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Finite element analysis solution applications to photoreceptor modules

Robert Hildebrand

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Finite Element Analysis Solution Applications to

Photoreceptor Modules

A Thesis submitted in Partial Fulfillment of the Requirements for the

MASTER OF SCIENCE

In Mechanical Engineering at Rochester Institute of Technology, 1999

By

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Signed,

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Robert Hildebrand

8/25/99

(Date)
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NOMENCLATURE

a_i  Arbitrary coefficients used in the derivation of Finite Elements
a    Belt tension per belt thickness
A    Boundary Condition set over a domain
b    Boundary Vector extracted from U because the values are known
b^e  Boundary Vector for local element
E    Modulus of Elasticity
F    Force Vector for the Global system
F^e  Force Vector for a local element
g    Gravity
GV   term used in Gradient Theory to depict relationship of force over a domain to force over a boundary using normal vectors
h    Width of each local element and its multiples
I    Moment of Inertia
J    The Jacobian, used in the definition and derivation of K
K    Stiffness Matrix of the Global system
K^e  Stiffness Matrix of a local element
l    The height of the belt elements and its multiples
L    The length of the belt
m    Mass
M    Mass Matrix of the Global system
M^e  Mass Matrix of a local element
n    Denotes normal direction
n    Normal vector
P    Dummy force used in explanation of multidimensional analysis
R    Residual of the Galerkin Finite Element Method
t    Belt thickness
T    Belt Tension over both thickness and width
T'   Resolved Force on a given Roll due to T and wrap angle
U    Deformation result for a set of nodes
Û   Polynomial approximation of U
U^e  Deformation of a local element node
U'   First derivative of deformation
U''  Second derivative of deformation
w    Unloaded belt edge distance, width
x    Local coordinate frame denotation for distance along a beam
X,Y,Z The Global Coordinate Frame components, DOF 1,2,3 respectively
φ    Phi is the wrap angle of a given roller
Γ    Gamma is the Boundary of the Domain
η    Local coordinate frame component for Belt Elements
ν    Poisson’s ratio
θ    Theta is one half of Phi and used in determining Roll Load resolution vectors
ρ    Density
Ω    Signifies the domain of an area or section to be integrated
Ψ    Represents the shape functions used to define the deflection characteristics of
      Finite Elements
ζ    Local coordinate frame component for Belt Elements
Abstract

One of the primary components in a Xerox copier is the print engine. The center of this engine is comprised of a photoreceptor, which is a roller/belt module mounted to a frame. The belt revolves around the module acquiring and transposing toner to sheets of paper as they come into contact with the module. The initial design of these modules can often lead to registration and print quality problems later in the assembly and application phases of design. The current analysis procedure includes lengthy commercial FEA packages that require high designer investment. For this reason, many new ideas are never given the opportunity to develop.

The implementation of a low investment analysis step which is designed to reveal problems with a design’s general formulation could save the corporation both time and money. The means of statically approximating designs before they are modeled in commercial FEA packages could allow for more module configurations to be analyzed and considered. This low investment means of approximation has been developed here.

A user friendly Excel spreadsheet based generic photoreceptor module analyzer is derived, explained, and correlated in the ensuing analysis. Although approximate, the ability to compare designs and choose the best one for the application makes this analysis successful. The generic modeling capability is automated such that user interaction is minimal and navigation is relatively simple. Also included in this thesis is a step by step instruction set for inputting module parameters and running the program.

A Nastran FEA model was constructed and correlated to this solver, which was shown to retain the correct order of magnitude (micron level) and overall deformation shape. Future adjustments and other software capabilities are also discussed.
The Rochester Institute of Technology's BSMS course study often provides increased opportunity to those in the program. Since the timeline for graduate studies is combined with the undergraduate requirements of co-op, there evolves an overlap of ideals. In this specific case, those ideals were exploited to encompass both a co-op experience and the completion of the Masters Thesis requirement. Stemming from the connection of the RIT faculty and the Xerox Corporate Research and Technology Center in Webster, NY, I was offered a position in the Media Receiving and Handling division of CR&T. The official start date of this endeavor was Nov. 25th 1997. Since that time, many things have changed. Two full co-ops and a full year of part time work at Xerox have been completed and the group that I was working for has now excepted my permanent employment. This connection is important in that it differentiates this thesis from the norm in a number of ways. As described in this section, the thesis material itself becomes the property of the Xerox Corporation. Further, this work acts as the first version to a series of solvers in the area of photoreceptor analysis and registration measurement. For this reason, a number of the assumptions and guidelines for the output of this work differ from the literature review/typical thesis project and write-up. I'd like to take this opportunity to thank all of those who not only helped me along with this thesis, but also at Xerox. The dignity and respect which they have shown, along with the undying resolve to solve the unsolvable has taught me much about what it means to be an engineer. More specifically, I'd like to thank William Nowak, for consistently guiding my journey and never settling for anything less than my very best.
Due to the technical nature of this analysis and software, it was decided that the engineering rights of the Xerox Corporation might come into conflict with the publishing of the results as a thesis. For this reason, all values used in this analysis, as well as materials, interactions, and purposes have been modified in such a way as to protect the interests of the corporation. However, all values that were modified were done so in a fashion as to not effect the theoretical process nor application. The actual solution specimens and empirical results thereof, remain the proprietary property of the corporation. Further, the results of this analysis, as well as the right to the usage of this software is restricted to the Xerox Corporation.

File Compatibility and Tutorials

The files created for this analysis are consistent with the Microsoft Office 97 for Windows NT version. All Documentation is based in Word while computational derivations and solutions are based in Excel. However, it is theorized that eventually, there will exist a full set of Matlab solution sets available for use, with the Excel version simply acting as an interactive tutorial. In either case, both solution sets contain text add-ins and step by step instructions for the use of the respective programs. These tutorials can be found in later sections of this report.
1. Introduction

Finite Element analysis has been the long-standing leader of the tools used to maximize the analysis process and quality of Xerography. These tools have encompassed both limited hand calculations on a first order basis and the extravagant usage of complex software systems such as MSC/NASTRAN for the solution of these problems. Provided that there exists a prominent spectrum of investment between these ends, it becomes the focus of this thesis to not only determine the most efficient compromise of these methods, but to enact a self contained, automated solution sequence capable of adapting itself to repetitive systems with respect to photoreceptor modules. If this adaptation were successful, as will soon be proven, the relative reduction in engineering investment required for Xerographic photoreceptor analysis would allow for increased exploration of future technologies, hence greatly benefiting the corporation.

The process of xerography depends heavily on the transfer of toner between substrates. In most cases, these substrates are a photoreceptor belt and paper. The motion quality of this belt will inevitably impact the quality of this transfer of toner. The tighter the tolerance on the belt motion, the better the quality of the resulting print. Further, the motion of the belt is dominated by the system that drives it, called a photoreceptor module. This module is responsible for controlling the paper path during the image cycle. For instance, the photoreceptor is comprised of a Mylar composite belt wrapped around a set of rollers. These rollers vary in function from drive, steering, tension, stripping, and encoding among others. There also exists a metal structure that holds these rollers in place, called a frame. This module is responsible for receiving the image impression, retaining the correct amount and color of toner, and finally for transferring the image to a sheet of paper. The system is self-cleaning and runs at various rates of speed. It is the response of this photoreceptor, along with the inputs of surrounding subsystems that contribute most to belt motion. Hence, the connection between producing a quality product and the modeling of the photoreceptor module is made.

The most common structural analysis techniques today use FEA software packages. One such FEA code is called Nastran, created by the MacNeal-Schwendler Corporation. Nastran uses a data input deck that is often derived from a pre/post processor CAD package. This
processor allows for the selection of element types and properties as well as the order of the analysis solution. The resulting data deck reflects the entire system as a discretization of the real continuous system, complete with loading and boundary condition information. The software then solves this data deck. The solution can range from static to dynamic, linear and non-linear analyses while outputting as little or as much nodal and elemental information as the user specifies. These types of programs tend to be extremely versatile with their modeling capability but rather user intensive with both modeling the geometry, applying correct properties and boundary conditions and then the acquisition and post processing of results. Further, it takes years of training and experience to master the use of these tools. As the analogy to a chain link is so often sighted, the chain itself is only as strong as its weakest link. Here too, this is the case, noting that the unseasoned engineer is that link himself. This often restricts the engineering team to a limited number of design choices due to dissipating levels of resource availability. Further, the advent of new technologies and ideas are often left unexplored for this very same reason. Perhaps this level of investment and interaction is not necessary. A more in depth look at photoreceptors, their behavior, and the way in which they are analyzed is therefore required.

The close examination of the analysis process of photoreceptors reveals a number of interesting points. Primarily, on a generic level, nearly all photoreceptors retain common properties. The function and general architecture, despite overall dimension and subsystem connection points is the same. It is this consistency that allows for the breakthrough of new ideas in p/r modeling. In the past, once architecture had been adapted, higher order in depth local analyses would dominate the engineering process. However, the existence of a photoreceptor specific modeling tool that provided a low level FEA solution set, by which architectures could be evaluated before being approved or denied, could truly benefit the process as a whole. It is this generic modeler that has been developed and will be explored in this thesis. For the convenience of the reader and for more efficient navigation of the information contained in this thesis, a flowchart of ideas and their coordinating sections is included in Figure 1.1.

The current configuration of the analysis is as follows. The photoreceptor module was modeled in three stages. 1) An existing frame design was modeled and analyzed to ensure that it would not interfere in roller and belt response in both the static and dynamic domains.
2) Once established, the roller and belt configuration was modeled utilizing simple Finite Elements with variable definition coefficients. In other words, the location, size, material, and all other properties could be varied automatically. 3) Then, the system was sent into a static three-dimensional solver that consists of an infinite number of loading applications. Each node in the system was labeled and configured with a dummy load and interaction subset, which, upon request, could be modified. The input and output sections of the analysis were automated and configured into a tutorial type spreadsheet using Microsoft Excel. The results of this analysis include the deflection of the rolls and belt nodes both out-of-plane and in-plane process and lateral directions. By accomplishing this, although rough as far as absolute accuracy is concerned, general trend and function/effect analysis can be delivered. In addition, a similar system to the static configuration was compiled and tested dynamically. Noting that the dynamic response of the system did not require as many nodes as the static analysis, a basic system was created and the Eigen values were extracted. These Eigen values can be utilized to great extent in the controlling and overall design arenas for reasons not able to be described here.

