Efficient job scheduling for a cellular manufacturing environment

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EFFICIENT JOB SCHEDULING FOR A CELLULAR MANUFACTURING ENVIRONMENT

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Abstract

An important aspect of any manufacturing environment is efficient job scheduling. With an increase in manufacturing facilities focused on producing goods with a cellular manufacturing approach, the need arises to schedule jobs optimally into cells at a specific time. A mathematical model has been developed to represent a standard cellular manufacturing job scheduling problem. The model incorporates important parameters of the jobs and the cells along with other system constraints. With each job and each cell having its own distinguishing parameters, the task of scheduling jobs via integer linear programming quickly becomes very difficult and time-consuming. In fact, such a job scheduling problem is of the NP-Complete complexity class. In an attempt to solve the problem within an acceptable amount of time, several heuristics have been developed to be applied to the model and examined for problems of different sizes and difficulty levels, culminating in an ultimate heuristic that can be applied to most size problems. The ultimate heuristic uses a greedy multi-phase iterative process to first assign jobs to particular cells and then to schedule the jobs within the assigned cells. The heuristic relaxes several variables and constraints along the way, while taking into account the flexibility of the different jobs and the current load of the different cells. Testing and analysis shows that when the heuristic is applied to various size job scheduling problems, the solving time is significantly decreased, while still resulting in a near optimal solution.

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1 Introduction

1.1 Problem Statement

In today's fast-paced and ever-changing society, significant value is placed on efficiency, timing, and cost. Globalization is here to stay and will continue to impact the way companies around the world conduct business. To remain competitive in comparison to lower-cost manufacturers around the globe, more and more U.S. manufacturing facilities are moving away from departmental manufacturing and turning towards cellular manufacturing approaches, as shown in Figure 1.1, to improve efficiencies.

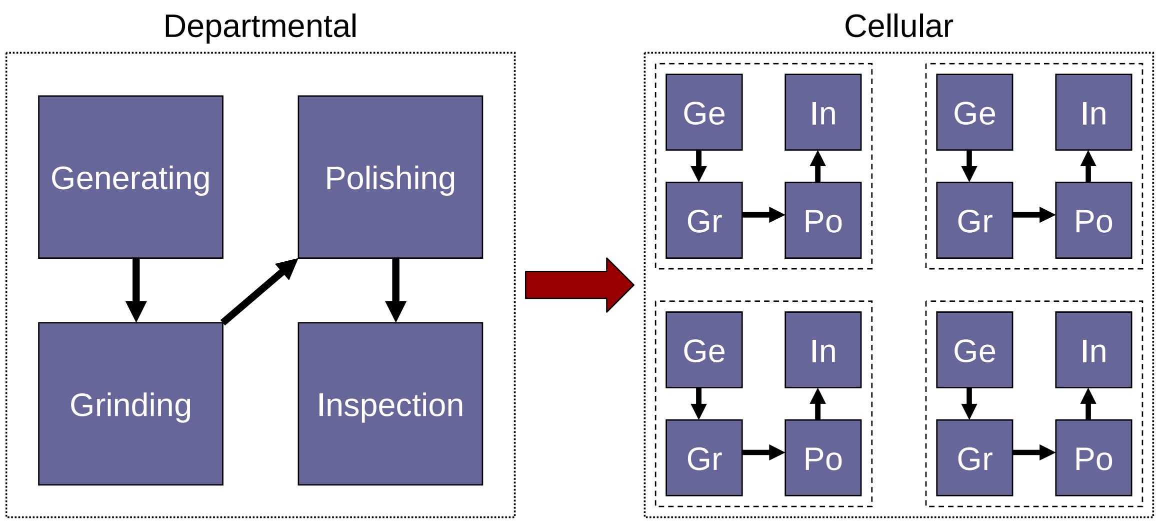


Figure 1.1: Shift from Departmental to Cellular Manufacturing

Within a departmental manufacturing environment, a job needs to travel through several different work centers, each dedicated to completing a single step in the overall process of manufacturing the job. This type of manufacturing setup lends itself to batch and queue processing, resulting in jobs with excessive travel times and waiting times and thus longer than necessary overall lead times. On the other hand, in a cellular manufacturing environment, several work cells comprise the manufacturing space. Each cell has the capability to complete each step in the process necessary to manufacture a job. Once a job begins in a cell, each step in the overall process ensues until the job is complete. This type of manufacturing setup promotes flow, resulting in minimal waiting times and travel times, shorter lead times, and better customer responsiveness.

Optimax Systems, located in Ontario, NY, is an innovative manufacturer of precision optics. They provide optical products, such as aspheres, cylinders, prisms, spheres, and optical coatings. Optimax typically provides precision optics to customers with a standard lead time between 6 weeks and 10 weeks. However, Optimax also offers an expedited service that provides their customers optics in as little as one week at a premium price. Two years ago, Optimax operated in a departmental manufacturing environment. Each department specialized in one step of the process of making an optic. For example, there was a grinding department that strictly focused on grinding the piece of glass. After grinding, the piece would head to the polishing department to be polished. This movement between the departments would continue until the optic completed each assigned step in the designated process. This method of producing an optic was a huge inefficiency. Each job would sit and wait on a shelf in a queue to be processed at each department. Departments were not strategically located by distance, so when a job was finished at one department, an employee had to walk the job to the next department and set the job on the new department’s shelf. Furthermore, employees did not know which job in the queue should be processed next. There was excessive and unnecessary work accounted for in the process time, waiting time, and travel time. As the business continued to grow, this type of manufacturing approach began to compromise Optimax's key strategies. It was necessary for Optimax to improve their approach to continue to be an important player in the optics industry.

Recently, Optimax has undergone an enormous facelift. They have gone from departmental, batch and queue processing to cellular, flow-focused manufacturing. Instead of having several departments that only complete one step in the process, Optimax now has several cells that complete all or most steps in the process. The next step in Optimax's transition is to optimize the scheduling of the jobs to the cells. Optimax has approximately 15 cells where jobs can be scheduled. Each cell has different parameters, including employee skills, equipment cost, and capacity. Each job has different parameters, such as potential profit, due date, specifications, and production requirements. Optimax is in need of a job scheduling tool that allows for real-time scheduling, based on current jobs as well as forecasted jobs. Furthermore, with frequent expedited orders, the job scheduler must be able to dynamically handle the addition of these jobs in a short period of time where capacity may be limited. The research performed in this thesis will aim to represent the job scheduling problem that is currently faced by Optimax and many other companies that operate in a cellular manufacturing environment. It will allow cellular manufacturers to more optimally schedule jobs throughout their facility to keep up with their key strategies and the ever-changing needs of their customers.

Most manufacturing facilities require some tool or technique to efficiently schedule jobs through the facility, regardless of the manufacturing method. Inefficient scheduling of jobs can compromise timeliness, quality, inventory, and most importantly profits. A tool that schedules jobs in a cellular manufacturing environment would be valuable to many facilities turning to this approach. The tool would allow manufacturers to attempt to minimize cost. Potential byproducts of the tool would include the ability to increase profits, while improving on-time delivery and customer responsiveness. Additionally, the tool would provide additional forecasting capabilities. Manufacturers could look ahead to see what type of cells have extra capacity or little capacity and quote jobs accordingly, attempting to always keep a near full-capacity facility. Figure 1.2 displays the main concept of such a tool.

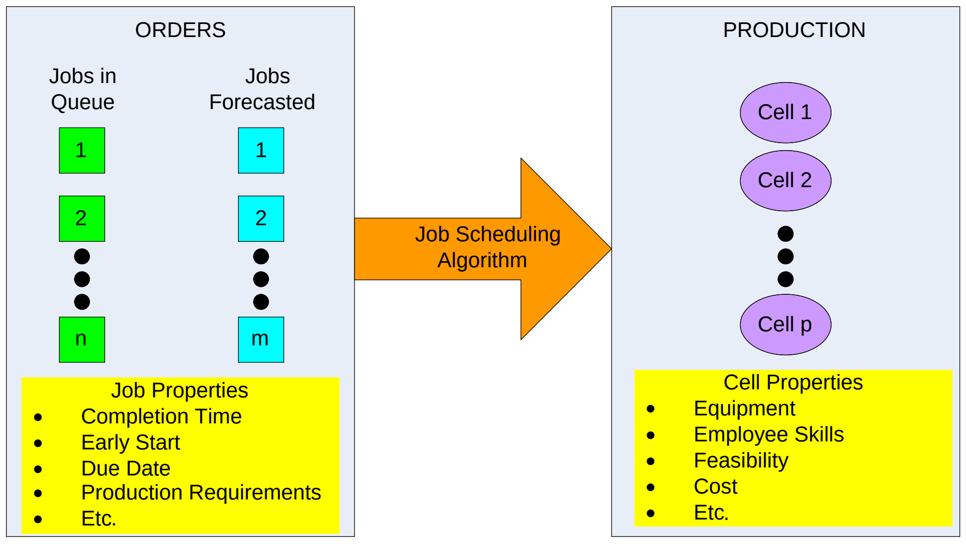


Figure 1.2: Job Scheduling Main Concept

There are several jobs, which are either in the work queue or forecasted to be produced in the future. Each job has different properties or parameters as shown, which may include completion time, early start date, due date, and production requirements. Similarly, there are several different cells each with different properties, including equipment, employee skills, feasibility, and cost. The goal of the job scheduler, through the use of a job scheduler algorithm, is to optimally assign the jobs to the cells at a specific time based upon an objective function that aims to maximize or minimize a specific set of criteria.

Figure 1.3 is an example of a potential output that the job scheduler could develop. Each job is assigned to a specific cell at a specific time. The duration of the job is based upon the estimated completion time, as assigned by the process engineering department or the manufacturing department. The chart shows the manufacturing facility the blocks of time dedicated to producing jobs, as well as the blocks of time where there is availability to potentially book an order for a job.

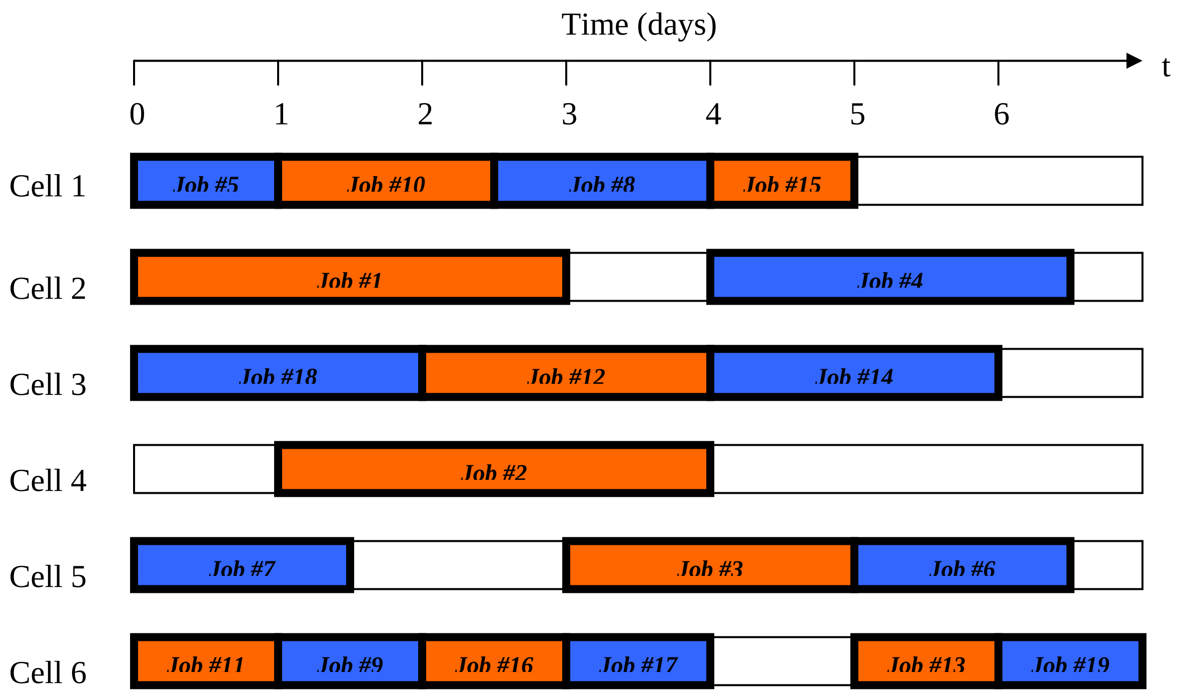


Figure 1.3: Potential Job Scheduler Output

This output is not only helpful to the production control department, which may schedule the jobs, but to the sales team as well. Since the job scheduler takes into account forecasted jobs also, this type of output can also be used as a forecasting tool. When the sales team is preparing a quote, they can look at the most up-to-date job schedule, to see the plant capacity, and more specifically cell capacity, at any particular time. If the plant or a specific cell has low availability at a particular time that a customer wants to place an order, the sales team may choose to present a high quote to the customer. On the other hand, if the plant or cell has high availability at a particular time, they may present a low quote to ensure that they receive the job and keep the facility running at an acceptable level. Therefore, if the facility receives an order during a low availability time period at a higher price, the additional profit outweighs the extra cost, such as overtime, to complete the job on time. Conversely, if the facility gains an order during a high availability time period at a lower price, the sacrifice in profit is outweighed by keeping the workers busy and not having to pay them without positive cash flow.

Providing such a schedule is not as simple as just placing jobs into cells at any time. Several factors must be considered to ensure that all parameters of the problem are met. For example, every job that needs to be scheduled has an associated due date driven by the customer and agreed upon by the manufacturing company. Similarly, production of a job may not be able to begin until a certain date, due to raw material or tooling needs. Job to cell feasibility comes into play, as each cell may not have equal capabilities, based on personnel and equipment, to produce a job. Therefore, each job will have a corresponding list of potential feasible cells that the job can run in. The potential job scheduler output also shows a few other important parameters of a cellular manufacturing environment. Jobs are scheduled in only one cell, only one job is scheduled in a cell at any particular time, and once a job is scheduled to begin it is produced without pre-emption. All parameters of the different jobs and different cells, as well as the parameters of cellular manufacturing can be described technically by a mathematical model, to be displayed and discussed in more detail in Chapter 2. Through the use of integer linear programming an optimal solution to a defined objective function can be achieved.

The mathematical representation of a cellular job scheduling problem is intrinsically complex. To truly characterize the actual size of the job scheduling problem faced by manufacturing facilities, such as Optimax, a representative number of jobs, cells, and time periods are needed to provide a truly beneficial schedule. Several parameters of the job and the cells must also be considered for added effectiveness. Furthermore, due to the dynamic nature of manufacturing facilities, the scheduling method must be a relatively quick process to be valuable. At every point that a new order arrives, the job schedule must be rerun to accommodate the new job to supply the new order. It is imperative that the job scheduling is as close to real-time scheduling as possible. However, the size and complexity of the job scheduling problem significantly impacts the time to solve for the optimal solution using integer linear programming via the mathematical model. In fact, as David W. Sellers wrote in "A Survey of Approaches to the Job Shop Scheduling Problem", job scheduling problems are of the NP-Complete complexity class [24]. It is practically impossible to investigate every potential feasible solution, except in the easiest problem sets. More specifically, Garey and Johnson showed that the multiprocessor scheduling problem is NP-Complete through a polynomial transformation from partition problems [9]. The job scheduling problem to be investigated in this thesis is a more specific case of the multiprocessor scheduling problem in which the multiple cells represent multiple processors and tasks are represented by jobs with their corresponding length (completion time) and deadline (due date).

Therefore, the problem to be addressed in this thesis is realized. There is a need to develop a job schedule for a cellular manufacturing facility. A mathematical model allows for the cellular job scheduling problem to be represented, while integer linear programming allows for the job schedule solution. However, the job schedule must be realized in real-time due to the dynamic nature of manufacturing. Yet, due to the size and complexity of the job scheduling problem as represented by the mathematical model, the integer linear program cannot recognize even a feasible solution schedule, let alone an optimal solution schedule, in a reasonable time frame to be of any value to the manufacturing facility.

Since conventional optimization techniques cannot be used to solve the cellular job scheduling problem faced by companies such as Optimax, it will be necessary to develop alternative methods and apply them to the mathematical model to more efficiently solve the scheduling problem to be of better use to the manufacturing facility. The work in this thesis involves the creation of a mathematical model that represents a cellular job scheduling problem and further work proves the complexity class of the problem. The only way to guarantee an optimal solution is to completely enumerate all points in the problem. Since this is not an acceptable alternative, due to time concerns, the research in this thesis will investigate and examine heuristic methods that will create more efficiency in the schedule solving process. The heuristic methods will aim to take advantage of the structure of the problem detailed in Chapter 2 to allow for more efficiency in the solving process. The heuristic methods will reduce the search space, thereby attempting to reduce the overall solving time, while aiming to meet all of the constraints of the problem. The attempted result is a near optimal solution schedule to a NP-Complete job scheduling problem in a significantly decreased, acceptable amount of computation time.

1.2 Literature Review

A tremendous amount of research and work has been done related to job scheduling. A variety of heuristic procedures and classical optimization tools have been used to solve several different job scheduling problems. In this literature review, several methods of solving a wide range of difficult and complex job scheduling problems have been investigated. The summary section will provide a brief explanation of the research direction of this thesis.

1.2.1 Scheduling Methods

Much research has been done on job scheduling using decision rules. Research was performed to determine optimal earliest time to start processing jobs [12]. Each job must be processed on the same machine, with random time duration. Each job has its own due date and a penalty for not meeting the due date, but also has an associated inventory cost for being completed before the due date. Heuristics used in this problem focus on decision-making regarding the random operations and cost parameters. The average processing time combined with the central limit theorem is used to determine the probability that a job will meet the due date. These probabilities and decision-making rules can be used to determine the associated costs of completing the job early or late.

