired discrete texture map can be generated by propagating a beam without a random phase (NR) through the system to the focal plane, propagating the beam with a random phase (R) through the system, and then evaluating the ratio.

The texture map can be generated in a simpler way if the mean, variance, and autocorrelation of the pattern are known. The mean of the texture map can be approximated by taking the ratio of the total intensity of the random and nonrandom beam after propagation through the aperture. To prove this result, we must first define the mean of the texture map, $\bar{t}$, as

$$\bar{t} = \frac{1}{N} \sum_i \sum_j \frac{|b_{f,R}(x'_i, y'_j)|^2}{|b_{f,NR}(x'_i, y'_j)|^2}. \quad (19)$$

Since the act of propagation in the Fraunhofer region is proportional to the FT, the total energy contained in the beam at the focal plane is equal to the product of the total energy contained in the initial beam and the aperture. By backpropagating the beam, one can show that the ratio of the total energy in the random and non-random beams at the focal plane (f) is given by

$$\frac{\sum_i \sum_j \varepsilon_{f,R}(x'_i, y'_j)}{\sum_i \sum_j \varepsilon_{f,NR}(x'_i, y'_j)} = \frac{\sum_i \sum_j |FT\{b_{f,R}(x'_i, y'_j)\}|^2}{\sum_i \sum_j |FT\{b_{f,NR}(x'_i, y'_j)\}|^2} = \frac{\sum_i \sum_j |b_{AA,R}(x'_i, y'_j)|^2}{\sum_i \sum_j |b_{AA,NR}(x'_i, y'_j)|^2}$$

Figure 4. After reflection from a rough surface, the beam is propagated twice. The first propagation distance, from the point of reflection to the aperture/lens system, is z_1 and the second, from the aperture/lens system to the focal plane, is z_2 . The act of propagation in the Fraunhofer region is proportional to the Fourier transform.

$$= \frac{\sum_i \sum_j |A(x'_i, y'_j) b_{BA,R}(x'_i, y'_j)|^2}{\sum_i \sum_j |A(x'_i, y'_j) b_{BA,NR}(x'_i, y'_j)|^2}, \quad (20)$$

where $A(x'_i, y'_j)$ is the aperture, $b_{BA}(x'_i, y'_j)$ is the beam at the input side of the aperture, and $b_{AA}(x'_i, y'_j)$ is the beam at the output side of the aperture. Parseval's theorem states that

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |f(x, y)|^2 dx dy = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |FT\{f(x, y)\}|^2 d\xi d\eta. \quad (21)$$

The ratio becomes

$$\frac{\sum_i \sum_j |A(x'_i, y'_j) b_{BA,R}(x'_i, y'_j)|^2}{\sum_i \sum_j |A(x'_i, y'_j) b_{BA,NR}(x'_i, y'_j)|^2} = \frac{\sum_i \sum_j |b_{f,R}(x'_i, y'_j)|^2}{\sum_i \sum_j |b_{f,NR}(x'_i, y'_j)|^2}. \quad (22)$$

If the irradiance is approximately constant over the aperture, we can replace $|b_{f,NR}(x'_k, y'_l)|^2$ with a constant, C , which results in

$$\frac{\sum_i \sum_j |b_{f,R}(x'_i, y'_j)|^2}{\sum_i \sum_j |b_{f,NR}(x'_i, y'_j)|^2} \approx \frac{1}{NC} \sum_i \sum_j |b_{f,R}(x'_i, y'_j)|^2 = \bar{t}, \quad (23)$$

where N is the total number of points in the array. Figure 5 shows the mean calculated using a full beam propagation versus the mean calculated using the approximation technique. The means are essentially identical even though the beam took up only a small portion of the array and varied quite a bit over the aperture. This indicates that the condition of the beam being approximately constant is not that hard to meet.

To generate a texture map with the correct statistics, we also need to know the signal-to-noise ratio. If the beam is approximately constant over the aperture, the signal-to-noise ratio is given by

$$t_{S/N} = \frac{\bar{t}}{\sigma_t} \approx \frac{\frac{1}{C} \overline{|b_{f,R}(x', y')|^2}}{\sqrt{\frac{1}{C^2} \sigma_{|b_{f,R}(x', y')|^2}}} = \frac{\overline{|b_{f,R}(x', y')|^2}}{\sigma_{|b_{f,R}(x', y')|^2}} = 1, \quad (24)$$

Figure 5. The graph on the left shows the mean as a function of aperture size using the approximation technique (blue diamonds) versus actually generating the texture map image using beam propagation and calculating the mean of the texture map image (red triangles). The image on the right is a sample texture map.

where σ_t is the standard deviation of the texture map. Finally, the generation of a new random texture map for each pulse reduces to taking the aperture times a new random phase and propagating a distance z given by

$$z = \frac{1}{N\lambda^2 t} \sum_i \sum_j |FT \{A(x'_i, y'_j) \text{ random}(x'_i, y'_j)\}|^2, \quad (25)$$

where $\text{random}(x'_i, y'_j)$ is a random phase array. The resulting signal-to-noise of the texture map will be one and the autocorrelation will depend on the aperture size as described by Equation 15. A sample texture map is shown in Figure 5.

