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**Integrating Visualization into the Modeling
Of Business Simulations**

3647 words

10 Figures

4 Tables

by

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This article demonstrates the advantages of using visualization as part of the modeling process. Several examples are given to show how visualization can help developers to more completely understand the range of behaviors for their algorithms. Specifically, the Cobb Douglas function and Gold Pray demand system are examined using a tool that combines mathematical modeling with visualization capabilities.

KEYWORDS: algorithm development; algorithm behavior; business simulation; Mathematica, modeling; visualization

Over the past 20 years, model builders have worked diligently to improve the algorithms that drive business simulations. Researchers have published numerous papers on different aspects of improving the realism and the reliability of business games. Still, many designers, even after developing equations, struggle with: (i) how to select starting values, (ii) how to gauge sensitivity of the parameters used in the model and (iii) how to ensure the system is robust. This paper illustrates the use of visualization techniques and tools for designers to gain a better understanding of existing or developing models.

The paper begins with a brief summary of the literature on algorithm development and then gives a general description of a modeling and visualization tool called Mathematica® (Wolfram, 1993). Finally, it demonstrates the use of visualization through an analysis of three commonly used demand models: (i) the Cobb Douglas Power Function, (ii) the Gold and Pray Demand System (1984), and (iii) the Product Attribute Model of Gold and Pray (1997) modified to include an interaction or cross-elasticity effect between independent variables. Mathematica® is utilized to identify both the stability and lack of stability of a system of equations. The paper concludes with some suggestions on the use of the software package and offers caveats associated with the methodological approach suggested by the authors.

Business simulation algorithms

A review of the modeling literature illustrates that there has been considerable work in algorithm enhancement of business games. In the operations arena, Thavikulwat (1993) proposed a linear set of equations to model production processes. Gold and Pray (1989), Gold (1990), and Gold (1992) developed models for cost and production functions embodied in business games. Pray and Methe (1991) put forth a model for new product development with generalized demand and production functions.

Quality modeling became popular in the 1990s with the work of Thavikulwat (1992), Mergen and Pray (1992), Teach (1992), and Teach and Schwartz (2000). All of these authors demonstrated methods and algorithms for modeling quality that could be added to existing or to new simulations.

In the area of marketing, many articles have been written about demand modeling including how to model price and non-price issues. Pray and Gold (1982) investigated the demand robustness of a number of commonly used business games. Articles soon followed by Teach (1984), Gold and Pray (1984), Goosen (1986) and Decker, LabBarre and Adler (1987) that moved the modeling of demand to a higher level. Further extensions by Golden (1987), Lambert and Lambert (1988) and Thavikulwat (1988, 1989) tested the reliability of various models and raised new issues about how demand should be modeled. Market segmentation was addressed formally by Teach (1990), Carvalho (1991, 1995) and Gold and Pray (1997, 1998).

This brief review shows that the leading business game designers have shared their design contributions with the field. It is also the case, however, that all of the algorithms described in the literature are just mathematical models and thus have certain limitations. Some algorithms are highly sensitive to the starting parameters selected. Others require the decision variables to be constrained in a narrow range for the simulation to behave in

a manner that is consistent with theory. Some models have discontinuities, which can also yield unreasonable results.

The methodology to be presented addresses these limitations by offering a relatively easy “visual method” of testing and verifying the overall effectiveness of an algorithm and providing insights into where difficulties may arise with actual usage. The drudgery of hours of mathematical sensitivity analysis can be avoided by including visualization in the modeling process.

Integrating Visualization into the Modeling Process

Visualization is the rendering of complex data in a visual image that is understandable for human observers. The advent of computer technology has allowed visualization to impact a wide variety of areas from aircraft design to advanced physics. Businesses are also turning to dedicated software visualization packages (e.g. OpenViz®, Visionary®) to help them understand and assess their processes or performance.

Integrating visualization into the modeling process is relatively straightforward. While it is certainly possible to develop the mathematical model and visualization in separate software packages, it is probably easiest to use an application package that permits both. Spreadsheet programs like Microsoft Excel will do both, but there are several mathematical packages (e.g. Maple®, Mathematica® or MatLab®) that are expressly designed to facilitate simultaneous mathematical modeling and visualization. Although each one has its merits, we will demonstrate the advantages of combining visualizations and modeling with Mathematica®.

Mathematica® by Wolfram Research is a software tool that allows the creation, solution, visualization and distribution of complex mathematical models. The interface is an electronic notebook where one can include ideas, partial results, and graphics. Users

develop mathematical models by evaluating individual lines of code, thus creating partial results that can be combined over and over again to develop more complex models.

Visualization functions are available at every step of the development process to help a model builder verify the behavior of the model. Because of the variety of tools and functions available, developers will frequently discover unanticipated behaviors that can enhance their understanding or help them avoid future problems with a model.