A team of researchers studied the development and implementation of a job scheduler at a glass factory, where each job has a precedence constraint, an urgency constraint, a due date, early date, and a late date [3]. Each job is made up of one or more operations. An initial feasible solution is developed by quickly satisfying all precedence and resource constraints, while intending to maximize machine utilization and minimize work in process. After initialization, the jobs are assigned to machines by a priority criterion and then an improving phase follows. Additional heuristics such as round robin, parallel tasks, and work in next queue were investigated. A modified due date method was decided upon which sufficiently satisfied due dates and work in progress.

Scheduling rules were developed for job shops that do not assume that the cost of tardiness per unit is the same for each job and that the holding cost is not proportional to the flowtime of the job [17]. A weighted slack rule was used that attempts to minimize the maximum weighted tardiness and weighted variance of tardiness of jobs. A weighted flow due date rule was also used, which attempts to yield the minimum values for the maximum flow time and weighted variance of flow time of jobs. Another team investigated the inapproximability of the no-wait job scheduling problem using the makespan criterion [11]. In this type of environment there is no waiting allowed between the executions of consecutive operations of the same job. Once a job is started, it must be completed operation by operation, without pre-emption. It was found that the polynomial time approximation scheme does not exist.

Decision rules are beneficial to job scheduling because they are relatively easy to comprehend, fairly simple to relate to the problem, and normally improve solving time. However, with problems of larger magnitude and complexity, the advantages of decision rules tend to diminish. Decisions rules do not provide optimal solutions to problems, and typically the more difficult the problems become, the further the decision rule solution is from the optimal. Thus, developing a decision rule is not an ideal choice for a heuristic to aid in solving the cellular job scheduling problem.

Mathematical programming is another scheduling method. Linear programming and mixed integer programming are more specific methods that fall into this category. Mathematical programming is advantageous because complete enumeration of the problem can be achieved resulting in a true optimal solution. Researchers investigated manufacturing systems where a high variety of products of different volumes must be produced on a tight due date [15]. They used the feasibility function to schedule jobs in a multi-machine random job shop. The objective is to balance the number of tardy and early jobs, which will reduce the difference between the maximum and minimum lateness of jobs. A simulation model with a multi-agent architecture was developed to allow for comparison of a researched feasibility function method versus common scheduling rules. The results show that the feasibility function is very beneficial for job scheduling.

An optimization-oriented method was used for simulation-based job scheduling, which integrated capacity adjustment [4]. The goal of this method is to eliminate tardy jobs within a manufacturing facility. The proposed method integrates parameter-space-search-improvement into the scheduling procedure. To gain a near optimal solution, a local search is completed to shorten the computation time. The method was tested using data from a practical large-scale system, but it was found that the computation time was still too long.

CPLEX-computed job schedules were compared with the self-tuning dynP job scheduler [13]. The dynP scheduler dynamically changes the active scheduling policy, so to accurately reflect changing characteristics of waiting jobs. For the CPLEX method, an integer problem was developed. Time scaling was applied, which allowed the schedule to be computed on a larger than one second precise scale. The results of this comparison showed that both methods provided very similar solutions. However, the self-tuning dynP scheduler provided the solutions in much less time than the CPLEX method. A polynomial algorithm was used for two-job shop scheduling with scheduling flexibility [22]. The routing of the job is not fixed but it must be determined from several alternatives. The developed algorithm is based on a geometric approach and uses dynamic programming to construct a network which helps to determine the optimal solution. This algorithm can be applied on any regular minimizing objective function. The algorithm can also be changed to work with multi-resource operations.

Mathematical programming methods are beneficial because they allow for obtaining an optimal solution. However, with complete enumeration on a NP-Complete problem, the solving time associated with classical optimization for a linear program or mixed integer program would be excessively long. The disadvantage of the lengthy time to solve for the optimal far outweighs the benefit created by obtaining the optimal solution.

1.2.2 Solving Methods

Solving the cellular job scheduling problem is not a simple task. Again the multiprocessor problem has been shown to be NP-Complete through a polynomial transformation from partition problems. Manufacturing facilities operate in dynamic environments. Orders can be received at any moment and the manufacturing floor must be able to react to accommodate the new job from the new order. Therefore, it is not an acceptable alternative to completely enumerate all points in the problem. The manufacturing facility needs the solution schedule in real-time. The solving method must provide a near-optimal solution in an acceptable amount of time. Several solving methods, including genetic algorithms, search methods, neighborhood relations, and greedy approaches, aim to solve the job scheduling problem with mixed results.

Heuristic hybridization and genetic search were used as a procedure to computationally provide a feasible solution to a job scheduling problem [18]. The problem was adapted to a genetic algorithm by the Active-Schedule Generation and a Priority-List algorithm, with a hopping scheme. An Evolutionary Intracell Scheduler (EVIS) provided iterative schedule improvement, resulting in near optimal solutions in reasonable computation time. Another approach used a multi-pass heuristic approach combined with a genetic algorithm [25]. The steps in the process included dispatch, initialization, evaluation, and then a loop which consisted of selection, mating, mutating, evaluation again, and replacement. The computational time was proved to be significantly less. Another genetic algorithm proposed for the job scheduling problem involved release and due-dates, with various tardiness criteria as objectives [6]. Different priority rules, such as first in first out, shortest process time, and critical ratio are used to improve the decision process. A permutation was developed which prioritizes any two operations involved in the problem. They found that the capabilities of a genetic algorithm decrease with an increasing problem size. With the help of a multi-stage decomposition, the search space is reduced and the genetic algorithm works well.

Co-evolution and sub-evolution processes were introduced into a genetic algorithm to tackle job scheduling [10]. Co-evolution was used to provide makespan and idle time schedule criteria as the fitness functions of the operation-based genetic algorithm. Subsequently, to provide high diversity for chromosome population, sub-evolution was used so that the total job waiting time schedule constraint is the fitness function for the genetic algorithm. With modifications to the standard deviation and average of the computational results, this method shows robustness in solving the job scheduling problem. Another genetic algorithm combined with a data mining based meta-heuristic was proposed to solve the job scheduling problem [7]. This genetic algorithm generates a learning population of feasible solutions, which are then mined by the mean of classifier systems. The mining step produces decision rules that are transformed into a meta-heuristic allowing for the efficient scheduling of operations to machines.

To build upon the efficiency of genetic algorithms, a team of researchers proposed a hybrid heuristic genetic algorithm [8]. Scheduling rules, such as shortest processing time and most work remaining were integrated into the genetic evolution process. To improve the solution performance, the neighborhood search technique was adopted as a supplementary procedure. The new hybrid genetic algorithm was proved to be effective and efficient in comparison to other methods, including the neighborhood search heuristic, simulated annealing, and traditional genetic algorithm. An immune algorithm method was proposed that goes through a series of steps, including initialization of antibodies, initialization of antigens, evaluation, generation, and calculation [23]. The binary strings will gather to the point where the good value of the fitness function is found. In comparison to genetic algorithms, the proposed immune algorithms provide solutions in faster computation times.

A job scheduling method was investigated using group constraints, which means that a job schedule for each line is decided upon and jobs dealing with the same process must be grouped [14]. The research included a rapid generation of an initial feasible solution by analyzing job flexibility according to an influential degree of a whole plan. Improvement rules were used in combination with a tabu search, which resulted in improvement of the total evaluation and confirmed effectiveness.

A stochastic strategy was developed for solving the job scheduling problem [16]. A tabu search was proposed and formalized to get a near optimal solution. The procedure is based on an iterative "neighborhood search." The tabu search keeps track of not only short term information, but long term information as well. Two strategies, intensification and diversification, are used to efficiently solve the problem in polynomial time. Another search technique is based upon relaxing and then imposing the capacity constraints on a few critical operations [24]. Subsequently, this technique is incorporated into a fast tabu search algorithm. Results from this technique show that the approach is very effective, by improving upon a range of test problems.

A heuristic was developed based on the tree search procedure for job scheduling to minimize total weighted tardiness [5]. Each job has specific due dates and delay penalties. A schedule is determined by minimizing the maximum tardiness subject to fixed sub-schedules solved at each node of the search tree and the successor nodes are generated, where the sub-schedules of the operations are fixed. Therefore, a schedule is obtained at each node and the sub-optimum solution is determined among the obtained schedules. Results show that the algorithm can find sub-optimum solutions with minimal computation time.

An extension of the job scheduling problem was studied, where the job routings are directed acyclic graphs that can model partial orders of operations and that contain sets of alternative subgraphs consisting of several operations each [19]. A tabu search and a genetic algorithm are used as heuristics, based upon two common subroutines. The first inserts a set of operations into a partial schedule and the other improves a schedule with fixed routing alternatives. The first subroutine relies on an efficient insertion technique, while the second subroutine is a generalization of standard methods for job scheduling. Results show that the methods proposed provide optimal solutions for three open problems.

Methods were researched for manufacturing environments with random job arrivals, non-deterministic processing times, and unpredictable events, such as machine breakdowns. A complete multi-agent framework, including Lagrange multipliers, is used to schedule jobs in this type of flexible workplace [1]. This approach combines real-time decision making with predictive decision making, which can combat various different scheduling problems. Another multi-agent scheduling method integrates earliness and tardiness objectives for a flexible job shop, consistent with the just-in-time manufacturing philosophy [27]. A job-routing and sequencing mechanism distinguishes jobs with one operation left and jobs with multiple operations left. The results of the research show that the proposed multi-agent scheduling method outperforms existing scheduling methods.

A job scheduling problem was researched, in which each job must process one task on *m* machines [2]. The determination of the longest paths is the critical computation. Heuristics are used by employing a neighborhood relation. To obtain a neighbor, a single arc from a longest path is reversed and so these transition steps guarantee a feasible schedule. Using logarithmic cooling schedules, the problem can be solved within polynomial time.

A greedy heuristic was developed for the flexible job scheduling problem, which is concerned with the assignment of operations to machines, as well as the sequence of the operations [21]. The first job is fixed to start the polynomial algorithm. The next job, with associated operations, is combined with the first job. The combinations are organized in a Gantt chart according to the optimal schedule. The algorithm continues until all jobs are formed into appropriate combinations, which gives the optimal job to machine assignment.

A heuristic schedule was used based upon asymptotic optimality in probability for open shops with job overlaps [20]. This approach focuses on scheduling applications where parallel processing within a job is possible. The objective is to output an optimal schedule while minimizing the summation of completion times of the jobs. The heuristic orders the jobs by the average processing time of the operations of the job. A lower bound on the optimal cost of each job is also introduced. The lower bound is used to prove asymptotic optimality in probability of the heuristic when the processing times are independently and identically distributed from any distribution with a finite variance.

Genetic algorithms, search methods, neighborhood relations, and greedy approaches are all nice methods to solving the job scheduling problem. However, there is no guarantee that any of these methods will achieve the goal of solving for a near-optimal solution to a NP-Complete problem in an acceptable computation time to be of use to a dynamic manufacturing environment. Therefore, this thesis will develop a heuristic that can be applied to a mathematical model that represents a cellular job scheduling problem. The heuristic will take advantage of the structure of the model to solve more efficiently, while maintaining an acceptable level of optimality. The work will aim to leverage several aspects of the mathematical model as well as specific characteristics of jobs and cells contained in the scheduling problem to improve the efficiency of solving the cellular job scheduling problem detailed in the mathematical model in Chapter 2.

2 Formulation

2.1 Mathematical Model

In this chapter, a developed mathematical model is presented to solve the job scheduling problem that is representative of a cellular manufacturing environment. The creation and design of the mathematical model is crucial to the types of heuristics that can be applied to the problem. The research that is completed for the thesis will be based upon this model. This mathematical model will schedule jobs in queue, as well as forecasted jobs, to the best possible cell for production at the best possible time(s), according to an objective function. It also describes important factors for jobs and cells, using input parameters and constraints.

Notation

$$j = job(1, 2, ..., n, n+1, n+2, ..., n+q) \qquad (2.1)$$

- jobs in queue $(1, 2, ..., n)$
- jobs forecasted $(n+1, n+2, ..., n+q)$

$$c = cell(1, 2, ..., p) \qquad (2.2)$$

$$t = time(1, 2, ..., r) \qquad (2.3)$$

Decision Variables

X_{jc} = {1 if *job j* is assigned to *cell c*; 0 otherwise} (2.4)

Y_{jct} = {1 if *job j* is processed in *cell c* at *time t*; 0 otherwise} (2.5)

S_j = *time t* that *job j* starts (2.6)

F_j = *time t* that *job j* finishes (2.7)

Input Parameters

$d_j =$ *time t* that *job j* is due (2.8)

$e_j =$ earliest *time t* to start *job j* (2.9)

ct_j = length of time to complete *job j* (2.10)

f_{jc} = *job j* to *cell c* feasibility (2.11)

- (1…feasible, 0…infeasible)

$m_{jc} =$ cost per unit time to produce *job j* in *cell c* (2.12)

Objective Function

$$Minimize\ Z = \sum_j \sum_c \left(X_{jc} * m_{jc} * ct_j \right) \tag{2.13}$$

Constraints

$$\sum_c X_{jc} = 1 \quad \forall\, j \tag{2.14}$$

$$X_{jc} \le f_{jc} \quad \forall\, j, c \tag{2.15}$$

$$\sum_t Y_{jct} \le TIME * \left(X_{jc} \right) \quad \forall\, j, c \tag{2.16}$$

$$\sum_j Y_{jct} \le 1 \quad \forall\, c, t \tag{2.17}$$

$$t * Y_{jct} + TIME * \left(1 - Y_{jct} \right) \ge S_j \quad \forall\, j, c, t \tag{2.18}$$

$$F_j \geq t * Y_{jct} \quad \forall j, c, t \tag{2.19}$$

$$\sum_c \sum_t Y_{jct} = ct_{jc} \quad \forall j \tag{2.20}$$

$$e_j \leq S_j \quad \forall j \tag{2.21}$$

$$F_j \leq d_j \quad \forall j \tag{2.22}$$

$$F_j - S_j = ct_j - 1 \quad \forall j \tag{2.23}$$

The mathematical model is clearly represented by three indicies; job, cell, and time. The job notation, in 2.1, describes the list of jobs to be scheduled, with accompanying actual job numbers. The cell notation, in 2.2, describes the list of cells that jobs can be scheduled in, with accompanying cell names. The time notation, in 2.3, describes the length of the discetized time periods, with accompanying time units. The three indicies will be used to schedule a job to a specific cell over specific time periods.

The mathematical model involves four decision variables. The assignment variable, shown in 2.4, is a two-dimensional (job, cell) binary variable that is equal to 1 if a job is assigned to a specific cell or 0 otherwise. Similarly, the schedule variable, shown in 2.5, is a three-dimensional (job, cell, time) binary variable that is equal to 1 if a job is assigned to a specific cell during a particular time. Otherwise the value of the variable is equal to 0. The start time variable, shown in 2.6, gives the time period that a job is scheduled to begin production, while the finish time variable, shown in 2.7, gives the time period that a job is scheduled to complete production.

The first job parameter that will be included in the mathematical model is due date, shown in 2.8. Due date is one of the main driving forces behind the scheduling of jobs.

Simply put, it provides a worst-case date for when the job must be completed that aligns with the needs of the customer. The customer expects the job to be delivered in accordance with the due date. If the job is not delivered on time it is likely that the company's reputation can be damaged or profits can be sacrificed. Therefore, due dates supply a simple to understand baseline date that is to be met for each job.

Early start date, shown in 2.9, is another job parameter that will be included in the mathematical model. Early start date operates in a similar manner to due date. It provides a best-case date for when a job can actually begin manufacturing. Early start date is a critical job parameter mainly for a few reasons. First, it comes into play with forecasted jobs that have yet to be confirmed for production. Manufacturing facilities do not want to begin production of a forecasted job, until there is a better understanding of whether the job will truly come to fruition. Secondly, inventory concerns come into play. It costs time and money to store products in inventory on both the producer and customer sides. The producer doesn't want the job to be completed too early, resulting in a significant finished goods inventory cost. Similarly, the customer doesn't want the product too soon before it is needed, resulting in additional storage costs. Finally, the early start date is put in place due to the availability of specialized tools and materials. Typically, there is some sort of lead time associated with the delivery of raw materials or tools needed for the production of a job. Obviously, the job cannot begin until the necessary materials and tools are available to the cell.

The final job-specific parameter to be included in the mathematical model is completion time, shown in 2.10. The completion time is defined as the number of time periods that a job will take for full production. For the purposes of this research, completion time will be of a deterministic nature. Typically, the completion time of a particular job would be determined by historical manufacturing data and information associated with similar past jobs. The completion time is important because is provides the block of time that a job must be scheduled for within a cell.

One cell input parameter that will be included in the mathematical model is known as feasibility, shown in 2.11. Each job is either feasible or infeasible with each of the different cells. This method provides a simple, straight forward approach to assigning feasibility of a job to a cell. Additionally, this method allows for compiling several different parameters into one parameter. The feasibility looks at many job parameters and cell parameters to determine the feasibility relationship between each job and each cell. At a minimum, the feasibility parameter takes into account specifications and production requirements of a job and the equipment and employee skills of a cell. If the necessary specifications and production requirements of a job match the equipment and employee skills that are located in a cell, the job is feasible for production in that particular cell. On the other hand, if the specifications and production requirements of a job don't align with the equipment and employee skills of a cell, that relationship is infeasible.