These three components make up the final analysis and theory development for this thesis, with emphasis on the static 3-D analysis. The intended value of this thesis is the ability of Xerox designers and engineers to test and validate options that wouldn't otherwise be attempted given the restraints on time and resource allocation. A quick and dirty solution such as this model delivers could significantly improve the design process where photoreceptors are concerned. For instance, if it were contemplated that a subsystem be moved to a new location, or the general cross sectional look to the module as a whole be modified, this program would recalculate the analysis instantaneously. The engineer can then view these results in both mathematical and graphical domains to determine whether the change was significant or not. Based on this, the full Finite Element model would or would not be created. If not, the time and expense of that analysis would be saved. If the model were indeed created, this analysis would simply take its place as one of the design process mile markers that verify that the design is headed for the desired result.
Figure 1.1. Thesis Navigation Flowchart
2. The Photo Receptor Module, Function and Components

The photoreceptor module and relating components all play significant rolls in the development of quality prints. In order to construct a basic understanding of the need of this type of analysis, consider a hypothetical print engine, shown in Figures 2.1 and 2.2.

Figure 2.1. The p/r module as it sits in the paper path [ref 2]
2.1. Sources of Misregistration

The print engine shown in Figure 2.1 shows the path by which the paper is fed through the copier. Along this path, there are a number of proprietary actions that take place. Among those that can be talked about, is the interaction of the paper and the photoreceptor. The formal definition of the photoreceptor is that system along with all of its subsystems that contracts and transfers toner to the paper as it is passed through the print engine. The subsystems which account for this contraction and transferal are not of interest here. What is of interest is that any relative movement between these substrates will cause significant
discrepancy in the quality of the prints developed. Consider a sheet of paper that is to contract toner from a belt it comes into contact with. Both the sheet and belt are moving at some constant velocity $U$. This situation is shown in Figure 2.3. Given that the relative movement of these substrates is zero, it can be assumed that the toner which was placed on the belt, will be placed on the paper in an identical configuration. Further consider a change in the nominal velocity of the paper, for instance due to drag. This delta $U$ will cause a sort of traffic jam of toner that worsens as the paper length is transgressed. This situation is shown in Figure 2.4. Secondly, this type of error can be due to velocity direction as well as magnitude as shown in Figure 2.5.

Although changes in velocity do occur in real print engines, velocity is not the only contributor to poor toner transition. Consider distorting the interaction by deflecting the belt surface. Regardless of the dimension in which the deflection lies, the resulting print quality is severely decreased. It should also be noted that these relative changes in both position and velocity could be corrected, provided that their existence is known. If it were possible to restrict the number of relative discrepancies in a design, the corporation would stand to gain prodigiously by taking advantage of that design. The relative position errors and hypothetical resulting prints are shown in Figures 2.6 through 2.8.

![Diagram](image)

Figure 2.3. Constant velocity belt and paper interaction
Inconsistent substrate velocity causes process misregistration errors often seen as smearing and banding.

Figure 2.4. Discrepancy in relative substrate velocity magnitude

Two way bias motion of the paper causes misregistration in the lateral direction.

Figure 2.5. Discrepancy in relative velocity direction
Direction of Belt Motion \( V = U \)

Out-of-plane deflection in the belt causes a projection of the image to be transferred much like a circle appears as an ellipse from a non-perpendicular angle.

Figure 2.6. Discrepancy in relative out-of-plane belt deflection

Direction of Paper Motion \( V = U \)

Velocity matches but relative position becomes transient. This could cause a multitude of registration errors.

Toner to be transferred

In-plane process motion most closely simulates changes in process velocity. These types of errors, over time, create areas of high and low toner much like traffic flows past a road hazard

Figure 2.7. Discrepancy in relative in-plane process belt deflection
Direction of Belt Motion \[ \text{Velocity} = U \]

Direction of Paper Motion \[ \text{Velocity} = U \]

In-plane lateral motion closely simulates changes in lateral velocity. These types of errors appear as waviness or skew in the quality of the print.

Velocity matches but relative position becomes transient. This could cause a multitude of registration errors.

Figure 2.8. Discrepancy in relative in-plane lateral belt deflection

Although the analysis at hand is limited to the static response of the system, the active control, steering and tension predominantly, used in the photoreceptor and its subsystems constantly provides new and updated boundary and forcing conditions. With this in mind, when the deflection shown in Figures 2.6 through 2.8 become transient, changes in velocity occur and thus become sources of misregistration error. Without question, the need for proper and accurate analysis tools that can evaluate the magnitude and direction of these errors is evident. Further, if these tools required less user input, the efficiency of such a process could lead to more aggressive exploitation of the process itself and the accelerated improvement of print quality with respect to substrate motion. Now that this need is understood, the methodology and process of analyzing these systems can be explored.

2.2. Analysis Methods and Description

The analysis of the photoreceptor and its subsystems must be thorough and take into account all aspects of the system’s function. A primary assumption on this level is that the subassemblies that accompany the photoreceptor but are not actually mounted to the photoreceptor do not influence the response of the module and thus are not accounted for.
here. Furthermore, those systems within the photoreceptor, such as the tension mechanism and steering subassembly, are treated as stationary objects without mass or inertia. No dynamic effects are taken into account. In effect, the resulting system is a completely independent roller/belt system. The results of the analysis will therefore predict how the rolls and belt alone respond to various loads and excitation.

In order to simplify the calculations by eliminating rigid body mode singularities, and to restrict an otherwise free movement of the belt, the rotational degree of freedom (DOF 6) of the roll nodes is assumed to be zero. Since this system is considered static and without drag, slip or other response factors, the rotational degree of freedom will only add to the matrix size.

In order to further secure this methodology of isolationism, the structure that the rolls are mounted to must also be analyzed for its contribution to the system response such that it may be neglected. Once established, the roll/belt system must be analyzed both statically and dynamically to ensure that the variable design returns the desired result. Although only the static analysis represents the primary goal of this thesis, both of these analyses run with the understanding that the representation of the system is of coarse mesh density and that the expected results are of "comparative use only" accuracy. However, a model such as this can offer a great deal of timesaving due to a clearer design path prediction when multiple ideas can be investigated quickly.

The methodology of the analysis of the roll/belt system parallels that of the Finite Element Method for an individual element. In this method, there exists a "master" or control element. By ensuring that the same element equation is used across a domain, the derivation of each individual element need only occur once. However, this savings in calculation time is dependent on the transformation equations that project the actual element into the space of the master element. These transformations relate the shape and boundary condition of each element to both a local and global set of coordinates. Once this relation is established, the actual element is transformed into the master, solved using the predetermined analysis strategy, and then brought back into the domain of the original element as a solution set.

In the same manner, each subsystem may also be treated in this way. By ensuring that each subsystem's analytical format is identical, the transformations will again work correctly. The static model uses this relationship as a template for its roll and belt sections as a whole.
It further breaks into the individual dimensions of each analysis to relate the fact that the analytical derivations for element response are not a function of dimension but of element definition. In other words, if each dimension of an element is represented in a similar fashion, then the numeric derivation and calculation of results for each will lie in parallel and needs only occur once.

Dynamic analyses utilize these interactions in a modified sense. Although the number of nodes required is decreased, the number of degrees of freedom in dynamic models is expanded with respect to static analyses. This causes the calculation time to increase significantly. In order to retain the capacity for high degree of freedom solutions in both types of analysis, the repetitive nature of the representation of the system is exploited. The local matrix solutions are combined into a global solution sequence. Then, the equation of motion matrices are broken into partitions for easier mathematical manipulation. These partitions are treated as individual components of the parent matrix and then solved individually on their own and culminated back into the whole. The end result is the "cheating" of the software's computational limitations to matrix size and definition. This process is explained in depth in Section 4.0, the description of the spreadsheet solution.

In a global sense, the analysis of this fixture coincides with standard practice, at least at the Xerox Corporation. A complex mix of inputs is slowly unraveled until a base structure can be obtained. This base structure is then analyzed first until it is understood and then built upon over time. It is this base structure which is analyzed by the methodology above. It is also the intent of this analysis path to provide the ability of inclusion, at some later date, of exterior systems that become increasingly understood. This type of agility in assumption and process becomes the basis for the development of unique "in house" analysis methods. The analysis suite currently being developed aspires toward this end.

3. Step by Step Analysis Theory

The following section includes the methodology of the analysis exploited into its various subsets. The prerequisites for using the analysis are described first, Section 3.1, followed by a review of the Finite Element Method, Section 3.2, in its classical format. The belt element derivation subset reveals the modifications to the classical FE method for this application.
The derivation of the elements chosen for this analysis along with the circumstances that lead to the selection is included in Section 3.3. Sections 3.4 through 3.6 describe the repeatability of the modeled components, dimensional considerations, and belt/roll interaction respectively. Finally, in Section 3.7, a *Generic Roll Analyzer* is described that allows for off line optimization of roll designs.

3.1. Frame prerequisite for analysis validity

As with any analysis, or collegiate level course offering for that matter, there must always exist a set of prerequisites that must be met in order to proceed. This analysis does not strive to be different in this arena. In order to simplify the number of elements in the model itself and to obtain the assumed interaction of the subsystems that make up a photoreceptor, it must be assumed that the roll ends are connected to ground. This assumption continues in that it says nothing as to the boundary conditions located at this juncture, but simply that the boundary condition or loading at one point in the system must only transmit these restraints through the roller belt elements and not the frame structure that supports it. In order to accommodate this requirement, the frame must have a minimal relative deflection at roller connection points under appropriate loading and mounting configurations.

The frame that supports the rollers in the photoreceptor is typically designed such that the static response is minimal and the dynamic response is at a high enough frequency as to not interfere with any of the subsystems. Although the structure itself has little "real" effect on the rolls and belt in the present design, it is important to note what assumptions are necessary about frame design such that the concurrent analysis on the rest of the system remains valid.

The first important subset of the frame analysis is relative deflection. This type of analysis will depend heavily on the type and number of mounting locations used to place the p/r module into the actual Xerox machine. There exists two types of mounting that are probable enough to be explored here, cantilevered and simply supported. Both types of mounting are common and there exists a strong analytical base for the exploitation of this type of analysis. Each mounting configuration will be tested for deflection under its own weight. If the deflection is considered significant, i.e. the relative position of the rollers
becomes transient, then the frame must be considered in any roll/belt level analysis. However, if the deflection does not show significant motion, then the roll/belt system may be considered "isolated" from the frame. It is this isolation condition which is required for the subsequent static and dynamic analyses to run correctly. It is therefore the objective of this section to interpret the conditions that cause significant frame deflection.

