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EVOLUTION OF BINARY SUPERMASSIVE BLACK HOLES VIA CHAIN REGULARIZATION

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Abstract: A chain regularization method is combined with special purpose computer hardware to study the evolution of massive black hole binaries at the centers of galaxies. Preliminary results with up to $N = 0.26 \times 10^6$ particles are presented. The decay rate of the binary is shown to decrease with increasing N , as expected on the basis of theoretical arguments. The eccentricity of the binary remains small.

Coalescence of binary supermassive black holes is potentially the strongest source of gravitational waves in the universe [1]. The coalescence rate is limited by the efficiency with which massive binaries can interact with stars and gas in a galaxy and reach the relativistic regime at separations of $\sim 10^{-3}$ pc. Exchange of energy between a binary black hole and stars should also leave observable traces in the stellar distribution, perhaps allowing us to infer something about the merger history of galaxies from their nuclear structure [2]. Henry Kandrup worked on this problem shortly before his death. In “Supermassive Black Hole Binaries as Galactic Blenders” [3], Kandrup *et al.* investigated the effects of a massive binary on the stellar orbits near the center of spherical and nearly spherical galaxies. They showed that the periodically-varying potential due to the binary, coupled with the fixed potential from the galaxy, was effective at inducing chaos in the stellar orbits, leading to diffusion in both energy and configuration space and to ejection of stars from the nucleus. This study was a complement to earlier studies based on scattering experiments [4,5] in which the potential of the galaxy was ignored.

Another approach to the binary black hole problem is via direct N -body techniques [6-8]. This approach is computationally challenging because of the need to handle close interactions between the star- and black hole particles with high precision. In addition, large particle numbers are required to avoid the effects of spurious relaxation [9,10]. Here, we present preliminary results of N -body integrations of the binary black hole problem, in which close interactions between the black holes and stars are handled via the Mikkola-Aarseth chain-regularization algorithm [11,12]. Recently Aarseth [13] described an application

of a time transformed leapfrog scheme [14] to this problem. We prefer the chain algorithm since it has proved itself in numerous applications, including one very similar to the current problem [15]. We incorporate the chain algorithm into a general-purpose N -body code by including the effects of nearby stars as perturbers to the chain. We present some preliminary results of binary black hole evolution computed via this algorithm on a special-purpose GRAPE-6 computer with particle numbers up to 0.26×10^6 .

Our basic N -body algorithm is an adaptation of the NBODY1 code of Aarseth [16] to the GRAPE-6 special purpose hardware. The code uses a fourth-order Hermite integration scheme with individual, adaptive, block time steps [17]. For the majority of the particles, the forces and force derivatives were calculated via a direct-summation scheme using the GRAPE-6.

Close encounters between the massive particles (“black holes”), or between black holes and stars, create prohibitively small time steps in such a scheme. To avoid this situation, we regularized the critical interactions as follows. Let \mathbf{r}_i , $i = 1, \dots, N$ be the position vectors of the particles. We first identify the subset of n particles to be included in the chain; the precise criterion for inclusion is presented below, but in the late stages of evolution, the chain always included the two black holes as its lowest members. We then search for the particle which is closest to either end of the chain and add it; this operation is repeated recursively until all n particles are included. Define the separation vectors $\mathbf{R}_i = \mathbf{r}_{i+1} - \mathbf{r}_i$ where \mathbf{r}_{i+1} and \mathbf{r}_i are the coordinates of the two particles making up the i th link of the chain. The canonical momenta \mathbf{W}_i corresponding to the coordinates \mathbf{R}_i are given in terms of the old momenta via the generating function

$$S = \sum_{i=1}^{n-1} \mathbf{W}_i \cdot (\mathbf{r}_{i+1} - \mathbf{r}_i). \quad (1)$$

Next, we apply KS regularization [18] to the chain vectors, regularizing only the interactions between neighboring particles in the chain. Let \mathbf{Q}_i and \mathbf{P}_i be the KS transformed \mathbf{R}_i and \mathbf{W}_i coordinates. After applying the time transformation $\delta t = g \delta s$, $g = 1/L$, where L is the Lagrangian of the system ($L = T - U$, where T is the kinetic and U is the potential energy of the system). We obtain the regularized Hamiltonian $\Gamma = g(H(\mathbf{Q}_i, \mathbf{P}_i) - E_0)$, where E_0 is the total energy of the system. The equations of motion are then

$$\mathbf{P}'_i = -\frac{\partial \Gamma}{\partial \mathbf{Q}_i}, \quad \mathbf{Q}'_i = \frac{\partial \Gamma}{\partial \mathbf{P}_i} \quad (2)$$

where primes denote differentiation with respect to the time coordinate s . Because of the use of regularized coordinates, these equations do not suffer from singularities, as long as care is taken in the construction of the chain.

