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ASSESSMENT OF ACCEPTANCE SAMPLING PLANS USING POSTERIOR DISTRIBUTION FOR A DEPENDENT PROCESS

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ABSTRACT

In this study, performance of single acceptance sampling plans by attribute is investigated by using the distribution of fraction nonconformance (i.e., lot quality distribution (LQD)) for a dependent production process. It is the aim of this study to demonstrate that, in order to emphasize consumer risk (i.e., the risk of accepting a bad lot), it is better to evaluate a sampling plan based upon its performance as assessed by the posterior distribution of fractions nonconforming in accepted lots. Similarly, it is the desired posterior distribution that sets the basis for designing a sampling plan. The prior distribution used in this study is derived from a Markovian model of dependence.

KEY WORDS: Acceptance sampling, dependent production processes, lot quality distribution (LQD), posterior distribution, mean squared nonconformance.

INTRODUCTION

In production processes where the individual items are produced and formed into lots sequentially, the dependence in quality of successive items significantly affects the variance measures. The resultant dependence bias often distorts the performance of the conventional acceptance sampling plans. In order to prevent the failure of such plans, there is a need to incorporate the serial dependence in the process as an integral part of the sampling plan. This problem is substantiated in a long list of studies (Mergen [9], Holmes and Mergen [6], Chen and Chou [1], Tang and Cheong [11]).

To address the outgoing quality in a dependent process, our study starts with a custom tailored dependence distribution as the prior. This distribution describes a stochastic process that is representative of many real applications. Using such a prior we first develop a posterior distribution and then a convenient indicator to assess the outgoing quality in the accepted lots under a given acceptance sampling plan. Integrating the quality distribution into an acceptance decision, our measure economically summarizes the information about outgoing quality levels. This model is applicable to situations where quality is measured by dichotomous quality conformance. The posterior distribution is derived using a two-state Markovian process. By recognizing the serial nature of the production process, the Markovian model directly incorporates sequential dependence as an integral part of the acceptance sampling plan.

In the following, first we summarize the background studies and the development of our prior distribution. Then, we demonstrate the computation of the posterior distribution and outgoing quality indicator. In the subsequent section we provide three examples. One example is a synthetic application, the second is based on data from practice, and third attempts to demonstrate the level of inaccuracy from erroneous independence assumption. Finally, we close the article with a conclusion.

BACKGROUND

The focus of this line of research has been capturing the dependence with a proper distribution or an adjustment to a selected conventional distribution. For example, recognizing the sequentiality problem, Mergen [9] and Holmes and Mergen [6] discuss a lot quality distribution for an item to item dependence in a production process. Specifically, they introduce a new measure for outgoing quality in accepted lots using a lot quality distribution. Chen and Chou [1] propose a linear cost model for continuous sampling plans under a dependent production process. Similarly, Tang and Cheong [11] develop a control scheme for detecting changes in fraction nonconforming in processes with correlation. In additional work, the significance of dependence in processes has been demonstrated in Deligonul and Mergen [4]. Also, Deligonul and Mergen [4] examine the impact of correlated data on the control limits for p-chart.

Along this stream of research a group of studies has considered a Bayesian approach to the dependence problem. Those studies, however, start from generic priors (Wetherill [12], Hald [5], Chiu [2], Jaraidei et.al. [8], Jaraidei and Asoudegi [7]). Although this venue is a welcome addition to the acceptance sampling literature, it does not give the selection of prior the attention it devotes to sampling design and its outcomes (e.g,Chun and Rinks, [3]. Most priors in this line of research are selected by criteria based on mathematical convenience. Typically, the question of suitability and convergence characteristic of the priors receive scant attention, if any at all.

MODEL

Prior distribution of fraction nonconformance:

The first step in this study is the development of a prior distribution for fraction nonconformance in dependent production processes. To this end, the following first order Markovian model, which is based on our prior work, is used to depict the dependence behavior of the production process (see Mergen [9], Mergen and Holmes [10], and Deligonul and Mergen [4]).