The final parameter, cost, shown in 2.12, provides a cost per time unit of manufacturing a particular job in a specific cell. Cost incorporates several different smaller costs associated with the manufacturing of a job in a cell. For example, each cell has an employee wage cost associated with it. Some cells have multiple employees and/or high-skilled employees that increase the wage cost. In addition, there is a burden cost associated with each cell that may incorporate equipment cost and square footage cost. The cost parameter also serves as an extension of the feasibility parameter. Although the feasibility parameter is binary, job to cell feasibility is actually not so cut and dry. For a particular job, there are some cells that are very good matches for production, there are some cells that are impossible for production, and there are some cells in between that could produce the job if necessary. Thus, the cost parameter comes into play with the cells in between to allow for some continuity within feasibility. For instance, Job A matches the parameters of Cell X very well. More often than not, Job A should be scheduled for production in Cell X. However, Job A could be scheduled to Cell Y, if Cell X is full and production is absolutely necessary by a certain date. The cost parameter associated with the Job A to Cell Y relationship can be inflated to an appropriate level to allow Job A to be scheduled to Cell Y, but simultaneously ensures that it happens only if absolutely necessary.

The goal of any firm or company should be to maximize profit. However since profit is difficult to represent from a scheduling perspective, the objective, shown in 2.13, in this model is to schedule the jobs accordingly to minimize the overall cost associated with producing the set of jobs in their assigned cells.

There are several conditions or constraints that must be met while scheduling the jobs, in accordance with cellular manufacturing principles. First each job must be produced entirely within only one cell, as represented in 2.14 and known as the “one cell only” constraint. The assigned cell for a particular job must be a feasible cell, as represented in 2.15 and known as the “cell feasibility” constraint. If a job is not assigned to a particular cell, it can’t be scheduled in that cell, as represented in 2.16 and known as the “schedule only if assigned” constraint. TIME is defined as the value of the latest time in the set of time periods. Furthermore, within one cell, only one job can be worked on at any particular time, as represented in 2.17 and known as the “one job at a time” constraint. The starting time of a job is determined using the “starting time” constraint, shown in 2.18, while the finishing time of a job is determined using the “finishing time” constraint, shown in 2.19. A job must be scheduled for the entirety of the designated completion time, as represented in 2.20 and known as the “scheduled time equals completion time” constraint, while not being scheduled before its early start date, as represented by 2.21 and known as the “early start date” constraint, or after its due date, as represented by 2.22 and known as the “due date” constraint. Once a job is scheduled for a particular time, it must remain in the cell until completion, in a sequential manner, as represented in 2.23 and known as the “sequential time” constraint.

For the purpose of this thesis, the mathematical model has been formulated using a software program known as Optimization Programming Language (OPL), version 3.7, from a company called ILOG. The problem will then be solved using OPL and a solution tool known as CPLEX. The baseline OPL model along with a glossary of terms to allow for easy translation can be found in Appendix A.

2.2 Job Scheduling Problem Sizes

There are an endless number of job scheduling problems that arise from the numerous combinations of input parameters, as well as the quantity of jobs, cells, and time periods. To address this concern, for the purpose of this work, the job scheduling problems will reflect the general state of job scheduling problems at facilities that operate in a cellular manufacturing environment, such as Optimax Systems, Inc., described in Chapter 1.

Typically, at any given time, the manufacturing facility is approximately operating at 85% capacity. This means that the jobs currently planned to be produced occupy 85% of the facility's physical work time to complete the jobs. Of course, this number is not constant and can fluctuate higher and lower depending on the market demand for goods.

A 10-week or 2.5-month time frame looking forward portrays the window of time that most facilities are concerned with to be scheduled. This allows for scheduling of jobs with a 6-10 week lead time. It also allows for scheduling of expedited jobs that must be scheduled with shorter lead times, potentially delaying other jobs.

The completion time of jobs is dependent on the quantity of parts in the job and the difficulty of the job. Simple jobs may take as little as one day to complete, while more difficult jobs can take upwards of 5 days or a full work week for completion. Some jobs can begin to be produced as soon as the order is confirmed. However, some jobs must be delayed due to material, tool, inventory, or forecasting reasons. The early start date takes these concerns into account, while adjusting the due date to provide a reasonable window of time for completion for any particular job.

Cell break points of 5 cells, 10 cells, and 15 cells will be used for experimentation. Obviously, jobs are not feasible to all cells, and a job may be a better fit for a certain cell than another cell. On average, jobs are allocated as feasible to 40% of the cells. This does not mean that each job is feasible to 40% of the cells, but overall 40% of the cells are feasible for all the jobs. For example, in the case of a 5-cell problem, Job 1 is feasible to only one cell, but Job 2 is feasible to three cells. For Job 2, each of the three feasible cells may not be equally feasible. This is where cost comes into play. The most difficult feasible cell will be allocated a higher cost in comparison to the easier feasible cells.

To bring it all together, 12 different job scheduling cases, as shown in Table 2.1, will be run as a set of different experiments, based on problem size. Along the left side of the table is the number of cells located within the problem. Along the top side of the table is the number of time periods, represented by days, located within the problem. The table shows the size of each case in terms of cell-days. Simply put, cell-days are calculated as the product of the number of cells and the number of days within the problem. This represents the total number of time slots that must be scheduled, or purposely not scheduled via the schedule variables. Typically the higher the number of cell-days, the more difficult the scheduling problem becomes. The size of the problem is shown in Table 2.2.

	(4 weeks)	(6 weeks)	(8 weeks)	(10 weeks)
	20 days	**30 days**	**40 days**	**50 days**
5 cells	100 cell-days	150 cell-days	200 cell-days	250 cell-days
10 cells	200 cell-days	300 cell-days	400 cell-days	500 cell-days
15 cells	300 cell-days	450 cell-days	600 cell-days	750 cell-days

Table 2.1: Job Scheduling Problem Cases

Cell-Days	**Problem Size**
100-200	Small
250-300	Medium
400-500	Large
600-750	Extra-Large

Table 2.2: Job Scheduling Problem Sizes

Since the completion times of jobs are normally between 1 day and 5 days, the completion times are assigned randomly between 1-5 days. Capacity is normally at about 85%. Therefore, the number of jobs that each case will have is calculated by multiplying the total cell-days by the average capacity (85%) and then dividing by the average completion time (3 days). Table 2.3 shows the number of jobs located within each scheduling problem case.

	(4 weeks)	(6 weeks)	(8 weeks)	(10 weeks)
	20 days	**30 days**	**40 days**	**50 days**
5 cells	29 jobs	43 jobs	57 jobs	71 jobs
10 cells	57 jobs	85 jobs	114 jobs	142 jobs
15 cells	85 jobs	128 jobs	170 jobs	213 jobs

Table 2.3: Number of Jobs in Job Scheduling Cases

Due to the large number of different experiments that were to be run, synthetic data was generated through random number techniques. Table 2.4 shows the methods to determine each of the input parameters.

Input Parameter	**Method of Generation**
Completion Time	Randomly assigns a completion time (1,2,3,4,5).
Due Date	Randomly assigns a due date; due dates skewed towards later time periods.
Early Start	Randomly assigns an early start date, based upon due date; early start dates skewed towards earlier time periods.
Feasible	Randomly allocates feasibility between each job and cell at a 40% chance of feasibility (1-feasible, 0-infeasible).
Cost	Randomly allocates cost (1,2,3,4,5) between each job and feasible cell.

Table 2.4: Input Parameter Determination

Table 2.5 displays approximate solving times for the problem sizes when attempting to solve optimally using integer linear programming via the developed mathematical model. Some problems take shorter or longer to solve than the given range, but a large majority of the problems fall within the range. The results reiterate the need to investigate a more efficient procedure to solve for problems of these sizes, especially the extra large problems, which are representative of the problems faced by companies such as Optimax. It is critical that a good solution be achieved in a reasonable timeframe to be of use to a dynamic manufacturing facility that requires real-time scheduling.

Problem Size	**Optimal Solution Approximate Solving Range**
Small	1 minute – 1 hour
Medium	1 hour – 1 day
Large	1 day – 3 days
Extra-Large	3 days – 1 week +

Table 2.5: Approximate Solving Times

3 Solution Methodology

3.1 Development and Evolution

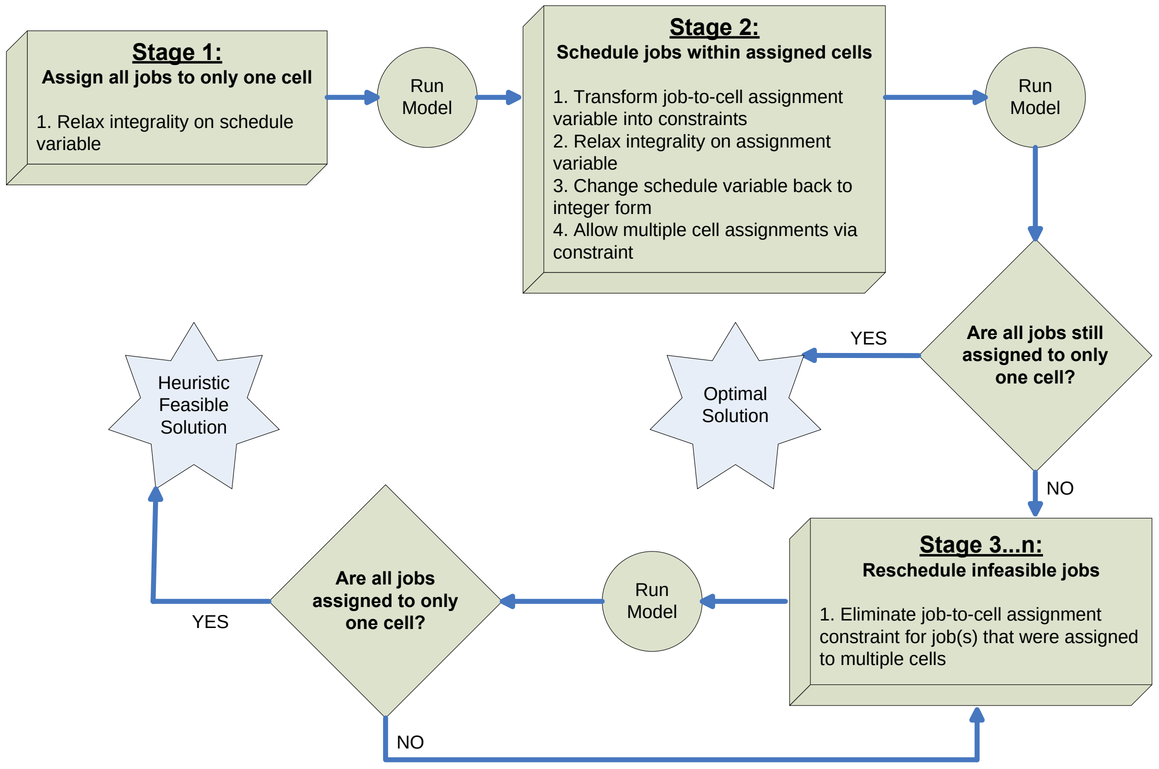
The formulation of the mathematical model significantly impacts the types of heuristics that can be applied to more efficiently solve the problem. The goal of the mathematical model was not only to represent the job scheduling problem of a cellular manufacturing facility, but to also allow the acceptance of different potential heuristic procedures. Once again, the objective of this job scheduling problem is to schedule all the jobs to a specific cell over a designated amount of time, while minimizing overall cost. In simplest form, only the schedule variable, $\boldsymbol{Y_{jct}}$, is necessary to deliver all the information to the manufacturing facility. The schedule variable shows exactly what job is assigned to what cell and at what time(s), through a binary notation. However, with the addition of the assignment variable, $\boldsymbol{X_{jc}}$, the problem can easily be broken down into two separate phases, assigning (jobs to cells) and scheduling (jobs to times within assigned cells). The ability to split the problem into separate phases, assigning and scheduling, enables the problem to be simplified through heuristic techniques. The heuristic will take advantage of the structure of the model to solve more efficiently, while maintaining an acceptable level of optimality. The developed heuristic will work to leverage the structure of the mathematical model of jobs and cells contained in the scheduling problem to improve the efficiency of solving the cellular job scheduling problem detailed in the mathematical model in Chapter 2.

A heuristic method does not just suddenly develop out of nowhere on its own. Instead the heuristic evolves from several different ideas through a process of repetitive trial and error, as well as significant experimentation. There are numerous ways to go about creating a heuristic, including relaxing constraints and relaxing integer variables. The size and difficulty of a job scheduling problem greatly impacts the capability of a heuristic when applied to the problem. The evolution of the heuristic to be detailed in this thesis started with attempts to solve small-sized problems and slowly progressed to solving larger-sized problems. The techniques developed in the smaller problems are adjusted and expanded upon so that they can be applied to the larger problems.

3.2 Small Problems

With 200 cell-days or less, small problems are the simplest class to be examined within this research. Small problems are likely the type of problem that a department area or small company, with smaller lead times, would face on a consistent basis. More often than not, small problems can be solved optimally through use of the baseline mathematical model, without the use of any heuristic procedures. Nevertheless, the computation time for solving optimally can range anywhere from a couple seconds to a couple minutes to a couple hours. By using just a few simple procedures, the problem-solving can be quickened and a feasible (potentially optimal) solution can be found in a fraction of the time. Figure 3.1 shows a simple heuristic method to find a solution to a small-sized problem by relaxing integrality of the assignment and schedule variables, as well as the “one cell only” constraint.

In stage 1, relax the integrality on the schedule variable, to allow all jobs to be assigned to one and only one cell in a small amount of time. All jobs are assigned to a cell, but are not scheduled at specific times in accordance with early start dates and due dates. The solution is far from feasible, yet still provides useful information to carry into the next stage. Transform each job-to-cell assignment variable into a constraint and add them to the mathematical model to be used in stage 2.



Stage 1:
Assign all jobs to only one cell
1. Relax integrality on schedule variable
Run Model
Stage 2:
Schedule jobs within assigned cells
1. Transform job-to-cell assignment variable into constraints
2. Relax integrality on assignment variable
3. Change schedule variable back to integer form
4. Allow multiple cell assignments via constraint
Run Model
Are all jobs still assigned to only one cell?
YES
Optimal Solution
NO
Stage 3...n:
Reschedule infeasible jobs
1. Eliminate job-to-cell assignment constraint for job(s) that were assigned to multiple cells
Run Model
Are all jobs assigned to only one cell?
YES
Heuristic Feasible Solution
NO

Figure 3.1: Heuristic Strategy #1 – Small Problem

For stage 2, change the schedule variable back to its original integer form. Instead relax the integrality on the assignment variable. In addition, relax the “one cell only” constraint, to now allow for multiple cell assignments per job. Since in stage 1, early start dates and due dates were not met, it is possible that all jobs assigned to a cell cannot be appropriately scheduled in that particular cell. Therefore, by relaxing the “one cell only” constraint, a job can be assigned and scheduled over two cells, if necessary.

After running the model again, if a solution is found where all jobs are assigned to only one cell, the solution is optimal. If one or more jobs are assigned to multiple cells, the solution is infeasible. Eliminate the job-to-cell assignment constraint added in stage 2 for the job(s) that are assigned to multiple cells and run the model again. Continue this process until all jobs are assigned to one and only one cell. At this point, a feasible solution is found, with the possibility that the solution is still optimal. Normally, this entire heuristic process takes no more than a few seconds depending on the magnitude of the problem.

Beneficially, this heuristic procedure provides an optimal or feasible solution in a short amount of computation time on a very consistent basis with problems of small magnitude. On the other hand, the heuristic can get caught in a large loop at stage 3, if jobs continue to get assigned to multiple cells. This leads to a longer computation time and backtracks to a more difficult problem. Furthermore, as the size of the problem at hand increases, the ability of this heuristic to provide a solution quickly diminishes. A more difficult problem spells more cells, more time, and more jobs. With an increase in the number of jobs, this heuristic has difficulty assigning all the jobs to one cell in stage 1. Additionally, as the number of cell-days increases, it is more difficult to schedule the jobs even if they can be assigned to distinct cells.

3.3 Medium Problems

In one way or another, medium problems experience a slight increase in the number of jobs, number of cells, or the number of time periods. Due to the increase of the dimensions of the problem, the strategy to acquire a solution must be adapted in relation to smaller problems. Medium problems still have a slim chance to be solved optimally, without any modifications to the baseline mathematical model. Nonetheless, the solving process could take several minutes or even several hours. By applying a three-stage heuristic procedure to this size problem, the solving time can be significantly decreased, while not sacrificing considerable optimality to the objective. The crucial part of this heuristic is obtaining an initial feasible solution to the adjusted problem at hand as soon as possible. After an initial feasible solution is found, useful bits of information from the adjusted feasible solution can be adapted to the next stage to speed along the overall solution process. Figure 3.2 shows the basic concept of the heuristic procedure.