Once most of the development has been completed, the notebook interface can be used to explore the model both numerically and visually. This exploratory mode of interaction can take the form of a set of “what-if” scenarios that allow the user to better comprehend the full complexity of the work. Charts, graphics and animations can be created automatically to contribute to the user’s understanding. Sharing such a model online is simple since the freely available MathReader® software allows anyone to explore and interact with a notebook.

Figure 1 shows portions of Mathematica® in action. A Cobb Douglas demand function is first defined, with parameters: a (scale parameter), s_1 (elasticity for price), s_2 (elasticity for marketing), price and mkt (marketing). Next, the function is evaluated with specific values so that the demand is 6000 units at the starting values. Varying only the price generates the two-dimensional demand plot in the figure.

Figure 1 about here

As with any piece of software, a new user to the software will have to spend some time learning both the notebook interface and the language used to write equations and functions. While the notebook interface is quite easy, the language of available commands is vast and can be intimidating. Fortunately, one can accomplish most analyses by exploring just the small subset of the language that is applicable to a specific problem. The documentation for Mathematica® is well written and illustrated, and

additional reference guides make developing significant models possible even for beginners.

Visual Modeling Examples

What follows is a demonstration of the visual modeling and exploratory techniques using three different demand models. In the first two cases, visualization is used to better understand the behavior of the Cobb Douglas and Gold and Pray demand models. In the last case, visualization is used to develop a new extension to the Gold and Pray demand model that handles cross elasticities.

Cobb Douglas market demand function: a stable function with constant elasticity

The first model analyzed is the Cobb Douglas function. This function was first deployed as a way to describe production functions in microeconomics but it can be modified easily to fit the demand side. To demonstrate the visual modeling aspects, we will simulate a simple demand function where price (P) and marketing (M) are the independent variables, and demand (Q) is the response or dependent variable.

The functional form is as follows:

$$Q = a P^{-e_p} M^{e_m} \quad [1]$$

where: the elasticities for price and marketing are e_p and e_m respectively; "a" is the scaling coefficient.

To verify the coding of the model we set the price elasticity (ϵ_p) at -1.2 , the marketing elasticity (ϵ_m) at $.3$, and the scaling coefficient " a " at 44257 . Sensitivity analysis on the Cobb Douglas function is demonstrated below.

The Cobb Douglas Demand Curve

Figure 2 shows a two-dimensional plot depicting a classic Marshallian demand curve. In this example, price was varied from \$10 to \$45 while marketing was held constant at \$500.

Note: the authors arbitrarily scaled Marketing to \$1000 units. Designers or users can scale coefficients either before use of the demand equation or through the scaling coefficient " a " to get the desired level of demand. The plot shows that demand is maximized at 17,500 and that the quantity demanded appears to be asymptotic to the x-axis.

Figure 2 about here

In the next illustration, Figure 3, the diminishing returns to marketing are clearly seen as we vary marketing from \$200 to \$3000 while holding price constant at \$25. It is interesting to note that demand reaches zero and that, for this model, it is possible to generate "negative demand" for very small levels of marketing. Such an observation is easy with a visual representation, but might be much more difficult otherwise.

Figure 3 about here

One way to look at systems of more than two variables is through three-dimensional plots. In the Figure 4, we vary both price and marketing. Figure 4 illustrates the non-linearity of the demand function and the relative stability of behavior over the range. What is interesting

to note is that at high prices, over \$40, even large dollar expenditures in marketing will **not** increase demand very much.

Figure 4 about here

Table 1 reflects some of the shortcomings of this demand function. The demand varies between 22,181 and 30,840 for a low price of \$10, whereas when the price is high, say \$40, it is reduced to 4203 and 5844 units, even with higher levels of Marketing.

Table 1 about here

Figure 5 reflects the nature of the total revenue function for this Cobb Douglas illustration. The visualization and tables suggest a possible shortcoming of the model- that high-priced niche strategies may not be successful with this demand function.

Figure 5 about here

Gold and Pray: a variable elasticity market demand function

Gold and Pray (1984) developed a variable elasticity demand model to overcome the shortcomings of using the Cobb Douglas system. Their system involves 10 equations and is described in detail with examples in Gold and Pray (1984). The algorithm to be simulated is as follows:

$$Q_t = g_1 P_t^{-(g_2 + g_3 P_t)} M_t^{+(g_4 + g_5 M_t)} \quad [2]$$

where: Q_t = market demand at time t .

P_t = average price at time t .