Let's suppose that large lots of size N are formed from a dependent production process. Further suppose that (d, g) and (d, b) are two states with d number of nonconforming items given that the last item produced (and lotted) was conforming and nonconforming, respectively. Let P(d: g, N) and P(d: b, N) denote the probabilities of d number of nonconforming items in a lot of size N where the last item was conforming and nonconforming, respectively. At any time p' indicates the fraction nonconforming (i.e., p'=d/N). Let x and y be the conditional process probabilities of having a conforming item given that the last item was conforming and nonconforming, respectively. Then the difference equations associated with the Markov Process depicting the number of nonconforming items are given as follows:

$$P(d:N) = P(d:b,N) + P(d:g,N) \tag{1}$$

where

$$P(d:b,N) = (1-x)P(d-1:g,N-1) + (1-y)P(d-1:b,N-1) \tag{2}$$

$$P(d:g,N) = xP(d:g,N-1) + yP(d:b,N-1) \tag{3}$$

for d = 1, 2, ….., N and N > d

and

$$P(0:b,N) = 0$$

$$P(0:g,N) = xP(0:g,N-1) \qquad \text{for } N>1$$

The prior distribution giving the probabilities associated with the lot quality is obtained from the solution of the above recursive equations, assuming that the Markov process starts, say, from the steady state probability of a conforming item, namely,

$$P(0:g,1) = \frac{y}{1 - x + y} \tag{4}$$

This choice of the above initial condition is a reasonable start up for the Markovian process given that the process is convergent. In literature there are alternative suggestions, such as antitetehic initiation procedures, for effective start up options. However, for our purposes the initial point in (4) is sufficient as it is rapid in convergence and reliable in use[1].

The solution of the difference equation yields the following distribution (see Mergen [9] and Mergen and Holmes [10] for details):

$$P(d)_N = \sum_{i=1}^{\mathrm{Min}(N-d,d-1)} \left[\binom{N-d-1}{N-d-i} x^{N-d-i} (1-x)^{i-1} y^{i} (1-y)^{d-i-1} \blacksquare C \right] + x^{N-2d-1} (1-x)^{d-1} y^{d} \blacksquare D$$

for d = 1,2, .., N-1 (5)

where

$$\binom{R}{Q} = 0 \qquad \text{if } R \le 0 \text{ and } Q < 0$$

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[1] We thank a referee for bringing this issue to our attention.

$$C = \frac{\binom{d-1}{d-i-1}(1-x)^2 + 2\binom{d-1}{d-i}(1-x)(1-y) + \binom{d-1}{d-i-1}(1-y)^2}{1-x+y} \qquad (6)$$

$$D = \frac{2\binom{N-d-1}{N-2d}x(1-x) + \binom{N-d-1}{N-2d}(d-1)\,x(1-y) + \binom{N-d-1}{N-2d-1}y(1-x)}{1-x+y} \qquad (7)$$

$$P(d) = \frac{yx^{N-1}}{1-x+y} \qquad \text{for } d = 0 \qquad (8)$$

$$P(d) = \frac{(1-x)(1-y)^{N-1}}{1-x+y} \qquad \text{for } d = N \qquad (9)$$

The above distribution depicts the probability of the number of nonconforming items. For the fractions nonconforming, associated probabilities are exactly the same if we switch from d with the range 0,1, 2,…, N to p' with 0/N, 1/N, 2/N, …, N/N. Therefore P(p': N) is obtained directly from the above distribution.

NOTATION TABLE

$P(d)_N$	=	probability distribution of nonconforming items in lots of size N
d	=	number of nonconforming items
N	=	lot size
x	=	probability that the next item will be conforming given that the last item was conforming
y	=	probability that the next item will be conforming given that the last item was nonconforming.
n	=	sample size
c	=	acceptance number

p'_d = fraction nonconforming (=lot quality = d/N)

<u>Posterior distribution conditioned upon acceptance:</u>

After obtaining the unconditional distribution of fractions nonconforming, we obtain the posterior distribution of the same random variable conditioned upon acceptance, that is P(p': A), where A denotes the event that the lot was accepted. Furthermore, we suppose the single acceptance sampling plan S(n, c) is adopted. Here n, c denotes sample size and acceptance number, respectively. If no more than c nonconforming items are found in a sample size n randomly drawn from a lot size N, the lot will be accepted as good. Given the sampling plan S(n, c), probability of acceptance, P(A: p', S), can be calculated by using binomial distribution.