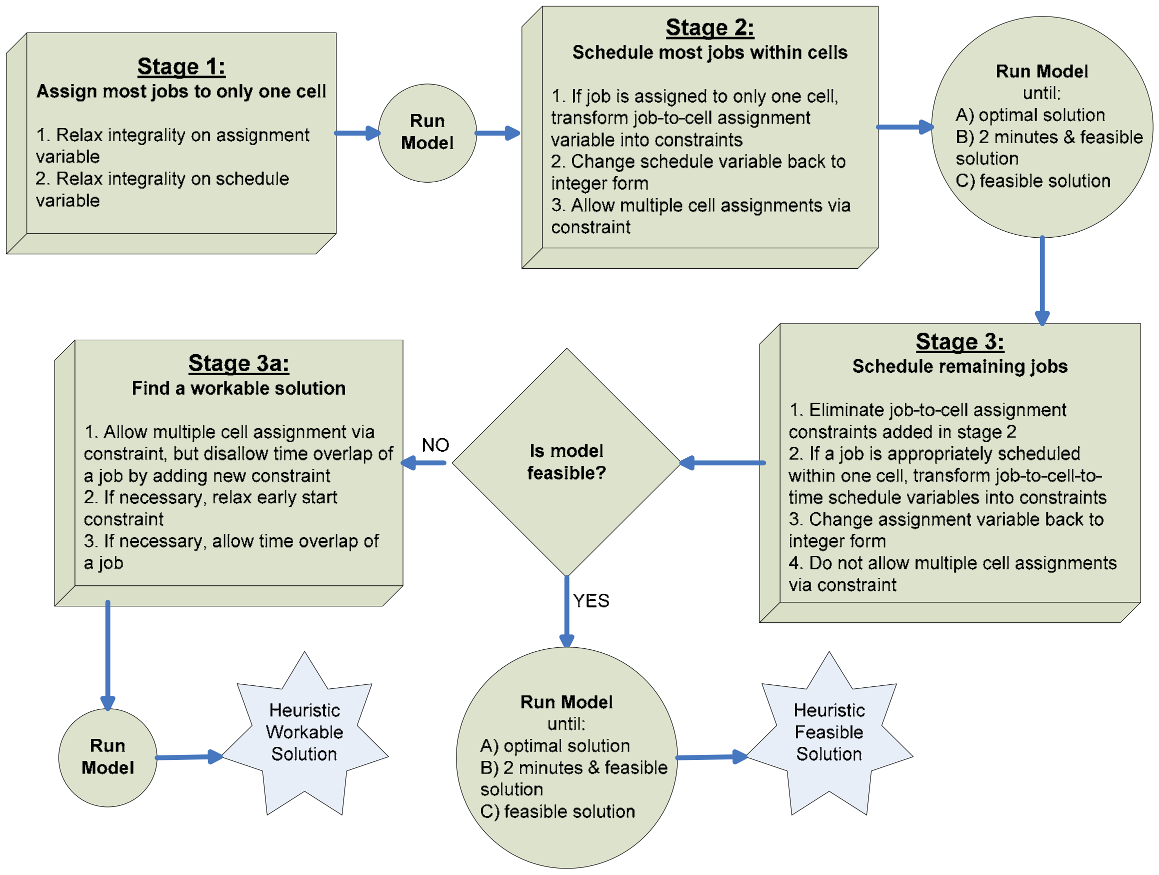

Stage 1:
Assign most jobs to only one cell
1. Relax integrality on assignment variable
2. Relax integrality on schedule variable
Run Model
Stage 2:
Schedule most jobs within cells
1. If job is assigned to only one cell, transform job-to-cell assignment variable into constraints
2. Change schedule variable back to integer form
3. Allow multiple cell assignments via constraint
Run Model until:
A) optimal solution
B) 2 minutes & feasible solution
C) feasible solution
Stage 3:
Schedule remaining jobs
1. Eliminate job-to-cell assignment constraints added in stage 2
2. If a job is appropriately scheduled within one cell, transform job-to-cell-to-time schedule variables into constraints
3. Change assignment variable back to integer form
4. Do not allow multiple cell assignments via constraint
Is model feasible?
NO
YES
Stage 3a:
Find a workable solution
1. Allow multiple cell assignment via constraint, but disallow time overlap of a job by adding new constraint
2. If necessary, relax early start constraint
3. If necessary, allow time overlap of a job
Run Model
Heuristic Workable Solution
Run Model until:
A) optimal solution
B) 2 minutes & feasible solution
C) feasible solution
Heuristic Feasible Solution

Figure 3.2: Heuristic Strategy #2 – Medium Problem

Stage 1 involves changing the assignment variable and the schedule variable from the integer form to the continuous form. Since this is no longer an integer program whatsoever, an optimal solution is quickly obtained in just a few seconds. Although this solution is far from a good solution for the true problem, it provides useful information to carry on to the next stage.

In stage 2, the assignment variables from the stage 1 solution are analyzed. If a job-to-cell assignment variable is equal to 1, it represents a high importance, relative to the objective function, to schedule that job within that cell. Thus the job-to-cell assignment actually becomes a constraint and is added within the model. This occurs for all job-to-cell assignment variables that are equal to 1. Usually between 60%-70% of jobs are assigned solely to one cell after stage 1.

Before the model is run again, the schedule variable is changed back to a binary integer variable. This is a step in the right direction towards the true mathematical model, as jobs now must be scheduled for an entire time period, instead of portions of a time period. Additionally, the "one cell only" constraint is modified to allow for multiple cell assignments. Therefore, jobs can be assigned to more than just one cell. By changing this constraint, a feasible solution is found significantly faster than by forcing all of the jobs to be scheduled to only one cell. Now the model can be run once again.

The model has now been turned into a partial integer program. Understandably, the solving process is more time-consuming. Nevertheless, since many of the jobs have already been assigned to a distinct cell, a feasible solution is obtained to the problem at hand, typically within about a minute. Next, a balancing act must occur as the longer program runs, the better the solution becomes, resulting in better information to carry into the next stage. After approximately 2 minutes, if the solution is not yet optimal, but is feasible, the model can be stopped and the procedure can continue with the best feasible solution. Two minutes was chosen for several reasons. First, time is not compromised significantly as two minutes is a very short amount of time for such a problem of this magnitude. Secondly, a feasible solution can typically be found within two minutes for this set of problems. Finally, after two minutes, the solution doesn't have much more room for improvement, but the time to achieve the improvement is significant. In the unlikely case that a feasible solution is not found within 2 minutes, allow the model to continue to run until a feasible solution is found.

In stage 3, the schedule variables from the stage 2 solution are analyzed. If a job is scheduled in only cell, the schedule variables for that job are transferred into the mathematical model in the form of constraints. After stage 2, about 75% of the jobs will be scheduled appropriately in one cell. This represents the eventual schedule for these jobs. However, before it can become the actual schedule, the remaining jobs must be scheduled. Since the schedule variables have been added to the mathematical model, all assignment variable constraints that were added in stage 2 can be removed. The remaining jobs are able to be scheduled to any feasible cell.

Before the model is run again, the mathematical model is changed back to its original form. The assignment variable is changed back to integer form. In addition, the "one cell only" constraint is changed back to allow for only one cell assignments. Now, the model can be run again in an attempt to find a good feasible solution to the true job scheduling problem. Typically, an optimal solution is found within one minute. The model can be stopped after two minutes with a feasible solution, in the unlikely case that the model hasn't yet found an optimal solution. The best feasible solution is used.

Due to the fact that several jobs have already been locked into place after stage 2, there is a chance that the problem is no longer feasible. Likely, one or two jobs could not be scheduled because other jobs were already scheduled to necessary time slots. In this case, adjustments must be made to achieve a workable solution. A workable solution comes in the form of allowing jobs to be scheduled over multiple cells, if necessary. This is a beneficial alternative, because the jobs are still completed on time, resulting in a satisfied customer. The "one cell only" constraint is again changed to allow for multiple cell assignments. However, another constraint, known as the "time overlap prevention" constraint, as shown in 3.1, must be added to prevent a job from being scheduled in two different cells at the same time.

$$\sum_{c} Y_{jct} \leq 1 \quad \forall\, j, t \tag{3.1}$$

The model is run again and a workable solution is likely found. In the very unlikely case that the problem is still infeasible, the early start constraint can be relaxed to allow for jobs to start earlier and/or the time overlap constraint can be eliminated to achieve a workable solution.

Positively speaking, the heuristic strategy described in this section achieves a feasible solution (majority of the time) or a workable solution, within a reasonable time frame, without forfeiting significant portions of the objective. This strategy addresses some of the concerns from the smaller problem strategy, which allows this heuristic strategy to be applied to slightly larger problems. In contrast, the medium scale problem heuristic strategy has a handful of downfalls that deduct from its usefulness. First and foremost, infeasibility has a slight chance of coming into play, since some scheduled jobs are locked into place before other jobs are scheduled. Though a workable solution can be achieved by relaxing constraints that do not impact delivery of jobs to customers, it can be very costly to the manufacturer to truly implement these relaxations on the manufacturing floor. There is a lack of definitiveness to this heuristic. Especially when the model is running within stage 2, an initial feasible solution is found at different times depending on the specific problem. While two minutes is used as the reference point, stopping the model for feasibility at different times can impact the final heuristic solution. Finally, once again, this heuristic strategy will have difficulty performing as the magnitude and difficulty of the job scheduling problem continues to amplify.

3.4 Large Problems

Once again, large problems increase in size over medium problems by adding more jobs, more cells, and more time. Cell-days range from 400 to 500 days, while the number of jobs is between 100 and 150 jobs. It is highly unlikely that a problem of this size can be solved optimally with integer linear programming in conjunction with the baseline mathematical model. With a larger, more difficult problem, creative techniques must be used to expand upon the heuristic strategies developed for smaller problems. As shown in Figure 3.3, an additional stage is added to create this heuristic strategy, while adjusting other techniques developed in the heuristic strategies designed for smaller problems.

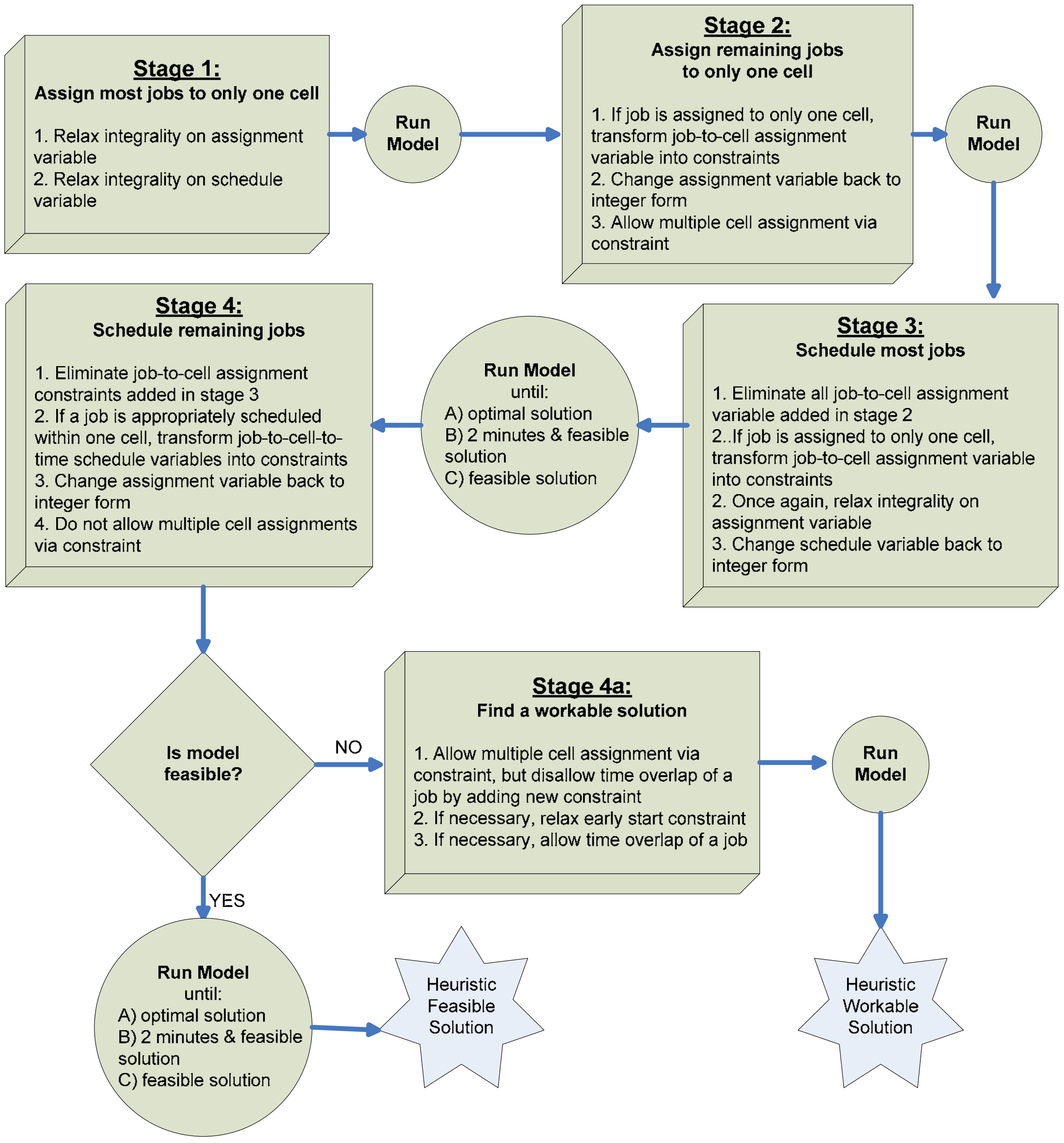

Stage 1:
Assign most jobs to only one cell
1. Relax integrality on assignment variable
2. Relax integrality on schedule variable
Run Model
Stage 2:
Assign remaining jobs to only one cell
1. If job is assigned to only one cell, transform job-to-cell assignment variable into constraints
2. Change assignment variable back to integer form
3. Allow multiple cell assignment via constraint
Run Model
Stage 3:
Schedule most jobs
1. Eliminate all job-to-cell assignment variable added in stage 2
2..If job is assigned to only one cell, transform job-to-cell assignment variable into constraints
2. Once again, relax integrality on assignment variable
3. Change schedule variable back to integer form
Run Model until:
A) optimal solution
B) 2 minutes & feasible solution
C) feasible solution
Stage 4:
Schedule remaining jobs
1. Eliminate job-to-cell assignment constraints added in stage 3
2. If a job is appropriately scheduled within one cell, transform job-to-cell-to-time schedule variables into constraints
3. Change assignment variable back to integer form
4. Do not allow multiple cell assignments via constraint
Is model feasible?
NO
Stage 4a:
Find a workable solution
1. Allow multiple cell assignment via constraint, but disallow time overlap of a job by adding new constraint
2. If necessary, relax early start constraint
3. If necessary, allow time overlap of a job
Run Model
YES
Run Model until:
A) optimal solution
B) 2 minutes & feasible solution
C) feasible solution
Heuristic Feasible Solution
Heuristic Workable Solution

Figure 3.3: Heuristic Strategy #3 – Large Problem

The heuristic strategy for this set of problems can be broken down into four main stages. The main difference between this heuristic and the previous heuristic is that all jobs are actually assigned to one and only one cell before any scheduling actually takes place. Again, it is critical that an initial feasible solution to the problem at hand is obtained as soon as possible, so that useful bits of the adapted feasible solution can be transferred to the next stage to speed up the overall solution process.

Stage 1 involves relaxing the integrality of the assignment variable and the schedule variable, to allow most jobs to be assigned to one cell in a short amount of computation time. After the model is run, if a job is assigned to only one cell, the corresponding job-to-cell assignment variable is transformed into a constraint and added to the model. From here, the assignment variable is changed back to an integer variable, to allow the remaining jobs to be scheduled. Additionally, the "one cell only" constraint is relaxed to allow for multiple cell assignments. Therefore, jobs can be assigned to more than just one cell. Jobs that were assigned to a specific cell in stage 1 are now flexible enough to move to another cell if necessary. By changing this constraint, a feasible solution is found significantly faster than by forcing all of the jobs to be scheduled to only one cell. Now the model can be run once again for stage 2.

The model has now been turned into a partial integer program and as expected, the solving process takes longer. Yet, many of the jobs have already been assigned to a distinct cell, so a feasible solution is normally obtained to the problem on hand within a minute or so. If an optimal solution has not been found after 2 minutes, the model can be stopped and the procedure can continue with the best feasible solution. More likely than not, all jobs will be assigned distinctly to one cell at the end of this stage.

In stage 3, all of the assignment variable constraints added in stage 2 are eliminated. Instead, all of the new assignments from the stage 2 solution are analyzed. If a job-to-cell assignment variable is equal to 1, the job-to-cell assignment is added as a constraint within the model. The schedule variable is changed to integer form, while the assignment variable is changed back once again to continuous form. Additionally, if necessary, jobs can actually move to a different cell than the one assigned to in stage 1 or stage 2. This is due to the fact that the model still allows for multiple cell assignments and the assignment variable is continuous, which allows it to happen at a lower cost to the objective.

Next, a balancing act must occur, as the longer program runs, the better the solution becomes, delivering better results to transfer to stage 4. After approximately 2 minutes, if the solution is not yet optimal, the model can be stopped (so long as there is a feasible solution) and the procedure can continue with the best feasible solution. Two minutes was chosen for similar reasons, as stated in the previous section regarding the medium problems.

In stage 4, the schedule variables from the stage 3 solution are analyzed. If a job is scheduled in only cell, the schedule variables for that job are transferred into the mathematical model in the form of constraints. After stage 2, normally over 90% of the jobs have been scheduled appropriately in one cell. This represents the eventual schedule for these jobs. However, before it can become the actual schedule, the remaining 10% of the jobs must be scheduled. Since the schedule variables have been added to the mathematical model, all assignment variable constraints that were added in stage 3 can be removed.

Prior to the model running, the mathematical model is changed back to its original form. The assignment variable is changed back to integer form and multiple cell assignments are again disallowed. Now, the model can be run again in an attempt to find a good feasible solution to the initial problem. Typically, an optimal solution is found well within 2 minutes. However, in the unlikely case that it can’t, the model should be stopped (so long as there is a feasible solution). The best feasible solution is used.