Since it is impractical to include all N particles in the chain, we must consider the effects of external forces on the chain members. Let \mathbf{F}_j be the perturbing acceleration acting on the j th body of mass m_j . The perturbed

system can be written in Hamiltonian form by simply adding the perturbing potential:

$$\delta U = \sum_{j=1}^n m_j \mathbf{r}_j \cdot \mathbf{F}_j(t). \quad (3)$$

Only one chain was defined at any given time. At the start of the N -body integrations, there was no regularization, and all particles were advanced using the variable-time-step Hermite scheme. The chain was “turned on” at the time when one of the particles (including possibly one of the black holes) achieved a time step shorter than t_{chmin} and reached a distance from one of the black holes smaller than r_{chmin} . Each star inside r_{chmin} radius was then added to the chain, and the two black holes were always included. The values of t_{chmin} and r_{chmin} were determined by carrying out test runs; we adopted $t_{chmin} \approx 10^{-5} - 10^{-6}$ and $r_{chmin} \approx 10^{-4} - 10^{-3}$ in standard N -body units.

The chain’s center of mass was a pseudoparticle as seen by the N -body code and was advanced by the Hermite scheme in the same way as an ordinary particle. However, when integrating the trajectories of stars near to the chain, it is essential to resolve the inner structure of the chain. Thus for stars inside a critical r_{crit1} radius around the chain, the forces from the individual chain members were taken into account. The value of r_{crit1} was set by the size of the chain to be $r_{crit1} = \lambda R_{ch}$ with R_{ch} the spatial size of the chain and $\lambda = 100$.

In addition, the equations of motion of the chain particles must include the forces exerted by a set of external perturber stars. Whether or not a given star was listed as a perturber was determined by a tidal criterion: $r < R_{crit2} = (m/m_{chain})^{1/3} \gamma_{min}^{-1/3} R_{ch}$ where m_{chain} represents the mass of the chain, m is the mass of the star, and γ_{min} was chosen to be 10^{-6} ; thus $r_{crit2} \approx 10^2 (m/m_{chain})^{1/3} R_{ch}$.

The membership of the chain changed under the evolution of the system. Stars were captured into the chain if their orbits approached the binary closer than R_{ch} . Stars were emitted from the chain if they got further from both of the black holes than $1.5R_{ch}$. The difference between the emission and absorption distances was chosen to avoid a too-frequent variation of the chain membership. When the last particle left the chain, the chain was eliminated and the integration turned back to the Hermite scheme, until a new chain was created.

In our numerical experiments, the typical number of chain members was 5 – 10. The number of perturber stars was typically 500 – 1000, and the number of stars inside r_{crit1} was 2000 – 5000. Using a minimum tolerance of 10^{-12} for the chain’s Bulirsch-Stoer integrator allowed us to reach typical relative accuracies of 10^{-9} in the chain integration. The relative accuracy in the conservation of the total energy of the system was determined by the Hermite scheme. For all of our numerical simulation the relative error in the total energy was less than 10^{-4} during the course of the integration.

For this set of experiments, we adopted Dehnen’s [19] density law,

$$\rho(r) = \frac{(3 - \gamma)M}{4\pi} \frac{a}{r^\gamma(r + a)^{4-\gamma}}, \quad (4)$$

with $\gamma = 0.5$, to describe the initial stellar distribution. The initial positions and velocities of the N stars were generated from the steady state phase space density $f(E)$ that reproduces the density law (4). Henceforth we adopt units such that the gravitational constant G , the total stellar mass M , and the Dehnen scale length a are equal to one. To this model we added two black hole particles each of mass 0.005. The black holes were placed symmetrically about the center of the galaxy, offset at a distance 0.1 from the center. The tangential velocities were set to ± 0.16 yielding nearly circular initial orbits for the two black holes about the center of the galaxy.