The second consideration of the frame analysis is the natural frequency or dynamic response. It is desired that the resonant portion of the frame's design not occur at frequencies near that of the excitations within the system. This part of the analysis is more difficult to correlate to generic systems due to the wide variety of designs. The results for this section of analysis will be used to verify the closed form simplification assumptions and left to the design team as an undetermined variable in the photoreceptor module.

The construction of a generic frame does not lend itself in this type of analysis because of the wide variety of frame designs. For this reason, an example frame, shown in Figure 3.0, was selected and used as a model for general analysis purposes. It is assumed that the reaction of this model will represent the relative level of significance that should be placed on frame design analysis prior to the construction of the full p/r module FEA model. A total of four models were constructed, two utilizing closed form techniques, and two using MSC/NASTRAN. The closed form models represent significant simplifications to the geometry in order to lessen the mathematical load. The verification steps for these assumptions, as well as the spreadsheet and FEA solutions for each subset can be found in the Appendix of this report under the Frame Analysis heading. The recommendation set to the user of this software as to the requirements that the frame must meet in order for the static and dynamic results to remain valid follows below.
The conclusions of the frame analysis, as shown in the Appendix, may seem muddled and unrelated. However, their interaction and global importance in the validity of using the software provided here will soon be apparent. It was first assumed that only two types of photoreceptor mounts would be considered, cantilevered and simply supported. It was then hypothesized that these systems could be split into a belt / roller and frame subsystem set. Provided the occurrence of this, it was deemed necessary to eliminate the influence of the frame on the belt / roller system from the analysis process. Given the results of this analysis, the influence of the frame can only be ignored under certain circumstances.

The influence of the frame can be neglected if the frame is mounted in a simply supported fashion and is not resonated during the operation of the photoreceptor. However, should this not be the case, an analysis of the frame in question must be performed to depict both the magnitude and direction of deflection induced at each roller centerline endpoint. Fortunately, if the frame does indeed interact with the belt / roll system, this does not inherently mean that a commercially available FEA model must be constructed. The implementation of this information as nodal boundary condition inputs will be discussed in
Section 4. For any cantilevered case system, a closed form 1DOF approximation may be used. Since the assumption that the free side plate remains relatively rigid, there can exist no in-plane relative deflection between points in the side plate. This result translates such that the end deflection found for the 1DOF system will in effect be the same deflection at each roll centerline endpoint at the free end. Again, it is this information which must be input into the static analysis as a boundary condition. For the resonating simply supported case, this analysis does not produce valid results. It is assumed that the investment required to construct a full blown FEA model of the frame in order to achieve the relative node deflections at various points in the side plates for input into a coarse estimation program such as this would be unrealistic. If there does exist a resonance in the frame which is excited during operation, the design of the frame should ultimately be revised before implementing any kind of roll or belt interaction calculation, regardless of origin.

3.2. Finite Element Methodology

The Finite Element Method is a mathematical tool for solving Boundary Value Problems (BVP's). Inherently, the results of such an analysis are only as good as the model and the technique used to analysis it. The BVP that is solved here is of the form shown in Eq. 3.1.

\[
L(u) + f = 0 \quad \text{in } \Omega \quad \text{Eq. 3.1a}
\]

\[
A(u) = 0 \quad \text{on } \Gamma \quad \text{Eq. 3.1b}
\]

Where:

- \( L \) is a differential operator
- \( u \) is the dependent variable
- \( f \) is the set of external forces
- \( \Omega \) is the domain under consideration
- \( A \) is the set of boundary conditions
- \( \Gamma \) is the boundary

Utilizing this, there are 4 basic steps inherent to the classical Finite Element Solution.
Step 1: Mesh the Domain

Meshing the domain involves the discretization of a continuous object in space. This discretization is necessary since the Finite Element Method requires the use of infinitesimally small segments. If a continuous body were to be split into these segments, there must be the assumption that the segments have the ability to react as or represent the whole accurately. In fact, not only is this the case, but a minute fraction of the infinite number of elements is needed to do this. The meshing technique takes this into account and provides coordinates to each point, or node, on the body which is of interest. The result is a piece-wise polynomial approximation of the body. The order of the polynomial is predetermined based on the type and function of the object, as well as the level of accuracy required by the model.

Along this methodology, the Finite Element Method returns exact solution values at the nodes themselves. Internodal results can be found using interpolation techniques, an advantage of higher order analysis. However, this can often lead to erroneous results. Therefore it is important to place a node at every critical position in the body. The mesh is the resulting distribution of nodes and the elements that connect them throughout the domain. The deflection of these elements is dominated by the shape functions, $\Psi$, which define them. A brief description of $\Psi$ and the relationship between the shape functions and nodal deflection is shown in Equation set 3.2.

\[
U(x, y) = \sum_{e=1}^{n_e} \hat{U}^e(x, y)
\]

where
\[
\hat{U}^e(x, y) = \sum \psi_i(x, y) U^e_i
\]

and
\[
\hat{U}^e(x, y) = \text{Polynomial Approx. of } U \text{ over element } e
\]

$U^e_i = \text{Nodal Value of } \hat{U}^e$

$\psi_i(x, y) = \text{Shape Function}$
Step 2: The Element Equation

The element equation is derived by utilizing the approximation of $U$, called $\hat{U}$, and a series of shape functions $\Psi$. By substituting the polynomial approx. of $U$ into the Partial Differential Equation and applying some kind of minimization, the Element Equation, Equation 3.3, can be reached.

$$\begin{align*}
[K^e][U^e] &= F^e \\
\text{where:} \\
[K^e] &\text{is the Element Connectivity Matrix} \\
[U^e] &\text{is the Displacement array evaluated at each node} \\
[F^e] &\text{is the Combined Forcing and Restraint condition of the system}
\end{align*}$$

Common types of minimization include functional forms such as the Raleigh-Ritz method and Differential Equation forms such as the Weighted Residual method. The functional form is a numeric calculation for the minimization of Global Energy. The Weighted Residual Method is a localized differential solution to the Equilibrium State of the system. It is this local differential equilibrium which is of interest here.

Returning to the FEM equation 3.1a and approximating $U$ as follows in Equation set 3.4, results in an equation for $R$.

$$U \equiv \hat{U} = \psi_0 + \sum_{i=1}^{n} \psi_i c_i$$

Results in

$$L(\hat{u}) + F = R \neq 0$$

The Residual "$R$" is the error imposed by the approximation. If this error is minimized, then the approximation can be said to accurately represent the system in question. The Weighted Residual method of doing this is represented in Equation 3.5 below.
The representation of phi (\(\phi\)) introduces another split in the analysis decision path. There exists three main forms of phi called the Galerkin, Least Square, and Collocation methods. The definitions of each are shown in Equation 3.6.

\[
\phi_j = \psi_j \quad \text{Galerkin Method} \quad \text{Eq. 3.6a}
\]

\[
\phi_j = \frac{\partial R}{\partial c_j} \\
\int_0^1 \phi_j R dx = \int_0^1 \frac{\partial R}{\partial c_j} R dx \quad \text{Least Squares Method Eq. 3.6b}
\]

\[
= \frac{1}{2} \frac{\partial}{\partial c_j} \int R^2 dx
\]

\[
\phi_j = \delta(x - x_j) \quad \text{Collocation Method} \quad \text{Eq. 3.6c}
\]

The Galerkin Method is the method of choice for this analysis due to its overall capacity for accuracy and repeatability. The Galerkin process is a basic step by step numerical manipulation that varies only the shape function \(\Psi\) from system to system or order of accuracy. The Combined Weighted Residual Galerkin FEM derivation process is shown for each element introduced in this analysis. Those chosen to be included in the solver appear later in this section while those derived but found inferior in efficiency to those chosen are provided in the Appendix under the heading FEM element derivations.

Step 3: Assembly and Application of Boundary Conditions

Taking the summation of the element equation over the entire domain, noting that \(\Gamma\) will have an effect wherever the edge of that domain is reached, results in the Eq. Set 3.7 below. The method of reducing the summation to a global matrix equation is an extension of the Galerkin Weighted Residual (GWR) Method. The GWR method achieves the system equation on a local or elemental level. The model element must therefore be extended to
each actual element in the system by way of the boundary conditions between them or on the edge of the domain. The result is the global matrix, comprised of element equations interconnected and overlapped to approximate the actual system much like standard building blocks are used to create multitudes of designs.

\[ \sum_{\epsilon=1}^{n_{\epsilon}} [K]_{\epsilon} U_{\epsilon} = \sum F_{\epsilon} \]

Becomes

\[ [K]U = F \]

where

\[ [K] = \text{Global Stiffness Matrix} \]

\[ U = \text{Displacement Vector (Nodal)} \]

\[ F = \text{Force Vector (Nodal)} \]

Step 4: Solve to Get the Displacement Vector \( U \)

Matrix manipulation techniques are implemented to achieve the set \( U \). This solution step only achieves the solution at the nodal points. In order to get a full set of solution values, displacement anywhere, gradients, and error estimations, post processing must be induced.

In order to successfully solve the matrix equations shown in the past few figures, the stiffness matrix must be inverted and multiplied by the boundary conditions. However, the solution restraints of Excel limit the invertable matrix size to 52x52. This number is actually published as 54x54 but some iterative sequencing in formula modification proved otherwise. Microsoft has confirmed that their publication was false. However, in either case, the matrix here is much too large to be solved as it is. Hence, the method of matrix partitioning comes into play. Matrix partitioning is the segmentation of a matrix into submatrices. Each submatrix is then treated as an element in the parent matrix. Having fewer elements, the parent may now be solved via any appropriate means. The submatrices are then solved using established subroutines. The process, taken from Pestel and Leckie [3], is shown in Figure.
A3.28. The end result is the circumvention of calculation limitations and the capability of solving much larger matrices. The arguments used and shown in the spreadsheet are completely linked into this sequence. The inclusion of these matrices was decided based on the simplification of embedded formulas, due to an increased number of steps, and the possible need for this information in the future.

3.3. Simple Component implementation

Once the need for a Finite Element solution was established, the selection process of the type of elements to use began. One of the fundamentals of this analysis was the use of simple components, thus allowing the use of simple elements to represent them. Although the temptation to use advanced and often extravagant analysis methods is often hard to resist, the basic, and in many ways, classical methods are often sufficient for the task. Another point of view might reveal that if a 15 stage mathematical sequence appears hard, one must only look to each stage in its own right to see that, taken one at a time, the sequence is nothing but a simple mathematical sequence. The analysis of photoreceptors is no exception. Appearing complex, with numerous subsystems and interactions, once broken down into simple components, the mathematical model prevails. Further, noting that in the static domain, the effects of the subsystems attached to the rolls is negligible, the system can be segmented into repeating systems of a section of belt attached to a roll. Once this system is generically modeled, it needs only to be copied into the correct orientation and restraint to represent the required system as a whole.