M_t = average marketing expenditure at time t .

g_k = market demand parameters k where $k = 1$ through 5

To solve for the parameters of the market demand equation the administrator must specify the desired exogenous elasticities of each independent demand variable at two different levels (i.e. P , M). The elasticity formulas are as follows:

$$Ep_t = g_2 + g_3 P_t (1 + \ln P_t) \quad [3]$$

$$Em_t = g_4 + g_5 M_t (1 + \ln M_t) \quad [4]$$

where: Ep_t = price elasticity at time t .

Em_t = marketing expenditure elasticity at time t .

Selecting two levels for each elasticity (Ep , and Em) and the corresponding demand variable (P and M) over a reasonable range gives two equations with two unknowns and allow simultaneous solution of the system parameters g_k (for $k=2,5$). The selection of g_1 determines the initial market size. A modeling tool makes solving and simplifying this system of equations relatively easy. Thus, beyond its visualization uses, Mathematica® offers many mathematical tools to aid in the development of the functional structure of a model. To demonstrate the demand system, Table 2 includes the values used to produce the subsequent visualizations.

Table 2 about here

With these parameters, the price elasticity of demand at the market level increases from an inelastic .95 to a highly elastic 3.00 when the price increases from \$25 to \$35. Likewise, the marketing elasticity declines as more money is allocated to marketing.

With these values and a scaling coefficient, market demand is about 6000 units when price is \$20 and marketing \$500. A visualization where price varies over the relevant range from \$25 to \$35 and marketing ranges from \$500 to \$1500 reveals some very interesting results.

The 3-D plot of Figure 6 demonstrates that the gross behavior of the Gold and Pray model is consistent with the theory reflected in the Cobb Douglas function. It is interesting to note that this variable elasticity behaves similarly to that in the Cobb Douglas model in that at the higher prices, demand is **not** very responsive to increases in marketing. Such a global view of the behavior of a function may be difficult to apprehend quickly without utilizing visualization.

Figure 6 about here

The Gold and Pray model can be further verified by using the modeling program to calculate the arc elasticities for price and marketing. Indeed, the line graph shown in Figure 7 demonstrates that the price elasticities are consistent with preestablished ranges and expectations. Higher price elasticities are associated with higher prices, *ceteris parabus*. However, the model and theory are in agreement only if the price and marketing values are constrained to be within the relevant ranges shown in Table 1.

Figure 7 about here

Table 3 shows that the elasticities for marketing over the range \$250 to \$1500 are consistent with the preestablished values shown in Table 1 and with marketing theory. However, above the \$1500 level, negative returns occur to marketing. These negative returns are one example of instability in the demand function.

Table 3 about here

Instability in the function

Mathematica® makes it readily apparent that the demand system is robust over the constrained range of decision inputs. Increasing the price and marketing out of the range reveals some extreme behaviors within the model.

Notice in Figure 8 that at prices from \$4 to \$20 the function behaves in a manner that is **inconsistent** with demand theory. The model behaves consistently with theory for all prices above \$20. What is interesting, however, is that at lower prices, say from \$5 to \$18 dollars, price increases cause demand to increase! Prices outside the designed range can make the model behave as an economic Giffen good.

Figure 8 about here

Figure 9 depicts quantity demanded with marketing varying from \$200 to \$3000 and price fixed at \$25. The marketing response appears to be consistent with expectations over the relevant range. But negative returns to marketing occur outside the upper limit of

\$1500. The negative returns may be helpful in some simulation situations. But the theoretical realism of negative returns can be challenged.

Figure 9 about here

Discovering unexpected behaviors in a model is one of the key benefits of using visualization techniques. A classic example is that of Anscombe (1973) who developed a data set where a naïve student of statistics might generate four identical two variable regression models. However, plotting the data and residuals reveals that three of the models have serious shortcomings including: curvature, outliers and extremely influential data points! There are many situations like Anscombe's where visualizing a mathematical function will provide the quickest and most clear indication of the function's behavior.

In our example, visualization has made clear that the Gold and Pray demand function behaves in a theoretically appropriate way, but only as long as the assumed range of price and marketing values are obeyed.

Using visualization to develop new models

Teach and Schwartz (2000) have developed a modification of the Teach (1984) model to allow for interactive effects between independent variables. As Teach points out, the original Gold and Pray (1997) attribute demand model does not allow for cross elasticities or for an interactive effect between say price and advertising. In this final example, visual modeling is used to demonstrate how the variable elasticity model of Gold and Pray can be easily extended to allow for interactive effects between combinations of independent variables. In this case, visualization is used as part of the development of this extension to the model.