We integrated the above model with three different particle numbers: $N = 16384$, 65536, and 262144; the latter is close to the maximum number of particles that can be handled in the GRAPE-6 memory. The integrations were carried out for 170 time units. Elapsed times were (2.5, 26, 105) hours for the three runs.

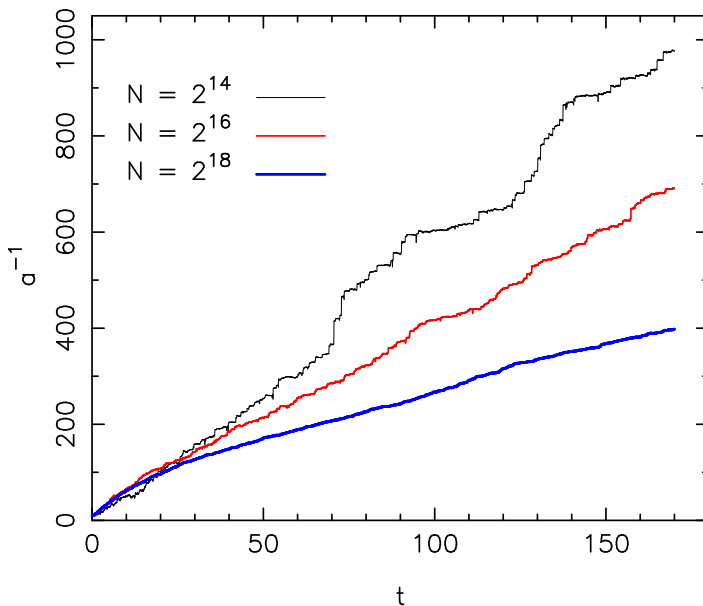


Figure 1. Evolution of the binary semi-major axis.

The orbits of the two black holes initially decay, and at a time of roughly 30 they form a bound pair. After this, the semimajor axis a of the binary shrinks as the two black holes interact with stars and eject them from the nucleus via the gravitational slingshot. The instantaneous decay rate is given approximately by

$$\frac{d}{dt} \left(\frac{1}{a} \right) = \frac{G\rho}{\sigma} H \quad (5)$$

where ρ and σ are the density and velocity dispersion of the stars, and H is a dimensionless constant of order 16 [4,5]. Because the stellar density changes with time as the binary ejects stars, the binary’s decay rate can in general be a complicated function of time. Two limiting cases are of interest [9]. When the particle number N is small, gravitational encounters are able to scatter stars into the “loss cone” around the binary at a higher rate than they are ejected. The density near the binary remains approximately constant and the decay follows $a^{-1} \sim t$. This is the “full loss cone” regime. When N is large, encounters between stars are weak, and the binary’s loss cone remains nearly empty. Decay of the binary is limited by the rate at which stars diffuse into the loss cone; since the diffusion time scales approximately as N , the binary decay follows $a^{-1} \sim t/N$. This is the “diffusion” regime. Real galaxies are expected to be in the diffusion regime [9].

Figure 1 shows the time evolution of the semi-major axis of the binary in each of the three integrations. The binary’s energy evolves as an approximately linear function of the time but with a prefactor that depends on N ; the approximate dependence is

$$\frac{1}{a} \approx \frac{160t}{N^{1/3}}. \quad (6)$$

This is intermediate between the $a^{-1} \propto t$ dependence of the full loss cone limit, and the $a^{-1} \propto t/N$ dependence in the diffusion limit. We conclude that, for the particle numbers considered here, replenishment of the binary’s loss cone is taking place but at a lower rate than the rate at which the loss cone is being emptied. Apparently, particle numbers in excess of $\sim 10^6$ are required if N -body integrations are to be completely in the diffusive regime characteristic of real galaxies. Such large particle numbers can in principle be handled with direct-summation codes like ours if coupled with parallel hardware [20].