3.3.1. Roll element Derivation

The element selection process for the rollers was initiated with the assumption that the rollers in the analysis act as Euler beams. Further, they are assumed to be isotropic in nature and of uniform geometrical definition. At this time, rotation, runout and conicity are not implemented in this analysis. The generic response equation for beams is shown in Equation 3.8. An example of a discretized roll into 4 beam elements is shown in Figure 3.1.
\[ (EI\dddot{u})'' + \rho A \dddot{u} = f(x,t) \]  
\[ \text{Eq. 3.8} \]

where:

- \( E \) is the Young’s modulus of the roller material
- \( I \) is the Area Moment of Inertia of the cross section of the beam
- \( EI \) is the Flexure Rigidity
- \( f \) is the applied force per unit length (transverse)

Figure 3.1. Example of Beam element with Local vs. Global numbering

*Note the case difference between the Local and Global \( x \) dimension.*

The Finite Element Method allows a displacement field to be derived by endpoint deformations,

\[ U = \sum \hat{U} \]
\[ \hat{U}^e = \sum \psi_i U_i^e \]

\[ \text{Equation set 3.9} \]

where \( \psi \) is the shape function of the element and \( U \) is the displacement of the node.

Utilizing the Galerkin Weighted Residual Method achieves the following:
\[ R = (EI \sum \psi^J U^J) + \rho A \sum \psi^J \dddot{U}^J - f \neq 0 \]

\[ \int_0^h \psi_j R \, dx = 0 \quad \text{for} \quad j = 1, 2, \ldots, n \]

\[ \int_0^h \psi_j (EI \sum \psi^J U^J) \, dx + \int_0^h \psi_j \rho A \sum \psi^J \dddot{U}^J \, dx - \int_0^h \psi_j f \, dx = 0 \]

Int by parts 2x

\[ \int_0^h \psi_j EI \sum \psi^J U^J \, dx + \int_0^h \psi_j \rho A \sum \psi^J \dddot{U}^J \, dx = \big[ - \psi_j (EI \sum \psi^J U^J) + \psi_j EI \sum \psi^J U^J \big]_0^h + \int_0^h \psi_j f \, dx \]

Where

\[ EL \sum \psi_j U_j' = V \]

\[ EI \sum \psi_j U_j = M \]

\[ \sum \int_0^h \psi_j EI \psi_j \, dx U_j' + \sum \int_0^h \psi_j \rho A \psi_j \, dx \dddot{U}^J = \big[ - \psi_j V + \psi_j M \big]_0^h + \int_0^h \psi_j f \, dx \]

Where

\[ \int_0^h \psi_j EI \psi_j \, dx U_j' = K^J_j \]

\[ \int_0^h \psi_j \rho A \psi_j \, dx \dddot{U}^J = M^J_j \]

\[ - \psi_j V \psi_j M \big]_0^h = b_j \]

\[ \int_0^h \psi_j f \, dx = f_j \]

\[ \sum K^J_j U_j' + \sum M^J_j \dddot{U}^J = b_j + f_j \]

\[ [K^J \dddot{U}^J + [M^J \dddot{U}^J = b + f^J \]

Stiffness Matrix K, Mass Matrix M, Boundary Vector b, and Force Vector f

For further information, one can refer to reference [1]

The resulting equation represents the full dynamic response of the beam element in matrix notation. Since the current application is static, the acceleration term \( \dddot{U}^J \) may be set to zero. This equation set will be revisited however when the dynamic portion of these analyses are encountered. In order to implement the above beam derivation, the order of the finite element to be used must be specified. Linear, quadratic and cubic element derivations follow here as examples of such elements to be used.
3.3.1.1. 1st order Roller Element implementation

The element shown in Figure 3.2 represents a first order deflection beam element. Following steps 1 and 2 of the Finite Element method process yields the shape functions for linear elements:

Step 1: Mesh generation and Function approximation

*Global Coord. X Local Node 1, U₁ Local Node 2, U₂
Local Coord. x

![Figure 3.2. Element definition for 1st Order Element of Length h](image)

Step 2. Determining the Element Equation

In standard form:
\[ U' = \sum \psi_i U_i \]

In mathematical form:
\[ U' = a_0 + a_1 x \]

Boundary conditions verified:
\[ x = 0 \rightarrow U' = U'_1 = a_0 \]
\[ x = h \rightarrow U' = U'_2 = a_0 + a_1 h \]
\[ a_0 = U'_1, \quad a_1 = \left( \frac{U'_2 - U'_1}{h} \right) \]

Returning to the mathematical form...
\[ U' = U'_1 + \left( \frac{U'_2 - U'_1}{h} \right) x \]
\[ = \left( 1 - \frac{x}{h} \right) U'_1 + \left( \frac{x}{h} \right) U'_2 \]
\[ U' = \psi_1 U'_1 + \psi_2 U'_2 = \sum_{i=1}^{2} \psi_i U'_i \]

Hence:
\[ \psi_1 = 1 - \left( \frac{x}{h} \right) \]
\[ \psi_2 = \left( \frac{x}{h} \right) \]

Equation set 3.11
3.3.1.2. 2\textsuperscript{nd} order Roller Element implementation

The element shown in Figure 3.3 represents a second order deflection beam element. Following the same approach as the linear shape function derivation, the quadratic element is defined as follows;

Step 1: Mesh generation and Function approximation

![Element definition for 2\textsuperscript{nd} Order Element of Length 2h](image)

Figure 3.3. Element definition for 2\textsuperscript{nd} Order Element of Length 2h
Step 2. Determining the Element Equation

\[ \hat{U} = a_0 + a_1 x + a_2 x^2 \]

Cast in the form

\[ \hat{U} = \sum_{i=1}^{3} \psi_i U_i^e \]

with Boundary Conditions

\[
\begin{align*}
    x = -h & \quad \hat{U} = U_1^e = a_0 - a_1 h + a_2 h^2 \\
    x = 0 & \quad \hat{U} = U_2^e = a_0 \\
    x = h & \quad \hat{U} = U_3^e = a_0 + a_1 h + a_2 h^2 \\
    a_0 &= u_2^e \\
    a_1 &= \frac{U_3^e - U_1^e}{2h} \\
    a_2 &= \frac{U_1^e + U_3^e - 2U_1^e}{2h^2}
\end{align*}
\]

returning to the mathematical form

\[ \hat{U} = \sum_{i=1}^{3} \psi_i U_i^e \]

results in;

\[
\begin{align*}
    \psi_1 &= -\frac{1}{2} \left( \frac{x}{h} \right) \left( 1 - \frac{x}{h} \right) \\
    \psi_2 &= 1 - \left( \frac{x}{h} \right)^2 \\
    \psi_3 &= \frac{1}{2} \left( \frac{x}{h} \right) \left( 1 + \frac{x}{h} \right)
\end{align*}
\]
3.3.1.3. $3^{rd}$ order Roller Element implementation

The element shown in Figure 3.4 represents a third order deflection beam element. Once again following the same approach, the cubic element is defined as follows:

Step 1: Mesh generation and Function approximation

*Global Coord. $X$

Local Coord. $x$

$U_1^e = U_1$

$U_2^e = U_1^e$ \textit{Coincident Elements}

$U_3^e = U_1$ \textit{Must Share Slopes}

$U_4^e = U_2^e$

Figure 3.4. \textit{Element definition for $3^{rd}$ Order Element of Length $h$}
Step 2. Determining the Element Equation

\[ \dot{U}' = a_0 + a_1 x + a_2 x^2 + a_3 x^3 \]
\[ \frac{\partial \dot{U}'}{\partial x} = a_1 + 2a_2 x + 3a_3 x^2 \]

Cast in the form:

\[ \dot{U}' = \sum_{i=1}^{4} \psi_i(x)U_i' \]

With Boundary Conditions:

\[ x = 0 \quad \begin{cases} \dot{U}' = U_1' = a_0 \\ \psi_1 = 0 \end{cases} \]
\[ x = h \quad \begin{cases} \dot{U}' = U_2' = a_0 + a_1 h + a_2 h^2 + a_3 h^3 \\ \psi_2 = 0 \end{cases} \]

Returning to:

\[ \dot{U}' = \sum_{i=1}^{4} \psi_i(x)U_i' \]

and solving for the shape functions yields:

\[ \psi_1(x) = 1 - 3\left( \frac{x}{h} \right)^2 + 2\left( \frac{x}{h} \right)^3 \]
\[ \psi_2(x) = x \left( 1 - \frac{x}{h} \right)^2 \]
\[ \psi_3(x) = 3\left( \frac{x}{h} \right)^2 - 2\left( \frac{x}{h} \right)^3 \]
\[ \psi_4(x) = x \left( \frac{x}{h} \right)^2 - \left( \frac{x}{h} \right)^3 \]

\[
\begin{array}{cccc}
derivative & 0 & 1 & 2 \\
\Psi_1 & 1-3*(x/h)^2+2*(x/h)^3 & (6/h^2)*(-x+x^2/h) & (6/h^2)*(-1+2*x/h) \\
\Psi_2 & x-2*(x^2/h)+x^2*(x/h)^2 & 1-4x/h+3*(x/h)^2 & (2/h)*(2+3*x/h) \\
\Psi_3 & 3*(x/h)^2-2*(x/h)^3 & (6/h^2)*(x-x^2/h) & (6/h^2)*(1-2*x/h) \\
\Psi_4 & -x*(x/h)+x^2*(x/h)^2 & -2x/h+3*(x/h)^2 & (2/h)*(-1+3*x/h) \\
\end{array}
\]

\[
[K^e] = \frac{El}{h^3} \begin{vmatrix} 12 & 6h & -12 & 6h \\ 6h & 4h^2 & -6h & 2h^2 \\ -12 & -6h & 12 & -6h \\ 6h & 2h^2 & -6h & 4h^2 \end{vmatrix}
\]
3.3.2. Belt Element Derivation

The element selection process for the belt elements was not as clear cut as that for the rollers. The geometry of the belt represents a flat plate, where its thickness is much less than any other dimension. Due to the thin x-section, the belt is adequately modeled as both membranes out-of-plane and shells in-plane. This ambiguity is further confused by the existence of competing families of quad elements. These families are broken down into sets of variable order elements with a general trend or ideal by which they are defined. To illustrate this, Figure 3.5 and 3.6 portray the differences between the most popular, the LaGrange and Serendipity families.