Because several jobs have already been locked into place after stage 3, there is the possibility that the problem is no longer feasible. It is probable that a few jobs could not be scheduled because other jobs were already scheduled to necessary time slots. In this case, adjustments must be made to achieve a workable solution. The same modifications as explained with heuristic strategy #2 can once more be used to tackle this setback.

The large problem heuristic is very similar to the medium problem heuristic. The additional stage permits jobs to move from cell to cell, while also limiting the size of the problem within each stage. Therefore, the heuristic allows for larger problems to be solved feasibly, a majority of the time, within a reasonable computation time. Yet again, however, this heuristic has its fair share of pitfalls. Although it can handle larger, more difficult problems, infeasibility is now an even greater possibility, because significantly more jobs are initially assigned, which leads to more jobs being scheduled before others. Furthermore, there is still an uncertainty around when stages should be stopped. The question arises, “When is a feasible solution good enough?” This is a very tough question to answer and leads to ambiguity and inconsistency within the final solution. As problems continue to become more difficult, due to increasing number of jobs, cells, and time periods, the method of scheduling jobs detailed in this section and the two previous sections will no longer be able to handle the more complicated problems.

3.5 Ultimate Heuristic

3.5.1 Background

Extra large problems consist of problems with 600-750 cell-days. The largest problem that will be examined contains 15 cells, 50 time periods, and 213 jobs. This scale problem is consistent of job scheduling problems face by several companies, including Optimax. The number of cells is suggestive of a full-size manufacturing facility with several different cells. The number of time periods is indicative of a business type in which completion lead times are typically in the 6-10 week timeframe. To solve a problem of this magnitude, a heuristic must innovatively be created that can handle the difficulty of the problem, but also addresses all of the concerns of the small, medium, and large problem heuristic strategies, previously presented. It is critical that the heuristic is able to schedule all jobs feasibly, with a definitive approach, in a rational sum of computation time.

Retrospectively, the previous heuristics go awry in a few critical areas. First, when jobs are initially assigned to only one cell, only two factors are considered, cost and cell time capacity (equivalent to total time periods). Due to the objective function attempting to minimize cost, the mathematical model attempts to assign all jobs to the corresponding least cost cell. Each job typically ends up being assigned to its least cost cell, unless the cell capacity is maxed out, in which case, one or more jobs must be moved to a different higher cost cell. This procedure is fine, but it overlooks three major aspects of the problem, due dates and early start dates for each job, and current load (sum of completion times for assigned jobs) for each cell. At this point, since the schedule variable is not of integer form, several jobs can be assigned to the same cell, even though it is impossible for all the jobs to meet the “early start” constraint and “due date” constraint. Moreover, if one cell is cheaper across the board in comparison to other cells, the load could be maxed out, while the other cells are just fractionally full. For the maxed out cell, once the schedule variable is integer again, reviving the early start dates and due date, it is very unlikely that all jobs assigned to the cell can actually be appropriately scheduled within the cell. One or more jobs are scheduled over two separate cells and must slide entirely out of the maxed out cell into another cell. However, in an attempt to decrease the overall difficulty of the problem, heuristic strategies #2 and #3 call for cementing appropriately scheduled jobs to cells at specific times through the use of additional constraints. This is troublesome because the jobs that need to move to another cell may not have another feasible timeframe, due to the fact that several jobs have already been scheduled and forced into place. Since it is hard to understand where exactly the conflict occurs, it is difficult to un-schedule a clashing job and thus infeasibility sets in. The following tables illustrate this phenomenon using a simple problem, with 5 jobs, 3 cells, and 5 time periods. Table 3.1 simply shows the scheduling grid that the job scheduler will attempt to fill with a feasible solution, while Table 3.2 displays the input parameters for the problem.

	TIME				
	1	2	3	4	5
Cell 1					
Cell 2					
Cell 3					

Table 3.1: Scheduling Grid

				COST		
	Comp. Time	Due Date	Early Start	Cell 1	Cell 2	Cell 3
Job 1	1	5	1	2	4	x
Job 2	2	3	1	2	1	x
Job 3	3	4	2	x	1	4
Job 4	3	5	1	2	x	5
Job 5	3	4	1	1	5	1

*x denotes infeasible job-cell relationship

Table 3.2: Input Parameters

Table 3.3 shows the initial assignments, denoted by the completion time of the job, which the model would have made according to heuristic strategies #2 and #3. The cell load is also calculated for each cell at the bottom of the chart.

	Cell 1	Cell 2	Cell 3
Job 1	1		
Job 2		2	
Job 3		3	
Job 4	3		
Job 5			3
Cell Load	**4**	**5**	**3**

Table 3.3: Initial Assignments (with completion time) and Cell Loads

At first glance the assignments look excellent as cost (16) is at a minimum. Each job is assigned to its least cost cell. Nevertheless, it is clearly noticeable that it is impossible to schedule both Job 2 and Job 3 within Cell 2 once early start dates and due dates are taken into account. In a larger problem, this conflict is not likely so apparent. Equally intriguing is the fact that Cell 2 also has the greatest cell load, which, regardless of the obvious conflict, makes it a greater candidate for a scheduling conflict once due date and early start date constraints come back into play.

Regardless, heuristic strategies #2 and #3 push forward and begin scheduling jobs using integer schedule variables, while now allowing multiple cell assignments. Table 3.4 shows the job schedule after this stage.

	TIME				
	1	**2**	**3**	**4**	**5**
Cell 1	Job2	Job4	Job4	Job4	Job1
Cell 2		Job2	Job3	Job3	Job3
Cell 3	Job5	Job5	Job5		

Table 3.4: Initial Scheduling Grid

Job 1, Job 4, and Job 5 are able to be scheduled appropriately within their respective assigned cells. As previously determined, both Job 2 and Job 3 were unable to be scheduled within Cell 2. Since the cost to send Job 2 to Cell 1 is less than the cost to send Job3 to Cell 3, one time unit of Job 2 is scheduled in Cell 1. Now to ease the difficulty of the problem, heuristic strategies #2 and #3 call for all jobs that are scheduled appropriately in only one cell be locked into place by a job-to-cell-to-time schedule constraint. Thus, Job 1, Job 3, Job 4, and Job 5 are cemented into their current place. Although the heuristics free Job 2 from any cell assignment, it is too late at this point. Job 2 no longer has a feasible time frame to be scheduled in. Cell 1 is the only other feasible cell for Job 2 and there is only one time period left within the cell. The scheduling problem is now infeasible. Jobs that were frozen in place captured time periods needed for Job 2. However, the original problem does have an optimal feasible solution, as shown in Table 3.5, with an objective value cost of 20. The heuristics could not achieve the optimal solution because more jobs were assigned to a cell than could be scheduled and furthermore, more flexible jobs were scheduled prior to less flexible jobs.

	TIME				
	1	**2**	**3**	**4**	**5**
Cell 1	Job2	Job2	Job4	Job4	Job4
Cell 2	Job1	Job3	Job3	Job3	
Cell 3	Job5	Job5	Job5		

Table 3.5: Optimal Solution Scheduling Grid

The take away point from this example is twofold. First, jobs that may be over-assigned to a particular cell, according to cell load, must be allowed ample possibility to move to another feasible cell. Additionally, a window of time can't be locked up with a job until there is a strong likelihood that the timeframe will not be needed by another less flexible job. This is a critical balancing act, because permanently scheduling a job too soon can lead to infeasibility, but not constraining a job leads to a more difficult problem to solve.

Significant uncertainty creeps into the problem-solving process when a substantial number of cell-days must be scheduled all at once. This is what leads to the guessing game of when to stop the model after a feasible solution is found. The larger the problem becomes, the more cell-days there are to be scheduled and the longer it takes to find a feasible solution. Therefore, this issue will not be alleviated until a new method is in place to limit the number of cell-days that must be scheduled at once, while still not prematurely permanently scheduling jobs into a set time frame.

3.5.2 Techniques

The techniques used in the ultimate heuristic address the concerns of the smaller sized problem heuristic strategies described previously. The ultimate heuristic uses relaxations to multiple variables and constraints to create a framework, which leads into an iterative greedy process that takes into account flexibility of the jobs and the current loads of the cells.

First, all jobs must be assigned to one and only one cell, using a greedy approach. This allows the scheduling of the jobs to occur at a much faster rate. The model does not need to be concerned with assigning and scheduling all jobs simultaneously. Instead the model can focus on scheduling the jobs within the assigned cells, and when necessary move a job to a different cell. When dealing with so many jobs, the assignment process must be completed over two stages. The first stage consists of relaxing integrality for both the assignment variable and the schedule variable. If the assignment variable for a particular job assigns the job to only one cell, the assignment variable is converted to a constraint within the model. Now, the assignment variable is returned to its original integer form and the model is run again, allowing multiple cell assignments via the "one cell only" constraint. Multiple cell assignments are allowed to speed up the solving process. Every job will now be distinctly assigned to only one cell. Yet again, each assignment variable is converted into a constraint. By relaxing the "one cell only" constraint, as shown in 3.2, a job will have the ability to move out of its assigned cell if it is absolutely necessary.

$$\sum_{i} X_{ji} \geq 1 \quad \forall\, j \tag{3.2}$$

Up until this point, not much is different from the previous heuristic strategies. This is a great method to assign jobs to cells because jobs are optimally assigned to the lowest cost cells. However, the model finds itself in a familiar situation, as scheduling the jobs within the cells is a whole other matter. There are 750 cell-days that must be scheduled within a problem containing 15 cells and 50 time periods. Once jobs begin to be scheduled, jobs will undoubtedly have to move to different cells to meet all the constraints. It is impossible for the mathematical model to handle so many schedule variables all at once. The previous heuristic strategies attempt to tackle this issue, but sometimes wind up with an infeasible solution. The "one job at a time" constraint, significantly contributes to the model being unable to schedule all the jobs in a timely manner. If each time within each cell, could handle multiple jobs the problem could be solved in a fraction of the time. It is imperative to find a method that iteratively relaxes this constraint to schedule jobs within cells without locking up critical time windows potentially needed by less flexible jobs.

In comes a new input parameter called for by the ultimate heuristic, known as normalized flexibility, as shown in 3.3. It is used to firmly schedule inflexible jobs prior to the most flexible jobs. Flexibility, shown in 3.4, is calculated based upon a job's completion time, early start date, due date, and number of respective feasible cells. Flexibility is entered into the model using a normalized scale. The normalized flexibility function is shown in 3.5. The greatest flexibility of any job within the problem is used as the divisor for normalizing all the flexibilities. Therefore, the normalized flexibility will be on the scale from 0-1. Normalization provides simplicity within the problem and offers consistency amongst all job scheduling problems. The flexibility calculations can take place automatically before the model is run. The normalized flexibility function along with Table 3.6 shows the flexibility and normalization calculation process.

$$n_j = \text{ normalized flexibility of } job\ j \qquad (3.3)$$

$$flexibility = \frac{(DueDate - EarlyStart + 1) * \# FeasibleCells}{CompletionTime} \qquad (3.4)$$

$$n_j = \frac{(DueDate - EarlyStart + 1) * \# FeasibleCells}{CompletionTime * MaxFlexibility} \qquad (3.5)$$

				FEASIBILITY					
	Comp. Time	Due Date	Early Start	Cell 1	Cell 2	Cell 3	# Feasible	Flexibility	Normalized
Job 1	1	5	1	1	1	x	2	10.0	1.00
Job 2	2	3	1	1	1	x	2	3.0	0.30
Job 3	3	4	2	x	1	1	2	2.0	0.20
Job 4	3	5	1	1	x	1	2	3.3	0.33
Job 5	3	4	1	1	1	1	3	4.0	0.40

*x denotes infeasible job-cell relationship

Table 3.6: Input Parameters with Normalized Flexibility

The table shows that Job 3 is the least flexible job, followed by Job2 and Job 4. Job 1 is the most flexible job as it takes only one time period to complete, while it has a large feasible time window for production and is feasible in two different cells. For the purpose of the ultimate heuristic, any job with a normalized flexibility of greater than 0.25 and a completion time equal to 1, will be known as a “flexible” job and will be scheduled after all other jobs have already been permanently scheduled. This is done by changing the “scheduled time equals completion time” constraint as shown in Equation 3.6. Jobs with a high flexibility have several options of where they can be scheduled. Likely, they can be scheduled anywhere across several different time periods and several different cells.

$$\begin{cases} \sum_{c}\sum_{t} Y_{jct} = 0 \quad \forall j & \text{if } n_j > 0.25 \text{ and } ct_j = 1 \\ \sum_{c}\sum_{t} Y_{jct} = ct_{jc} \quad \forall j & \text{otherwise} \end{cases} \tag{3.6}$$

Conveniently enough, it just so happens that the two most inflexible jobs (Job 2, Job3) within this set of data will be initially assigned to the same cell (Cell 2), as shown previously in Table 3.3, but both jobs are unable to be scheduled within the cell. As the example discovered, Cell 2 also has the greatest cell load after the assignment phase of all the cells, making it an obvious candidate for a scheduling conflict. Up until now, cell load has not been a factor examined while scheduling jobs. To put an end to this, the ultimate heuristic calls for a new decision variable, known as cell load, which will be introduced to the mathematical model after the assignment phase is complete. The cell load of a cell is calculated as the completion time of all the jobs currently assigned to the cell. To combat the issue of having to schedule so many cell-days at once, only one cell will be scheduled at a time within each stage of the ultimate heuristic. The cell load will provide the order of cells to be scheduled. The cell with the greatest load is scheduled first and so on. Leveraging cell load will allow over-assigned cells to export jobs to cells that can accommodate the jobs before time windows become locked up. The cell load is updated continuously with the completion of each stage. “Flexible” jobs with a normalized flexibility greater than 0.25 and a completion time equal to 1 are not included in the cell load because they are scheduled after all other jobs.

The ability to schedule only one cell per stage is created by splitting up the "one job at a time" constraint over the number of cells in the problem. For the cell to be scheduled next, as well as for all cells that have already been scheduled, the constraint allows only one job to be scheduled at a time. The remaining cells allow for as many jobs as possible to be scheduled at once. During any stage, the maximum number of cell-days truly being scheduled is equivalent to the number of time periods. This in itself is a tremendous simplification to the problem and should allow for a rapid optimal solution for each stage. Figure 3.4 shows the scheduling process according to the cell loads.

Summary:

# Jobs = 5

# Cells = 3

Cell Loads:

Cell Load [Cell 1] = 3

Cell Load [Cell 2] = 5

Cell Load [Cell 3] = 3

Constraints:

1) $$\sum_{j} Y_{j,1,t} \leq 5 \quad \forall t$$

2) $$\sum_{j} Y_{j,2,t} \leq 1 \quad \forall t$$

3) $$\sum_{j} Y_{j,3,t} \leq 5 \quad \forall t$$

Figure 3.4: Scheduling Cell by Cell According to Cell Load

After all the jobs have been assigned and the "flexible" jobs have been removed from being scheduled, the cell loads are calculated and are shown above. Since Cell 2 has the greatest cell load, it will be scheduled first. Therefore, within Cell 2, only one job can be scheduled at a time, as restricted by the second constraint. However, within Cell 1 and Cell 3, the constraint allows the maximum number of jobs (5) to be scheduled at any particular time. The model runs and the jobs appropriately scheduled within Cell 2 are permanently scheduled. The jobs that are not correctly scheduled are reassigned to the next best cell and the process continues with the cell with the next greatest load.

Figure 3.5 shows the flow diagram for the ultimate heuristic. The ultimate heuristic uses a greedy multi-phase iterative process to first assign jobs to particular cells and then to schedule the jobs within the assigned cells. The heuristic relaxes several variables and constraints along the way, while taking into account the flexibility of the different jobs and the current load of the different cells.