$$P(A : p'_d, S) = P_a = \begin{cases} 0 & \text{for } d=N \\ \sum_{i=0}^{c} \binom{n}{i} p'^{\,i}_d (1-p'_d)^{n-i} & \text{for } p'_d = (d-1)/N > 0, \text{ for } d=2, \ldots, N \\ 1 & \text{for } p'_d = 0 \end{cases} \tag{10}$$

Then the posterior distribution P(p': A, S) is obtained as,

$$P(p'_d : A, S) = P(p'_d) = \frac{P(A : p'_d, S)\, P(p'_d : N)}{\sum_{j=1}^{N} P(A : p'_j, S)\, P(p'_j : N)} \quad \text{for } p'_d = (d-1)/N > 0 \text{ for } d = 2, \ldots, N \tag{11}$$

The calculation implicitly assumes that during an inspection process fractions nonconforming are not affected. Although this assumption does not hold for all cases, it is reasonable for large lot sizes.

Computing the quality indicator MSNC:

The average outgoing quality limit (AOQL), which is the maximum value of average outgoing quality (AOQ) values as p' ranges from 0 to 1, is a possible single quantity of interest for assessing the outgoing quality (where AOQ can be computed as $AOQ_{p'} = p' P_a$ where P_a is the probability of accepting a lot with p' fraction nonconformance using a specific acceptance sampling plan). However, AOQL is a conservative measure in the sense that it inflates the maximum outgoing fraction nonconformance since it does not take into account the lot quality distribution. This in turn causes sample sizes getting bigger to maintain a desired outgoing quality. In fact, Holmes and Mergen [6][2], Jaraidei et al. [8], Jaraidei and Asoudegi [7], and others enrich the idea by treating p' as a random variable and calculate the expected value of the average outgoing quality values (EAOQ).

Although this seems to be an improvement over the AOQL, this indicator is not sensitive to the dispersion of the distribution. This is a significant defect against many fractions nonconformance distributions (i.e., lot quality distribution (LQD's)) observed from dependent production processes. Mergen [9] surprisingly reports that, in practice,

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[2] Mergen [9] and Holmes and Mergen [6] define EAOQ as,

$$EAOQ = \sum_{p'=0}^{1} p' P_a \, P(p' : N) \qquad \text{if the } P(p' : N) \text{ is discrete}$$

$$EAOQ = \int_0^1 p' P_a \, P(p' : N) \, dp' \qquad \text{if the } P(p' : N) \text{ is continuous.}$$

some LQD's exhibit higher right tails, which is significant for cases with strong dependence. In fact, this is one of the prime reasons differentiating the observed LQD's from Poisson or binomial types. This characteristic injects a downward bias in the expected value of the AOQ computations.

Here we propose an indicator combining the square of the mean with the variance of the posterior distribution. This quantity, termed mean squared nonconformance (MSNC), addresses both the expected value component and variability. In MSNC the expected value of p' is squared to bring it to the same scale with the variance. With its encompassing nature this measure captures a wider view of the quality characteristics by imitating the mean square quantity:

$$MSNC = E^2(p' : A) + Var(p' : A) \qquad (12)$$

MSNC, as computed from the posterior distribution, incorporates the dependence characteristics of the process and the dispersion of nonconformance fraction as an integral part of the sampling plan.

EXAMPLES

In order to demonstrate the approach proposed in this study, first a dependent process with x=0.95 and y=0.30 is utilized. Although this example is a synthetic situation, it describes the mechanics of our approach well. To demonstrate the computational process, the lot quality distributions, both prior and posterior, have been calculated together with MSNC as the indicator for outgoing quality under various inspection plans. The results are exhibited in the Table 1.

(Approximate location for Table 1)

In Table 1, the difference between prior and posterior distribution indicates the improvement in the outgoing lot quality through the use of a given sampling plan. As in this example, under dependence, EAOQ and MSNC converge in performance for lower values of variance. As the variance gets bigger, especially inflated by dependence, then MSNC more accurately represents the average outgoing quality. This happens because at higher variance levels, indicators that are based on expected value of the outgoing quality, such as EAOQ, will lag behind other indicators that incorporate variance.