Noting that the primary difference in these families is the use of a center node, the decision was made that the center node does not contribute significantly to the analysis at hand. The center of the serendipity element does indeed transfer both deflection and force through the element. However, this force is not allowed to be the maximum in the element. Considering the method by which the belt is nominally loaded, by an end deflection in contact with the rolls, this center would not be of specific need or interest. The use of the center node is most notably sacrificed in the external loading of the belt and post processing of the results. The external loading of the belt must be weighted to account for an alternating number of nodes across the element dimension. This nodal configuration for 2nd order quads is shown in Figure 3.6. Lastly, in order to achieve deflection information at the node center, post processing of the deflection results must occur. This not only results in a loss of accuracy in the element interior, but also restricts the resolution of the deflection plots in Microsoft Excel. Despite these limitations, the serendipity elements do have one primary advantage over their Lagrangian counterparts. The use of 8 nodes instead of 9 can significantly reduce computational time and increase efficiency in a large global solution. Given this situation, it was deemed appropriate to utilize the serendipity family for all belt element derivations.
Figure 3.5. Serendipity(l) and Lagrange(r) 2nd order quad element deflections [ref 1]
The belt section derivations have a tendency to get quite involved. For this reason, the following explanation to the thought process for the decisions made is included.

In photoreceptor modules, the belt material is usually a Mylar composite. Regardless whether the actual belt is Mylar or not however, the material properties of the belt, primarily the stiffness, are orders of magnitude lower than that of the rest of the system. This stiffness ratio causes numerical problems even in the most advanced Finite Element codes such as MSC/NASTRAN and HKS/ABAQUS because of the ill-conditioned global stiffness matrices which it creates. Hence, in order to accomplish a low level FEA model which can remain in the required accuracy band, a good deal of thought must be given to how this anomaly would be treated.

Two dimensional Finite Element theory yields that the deflection of a rectangular shaped object can be found using the same shape function-based four-step process as that used for the rollers. However, the unique properties of the belt cause out-of-plane deflection to become an issue when arbitrary loading configurations are accounted for. The out-of-plane deflection, due to the relative low stiffness, is assumed to act as a membrane restricted about all four edges of the rectangular element. The assumption does not allow that the actual

\[ \psi_1 = \frac{1}{4} (1 - 2 \xi^2 (1 - \eta^2) - \eta) \]
\[ \psi_2 = \frac{1}{4} (1 - 2 \xi^2 (1 - \eta^2) + \eta) \]
\[ \psi_3 = \frac{1}{4} (1 + 2 \xi (1 - \eta^2) - \eta) \]
\[ \psi_4 = \frac{1}{4} (1 + 2 \xi (1 - \eta^2) + \eta) \]
\[ \psi_5 = \frac{1}{4} (1 - 2 \eta^2 (1 - \xi^2) - \xi) \]
\[ \psi_6 = \frac{1}{4} (1 - 2 \eta^2 (1 - \xi^2) + \xi) \]
\[ \psi_7 = \frac{1}{4} (1 + 2 \eta (1 - \xi^2) - \xi) \]
\[ \psi_8 = \frac{1}{4} (1 + 2 \eta (1 - \xi^2) + \xi) \]

Figure 3.6. Serendipity (bottom) and LaGrange (top) \( \Psi'' \) def. [ref 1]
system is only restricted on two of its four edges, when in contact with the rollers. This configuration, although not exactly representative of the real system, allows for order of magnitude accuracy of the belt deflection out of the element plane. As shown in Equation 3.14 [1], the deflection equations for all three dimensions are of the same form. The coefficient "a" allows for the change in relative stiffness either as a membrane or inplane isotropic resistance. Since the form of these equations is identical, the element deflection derivation need only be shown once. The resulting shape functions and stiffness matrices developed are of the same order and magnitude. Again, the only difference is the multiplier of dimensional stiffness.

\[-a\left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2}\right) = f(x, y)\]  

Eq. 3.14

In the interest of providing a global sense of element choice, and to show the relative efficiency of the decisions made, multiple order elements were derived. Linear, quadratic, cubic, and quadratic/cubic transition elements were developed. The choice of belt element to use will depend on the amount of accuracy needed along with the number of degrees of freedom allowed by the software package. For the sake of organization and the purely mathematical sense of the derivations, only the 2nd order element is shown here. Once again, the full set of belt derivation can be found in the Appendix section under the heading FEM Element Derivations.

Recalling the Galerkin Finite Element Method for achieving a matrix solution, the derivation of the belt elements can be modeled using a predetermined format, shown below in Equation set 3.15. The variable U represents the deflection in the dimension currently being analyzed. Recall that each dimension of the analysis is solved independently as a scalar and then combined during post processing. The error associated with this method is derived from a limited iteration scheme. Greater accuracy is inversely related to calculation time and iteration number, and was therefore minimized. For the transverse deflection analysis, \(f(x,y)\) is an out of plane force that may be specified as a function of location within the quad elements. The in plane analysis uses a shell interpretation of the same process. The resulting element equation is the same with the exception of the values of the stiffness matrix [\(K\)].
$U = \sum_{i=1}^{n} U^e$

$\hat{U}^e = \sum_{i=1}^{n} \psi_i U^e$

$-a\nabla^2 U = f(x, y)$

$R = -a\nabla^2 U - f(x, y) \neq 0$

$\int_{\Omega} \psi_i (-a\nabla^2 U - f(x, y)) d\Omega = 0$

$\int G\nabla^2 F = -\int \nabla G \cdot \nabla F d\Omega + \oint_{\partial \Omega} G \nabla F d\Gamma$

$a \int_{\Omega} \nabla \psi_i \cdot \nabla \hat{U} d\Omega - a \int \psi_i \left( \hat{n} \cdot \nabla \hat{U} \right) d\Gamma - \int \psi_i f(x, y) d\Omega = 0$

$\sum_{i} \int_{\Omega} \left( \frac{\partial \psi_i}{\partial x} \frac{\partial \psi_j}{\partial x} + \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_j}{\partial y} \right) d\Omega U^e_i = \int \psi_i f(x, y) d\Omega + a \oint \psi_i \left( n_x \frac{\partial \hat{U}}{\partial x} + n_y \frac{\partial \hat{U}}{\partial y} \right) d\Gamma$

element equation

$[K]_{i4} U^e_i = F^e_i + b^e_i$

$K^e_{ij} = a \int_{\Omega} \left( \frac{\partial \psi_i}{\partial x} \frac{\partial \psi_j}{\partial x} + \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_j}{\partial y} \right) d\Omega$

$F^e_i = \int \psi_i f(x, y) d\Omega$

$b^e_i = a \oint \psi_i \left( n_x \frac{\partial \hat{U}}{\partial x} + n_y \frac{\partial \hat{U}}{\partial y} \right) d\Gamma$

The term $d\Omega$ is expressed using the Jacobian "$J$"

$dx dy = |J| d\zeta d\eta$

$[J] = \begin{bmatrix}
\frac{\partial X}{\partial \zeta} & \frac{\partial X}{\partial \eta} \\
\frac{\partial Y}{\partial \zeta} & \frac{\partial Y}{\partial \eta}
\end{bmatrix}$

also

$\begin{pmatrix}
\frac{\partial \psi_i}{\partial x} \\
\frac{\partial \psi_i}{\partial y}
\end{pmatrix} = [J]^{-1} \begin{pmatrix}
\frac{\partial \psi_i}{\partial \zeta} \\
\frac{\partial \psi_i}{\partial \eta}
\end{pmatrix}$

Equation set 3.15
3.3.2.1. 2\textsuperscript{nd} order Serendipity quadrilateral element derivation

Figure 3.7 below portrays the global layout of elements for each belt section. The individual elements are of the form shown in Figure 3.8.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{Figure3_7}
\caption{Global layout of elements for each belt section.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{Figure5_8}
\caption{Individual Quadratic Belt Element definition}
\end{figure}
Recall from Equation set 3.15.

\[ K_\psi^{\psi} = a \int_{\Omega} \left( \frac{\partial \psi_i}{\partial x} \frac{\partial \psi_j}{\partial x} + \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_j}{\partial y} \right) J d\xi d\eta \]

In order to find the resulting elemental stiffness matrix, the Jacobian, \([J]\), must first be found. The elements of \(\text{"}J\text{"}\) are derived for the element layout, shown in Figure 3.7, in Equation set 3.16.

\[
\frac{\partial x}{\partial \xi} = \sum_{i=1}^{s} \frac{\partial \psi_i}{\partial x} x_i,
\]

\[
= \frac{1}{4} \left[ (2\zeta + \eta - 2\eta \zeta - \eta^2)x_1 + 4(\eta - 1)\zeta x_2 + (2\zeta - \eta - 2\eta \zeta + \eta^2)x_3 + 2(\eta^2 - 1)x_4 + 2(1 - \eta^2)x_5 + \right] - \frac{1}{4} \left[ (2\zeta - \eta + 2\eta \zeta - \eta^2)x_6 + 4(1 + \eta)\zeta x_7 + (2\zeta + \eta + 2\eta \zeta + \eta^2)x_8 \right]
\]

\[
= \frac{1}{4} \left[ 2\zeta(x_1 + x_3 + x_6 + x_8) + \eta(x_1 - x_3 - x_6 + x_8) + 2\eta \zeta(-x_1 + x_3 + x_6 + x_8) + \eta^2(-x_1 + x_3 + x_6 + x_8) + \right] - \frac{1}{4} \left[ 4\eta \zeta x_2 - 4\zeta x_3 + 2\eta^2 x_4 - 2x_4 + 2x_5 - 2x_6 - 4\zeta x_7 - 4\eta \zeta x_7 \right]
\]

\[
= \frac{1}{4} \left[ 4\eta^2 h + \eta \zeta(4x_2 - 4x_7) + \zeta(-4x_2 - 4x_7) + \eta^2(2x_4 - 2x_5) + 4h \right] = \eta^2 h + h - \eta^2 h
\]

\[
= h
\]