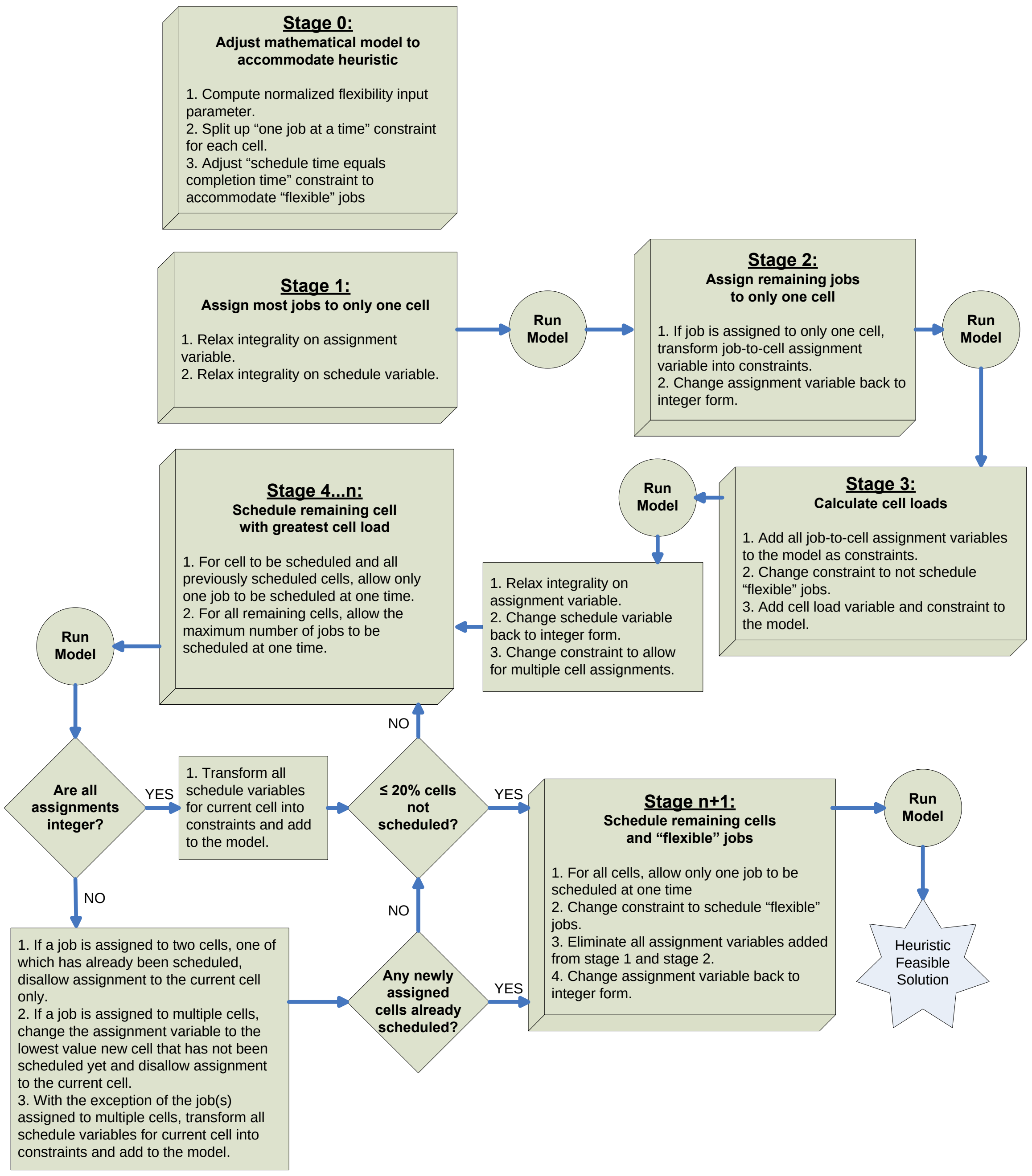

Stage 0:
Adjust mathematical model to accommodate heuristic
1. Compute normalized flexibility input parameter.
2. Split up "one job at a time" constraint for each cell.
3. Adjust "schedule time equals completion time" constraint to accommodate "flexible" jobs
Stage 1:
Assign most jobs to only one cell
1. Relax integrality on assignment variable.
2. Relax integrality on schedule variable.
Run Model
Stage 2:
Assign remaining jobs to only one cell
1. If job is assigned to only one cell, transform job-to-cell assignment variable into constraints.
2. Change assignment variable back to integer form.
Run Model
Stage 3:
Calculate cell loads
1. Add all job-to-cell assignment variables to the model as constraints.
2. Change constraint to not schedule "flexible" jobs.
3. Add cell load variable and constraint to the model.
Run Model
1. Relax integrality on assignment variable.
2. Change schedule variable back to integer form.
3. Change constraint to allow for multiple cell assignments.
Stage 4…n:
Schedule remaining cell with greatest cell load
1. For cell to be scheduled and all previously scheduled cells, allow only one job to be scheduled at one time.
2. For all remaining cells, allow the maximum number of jobs to be scheduled at one time.
Run Model
Are all assignments integer?
YES
1. Transform all schedule variables for current cell into constraints and add to the model.
≤ 20% cells not scheduled?
NO
YES
Stage n+1:
Schedule remaining cells and "flexible" jobs
1. For all cells, allow only one job to be scheduled at one time
2. Change constraint to schedule "flexible" jobs.
3. Eliminate all assignment variables added from stage 1 and stage 2.
4. Change assignment variable back to integer form.
Run Model
Heuristic Feasible Solution
NO
1. If a job is assigned to two cells, one of which has already been scheduled, disallow assignment to the current cell only.
2. If a job is assigned to multiple cells, change the assignment variable to the lowest value new cell that has not been scheduled yet and disallow assignment to the current cell.
3. With the exception of the job(s) assigned to multiple cells, transform all schedule variables for current cell into constraints and add to the model.
NO
Any newly assigned cells already scheduled?
YES

Figure 3.5: Ultimate Heuristic

Several of the techniques present within this heuristic have already been partially explained. To best describe the heuristic in its entirety and to illustrate the dynamics of the whole procedure, the same simple problem will be used, but the heuristic will be applied. Table 3.7 shows the input parameters for the problem.

				COST			FEASIBILITY					
	Comp. Time	Due Date	Early Start	Cell 1	Cell2	Cell3	Cell 1	Cell 2	Cell 3	# Feasible	Flexibility	Normalized
Job 1	1	5	1	2	4	x	1	1	x	2	10.0	1.00
Job 2	2	3	1	2	1	x	1	1	x	2	3.0	0.30
Job 3	3	4	2	x	1	4	x	1	1	2	2.0	0.20
Job 4	3	5	1	2	x	5	1	x	1	2	3.3	0.33
Job 5	3	4	1	3	5	1	1	1	1	3	4.0	0.40

*x denotes infeasible job-cell relationship

Table 3.7: Input Parameters with Normalized Flexibility

The following is the state of the OPL model and the OPL data after stage 0 is complete. The grayed contents are additions or changes made to the baseline mathematical model to accommodate the heuristic. A double backslash (//) denotes a comment within the programming code and therefore the entire following line is not actually used in the model.

OPL Model:

<u>Notation:</u>

```
int nbCell = ...;
range Cell 1..nbCell;

int nbTime = ...;
range Time 1..nbTime;

int nbJob = ...;
range Job 1..nbJob;
```

<u>Input Parameters:</u>

```
float+ feasible[Job,Cell]=...;
float+ completionTime[Job]=...;
float+ earlyStart[Job]=...;
float+ dueDate[Job]=...;
float+ cost[Job,Cell]=...;
float+ flexibility[Job]=...;
```

Decision Variables:

```
var int+ assignment[Job,Cell] in 0..1;
var int+ schedule[Job,Cell,Time] in 0..1;
var float+ start[Job] in 1..nbTime;
var float+ finish[Job] in 1..nbTime;
//var float+ cellLoad[Cell] in 1..nbTime;
```

Objective Function:

```
minimize

 sum(j in Job, c in Cell)

   (assignment[j,c] * cost[j,c] * completionTime[j])
```

Constraints:

```
subject to {

//one cell only
   forall (j in Job)
      sum (c in Cell)
         assignment[j,c] = 1;

//cell feasibility
   forall (j in Job & c in Cell)
         assignment[j,c]<=feasible[j,c];

//schedule only if assigned
   forall (j in Job, c in Cell)
      sum (t in Time)
         schedule[j,c,t] <= nbTime * assignment[j,c];

//one job at a time
   forall (t in Time)
      sum (j in Job)
         schedule[j,1,t] <= 1;

   forall (t in Time)
      sum (j in Job)
         schedule[j,2,t] <= 1;

   forall (t in Time)
      sum (j in Job)
         schedule[j,3,t] <= 1;
//scheduled time equals completion time
  forall (j in Job)
    if flexibility[j] > 0.25 & completionTime[j] = 1 then
      sum (c in Cell, t in Time)
        schedule[j,c,t]=completionTime[j]
//        schedule[j,c,t]=0
    else
      sum (c in Cell, t in Time)
        schedule[j,c,t]=completionTime[j]
    endif;

//starting time
  forall (j in Job & c in Cell & t in Time)
      t*schedule[j,c,t]+nbTime*(1-schedule[j,c,t]) >= start[j];

//finish time
  forall (j in Job & c in Cell & t in Time)
      finish[j]>=t*schedule[j,c,t];

//early start
  forall (j in Job)
      start[j]>=earlyStart[j];

//due date
  forall (j in Job)
      finish[j]<=dueDate[j];

//sequential
  forall (j in Job)
      finish[j]-start[j]=completionTime[j]-1;

//cell load
// forall (c in Cell)
//     sum (j in Job, t in Time)
//       schedule[j,c,t]=cellLoad[c];

};
```

OPL Data File:

```
nbCell = 3;
nbTime = 5;
nbJob = 5;

feasible = [[1,1,0],[1,1,0],[0,1,1],[1,0,1],[1,1,1]];
```

```
completionTime = [1,2,3,3,3];
earlyStart = [1,1,2,1,1];
dueDate = [5,3,4,5,4];
cost = [[2,4,0],[2,1,0],[0,1,4],[2,0,5],[1,5,1]];
flexibililty = [1.00,0.30,0.20,0.33,0.40];
```

During stage 0, the flexibility input parameter is added to the model and the normalized flexibility values for each job are added into the data file. The cell load variable and constraint is present in the model, but are commented out until stage 3, when they are needed. The same can be said for the constraint that does not schedule "flexible" jobs. Finally, as a whole, the "one job at a time" constraint is unchanged. However, it has been split up over the three cells for ease of use starting at stage 4.

Modified Variables for Stage 1:

```
var float+ assignment[Job,Cell] in 0..1;
var float+ schedule[Job,Cell,Time] in 0..1;
```

In stage 1, both the integrality on the assignment variable and the schedule variable is relaxed, in an attempt to assign as many jobs as possible in a short computation time. The model is run and the assignment variable results are shown below.

Variable Results from Stage 1: (assigning initial jobs)

```
assignment[1,1] = 1.0000
assignment[2,2] = 1.0000
assignment[3,2] = 1.0000
assignment[4,1] = 1.0000
assignment[5,1] = 0.6000
assignment[5,3] = 0.4000
```

New Constraints for Stage 2:

```
assignment[1,1] = 1;
assignment[2,2] = 1;
```

```
assignment[3,2] = 1;
assignment[4,1] = 1;
```

Modified Variables for Stage 2:

```
var int+ assignment[Job,Cell] in 0..1;
```

In stage 2, each assignment variable that is integer is converted to a constraint and added to the model. Job 5 is the only job from stage 1 not to be assigned solely to one cell. Therefore, assignment constraints will be added to the model for Jobs 1-4, as shown above, and the assignment variable is changed back to integer form before the model is run again.

Variable Results from Stage 2: (assigning remaining jobs)

```
assignment[1,1] = 1
assignment[2,2] = 1
assignment[3,2] = 1
assignment[4,1] = 1
assignment[5,3] = 1
```

New Constraints for Stage 3:

```
assignment[5,3] = 1;

forall (c in Cell)
  sum (j in Job, t in Time)
    schedule[j,c,t]=cellLoad[c];
```

Modified Constraints for Stage 3:

```
forall (j in Job)
  if flexibility[j] > .25 & completionTime[j] = 1 then
    sum (c in Cell, t in Time)
//      schedule[j,c,t]=completionTime[j]
      schedule[j,c,t]=0
  else
    sum (c in Cell, t in Time)
      schedule[j,c,t]=completionTime[j]
  endif;
```

New Variables for Stage 3:

```
var float+ cellLoad[Cell] in 1..nbTime;
```

Heading into stage 3, all jobs have been distinctly assigned to one cell, with Job 5 being constrained to Cell 3. The goal of stage 3 is to determine cell loads for each cell in order to provide the cell scheduling order. Therefore, the cell load variable and constraint are added to the model. This is its own stage for two main reasons. First, it takes significantly longer to determine the cell load when all jobs are not already assigned to one cell only. Secondly, the jobs that are “flexible” (flexibility > 0.25 and completion time = 1), can be removed from the cell load, by the slight modification to the constraint just shown. The commented line is removed and the line directly below is added to the model. “Flexible” jobs are still assigned to a cell to provide time gaps that offer added elasticity to the jobs that are scheduled first. The model is ready to be run again.

Variable Results from Stage 3: (calculating cell loads)

```
cellLoad[1] = 3.0000
cellLoad[2] = 5.0000
cellLoad[3] = 3.0000
```

Modified Constraints for Stage 4:

```
forall (t in Time)
    sum (j in Job)
      schedule[j,1,t] <= nbJob;

forall (t in Time)
    sum (j in Job)
      schedule[j,2,t] <= 1;
```

```
forall (t in Time)
    sum (j in Job)
      schedule[j,3,t] <= nbJob;

forall (j in Job)
    sum (c in Cell)
      assignment[j,c] >= 1;
```

<u>Modified Variables for Stage 4:</u>

```
var float+ assignment[Job,Cell] in 0..1;
var int+ schedule[Job,Cell,Time] in 0..1;
```

Before any cells can be scheduled, the schedule variable is changed back to integer form, while the assignment variable is once again relaxed to a continuous variable, in order to easier allow jobs to move to different cells if necessary. In doing so, the “one cell only” constraint is also relaxed. Currently, Cell 2 has the greatest load and thereby will be scheduled first. If during any stage the greatest cell load for an unscheduled cell is equivalent for two different cells, arbitrarily choose a cell to schedule next. The constraints are changed to allow only one job at a time to be scheduled in Cell 2. However, the maximum number of jobs can be scheduled at any time within Cell 1 and Cell 3. The modified constraints just shown permit this to happen.

<u>Variable Results from Stage 4:</u> (scheduling Cell 2)

```
cellLoad[1] = 4.0000
cellLoad[2] = 4.0000
cellLoad[3] = 3.0000

assignment[1,1] = 1.0000
assignment[2,1] = 0.2000
assignment[2,2] = 1.0000
assignment[3,2] = 1.0000
assignment[4,1] = 1.0000
assignment[5,3] = 1.0000
```

```
schedule[2,1,2] = 1

schedule[4,1,1] = 1
schedule[4,1,2] = 1
schedule[4,1,3] = 1

schedule[2,2,1] = 1

schedule[3,2,2] = 1
schedule[3,2,3] = 1
schedule[3,2,4] = 1

schedule[5,3,1] = 1
schedule[5,3,2] = 1
schedule[5,3,3] = 1
```

New Constraints for Stage 5:

```
assignment[2,1] = 1;

schedule[3,2,2] = 1;
schedule[3,2,3] = 1;
schedule[3,2,4] = 1;
```

Modified Constraints for Stage 5:

```
assignment[2,2] = 0;

forall (t in Time)
    sum (j in Job)
      schedule[j,1,t] <= 1;
```

The grayed variables from stage 4 show that Job 2 gets assigned to two different cells and thus winds up getting scheduled across the two cells. Consequently, the assignment variable constraint is changed to assign Job 2 to Cell 1, the next best cell option cost-wise. If a job is assigned to multiple other un-scheduled cells with an equivalent variable value, constrain the job to the cell with the smallest cell load. Another assignment variable constraint is added to disallow Job 2 to be assigned to Cell 2 again. If

Cell 1 had already been scheduled, simply disallow scheduling to Cell 2, without forcing the job to Cell 1. This occurs because it is likely that Cell 1, since it had already been scheduled, will not have room for another job. To avoid changing the assignment again after the next stage, this modified action is taken. The remaining jobs scheduled in Cell 2, not assigned to multiple cells, now become permanently scheduled through use of the schedule variable constraints. Notice that Job 1 was not scheduled anywhere because it is a "flexible" job. Since Cell 1 has the greatest cell load of the jobs not yet scheduled, it will be scheduled next and so the "one job at a time" constraint is applied for Cell 1. The current state of the schedule, after Stage 4 and before Stage 5, is shown in Table 3.8.

	TIME				
	1	**2**	**3**	**4**	**5**
Cell 1					
Cell 2		Job3	Job3	Job3	
Cell 3					

Table 3.8: State of Schedule after Stage 4

Variable Results from Stage 5: (scheduling Cell 1)

```
cellLoad[1] = 5.0000
cellLoad[2] = 3.0000
cellLoad[3] = 3.0000

assignment[1,1] = 1.0000
assignment[2,1] = 1.0000
assignment[3,2] = 1.0000
assignment[4,1] = 1.0000
assignment[5,3] = 1.0000

schedule[2,1,1] = 1
schedule[2,1,2] = 1

schedule[4,1,3] = 1
schedule[4,1,4] = 1
schedule[4,1,5] = 1
```

```
schedule[3,2,2] = 1
schedule[3,2,3] = 1
schedule[3,2,4] = 1

schedule[5,3,1] = 1
schedule[5,3,2] = 1
schedule[5,3,3] = 1
```

New Constraints for Stage 6:

```
schedule[2,1,1] = 1;
schedule[2,1,2] = 1;
schedule[4,1,3] = 1;
schedule[4,1,4] = 1;
schedule[4,1,5] = 1;
```

Modified Constraints for Stage 6:

```
forall (t in Time)
    sum (j in Job)
      schedule[j,1,t] <= 1;
```

All jobs were assigned to only one cell. Therefore, all schedule variables for jobs scheduled in Cell 1 are transformed into constraints and add to the model for the next stage. Job 2, which could not be scheduled entirely within Cell 2, was able to be scheduled appropriately within Cell1. Cell 3 is the last cell remaining to be scheduled and thus will be scheduled next. The “one job at a time” constraint is applied for Cell 3. In problems of larger size with more cells, it is not necessary to iteratively schedule the last 20% of the cells. Skip ahead to the final step in this case. The current state of the schedule, after Stage 5 and before Stage 6, is shown in Table 3.9.