The second example is a real case from industry. The data gathered from the manufacturing of a subassembly of an aircraft engine. Further details about this data can be found in Mergen and Holmes [10]. In Table 2 we exhibit the pertinent lot quality distribution obtained from a 100% inspection of 1487 lots of size 8 of the subassembly. MSNC values are calculated from the posterior distribution given various different sampling plans. In order to better show the utility of the proposed method we provide an application of the method on the same data, this time under the independence assumption. The independence is ensured by assigning the steady state value of the fraction nonconforming to x and y.

(Approximate location for Table 2)

Results in Table 2 show the results under the erroneous assumption of independence when the process is dependent. As can be seen, under the assumption of independence, MSNC values show that we tend to overestimate the outgoing fraction nonconformance when the acceptance numbers are small and underestimate it when the acceptance numbers are bigger. Either way would result in an increased cost to the producer and/or the user.

CONCLUDING REMARKS

This study proposes a new indicator, MSNC, to gauge the performance of single acceptance sampling plans for attributes by using the distribution of fraction nonconformance (i.e., lot quality distribution (LQD). The context of the study is a sequential sampling from a dependent production process by a model based on a two-state dependent process. It is widely documented that the quality dependence of a stream of items from a manufacturing process will frequently violate the randomness of the binomial process. In the presence of autocorrelation, back to back items will distort the acceptance sampling plan. The adverse effect of this is that it obviously elevates the risk for both producer and/or consumer. Moreover, frequent misspecification of outgoing quality will also discredit quality programs in the eye of -personnel and devalue the acceptance sampling plan.

Our recommendation of a special indicator, MSNC, for the outgoing quality of the lots provides a simple approach for a single sampling inspection. By incorporating dependence as an integral part of its measure, MSNC helps the efficiency of sampling plan. First, it improves the measure by incorporating more information by the use of a custom tailored prior distribution which in turn improves precision. Second, it accounts for variance, Var(p': A) for both small and large acceptance numbers (c values). This is particularly important when the dependence variance is substantial in the original stream of items..

The outgoing quality indicator, MSNC, will rise with increasing acceptance numbers and deflate with sample size for a given acceptance number in the plan. This is expected since the information content will be depressed in smaller sample sizes. Therefore MSNC reacts to the uncertainty in the anticipated way. Also, there might be other uses for MSNC. For example, MSNC, as calculated over the prior distribution, is the maximum value that can be observed with any sampling inspection plan. The difference between a MSNC-prior and a MSNC-posterior is a measure of the contribution of the inspection plan in quality assessment. This procedure exhibits an indirect value of the use of alternative acceptance plans.

As future research venues, we find value in the performance simulation of MSNC in wide range of conditions. This will clarify its applicability and test its conformance to various contexts. Second, it may be a useful direction to consider a MSNC for multi-attribute data. Alternatively, the concept can be used for acceptance sampling by variable measure as opposed to attribute data. Despite its potential complexity, this will broaden the implementation potential of the indicator. Lastly, the performance of MSNC under various sampling plans will be a valuable contribution to the practice.

In short, MSNC may be useful in designing acceptance sampling plans or evaluating existing ones from the point of view of consumer risk, and it differs from the existing quality level oriented measures on two accounts. First, MSNC takes into account that the process has certain dependency characteristics, and second, it adopts a Bayesian approach by employing a prior distribution for computing outgoing quality.