Equation set 3.16

\[
\frac{\partial x}{\partial \eta} = \sum_{i=1}^{s} \frac{\partial \psi_i}{\partial \eta} x_i
\]

\[
= \frac{1}{4} \left[ (2\eta + \zeta - 2\eta \zeta - \zeta^2)x_1 + 2(\zeta^2 - 1)x_2 + (2\eta - \zeta - \zeta^2 + 2\eta \zeta)x_3 + 4(\zeta - 1)\eta x_4 - \right] - \frac{1}{4} \left[ 4(1 + \zeta)\eta x_5 + (2\eta - \zeta + \zeta^2 - 2\eta \zeta)x_6 + 2(1 - \zeta^2)x_7 + (2\eta + \zeta + \zeta^2 + 2\eta \zeta)x_8 \right]
\]

\[
= \frac{1}{4} \left[ 2\eta(x_1 + x_3 + x_6 + x_8) + \zeta(x_1 - x_3 - x_6 + x_8) + 2\eta \zeta(-x_1 + x_3 - x_6 + x_8) + \right] - \frac{1}{4} \left[ \zeta^2(-x_1 - x_3 + x_6 + x_8) + 2\zeta^2 x_2 - 2x_2 + 8\eta \zeta x_4 - 4\eta x_4 - 4\eta x_5 - 4\eta \zeta x_5 + 2x_7 - 2\zeta^2 x_7 \right]
\]

\[
= \frac{1}{4} \left[ 8\eta \zeta h + \zeta^2(2x_2 - 2x_7) + \eta \zeta(4x_4 - 4x_7) - 2x_2 + 2x_7 \right] = 2\eta \zeta h - 2\eta \zeta h
\]

\[
= 0
\]
\[
\frac{\partial y}{\partial \zeta} = \sum_{i=1}^{s} \frac{\partial \psi_i}{\partial \zeta} y_i
\]
\[
= \frac{1}{4} \left[ (2\zeta + \eta - 2\eta\zeta - \eta^2) y_1 + 4(\eta - 1)\zeta y_2 + (2\zeta - \eta - 2\eta\zeta + \eta^2) y_3 + 2(\eta^2 - 1) y_4 + 2(1 - \eta^2) y_5 + \right]
\]
\[
= \frac{1}{4} \left[ (2\zeta - \eta + 2\eta\zeta - \eta^2) y_6 - 4(1 + \eta)\zeta y_7 + (2\zeta + \eta + 2\eta\zeta + \eta^2) y_8 \right]
\]
\[
= \frac{1}{4} \left[ 2(\zeta(y_1 + y_3 + y_6 + y_8) + \eta(y_1 - y_3 - y_6 - y_8) + 2\eta\zeta(-y_1 - y_3 + y_6 - y_8) + \eta^2(-y_1 - y_3 - y_6 + y_8) + \right]
\]
\[
= \frac{1}{4} \left[ 4\eta\zeta y_2 - 4\zeta y_2 + 2\eta^2 y_4 - 2y_4 + 2y_5 - 2\eta^2 y_4 - 4\zeta y_7 - 4\eta\zeta y_7 \right]
\]
\[
= \frac{1}{4} [8\eta\zeta - 8\eta\zeta]
\]
\[
= 0
\]

Equation set 3.16 cont.

\[
\frac{\partial y}{\partial \eta} = \sum_{i=1}^{s} \frac{\partial \psi_i}{\partial \eta} y_i
\]
\[
= \frac{1}{4} \left[ (2\eta + \zeta - 2\eta\zeta - \zeta^2) y_1 + 2(\zeta^2 - 1) y_2 + (2\eta - \zeta - \zeta^2 + 2\eta\zeta) y_3 + 4(\zeta - 1)\eta y_4 - \right]
\]
\[
= \frac{1}{4} \left[ (2\eta - \zeta + \zeta^2 - 2\eta\zeta) y_6 + 2(1 - \zeta^2) y_7 + (2\eta + \zeta + \zeta^2 + 2\eta\zeta) y_8 \right]
\]
\[
= \frac{1}{4} \left[ 2\eta y_1 + y_3 + y_6 + y_8) + \zeta(y_1 - y_3 - y_6 + y_8) + 2\eta\zeta(-y_1 + y_3 - y_6 + y_8) + \right]
\]
\[
= \frac{1}{4} \left[ \zeta^2(-y_1 - y_3 + y_6 - y_8) + 2\zeta^2 y_2 + 2y_2 + 8\eta\zeta y_4 - 4\eta y_4 - 4\eta y_5 - 4\eta y_8 + 2y_7 - 2\zeta^2 y_7 \right]
\]
\[
= \frac{1}{4} [4\zeta^2 l + 2(y_2 - y_2) + 2\zeta^2(y_2 - y_2)] = \zeta^2 l + l - \zeta^2 l
\]
\[
= l
\]

The resulting Jacobian matrix is therefore reduced to the form in Equation 3.17:

\[
J = \begin{bmatrix}
    h & 0 \\
    0 & l \\
\end{bmatrix}
\]

Eq. 3.17

Close examination reveals that this result parallels the actual element definition. This is a direct correlation of the assembly methodology of using repetitive components. By achieving this generic form for the Jacobian, its variable form can be placed into an automated spreadsheet solution. The variable form of the Jacobian is shown in Equation set 3.18. Also in this set are the magnitude and inverse representations of the matrix.
\[ J = \begin{bmatrix} \frac{\Delta x}{2} & 0 \\ 0 & \frac{\Delta y}{2} \end{bmatrix} \]

\[ |J| = hl \quad \text{Eq. 3.18} \]

\[ J^{-1} = \frac{1}{hl} \begin{bmatrix} l & 0 \\ 0 & h \end{bmatrix} \]

Once the Jacobian is established, the equation for elemental stiffness may be updated. This is shown in Equation set 3.19. The variable "a" defines the directional stiffness of the element as a function of orientation, i.e. membrane stiffness for out of plane [1].

\[
\frac{\partial \psi_i}{\partial x} = [J]^{-1} \begin{bmatrix} \frac{\partial \psi_i}{\partial \zeta} \\ \frac{\partial \psi_i}{\partial \eta} \end{bmatrix} \\
\frac{\partial \psi_i}{\partial y} = \frac{l}{hl} \frac{\partial \psi_i}{\partial \zeta} = \frac{1}{h} \frac{\partial \psi_i}{\partial \xi} \\
\frac{\partial \psi_i}{\partial y} = \frac{h}{hl} \frac{\partial \psi_i}{\partial \eta} = \frac{1}{l} \frac{\partial \psi_i}{\partial \eta} 
\]

Equation set 3.19

\[
K_{\psi}^\psi = a \int_\zeta \int_\eta \left( \frac{\partial \psi_i}{\partial x} \frac{\partial \psi_j}{\partial x} + \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_j}{\partial y} \right) |J| \partial \zeta \partial \eta \\
= a \int_\zeta \int_\eta \left( \frac{1}{h^2} \frac{\partial \psi_i}{\partial \xi} \frac{\partial \psi_j}{\partial \xi} + \frac{1}{l^2} \frac{\partial \psi_i}{\partial \eta} \frac{\partial \psi_j}{\partial \eta} \right) |hl| \partial \zeta \partial \eta 
\]

The next stage in stiffness matrix representation is the manipulation of the shape functions \( \Psi_i \). This manipulation requires that the partial derivatives of each shape function be taken with respect to the elemental coordinates \( \eta \) and \( \zeta \). For reference purposes only, it is noted that the shape functions within the element are symmetrically related. This relationship is what allows the elements to be derived independent of orientation and placement with respect to dimension. The generic forms of the shape functions are shown in Equation set
3.20 while the manual derivations of the partial derivatives of each is shown in Equation set 3.21.

For $i = 1, 3, 6, 8$ (corner nodes)

$$\psi_i = \frac{1}{4} (1 + \xi \eta_i)(1 + \eta \xi_i)(\xi \eta_i + \eta \xi_i - 1)$$

For $i = 2, 7$

$$\psi_i = \frac{1}{2} (1 - \xi^2)(1 + \eta \xi_i)$$  \quad \text{Equation set 3.20}

For $i = 4, 5$

$$\psi_i = \frac{1}{2} (1 + \xi \eta_i)(1 - \eta^2)$$
Equation set 3.21 - Psi function derivatives manual exploration

\[ \psi_1 = \frac{1}{4} (1 - \zeta)(1 - \eta)(-1 - \xi - \eta) \]
\[ = \frac{1}{4} (1 - \zeta - \eta + \eta \xi)(-1 - \xi - \eta) \]
\[ = \frac{1}{4} (-1 - \zeta - \eta + \zeta^2 + \eta \xi + \eta + \eta \xi + \eta^2 - \eta \xi - \xi^2 \eta - \xi \eta^2) \]
\[ = \frac{1}{4} (-1 + \zeta^2 + \eta^2 + \eta \xi - \xi^2 \eta - \xi \eta^2) \]
\[ \frac{\partial \psi_1}{\partial \xi} = \frac{1}{4} (2 \zeta + \eta - 2 \eta \xi - \eta^2) \]
\[ \frac{\partial \psi_1}{\partial \eta} = \frac{1}{4} (2 \eta + \zeta - 2 \eta \xi - \xi^2) \]

\[ \psi_2 = \frac{1}{2} (1 - \zeta^2)(1 - \eta) \]
\[ \frac{\partial \psi_2}{\partial \xi} = \frac{1}{2} (1 - \eta)(-2 \zeta) = (\eta - 1) \zeta \]
\[ \frac{\partial \psi_2}{\partial \eta} = \frac{1}{2} (\zeta^2 - 1) \]

\[ \psi_3 = \frac{1}{4} (1 + \zeta)(1 - \eta)(-1 + \xi + \eta) \]
\[ = \frac{1}{4} (1 + \zeta - \eta - \eta \xi)(-1 + \xi - \eta) \]
\[ = \frac{1}{4} (-1 + \zeta - \eta - \zeta^2 - \eta \xi + \eta - \eta \xi + \eta^2 + \eta \xi - \eta \xi^2 + \eta^2 \xi) \]
\[ = \frac{1}{4} (-1 + \zeta^2 + \eta^2 - \eta \xi - \eta \xi^2 + \eta^2 \xi) \]
\[ \frac{\partial \psi_3}{\partial \xi} = \frac{1}{4} (2 \zeta - \eta - 2 \eta \xi + \eta^2) \]
\[ \frac{\partial \psi_3}{\partial \eta} = \frac{1}{4} (2 \eta - \zeta - \zeta^2 + 2 \eta \xi) \]
\[ \psi_4 = \frac{1}{2}(1 - \zeta)(1 - \eta^2) \]
\[ \frac{\partial \psi_4}{\partial \zeta} = \frac{1}{2}(\eta^2 - 1) \]
\[ \frac{\partial \psi_4}{\partial \eta} = (\zeta - 1)\eta \]

\[ \psi_5 = \frac{1}{2}(1 + \zeta)(1 - \eta^2) \]
\[ \frac{\partial \psi_5}{\partial \zeta} = \frac{1}{2}(1 - \eta^2) \]
\[ \frac{\partial \psi_5}{\partial \eta} = -(1 + \zeta)\eta \]

\[ \psi_6 = \frac{1}{4}(1 - \zeta)(1 + \eta)(-1 - \zeta + \eta) \]
\[ = \frac{1}{4}(1 - \zeta + \eta - \eta^2)(-1 - \zeta + \eta) \]
\[ = \frac{1}{4}(-1 - \zeta + \eta + \zeta^2 - \eta^2\zeta - \eta - \eta^2\zeta + \eta^2 + \eta^2\zeta + \eta^2\zeta^2 - \eta^2\zeta) \]
\[ = \frac{1}{4}(-1 - \zeta^2 + \eta^2 - \eta\zeta + \eta\zeta^2 - \eta^2\zeta) \]
\[ \frac{\partial \psi_6}{\partial \zeta} = \frac{1}{4}(2\zeta - \eta + 2\eta^2\zeta - \eta^2) \]
\[ \frac{\partial \psi_6}{\partial \eta} = \frac{1}{4}(2\eta - \zeta + \zeta^2 - 2\eta\zeta) \]

\[ \psi_7 = \frac{1}{2}(1 - \zeta^2)(1 + \eta) \]
\[ \frac{\partial \psi_7}{\partial \zeta} = -(1 + \eta)\zeta \]
\[ \frac{\partial \psi_7}{\partial \eta} = \frac{1}{2}(1 - \zeta^2) \]