Figure 2 shows the eccentricity evolution of the binary. The value of e at the time when the hard binary first forms, $t \approx 30$, is substantially different in the different integrations due presumably to finite- N effects during the initial inspiral of the two black holes. Thereafter the eccentricity fluctuates in the case of the two small- N integrations, but decreases with time in the integration with the largest N . We compared the eccentricity evolution with the predictions of scattering theory. The rate of change of e is commonly written

$$\frac{de}{dt} = K \frac{d}{dt} \ln a^{-1} \quad (7)$$

where $K = K(e, a)$ [4,5]. The functional form of $K(e, a)$ is not well known; we adopted the expression given in [5] for a hard binary. We then wrote equation (7) as

$$e_i = e_{i-1} + K(e_{i-1}, a_{i-1}) \ln \left(\frac{a_{i-1}}{a_i} \right) \quad (8)$$

where the subscript denotes the time step. Combining equation (8) with the N -body results for $a(t)$, Figure 1, we could then predict the expected evolution

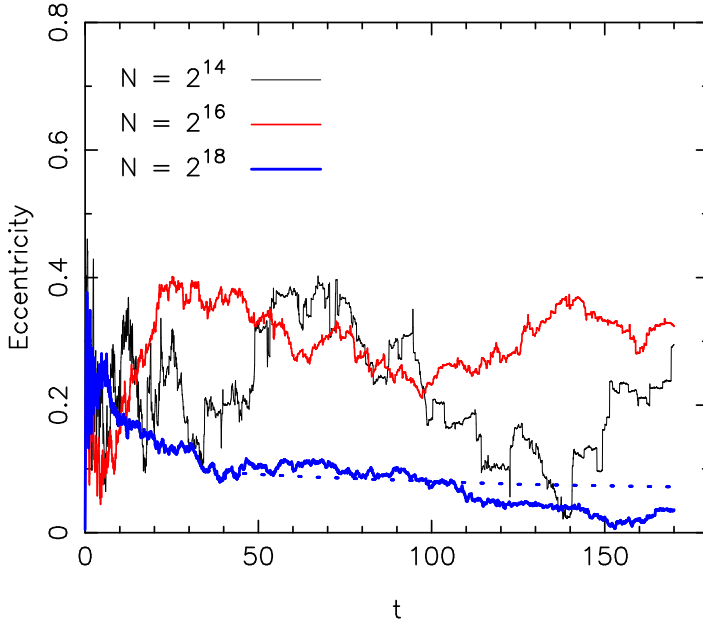


Figure 2. Evolution of the binary orbital eccentricity.

in e . The result for the largest- N integration is shown as the dashed line in Figure 2. There is reasonable agreement, but the eccentricity evolution even in the largest- N integration still exhibits substantial fluctuations. In this regard too, we are not yet in a regime where the evolution is similar to what would be expected in real galaxies.

A recent N -body study [13] found a much greater degree of eccentricity evolution, although the initial orbit of the binary was highly non-circular.

As the binary decays, it ejects stars from the nucleus and lowers the nuclear density. Figure 3 shows initial and final density profiles for the three integrations. The net effect of a black hole binary on the stellar distribution is commonly measured in terms of the “mass deficit,” defined as the mass in stars that was removed by the binary [2]. We find mass deficits of (1.94, 1.56, 1.17) in units of the combined black hole mass in the integrations with $N = (0.016, 0.065, 0.26) \times 10^6$. These values are of the same order as the mass deficits inferred in giant elliptical galaxies [2,21,22].

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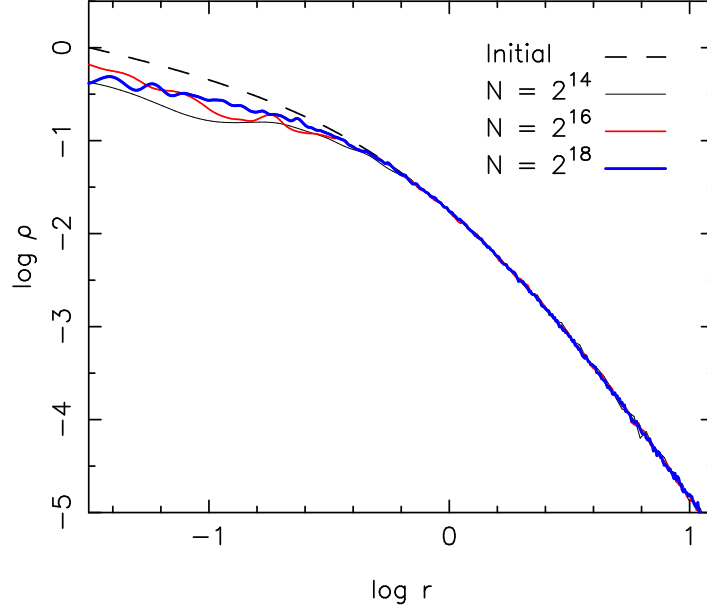


Figure 3. Initial (dashed line) and final density profiles.

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