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Table 1. Prior and Posterior Distribution and MSNC's for a Dependent Process for Lot sizes 20, 30, 40 for various sampling plans.

p'	PRIOR N = 20	POSTERIOR c = 1 N = 20 n = 2
0.00	0.3234	0.3391
0.05	0.1318	0.1368
0.10	0.1151	0.1194
0.15	0.0978	0.1003
0.20	0.0807	0.0813
0.25	0.0649	0.0638
0.30	0.0509	0.0485
0.35	0.0390	0.0359
0.40	0.0292	0.0257
0.45	0.0214	0.0179
0.50	0.0154	0.0121
0.55	0.0108	0.0079
0.60	0.0074	0.0050
0.65	0.0050	0.0030
0.70	0.0032	0.0017
0.75	0.0021	0.0009
0.80	0.0013	0.0005
0.85	0.0008	0.0002
0.90	0.0004	0.0001
0.95	0.0002	0.0000
1.00	0.0002	0.0000
EAOQ		0.1234
MEAN		0.1290
VAR		0.0210
MSNC		0.0380

For all cases x = 0.95, y = 0.30.

Table 1 (contd.)

p'	PRIOR N = 30	POSTERIOR c = 1 N = 30 n = 2	 c = 1 N = 30 n = 3
0.00	0.1937	0.2041	0.2124
0.03	0.1115	0.1148	0.1208
0.07	0.1075	0.1113	0.1164
0.10	0.1003	0.1032	0.1069
0.13	0.0904	0.0923	0.0943
0.17	0.0791	0.0799	0.0803
0.20	0.0675	0.0673	0.0663
0.23	0.0563	0.0553	0.0532
0.27	0.0460	0.0444	0.0416
0.30	0.0368	0.0348	0.0316
0.33	0.0290	0.0268	0.0235
0.37	0.0224	0.0201	0.0171
0.40	0.0170	0.0149	0.0121
0.43	0.0127	0.0107	0.0084
0.47	0.0094	0.0076	0.0056
0.50	0.0068	0.0053	0.0037
0.53	0.0048	0.0036	0.0024
0.57	0.0034	0.0024	0.0015
0.60	0.0023	0.0015	0.0009
0.63	0.0016	0.0010	0.0005
0.67	0.0010	0.0006	0.0003
0.70	0.0007	0.0004	0.0002
0.73	0.0004	0.0002	0.0001
0.77	0.0003	0.0001	0.0000
0.80	0.0002	0.0001	0.0000
0.83	0.0001	0.0000	0.0000
0.87	0.0001	0.0000	0.0000
0.90	0.0000	0.0000	0.0000
0.93	0.0000	0.0000	0.0000
0.97	0.0000	0.0000	0.0000
1.00	0.0000	0.0000	0.0000
EAOQ		0.1296	0.1141
MEAN		0.1350	0.1250
VAR		0.0160	0.0140
MSNC		0.0340	0.0300

For all cases x = 0.95, y = 0.30.

Table 1 (contd.)

p’	PRIOR N = 40	POSTERIOR c = 1 N = 40 n = 2	c = 1 N = 40 n = 3	c = 1 N = 40 n = 4
0.00	0.1160	0.1200	0.1262	0.1337
0.02	0.0854	0.0884	0.0928	0.0981
0.05	0.0907	0.0937	0.0980	0.1031
0.07	0.0914	0.0941	0.0979	0.1021
0.10	0.0884	0.0906	0.0936	0.0966
0.13	0.0828	0.0844	0.0863	0.0879
0.15	0.0754	0.0763	0.0771	0.0774
0.17	0.0670	0.0672	0.0670	0.0661
0.20	0.0582	0.0578	0.0568	0.0550
0.22	0.0496	0.0488	0.0470	0.0446
0.25	0.0415	0.0403	0.0382	0.0354
0.27	0.0342	0.0327	0.0303	0.0274
0.30	0.0277	0.0261	0.0237	0.0208
0.32	0.0221	0.0205	0.0181	0.0155
0.35	0.0174	0.0158	0.0136	0.0113
0.38	0.0135	0.0120	0.0101	0.0081
0.40	0.0104	0.0090	0.0073	0.0057
0.42	0.0078	0.0066	0.0052	0.0039
0.45	0.0058	0.0048	0.0037	0.0026
0.47	0.0043	0.0034	0.0025	0.0017
0.50	0.0031	0.0024	0.0017	0.0011
0.52	0.0022	0.0017	0.0011	0.0007
0.55	0.0016	0.0011	0.0007	0.0004
0.57	0.0011	0.0008	0.0005	0.0003
0.60	0.0008	0.0005	0.0003	0.0002
0.63	0.0005	0.0003	0.0002	0.0001
0.65	0.0004	0.0002	0.0001	0.0001
0.67	0.0002	0.0001	0.0001	0.0000
0.70	0.0002	0.0001	0.0000	0.0000
0.72	0.0001	0.0000	0.0000	0.0000
0.75	0.0001	0.0000	0.0000	0.0000
0.77	0.0000	0.0000	0.0000	0.0000
0.80	0.0000	0.0000	0.0000	0.0000
0.82	0.0000	0.0000	0.0000	0.0000
0.85	0.0000	0.0000	0.0000	0.0000
0.88	0.0000	0.0000	0.0000	0.0000
0.90	0.0000	0.0000	0.0000	0.0000
0.92	0.0000	0.0000	0.0000	0.0000
0.95	0.0000	0.0000	0.0000	0.0000
0.97	0.0000	0.0000	0.0000	0.0000
1.00	0.0000	0.0000	0.0000	0.0000
EAOQ		0.1325	0.1193	0.1061
MEAN		0.1370	0.1300	0.1220
VAR		0.0130	0.0110	0.0100
MSNC		0.0320	0.0280	0.0250