Eq set 3.21 cont.
\[
\psi_s = \frac{1}{4} (1 + \zeta)(1 + \eta)(-1 + \zeta + \eta) \\
= \frac{1}{4} (1 + \zeta + \eta + \eta \zeta)(-1 + \zeta + \eta) \\
= \frac{1}{4} (-1 + \zeta + \eta - \zeta + \zeta^2 + \eta \zeta - \eta + \eta \zeta + \eta^2 - \eta \zeta + \eta \zeta^2 + \eta^2 \zeta) \\
= \frac{1}{4} (-1 + \zeta^2 + \eta^2 + \eta \zeta + \eta \zeta^2 + \eta^2 \zeta) \\
\text{Eq set 3.21 cont.}
\]

\[
\frac{\partial \psi_s}{\partial \zeta} = \frac{1}{4} (2 \zeta + \eta + 2 \eta \zeta + \eta^2) \\
\frac{\partial \psi_s}{\partial \eta} = \frac{1}{4} (2 \eta + \zeta + \zeta^2 + 2 \eta \zeta)
\]

Once the partial derivatives are formulated, they must be input into the stiffness algorithm and solved as a function of stiffness matrix element location, i.e. ij designation. For mathematical efficiency purposes, the software program MAPLE V was used to derive these matrix values. The MAPLE V results are shown below as Equation sets 3.23-3.25. Please note that the notation used was as shown in Equation 3.22.

\[
y_{1z} = \frac{\partial \psi_1}{\partial \zeta} \\
\text{Eq. 3.22}
\]
\[
\begin{align*}
y_{1z} & := \frac{1}{4} \left( 2z + n - 2n z - n^2 \right) \\
y_{2z} & := z(n - 1) \\
y_{3z} & := \frac{1}{4} \left( 2z - n - 2n z + n^2 \right) \\
y_{4z} & := \frac{1}{2} (n - 2) \\
y_{5z} & := \frac{1}{2} \left( 1 - n^2 \right) \\
y_{6z} & := \frac{1}{4} \left( 2z - n + 2n z - n^2 \right) \\
y_{7z} & := -z(1 + n) \\
y_{8z} & := \frac{1}{4} \left( 2z + n + 2n z + n^2 \right) \\
y_{1n} & := \frac{1}{4} \left( 2n - n z - 2n z + z^2 \right) \\
y_{2n} & := \frac{1}{2} (z - 2) \\
y_{3n} & := \frac{1}{4} \left( 2n - z - z^2 + 2n z \right) \\
y_{4n} & := n(z - 1) \\
y_{5n} & := -n(1 + z) \\
y_{6n} & := \frac{1}{4} \left( 2n - z + z^2 - 2n z \right)
\end{align*}
\]
Recalling the elemental stiffness matrix equation, the Maple solution is shown in Figures 3.11 – 3.17.

\[
K_{ij}^e = a \int \left( \frac{\partial \psi_i}{\partial x} \frac{\partial \psi_j}{\partial x} + \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_j}{\partial y} \right) \left| J \right| d\zeta d\eta
\]

\[
= a \int \left( \frac{1}{h^2} \frac{\partial \psi_i}{\partial \zeta} \frac{\partial \psi_j}{\partial \zeta} + \left( \frac{1}{l^2} \frac{\partial \psi_i}{\partial \eta} \frac{\partial \psi_j}{\partial \eta} \right) \right) h l d\zeta d\eta
\]

Equation 3.24  MAPLE V solution verification and initial stiffness results
> k13:=int(int(((1/h^2)*y1z*y3z+(1/1^2)*(y1n*y3n)),z=-1..1),n=-1..1)

\[ k13 := \frac{1}{720} \left( \frac{37 \ell^2 + 68 h^2}{\ell^2 h^2} + \frac{17}{720} \frac{11 \ell^2 + 4 h^2}{\ell^2 h^2} \right) \]

> k14:=int(int(((1/h^2)*y1z*y4z+(1/1^2)*(y1n*y4n)),z=-1..1),n=-1..1)

\[ k14 := \frac{1}{720} \left( \frac{21 \ell^2 + 200 h^2}{\ell^2 h^2} - \frac{1}{720} \frac{69 \ell^2 + 440 h^2}{\ell^2 h^2} \right) \]

> k15:=int(int(((1/h^2)*y1z*y5z+(1/1^2)*(y1n*y5n)),z=-1..1),n=-1..1)

\[ k15 := \frac{1}{720} \left( \frac{-21 \ell^2 + 160 h^2}{\ell^2 h^2} - \frac{1}{720} \frac{69 \ell^2 + 160 h^2}{\ell^2 h^2} \right) \]

> k16:=int(int(((1/h^2)*y1z*y6z+(1/1^2)*(y1n*y6n)),z=-1..1),n=-1..1)

\[ k16 := \frac{1}{90} \frac{17 \ell^2 + 28 h^2}{\ell^2 h^2} \]

> k17:=int(int(((1/h^2)*y1z*y7z+(1/1^2)*(y1n*y7n)),z=-1..1),n=-1..1)

\[ k17 := \frac{2}{45} \frac{-5 \ell^2 + 3 h^2}{\ell^2 h^2} - \frac{1}{45} \frac{10 \ell^2 + 9 h^2}{\ell^2 h^2} \]

> k18:=int(int(((1/h^2)*y1z*y8z+(1/1^2)*(y1n*y8n)),z=-1..1),n=-1..1)

\[ k18 := \frac{23}{90} \frac{\ell^2 + h^2}{\ell^2 h^2} \]

> k22:=int(int(((1/h^2)*y2z*y2z+(1/1^2)*(y2n*y2n)),z=-1..1),n=-1..1)

\[ k22 := \frac{2}{45} \frac{5 \ell^2 - 6 h^2}{\ell^2 h^2} + \frac{2}{45} \frac{35 \ell^2 + 6 h^2}{\ell^2 h^2} \]

> k23:=int(int(((1/h^2)*y2z*y3z+(1/1^2)*(y2n*y3n)),z=-1..1),n=-1..1)

\[ k23 := \frac{1}{45} \frac{-5 \ell^2 + 6 h^2}{\ell^2 h^2} + \frac{1}{45} \frac{-35 \ell^2 + 9 h^2}{\ell^2 h^2} \]

> k24:=int(int(((1/h^2)*y2z*y4z+(1/1^2)*(y2n*y4n)),z=-1..1),n=-1..1)

\[ k24 := 0 \]

> k25:=int(int(((1/h^2)*y2z*y5z+(1/1^2)*(y2n*y5n)),z=-1..1),n=-1..1)

\[ k25 := 0 \]

> k26:=int(int(((1/h^2)*y2z*y6z+(1/1^2)*(y2n*y6n)),z=-1..1),n=-1..1)

\[ k26 := \frac{2}{45} \frac{-5 \ell^2 + 3 h^2}{\ell^2 h^2} - \frac{1}{45} \frac{10 \ell^2 + 9 h^2}{\ell^2 h^2} \]

Equation set 3.25  MAPLE V solution for elemental stiffness results
> k27 := int(int(((1/h^2)*y2z*y2z+(1/1^2)*(y2n*y2n)), z=-1..1), n=-1..1) ;
  k27 := \frac{-5}{45} \frac{\ell}{h^2} \frac{\ell}{h^2} + \frac{3}{45} \frac{h^2}{h^2} \frac{h^2}{h^2}

> k28 := int(int(((1/h^2)*y2z*y8z+(1/1^2)*(y2n*y8n)), z=-1..1), n=-1..1) ;
  k28 := \frac{2}{45} \frac{-5}{h^2} \frac{\ell}{h^2} + \frac{1}{45} \frac{10}{h^2} \frac{\ell}{h^2} + \frac{9}{h^2} \frac{h^2}{h^2}

> k33 := int(int(((1/h^2)*y3z*y3z+(1/1^2)*(y3n*y3n)), z=-1..1), n=-1..1) ;
  k33 := \frac{1}{720} \frac{43}{\ell^2} \frac{\ell}{h^2} + \frac{88}{720} \frac{\ell}{h^2} \frac{\ell}{h^2} + \frac{1}{720} \frac{373}{\ell^2} \frac{\ell}{h^2} + \frac{328}{720} \frac{\ell}{h^2} \frac{\ell}{h^2}

> k34 := int(int(((1/h^2)*y3z*y4z+(1/1^2)*(y3n*y4n)), z=-1..1), n=-1..1) ;
  k34 := \frac{1}{720} \frac{-21}{\ell^2} \frac{\ell}{h^2} + \frac{160}{720} \frac{\ell}{h^2} \frac{\ell}{h^2} + \frac{1}{720} \frac{69}{\ell^2} \frac{\ell}{h^2} + \frac{440}{720} \frac{\ell}{h^2} \frac{\ell}{h^2}

> k35 := int(int(((1/h^2)*y3z*y5z+(1/1^2)*(y3n*y5n)), z=-1..1), n=-1..1) ;
  k35 := \frac{1}{720} \frac{21}{\ell^2} \frac{\ell}{h^2} + \frac{200}{720} \frac{\ell}{h^2} \frac{\ell}{h^2} + \frac{1}{720} \frac{-69}{\ell^2} \frac{\ell}{h^2} + \frac{440}{720} \frac{\ell}{h^2} \frac{\ell}{h^2}