	TIME				
	1	2	3	4	5
Cell 1	Job2	Job2	Job4	Job4	Job4
Cell 2		Job3	Job3	Job3	
Cell 3					

Table 3.9: State of Schedule after Stage 5

<u>**Variable Results for Stage 6:**</u> (scheduling Cell 3)

```
cellLoad[1] = 5.0000
cellLoad[2] = 3.0000
cellLoad[3] = 3.0000

assignment[1,1] = 1.0000
assignment[2,1] = 1.0000
assignment[3,2] = 1.0000
assignment[4,1] = 1.0000
assignment[5,3] = 1.0000

schedule[2,1,1] = 1
schedule[2,1,2] = 1

schedule[4,1,3] = 1
schedule[4,1,4] = 1
schedule[4,1,5] = 1

schedule[3,2,2] = 1
schedule[3,2,3] = 1
schedule[3,2,4] = 1

schedule[5,3,1] = 1
schedule[5,3,2] = 1
schedule[5,3,3] = 1
```

<u>**New Constraints for Stage 7:**</u>

```
schedule[5,3,1] = 1;
schedule[5,3,2] = 1;
schedule[5,3,3] = 1;
```

<u>**Modified Constraints for Stage 7:**</u>

```
forall (j in Job)
  if flexibility[j] > 0.25 & completionTime[j] = 1 then
    sum (c in Cell, t in Time)
```

```
        schedule[j,c,t]=completionTime[j]
//      schedule[j,c,t]=0
  else
    sum (c in Cell, t in Time)
      schedule[j,c,t]=completionTime[j]
  endif;

//   assignment[1,1] = 1;
//   assignment[2,2] = 0;
//   assignment[2,1] = 1;
//   assignment[3,2] = 1;
//   assignment[4,1] = 1;
//   assignment[5,3] = 1;
```

<u>Modified Variables for Stage 7:</u>

```
var int+ assignment[Job,Cell] in 0..1;
```

Once again all the jobs are assigned to only one cell and so the job(s) scheduled in Cell 3 now become permanently scheduled through use of the schedule variable constraints. Cell 3 was the last cell to be scheduled, which means the heuristic moves to the final phase of scheduling the "flexible" jobs and any remaining jobs not yet scheduled. Therefore, the constraint is changed back to ensure that all jobs are scheduled. All of the assignment variable constraints are eliminated for two reasons. First, all of the inflexible jobs have already been scheduled and secondly, some "flexible" jobs will likely have to move to a different cell to find a feasible time window. Finally, the assignment variable is changed back to integer form. The model is run for the last time. The current state of the schedule, after Stage 6 and before Stage 7, is shown in Table 3.10.

	TIME				
	1	2	3	4	5
Cell 1	Job2	Job2	Job4	Job4	Job4
Cell 2		Job3	Job3	Job3	
Cell 3	Job5	Job5	Job5		

Table 3.10: State of Schedule after Stage 6

__**Variable Results for Stage 7:**__ (scheduling remaining jobs and “flexible” jobs)

cellLoad[1] = 5.0000
cellLoad[2] = 4.0000
cellLoad[3] = 3.0000

assignment[1,2] = 1
assignment[2,1] = 1
assignment[3,2] = 1
assignment[4,1] = 1
assignment[5,3] = 1

schedule[2,1,1] = 1
schedule[2,1,2] = 1

schedule[4,1,3] = 1
schedule[4,1,4] = 1
schedule[4,1,5] = 1

schedule[1,2,1] = 1

schedule[3,2,2] = 1
schedule[3,2,3] = 1
schedule[3,2,4] = 1

A feasible solution is achieved through the use of the heuristic. Due to the simplicity of the problem, it is clear that the solution is in fact optimal. The objective value is equivalent to 20. Table 3.11 shows how the jobs would be scheduled according to the heuristic.

	TIME				
	1	2	3	4	5
Cell 1	Job2	Job2	Job4	Job4	Job4
Cell 2	Job1	Job3	Job3	Job3	
Cell 3	Job5	Job5	Job5		

Table 3.11: Heuristic Feasible Solution Schedule

3.5.4 Summary

The ultimate heuristic takes advantage of several techniques to arrive at a near optimal solution in a practical amount of solving time. The separation of the assignment process and schedule process allows the overall schedule to be broken down into two logical phases. Once the jobs are assigned to cells, the iterative scheduling phase is able to begin. Postponing the scheduling of “flexible” jobs and scheduling the greatest load cells first, limit the number of cell-days that must be scheduled at one time, while allowing ample space for the movement of jobs to different cells or time periods, before locking other jobs into specific positions. This vastly improves the feasibility chances of the final heuristic solution, especially on larger problems.

To further improve the solving time of the problem using the heuristic, a time limit could be placed on each stage. At times, the solver gets stuck in one stage for an extended period of time with a very slight opportunity for improvement. A time limit would stop the solver and proceed with the best feasible solution. Likely, the overall solution will not be as good, but if timing is an important factor, this could be a direction to consider.

To additionally aid to the efficiency of the heuristic, it is beneficial to assign different costs to each feasible cell for a particular job. For example, if it is equally cost effective to produce Job 1 in Cell 1 and Cell2, the cost input parameter would be the same. However, what happens during the solving process is the solver cannot decide which cell to assign/schedule the job in. Based on the way the heuristic is set up and the current stage, it will assign/schedule the job in both cells, delaying a solution and/or increasing the likelihood of infeasibility. Therefore, it is recommended to provide a trivial difference between the costs for each feasible cell for a particular job.

For small problems, the ultimate heuristic is likely not the best bet for aiding in the solving process. Due to the fact that there are several jobs with only one or two feasible cells, the ability to move jobs is already limited. When jobs are being cemented into place before others, infeasibility can creep into the problem before the iterative scheduling process finishes. Therefore, for problems of less magnitude, as examined in the small problem section, heuristic strategy #1 is likely the better direction to head in because no jobs are prematurely locked into a specific position. It will take longer to solve the problem, but feasibility will not likely be an issue.

4 Results and Analysis

4.1 Small Problem Testing

Of the 30 small problems that were tested, only 16 problems could be solved optimally within an hour. One hour was used as the measuring point for the sensible purpose that a job schedule must be realized within a practical timeframe to be of usefulness. An additional eight problems were able to render a feasible solution within the one-hour window. The remaining problems went unsolved and thus the bound will be used as the baseline value. The bound is not the optimal solution, but instead it is the best possible solution. No solution can be better than the bound. Nevertheless, it is very likely that the bound and the true optimal solution differ by some immeasurable amount. Each of the 30 problems was also solved using the ultimate heuristic methods. Table 4.1 shows the solution results for this set of problems.

For the first test, the heuristic solution is compared to the best possible solution. In this case, it is either the optimal solution or the bound. If a problem has an optimal solution, it is used. Otherwise, the bound is used. Each of the 30 problems is used within this test. A one sample t-test is used to provide the analysis instead of a paired t-test, due to the fact that the heuristic solution is impacted by the true value of the optimal solution. For example, Problem A has an optimal solution of 100 and a heuristic solution of 105, while Problem B has an optimal solution of 200 and a heuristic solution of 205. Both heuristic solutions differ from their respective optimal solutions by 5, but problem B actually has the better heuristic solution because the deviation is less from a percentage standpoint. Problem A deviates by 5%, while problem B only deviates by 2.5%.

Therefore the test value will be as shown in 4.1. The results of the one-sample t-test are shown in Figure 4.1 along with the boxplot in Figure 4.2.

Trial	Cells	Time Periods	Jobs	Optimal Solution	Feasible Solution	Bound	Best Possible Solution	Heuristic Solution
1	5	40	57		363	358	358	360
2	5	40	57	442			442	442
3	5	20	29	200			200	202
4	5	20	29	243			243	243
5	5	30	43	259			259	263
6	5	30	43	315			315	317
7	5	20	29	243			243	245
8	10	20	57	271			271	275
9	5	40	57		482	470	470	473
10	5	40	57			404	404	412
11	5	30	43	261			261	264
12	5	30	43			249	249	249
13	5	40	57	416			416	422
14	5	20	29		224	220	220	222
15	5	40	57	390			390	391
16	10	20	57		392	378	378	385
17	5	20	29	196			196	202
18	5	30	43	272			272	280
19	10	20	57		328	316	316	321
20	5	30	43	279			279	279
21	5	20	29		229	227	227	229
22	5	20	29		238	236	236	257
23	5	30	43		425	424	424	425
24	10	20	57			299	299	315
25	10	20	57			311	311	321
26	5	20	29	272			272	272
27	5	20	29	207			207	214
28	5	40	57	382			382	385
29	5	30	43			263	263	264
30	5	30	43			338	338	343

Table 4.1: Small Problem Solution Results

$$\frac{\textit{Heuristic Solution} - \textit{Best Possible Solution}}{\textit{Best Possible Solution}} \qquad (4.1)$$

```
One-Sample T-Test: Deviation from Best Possible Solution

Test of μ = 0 vs not = 0


Variable              N      Mean     StDev   SE Mean     T       P
Deviation from Best   30  0.015485  0.018658  0.003406  4.55   0.000

95% Confidence Interval
(0.008518, 0.022452)
```

Figure 4.1: One-Sample T-Test for Deviation from Best Possible Solution

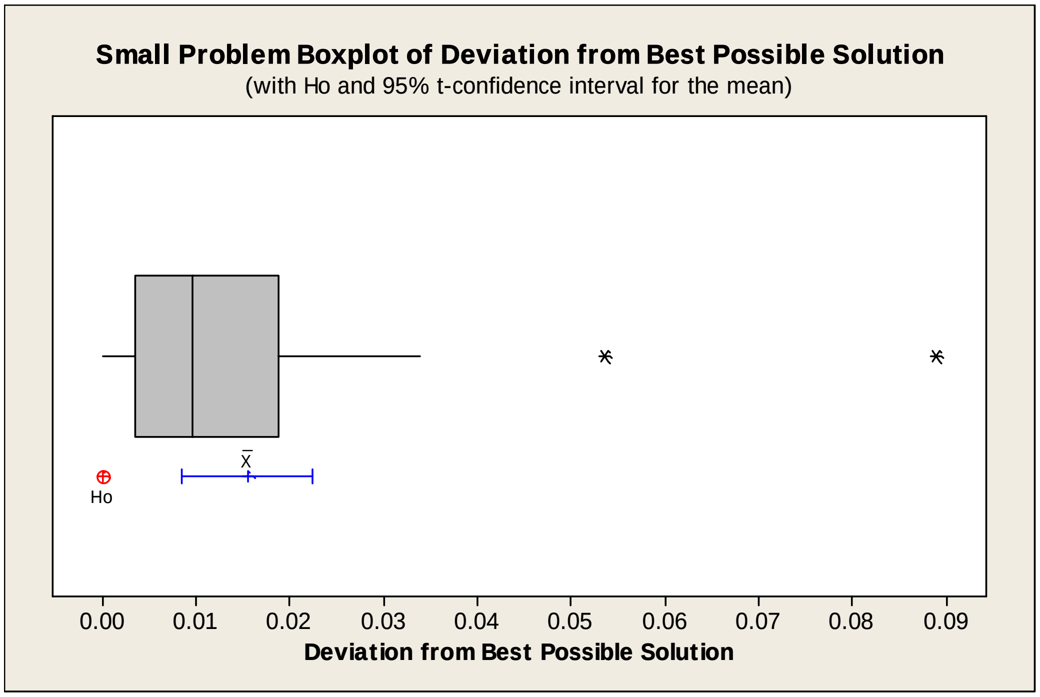


Figure 4.2: Small Problem Boxplot of Deviation from Best Possible Solution

For the second test, only the problems that had an optimal solution were analyzed. The heuristic solution is compared to the optimal solution. The test value for the 16 problems is as shown in 4.2. The results of the one-sample t-test are displayed in Figure 4.3.

$$\frac{\textit{Heuristic Solution} - \textit{Optimal Solution}}{\textit{Optimal Solution}} \tag{4.2}$$

One-Sample T-Test: Deviation from Optimal Solution

```
Test of μ = 0 vs not = 0


Variable                N      Mean     StDev   SE Mean     T       P
Deviation from Optimal  16  0.011559  0.011197  0.002799  4.13   0.001

95% Confidence Interval
(0.005592, 0.017525)
```

Figure 4.3: One-Sample T-Test for Deviation from Optimal Solution

The final test for this set looks only at problems that found a real solution. Problems with an optimal solution or a feasible solution are used within the test. The heuristic solution is compared to the optimal solution or feasible solution for each problem. The test value is as shown in 4.3. The results of the one-sample t-test are displayed in Figure 4.4.

$$\frac{\textit{Heuristic Solution} - \textit{Real Solution}}{\textit{Real Solution}} \tag{4.3}$$

One-Sample T-Test: Deviation from Real Solution (Optimal or Feasible)

```
Test of μ = 0 vs not = 0


Variable             N      Mean     StDev   SE Mean     T       P
Deviation from Real  24  0.007905  0.021020  0.004291  1.84   0.078

95% Confidence Interval
(-0.000971, 0.016781)
```

Figure 4.4: One-Sample T-Test for Deviation from Real Solution

Based upon the first test, there is a significant deviation between the heuristic solution and the best possible solution. However, as the 95% confidence interval shows, the deviation is not great. On average, the deviation is only 1.5%, with a 95% confidence upper limit of 2.2%. The boxplot in Figure 4.2 shows that the majority of the problems deviate by less than 2%, while a few problems with greater deviations are slightly skewing the results. As the problems without an optimal solution are faded out, the numbers improve. The second test shows that when an optimal solution was found, there is still a slight deviation from the optimal solution with the heuristic solution. Again, however, the difference is minimal, with an approximate mean of 1.1%. In the third test as problems with feasible solutions are added, the deviation decreases once again. In fact, more often than not, the heuristic solution is lower than the feasible solution. When the best real solution is compared to the heuristic solution, with 95% confidence, there is not a significant deviation between the heuristic solution and the real solution.

Though the heuristic solution may be slightly greater than the optimal solution, the solving time that it takes to achieve the heuristic solution is substantially less than the respective time for the optimal solution. Table 4.2 shows the computation time for each heuristic solution, as well as the computation time for the optimal solutions, where applicable.

Trial	Optimal Solution Solving Time (sec)	Heuristic Solution Solving Time (sec)
1		9
2	248	12
3	50	2
4	7	2
5	695	12
6	21	8
7	13	1
8	63	7
9		16
10		21
11	111	6
12		7
13	1591	11
14		2
15	110	49
16		14
17	94	2
18	59	7
19		12
20	1473	6
21		2
22		2
23		26
24		14
25		11
26	79	2
27	18	1
28	798	8
29		14
30		10

Table 4.2: Small Problem Solving Time Results

The following paired t-test in Figure 4.5 shows the significance of the difference in solving time for the heuristic solution in comparison with the solving time for the optimal solution. The 95% confidence interval shows that the true mean difference in solving time is anywhere between approximately one minute and 10 minutes. Keep in mind, that the results are based upon small problems. As the problems get larger, the difference in solving time will becomes very difficult to measure, because the time to solve optimally is so long.

Paired T-Test and Confidence Interval:

Optimal Solution Solving Time vs. Heuristic Solution Solving Time

```
Paired T for Optimal Solution Solving Time - Heuristic Solution Solving Time

                   N     Mean    StDev  SE Mean
Optimal Solution  16  339.375  522.173  130.543
Heuristic Soluti  16    8.500   11.460    2.865
Difference        16  330.875  521.602  130.401


95% Confidence Interval for mean difference: (52.933, 608.817)
T-Test of mean difference = 0 (vs not = 0): T-Value = 2.54  P-Value = 0.023
```

Figure 4.5: Paired T-Test for Optimal Solving Time vs. Heuristic Solving Time

4.2 Medium Problem Testing

Fifteen problems were analyzed in the medium size class. However, only two problems could even find a feasible solution, while none of the problems were able to achieve an optimal solution even after running overnight or for several hours. Therefore, from here on out, each heuristic solution will be compared against the respective bound for the problem, with the test value being as shown in 4.4.

$$\frac{\textit{Heuristic Solution} - \textit{Bound}}{\textit{Bound}} \qquad (4.4)$$

Obviously, the deviation from the bound will be greater than or equal to the deviation from the true optimal. Since the optimal solution is extremely difficult to solve for, the bound will provide a conservative baseline to gauge the efficiency of the heuristic. The results of the medium problems are shown in Table 4.3. The results of the one-sample t-test are displayed in Figure 4.6.

Trial	Cells	Time Periods	Jobs	Feasible Solution	Bound	Heuristic Solution	Heuristic Solving Time (sec)
1	5	50	71		502	508	62
2	5	50	71		503	516	51
3	15	20	85		309	312	24
4	15	20	85		307	325	84
5	15	20	85		258	263	32
6	15	20	85		307	308	36
7	10	30	85		426	426	50
8	10	30	85		560	561	53
9	5	50	71		459	469	75
10	10	30	85		473	473	38
11	15	20	85		298	308	45
12	5	50	71		567	573	33
13	10	30	85	444	442	442	37
14	10	30	85		396	405	51
15	5	50	71	500	498	502	27

Table 4.3: Medium Problem Solution Results

One-Sample T-Test: Deviation from Bound

```
Test of μ = 0 vs not = 0


Variable              N      Mean     StDev   SE Mean     T      P
Deviation from Bound  15  0.015150  0.015998  0.004131  3.67  0.003

95% Confidence Interval
(0.006291, 0.024009)
```

Figure 4.6: One-Sample T-Test for Deviation from Bound

Once again the t-test shows that when using 95% confidence, there is a significant deviation of the heuristic solution from the bound. However, the deviation is obviously very minimal, with the mean hovering around 1.5%. The heuristic solution's deviation from the true optimal solution is undoubtedly even lower. As Table 4.3 shows, the time to solve heuristically is less than two minutes for each trial. Combining the slight deviation with the minimal solving time, leads one to believe that the heuristic provides an efficient procedure to solve problems of this size. The boxplot in Figure 4.7 shows that one problem has a deviation of approximately 6%, but most of the remaining problems have a deviation of less than 3%.