For all cases x = 0.95, y = 0.30

Table 2. Comparison of the Posterior Distributions for Independent and Dependent Case.

		INDEPENDENT			DEPENDENT		
N	c	E(p’)	Var(p’)	MSNC	E(p’)	Var(p’)	MSNC
3	0	0.104	0.011	0.021	0.041	0.010	0.012
3	1	0.139	0.014	0.033	0.084	0.023	0.030
3	2	0.152	0.016	0.039	0.119	0.037	0.051
4	0	0.092	0.010	0.018	0.031	0.007	0.008
4	1	0.130	0.013	0.030	0.067	0.017	0.021
4	2	0.047	0.015	0.037	0.099	0.028	0.038
4	3	0.053	0.016	0.039	0.125	0.040	0.056
5	0	0.081	0.009	0.015	0.024	0.005	0.006
5	1	0.121	0.012	0.027	0.054	0.012	0.015
5	2	0.141	0.014	0.034	0.083	0.022	0.029
5	3	0.151	0.015	0.038	0.109	0.032	0.044
5	4	0.154	0.016	0.040	0.129	0.041	0.058
6	0	0.071	0.008	0.013	0.019	0.004	0.004
6	1	0.112	0.011	0.024	0.045	0.010	0.012
6	2	0.135	0.013	0.036	0.071	0.017	0.022
6	3	0.147	0.014	0.037	0.095	0.026	0.035
6	4	0.152	0.016	0.039	0.115	0.034	0.047
6	5	0.154	0.016	0.040	0.131	0.043	0.060
7	0	0.063	0.007	0.011	0.015	0.003	0.003
7	1	0.104	0.010	0.021	0.038	0.008	0.009
7	2	0.128	0.012	0.028	0.061	0.014	0.017
7	3	0.143	0.014	0.034	0.083	0.021	0.028
7	4	0.150	0.015	0.038	0.103	0.029	0.040
7	5	0.153	0.016	0.039	0.120	0.037	0.051
7	6	0.154	0.016	0.040	0.133	0.044	0.062
8	0	0.056	0.006	0.009	0.012	0.002	0.002
8	1	0.097	0.009	0.018	0.032	0.006	0.007
8	2	0.122	0.011	0.026	0.053	0.011	0.014
8	3	0.138	0.013	0.032	0.074	0.017	0.022
8	4	0.147	0.015	0.037	0.092	0.024	0.032
8	5	0.152	0.016	0.039	0.109	0.031	0.043
8	6	0.154	0.016	0.040	0.123	0.038	0.053
8	7	0.154	0.016	0.040	0.134	0.045	0.063

x=0.95 y=0.274 Prior distribution: E(p’)=0.154, Var(p’)=0.059

TABLE CAPTIONS

(Mergen and Deligonul)

Table 1. Prior and Posterior Distribution and MSNC's for a Dependent Process for Lot sizes 20, 30, 40 for various sampling plans.

Table 2. Comparison of the Posterior Distributions for Independent and Dependent Case.