3.3. Scintillation

A similar approach to the speckle solution is being investigated for scintillation. The full beam propagation by Nelson¹² includes phase screens to simulate turbulence effects. This technique is being examined to see if it can be used to generate a texture pattern with the correct statistics and if any shortcuts can be used. An advantage of this technique is that the same phase screens can be used for both the down and up path allowing simulation of enhanced backscatter effects.

4. CONCLUSIONS AND RECOMMENDATIONS

The model as outlined in this paper will provide the user community with a needed modeling capability not currently available. Besides IRMA, this work will be the only simulation code capable of producing synthetic imagery. While both models model the speckle return from surfaces, the DIRSIG model will add significant capability. IRMA does not model the aerosol return. Without this capability, IRMA cannot model LADAR systems that detect, track, and quantify gases in the atmosphere. IRMA does not include atmospheric turbulence effects. Furthermore, it cannot model bistatic systems. Additionally, IRMA's LADAR channel does not account for multiple bounce. Finally, unlike DIRSIG, IRMA is not inherently a spectral model, each wavelength must be run separately. On the other hand, IRMA does model phenomenology that will not be treated in DIRSIG, such as coherent detection.

As stated previously, this work is meant to be the first step in the development of a comprehensive LADAR model. Raman scattering and fluorescence could be added to the model with a moderate amount of effort. By keeping track of relative phase, a coherent detection model could be developed with a fairly substantial amount of effort. Finally, the speckle model could be improved by accounting for partially developed speckle, speckle due to partially coherent light, and the relationship between the speckle pattern as a function of wavelength.

DISCLAIMER

The views expressed in this article are those of the author and do not reflect the official policy or position of the United States Air Force, Department of Defense, or the U.S. Government.

REFERENCES

1. J. Schott, S. Brown, R. Raqueno, H. Gross, and G. Robinson, "An advanced synthetic image generation model and its application to multi/hyperspectral algorithm development," *Canadian Journal of Remote Sensing* **25**(2), pp. 99–111, 1999.
2. D. Longtin, M. Cheifetz, J. Jones, and J. Hummel, "Backscat lidar simulation version 4.0: Technical documentation and user's guide," Tech. Rep. AD-A285 851, Sparta, Inc., 24 Hartwell Avenue, Lexington, MA 02173, June 1994.
3. M. Schmitt, B. McVey, B. Cooke, and G. Busch, "Comprehensive system model for CO₂ DIAL," in *Gas and Chemical Lasers*, R. Sze, ed., **2702**, pp. 95–103, SPIE, (San Jose, CA), Jan. 1996.
4. J. W. E. Wilcox, *Development and Experimental Verification of a High Resolution, Tunable Lidar Computer Simulation Model for Atmospheric Laser Remote Sensing*. PhD thesis, University of South Florida, December 1995.
5. J. Streicher, I. Leike, and C. Werner, "Aliens: Atmospheric lidar end-to-end simulator," in *5th International Symposium on Atmospheric and Ocean Optics*, V. Zuev and G. Matvienko, eds., **3583**, pp. 380–386, SPIE, (Tomsk, Russia), June 15–17 1998.
6. D. Vechinski, M. Richards, J. Edwards, J. Touma, and M. Wellafre, "Multi-sensor modeling and analysis (msma) irma 5.0 user's guide," Tech. Rep. N-TR-00-073, Nichols Research, 1130 Eglin Parkway, Suite A, Shalimar, FL 32579, November 2000.
7. R. Measures, *Laser Remote Sensing: Fundamentals and Applications*, Wiley-Interscience, 1984.
8. G. Mégie, "Measurement of the atmospheric pressure and temperature by lidar differential absorption: Influence of the width of laser emission," *Applied Optics* **19**(1), pp. 34–43, 1980.
9. R. Beland, *Atmospheric Propagation of Radiation*, vol. 2 of *The Infrared and Electro-Optical Systems Handbook*, SPIE Optical Engineering Press, 1993.
10. J. Goodman, *Statistical Optics*, John Wiley and Sons, 1985.
11. L. C. Andrews and R. L. Phillips, *Laser Beam Propagation through Random Media*, SPIE Optical Engineering Press, 1998.
12. D. Nelson, D. Walters, E. MacKerrow, M. Schmitt, C. Quick, W. Porch, and R. Petrin, "Wave optics simulation of atmospheric turbulence and reflective speckle effects in CO₂ lidar," *Applied Optics* **39**(12), pp. 1857–1871, 2000.
13. D. Fried, "Optical resolution through a randomly inhomogeneous medium," *J. Opt. Soc. Am.* **56**, pp. 1372–1379, 1966.
14. J. Dainty, *Laser Speckle and Related Phenomena*, vol. 9 of *Topics in Applied Physics*, Springer-Verlag, 1984.
15. J. Goodman, *Introduction to Fourier Optics*, McGraw Hill, 1996.