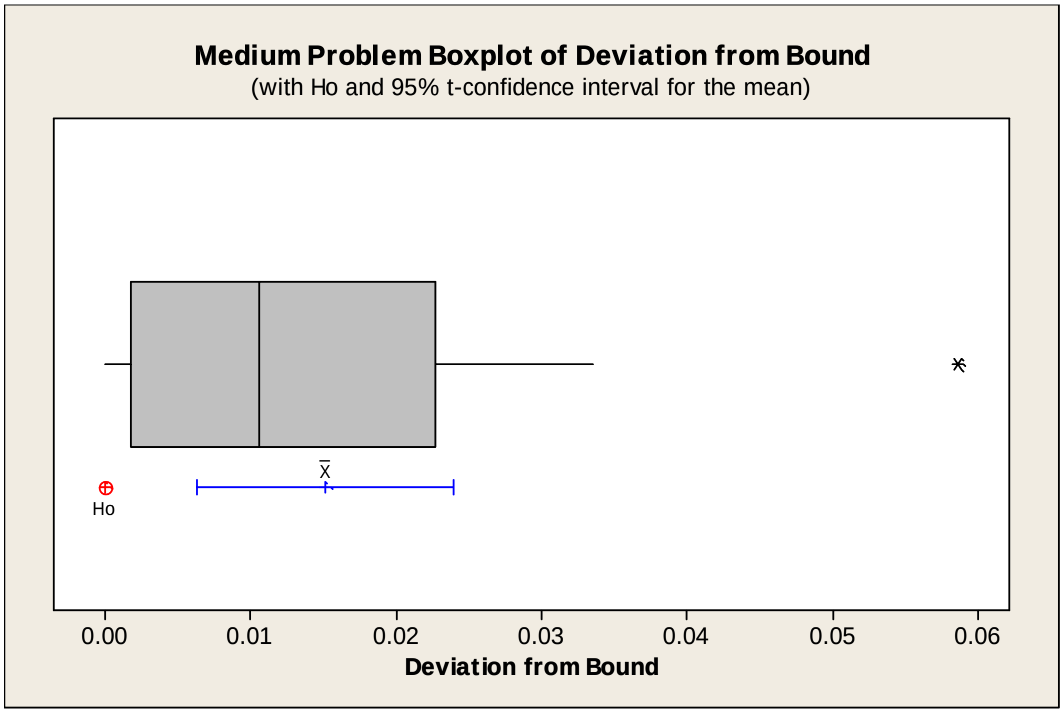


Figure 4.7: Medium Problem Boxplot of Deviation from Bound

4.3 Large Problem Testing

Large problems are past the solving breakpoint, making them impossible to even solve feasibly in any sort of reasonable timeframe. Thereby, once more the heuristic solutions will be compared against the bound for the respective problems. Table 4.4 displays the test results for the large problems. The results of the one-sample t-test are shown in Figure 4.8.

Trial	Cells	Time Periods	Jobs	Bound	Heuristic Solution	Heuristic Solving Time (min:sec)
1	10	50	142	711	714	4:50
2	10	50	142	651	663	1:40
3	10	50	142	702	706	2:01
4	10	40	114	553	558	2:00
5	10	40	114	565	569	1:53
6	10	40	114	613	614	2:34
7	15	30	128	605	615	10:47
8	15	30	128	510	510	5:21
9	10	40	114	524	538	2:31
10	10	40	114	516	527	1:02
11	10	50	142	716	725	2:08
12	10	40	114	647	674	4:17
13	15	30	128	510	514	4:59
14	10	50	142	679	700	4:07
15	15	30	128	528	528	7:29

Table 4.4: Large Problem Solution Results

One-Sample T-Test: Deviation from Bound

```
Test of μ = 0 vs not = 0


Variable             N      Mean     StDev   SE Mean      T      P
Deviation from Bound  15  0.013583  0.012295  0.003175   4.28  0.001

95% Confidence Interval
(0.006774, 0.020392)
```

Figure 4.8: One-Sample T-Test: Deviation from Bound

The t-test shows a significant deviation for the heuristic solution from the bound. Yet, the deviation is very slight, with a mean deviation of 1.4% and a 95% confidence upper limit mean of just over 2.0%. Taking into account the certainty that the heuristic solution deviation is less from the optimal solution, the heuristic provides a proficient method to arrive at a good feasible solution for large problems. The solving time using the heuristic is typically no more than 10 minutes. The boxplot in Figure 4.9 displays the spread of the deviation for all of the test problems.

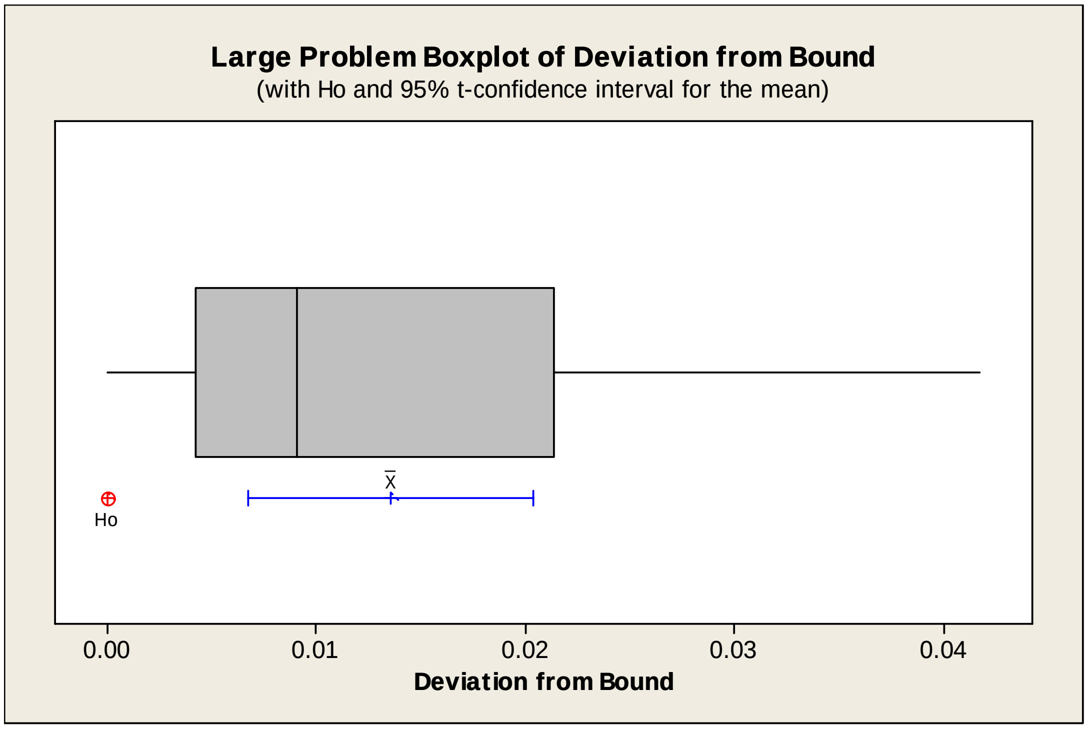


Figure 4.9: Large Problem Boxplot of Deviation from Bound

4.4 Extra Large Problem Testing

The extra large problems are the most difficult problem that were tested and will provide a good gauge as to how efficient the heuristic is for problems similar to real-life applications and true industry scenarios. Yet again, extra large problems are a far cry from being solved optimally, so the bound is used as the comparison standard. Table 4.5 displays the solution and timing results for the extra large problems. The results of the one-sample t-test are displayed in Figure 4.10.

Trial	Cells	Time Periods	Jobs	Bound	Heuristic Solution	Heuristic Solving Time (min:sec)
1	15	50	213	855	867	5:59
2	15	50	213	819	854	8:09
3	15	40	170	768	796	6:31
4	15	50	213	898	906	11:21
5	15	50	213	834	849	9:47
6	15	50	213	899	906	8:41
7	15	40	170	773	780	8:06
8	15	40	170	695	706	8:15
9	15	40	170	728	737	11:50
10	15	40	170	709	710	8:40

Table 4.5: Extra Large Problem Solution Results

One-Sample T-Test: Deviation from Bound

```
Test of μ = 0 vs not = 0


Variable              N      Mean     StDev   SE Mean     T      P
Deviation from Bound  10  0.016659  0.013034  0.004122  4.04  0.003

95% CI
(0.007335, 0.025983)
```

Figure 4.10: One-Sample T-Test for Deviation from Bound

Though the problems are getting much larger, the deviation from the bound does not seem to be rising at the same rate. Again, there is a statistical significant deviation, but it very minimal, averaging only 1.7% for the largest size problem set. The 95% confidence upper limit has the mean deviation at only 2.6%. The extra large problems are consistently able to be solved with the aid of the heuristic in less than 12 minutes. The boxplot in Figure 4.11 illustrates the reliability of the heuristic in providing a good feasible solution.

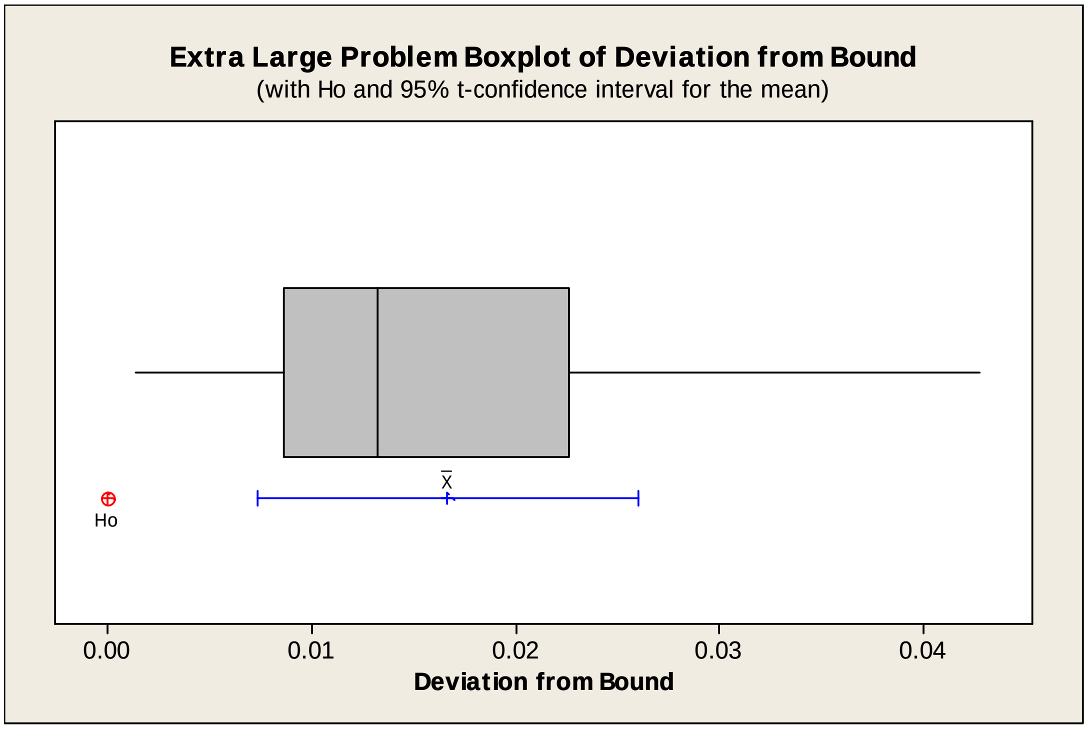


Figure 4.11: Extra Large Problem Boxplot of Deviation from Bound

4.5 Overall Problem Testing

Since the mean deviation does not change much between the size of problems, all of the data can be combined together to provide a more powerful test. The following t-test uses all 70 data points from each of the different problem size classes. For the small problems, in which an optimal solution was found, the optimal solution is used as the comparison standard. For all other problems, the bound is using as the measuring point.

Therefore, the formula for the test value will be the same as 4.4. The results of the one-sample t-test are displayed in Figure 4.12.

One-Sample T-Test: Deviation from Best Possible

```
Test of μ = 0 vs not = 0


Variable              N      Mean     StDev   SE Mean     T      P
Deviation from Best  70  0.015173  0.015874  0.001897  8.00  0.000

95% Confidence Interval
(0.011388, 0.018958)
```

Figure 4.12: One Sample T-Test for Deviation from Best Possible Solution

As expected, the combined results do not differ greatly from the problem size separated results. The mean deviation for the heuristic solution from the best possible solution is approximately 1.5% and has a 95% confidence interval between 1.1% and 1.9%.

5 Conclusions and Future Work

The ultimate heuristic created in this thesis offers a framework for substantial future work and the possibility of making the heuristic dynamic and even more efficient. The flexibility rating is an interesting concept that could be examined substantially more in depth. Questions arise such as, "What is the best way to determine the flexibility of a job?" and "At what breakpoint should a job be considered 'flexible'?". Postponing the scheduling of more or less "flexible" jobs could dramatically impact the efficiency and/or feasibility of the heuristic. The cell load is also another technique that could be pondered further. Is it best to always schedule the greatest load cell next or would another order provide better, more efficient results? Furthermore, there may be a better way to calculate the cell load. Currently, the cell load is equivalent to the time units to be scheduled in the cell. Perhaps the cell load should also take into account the specific jobs assigned to each cell and at what times, based on early start dates and due dates, each job could feasibly be scheduled. The cell with the greatest load is not always the most difficult to schedule.

This thesis has investigated heuristic methods to aid in the solution process of cellular job scheduling problems. Several heuristic procedures were proposed to help provide a near optimal solution in acceptable computation time, concluding with an ultimate heuristic that can be applied to several different size problems. The ultimate heuristic applies several techniques to the mathematical model to improve the solving process, while not deviating far from the true optimal solution. Techniques such as relaxing integrality on variables and relaxing constraints at timely positions within the solving process were vital to the overall efficiency of the heuristic. Incorporating a new input parameter known as flexibility and also considering current cell load during the iterative scheduling procedure was critical in maintaining feasibility throughout the solving process. Undoubtedly, as the results show, the ultimate heuristic provides a quality solution to a wide variety of difficult job scheduling problems within a tractable amount of solving time.

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APPENDIX A: Optimization Programming Language Model

<u>Notation:</u>

```
int nbJob = ...;
range Job 1..nbJob;

int nbCell = ...;
range Cell 1..nbCell;

int nbTime = ...;
range Time 1..nbTime;
```

<u>Input Parameters:</u>

```
float+ feasible[Job,Cell]=...;
float+ completionTime[Job]=...;
float+ earlyStart[Job]=...;
float+ dueDate[Job]=...;
float+ cost[Job,Cell]=...;
```

<u>Decision Variables:</u>

```
var int+ assignment[Job,Cell] in 0..1;
var int+ schedule[Job,Cell,Time] in 0..1;
var float+ start[Job] in 1..nbTime;
var float+ finish[Job] in 1..nbTime;
```

<u>Objective Function:</u>

```
minimize
  sum(j in Job, c in Cell)
    (assignment[j,c] * cost[j,c] * completionTime[j])
```

<u>Constraints:</u>

```
subject to {

//one cell only
  forall (j in Job)
    sum (c in Cell)
      assignment[j,c] = 1;
//cell feasibility
  forall (j in Job, c in Cell)
    assignment[j,c]<=feasible[j,c];

//schedule only if assigned
  forall (j in Job, c in Cell)
    sum (t in Time)
      schedule[j,c,t] <= nbTime * assignment[j,c];

//one job at a time
  forall (c in Cell, t in Time)
    sum (j in Job)
      schedule[j,c,t] <= 1;

//schedule time = completion time
  forall (j in Job)
    sum (c in Cell, t in Time)
      schedule[j,c,t]=completionTime[j];

//starting time
  forall (j in Job, c in Cell, t in Time)
      t*schedule[j,c,t]+nbTime*(1-schedule[j,c,t]) >= start[j];

//finishing time
  forall (j in Job, c in Cell, t in Time)
    finish[j]>=t*schedule[j,c,t];

//early start
  forall (j in Job)
    start[j]>=earlyStart[j];

//due date
  forall (j in Job, t in Time)
    sum (c in Cell)
      t*schedule[j,c,t]<=dueDate[j];

//sequential
  forall (j in Job)
    finish[j]-start[j]=completionTime[j]-1;

};
```

APPENDIX B: Glossary of OPL Terms

Notation:

Job – job notation; represented by (j)

Cell – cell notation; represented by (c)

Time – time notation; represented by (t)

int –integer value

nbJob – total number of jobs

nbCell – total number of cells

nbTime – total number of time periods

range Job 1..nbJob – creates array of jobs from 1 to the total number of jobs

range Cell 1..nbCell – creates array of cells from 1 to the total number of cells

range Time 1..nbTime – creates array of time periods from 1 to the total number of time periods

Input Parameters:

float+ **–** continuous parameter value

feasible[Job,Cell] – normalized feasibility parameter represented by job and cell

completitionTime[Job] – completion time parameter represented by job

earlyStart[Job] – early start date parameter represented by job

dueDate[Job] – due date parameter represented by job

cost[Job,Cell] – cost parameter represented by job and cell

<u>Decision Variables:</u>

var int+ **–** integer variable value

var float+ **–** float variable value

assignment[Job,Cell] in 0..1 – forces the assignment variable to be in the range of 0-1

schedule[Job,Cell,Time] in 0..1 – forces the schedule variable to be in the range of 0-1

start [Job] in 1..nbTime – forces the start variable to be in the range of 1-max time

finish[Job] in 1..nbTime – forces the finish variable to be in the range of 1-max time

<u>Objective Function:</u>

minimize – calls for minimization of the objective function

sum (j in Job, c in Cell) – sum operation over all jobs and all cells

<u>Constraints:</u>

subject to { } – calls for constraints of the problem

forall (j in Job) – perform operation for all jobs

forall (c in Cell) – perform operation for all cells

forall (t in Time) – perform operation for all times

sum (j in Job) – sum operation over all jobs

sum (c in Cell) – sum operation over all cells

sum (t in Time) – sum operation over all times