The final system is described via a set of simple examples. To determine the market- and firm-level demand for each segment, Equation 5 is used. Price is set to vary between \$25 and \$35 and advertising/marketing expenditures between \$500 and \$1200. A new variable is set up to account for the interactive effect, which is the ratio of marketing to price. This new variable includes the joint effects of price and marketing on demand. The elasticities on the ratio (R_t) are controlled over a range from 0 to -1.0.

$$Q_t = g_1 P_t^{-(g_2 + g_3 P_t)} M_t^{+(g_4 + g_5 M_t)} R_t^{-(g_6 + g_7 R_t)} \quad [5]$$

where: Q_t = market demand at time t .

P_t = harmonic average price of all products at time t .

M_t = average marketing expenditure for all products at time t .

R_t = average ratio of marketing divided by price (M_t/P_t) of all products for at time t .

g_k = market demand parameters k .

The range of parameters is displayed in Table 4.

Table 4 about here

In Figures 10 we replicate the simulation presented in Figure 6 but add a new element: a ratio variable of marketing to price which allows for an interactive effect between price and marketing.

Figure 10 about here

A visual comparison of Figure 10 with Figure 6 illustrates how the interactive effect of price and marketing change the entire shape of the response function – from concave to convex from the origin. The interactive impact has some interesting properties. Now if a firm has a low price (\$25) and markets extensively (\$1000), its quantity demanded increases from 7500 to over 15,000 units! Furthermore it appears that “branding” may be modeled with the interactive variables. Notice that firms with high prices (\$32) and large amounts of marketing (\$1200) can still have a high level of demand, over 11,000 units compared to 5,200 from Figure 6.

Summary and conclusions

The purpose of this paper was to demonstrate a visualization technique that is useful to designers of business simulations. Certainly, the idea of using visualization to supplement modeling is a simple one. However, the benefits of employing visualization can be considerable. Three examples were presented to demonstrate some of the capabilities and benefits of a visual approach to modeling. In one example, visualization helped identify the shortcomings of the Cobb Douglas function as a demand model. Then, the Gold and Pray demand system was shown to resolve some of these shortcomings (i.e. allowing the elasticities to vary) but was demonstrated to be highly unstable outside the preset parameters. In the final illustration, visualization was used to create a new extension of an existing model. Specifically, the Gold and Pray demand system was enhanced to include a new variable (the ratio of marketing to price) to account for cross-elasticity effects. The visualizations suggest that the interactive effects may be readily handled by this simple modification to the Gold and Pray demand system.

Of course, a visualization by itself is not enough to communicate all the nuances of a complex business demand function. As in the examples presented here, combining visualization with tables of numbers may provide the modeler with enough information to address many questions.

Because of the widespread availability of powerful software tools, visualization is now being applied to a large number of problems. For business simulation designers, tools that combine mathematical modeling with visualization make possible a faster and more interactive design process. In turn, such a process can enable new insights into existing models, or allow rapid inclusion of new economic issues into modern business simulations.

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Thomas F. Pray is a professor of the College of Business, Rochester Institute of Technology. He is a past president and fellow of the national Association for Business Simulation and Experiential Learning (ABSEL). Much of his published research is in the modeling of demand algorithms, quality modeling, and new product development issues in computerized business simulations. He is an active business consultant, who conducts business decision simulation seminars nationally and internationally.

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Figure 1: Working with Mathematica

Figure 2: The Cobb Douglas demand curve decreasing with price

Figure 3: The Cobb Douglas demand curve increasing with marketing

Figure 4: The Cobb Douglas demand surface

Figure 5: The Cobb Douglas revenue surface

Figure 6: The Gold Pray demand surface

Figure 7: Price Elasticity

Figure 8: Instability of the demand function when Price is out of range

Figure 9: Instability in the demand function when Marketing is out of range

Figure 10: Gold Pray Demand function with cross-elasticity

		Marketing →			
Price ↓		\$0	\$1000	\$2000	\$3000
	\$10	0	22,181	27,308	30,841
	\$20	0	9,655	11,867	13,424
	\$30	0	5,935	7,307	8,252
	\$40	0	4,203	5,174	5,843

Table 1: Demand as a function of Price and Marketing with the Cobb Douglas Function

	Starting value	Final value
Price (P)	\$25	\$35
E_p	.95	3.00
Marketing (M)	\$500	\$1500
E_m	0.4	0.15

Table 2: Parameters used in Gold Pray model visualizations

Marketing (\$1000)	Elasticity
200	.500
450	.421
700	.335
950	.241
1200	.154
1450	.061
1700	-.035
1950	-.132

Table 3: Marketing Expenditures and Marketing Elasticities

	Starting value	Final value
Price (P)	\$25	\$35
E_p	0.95	3.00
Marketing (M)	\$500	\$1500
E_m	0.4	0.15
Ratio (M/P)	$(500/25)=20$	$(1200/35)=34.28$
E_r	0.0	1.0

Table 4: Parameters used in the Gold Pray cross elasticity visualization