> k36 := int(int(((1/h^2)*y3z*y6z+(1/1^2)*(y3n*y6n)), z=-1..1), n=-1..1) ;
  k36 := \frac{23}{90} \frac{\ell}{h^2} \frac{\ell}{h^2}

> k37 := int(int(((1/h^2)*y3z*y7z+(1/1^2)*(y3n*y7n)), z=-1..1), n=-1..1) ;
  k37 := \frac{2}{45} \frac{-5}{h^2} \frac{\ell}{h^2} + \frac{1}{45} \frac{10}{h^2} \frac{\ell}{h^2} + \frac{9}{h^2} \frac{h^2}{h^2}

> k38 := int(int(((1/h^2)*y3z*y8z+(1/1^2)*(y3n*y8n)), z=-1..1), n=-1..1) ;
  k38 := \frac{1}{90} \frac{17}{h^2} \frac{\ell}{h^2}

> k44 := int(int(((1/h^2)*y4z*y4z+(1/1^2)*(y4n*y4n)), z=-1..1), n=-1..1) ;
  k44 := \frac{8}{45} \frac{3}{h^2} \frac{\ell}{h^2} + \frac{10}{45} \frac{h^2}{h^2} \frac{h^2}{h^2}

> k45 := int(int(((1/h^2)*y4z*y5z+(1/1^2)*(y4n*y5n)), z=-1..1), n=-1..1) ;
  k45 := \frac{8}{45} \frac{-3}{h^2} \frac{\ell}{h^2} + \frac{5}{45} \frac{h^2}{h^2} \frac{h^2}{h^2}

Equation set 3.25  MAPLE V solution for elemental stiffness results cont.
> \( k_{46} := \int \left( \int \left( \frac{(1/h^2)*y_4z*y_6z + (1/l^2)*y_4n*y_6n)}{z=-1..1}, n=-1..1 \right) \right) \)

\[
k_{46} := -\frac{1}{720} \frac{21 \rho^2 + 200 h^2}{\rho^2 h^2} - \frac{1}{720} \frac{-69 \rho^2 + 440 h^2}{\rho^2 h^2}
\]

> \( k_{47} := \int \left( \int \left( \frac{(1/h^2)*y_4z*y_7z + (1/l^2)*y_4n*y_7n)}{z=-1..1}, n=-1..1 \right) \right) \)

\[
k_{47} := 0
\]

> \( k_{48} := \int \left( \int \left( \frac{(1/h^2)*y_4z*y_8z + (1/l^2)*y_4n*y_8n)}{z=-1..1}, n=-1..1 \right) \right) \)

\[
k_{48} := -\frac{1}{720} \frac{-21 \rho^2 + 160 h^2}{\rho^2 h^2} - \frac{1}{720} \frac{69 \rho^2 + 160 h^2}{\rho^2 h^2}
\]

> \( k_{55} := \int \left( \int \left( \frac{(1/h^2)*y_5z*y_5z + (1/l^2)*y_5n*y_5n)}{z=-1..1}, n=-1..1 \right) \right) \)

\[
k_{55} := \frac{8}{45} \frac{3 \rho^2 + 10 h^2}{h^2 \rho^2}
\]

> \( k_{56} := \int \left( \int \left( \frac{(1/h^2)*y_5z*y_6z + (1/l^2)*y_5n*y_6n)}{z=-1..1}, n=-1..1 \right) \right) \)

\[
k_{56} := -\frac{1}{720} \frac{-21 \rho^2 + 160 h^2}{\rho^2 h^2} - \frac{1}{720} \frac{69 \rho^2 + 160 h^2}{\rho^2 h^2}
\]

> \( k_{57} := \int \left( \int \left( \frac{(1/h^2)*y_5z*y_7z + (1/l^2)*y_5n*y_7n)}{z=-1..1}, n=-1..1 \right) \right) \)

\[
k_{57} := 0
\]

> \( k_{58} := \int \left( \int \left( \frac{(1/h^2)*y_5z*y_8z + (1/l^2)*y_5n*y_8n)}{z=-1..1}, n=-1..1 \right) \right) \)

\[
k_{58} := -\frac{1}{720} \frac{21 \rho^2 + 200 h^2}{\rho^2 h^2} - \frac{1}{720} \frac{-69 \rho^2 + 440 h^2}{\rho^2 h^2}
\]

> \( k_{66} := \int \left( \int \left( \frac{(1/h^2)*y_6z*y_6z + (1/l^2)*y_6n*y_6n)}{z=-1..1}, n=-1..1 \right) \right) \)

\[
k_{66} := \frac{1}{720} \frac{43 \rho^2 + 88 h^2}{\rho^2 h^2} + \frac{1}{720} \frac{373 \rho^2 + 328 h^2}{\rho^2 h^2}
\]

> \( k_{67} := \int \left( \int \left( \frac{(1/h^2)*y_6z*y_7z + (1/l^2)*y_6n*y_7n)}{z=-1..1}, n=-1..1 \right) \right) \)

\[
k_{67} := -\frac{1}{45} \frac{5 \rho^2 + 6 h^2}{\rho^2 h^2} + \frac{1}{45} \frac{-3 \rho^2 + 9 h^2}{\rho^2 h^2}
\]

> \( k_{68} := \int \left( \int \left( \frac{(1/h^2)*y_6z*y_8z + (1/l^2)*y_6n*y_8n)}{z=-1..1}, n=-1..1 \right) \right) \)

\[
k_{68} := \frac{1}{720} \frac{37 \rho^2 + 68 h^2}{\rho^2 h^2} + \frac{17}{720} \frac{11 \rho^2 + 4 h^2}{\rho^2 h^2}
\]

> \( k_{77} := \int \left( \int \left( \frac{(1/h^2)*y_7z*y_7z + (1/l^2)*y_7n*y_7n)}{z=-1..1}, n=-1..1 \right) \right) \)

\[
k_{77} := \frac{2}{45} \frac{5 \rho^2 + 6 h^2}{\rho^2 h^2} + \frac{2}{45} \frac{3 \rho^2 + 6 h^2}{\rho^2 h^2}
\]

Equation set 3.25 MAPLE V solution for elemental stiffness results cont.
> k78 := int(int(((1/h^2) * y7z*y8z+ (1/1^2) * (y7n*y8n)), z=-1..1), n=-1..1);

\[ k78 = -\frac{1}{45} \frac{\ell^3 + 6 \ell^2 h}{\ell^2 h^2} + \frac{1}{45} \frac{-35 \ell^3 + 9 \ell^2 h^3}{\ell^2 h^2} \]

> k88 := int(int(((1/h^2) * y8z*y8z+ (1/1^2) * (y8n*y8n)), z=-1..1), n=-1..1);

\[ k88 = \frac{1}{720} \frac{43 \ell^3 + 88 \ell^2 h^2}{\ell^2 h^2} + \frac{1}{720} \frac{373 \ell^3 + 328 \ell^2 h^2}{\ell^2 h^2} \]

> h := 44;

> k11;

\[ k11 = \frac{1}{1393920} \frac{43 \ell^2 + 170368 \ell^2}{\ell^2} + \frac{1}{1393920} \frac{373 \ell^2 + 635008 \ell^2}{\ell^2} \]

> k12;

\[ k12 = \frac{1}{87120} \frac{5 \ell^2 + 11616 \ell^2}{\ell^2} + \frac{1}{87120} \frac{-35 \ell^2 + 17424 \ell^2}{\ell^2} \]

> k13;

\[ k13 = \frac{1}{1393920} \frac{37 \ell^2 + 131648 \ell^2}{\ell^2} + \frac{17}{1393920} \frac{11 \ell^2 + 7744 \ell^2}{\ell^2} \]

> k14;

\[ k14 = \frac{1}{1393920} \frac{21 \ell^2 + 387200 \ell^2}{\ell^2} - \frac{1}{1393920} \frac{-69 \ell^2 + 851840 \ell^2}{\ell^2} \]

> k15;

\[ k15 = \frac{1}{1393920} \frac{-21 \ell^2 + 309760 \ell^2}{\ell^2} - \frac{1}{1393920} \frac{69 \ell^2 + 309760 \ell^2}{\ell^2} \]

> k16;

\[ k16 = \frac{1}{174240} \frac{17 \ell^2 + 54208 \ell^2}{\ell^2} \]

> k17;

\[ k17 = \frac{1}{43560} \frac{-5 \ell^2 + 5808 \ell^2}{\ell^2} - \frac{1}{87120} \frac{10 \ell^2 + 17424 \ell^2}{\ell^2} \]

> k18;

\[ k18 = \frac{23}{174240} \frac{\ell^2 + 1936 \ell^2}{\ell^2} \]

Equation set 3.25 MAPLE V solution for elemental stiffness results (end) and a simplified version provided a lateral element division length of h = 44 mm
Equation set 3.25 Simplified elemental stiffness equations provided a lateral element division length of $h = 44\text{mm}$ cont.
Equation set 3.25 simplified elemental stiffness equations provided a lateral element division length of $h = 44$mm cont.
The actual values for the $k$ matrix are belt section specific and are specified as such in the spreadsheet solution. The element width factor, $h$, of 44mm was entered to show the simplification gained provided the width of the photoreceptor remains constraint. In order for this condition to be of use, the software set would need to be copied and edited to reflect this constraint. This type of editing is discussed in Section 4.0.

### 3.3.3. Modifications to the belt element connectivity and solution

As was shown in the belt element derivation, the element stiffness matrix is derived using equation sets 3.15 and 3.19. The stiffness algorithms are repeated here in Equation set 3.26.

$$
K_{ij}^n = a\int \int \left( \frac{\partial \psi_i}{\partial x} \frac{\partial \psi_j}{\partial x} + \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_j}{\partial y} \right) l |\delta\zeta \delta\eta|
$$

Equation set 3.26

$$
K_{ij}^n = a\int \int \left( \frac{1}{h^2} \frac{\partial \psi_i}{\partial \zeta} \frac{\partial \psi_j}{\partial \zeta} + \frac{1}{l^2} \frac{\partial \psi_i}{\partial \eta} \frac{\partial \psi_j}{\partial \eta} \right) hl |\delta\zeta \delta\eta|
$$

As shown on page 464 of Reddy, the theoretical application of this equation is only part of the full coupling required for the plane stress element to react correctly. The Reddy equation shows the directional coupling as follows. The $K^{11}$ and $K^{22}$ stiffness elements in Reddy are of the same format as that which is used in this application. The $K^{11}$ matrix would apply to the x or lateral stiffness set while the $K^{22}$ matrix applies to the y or process stiffness set. The obvious conclusion and reaction to this statement reveals that stiffness matrices $K^{12}$ and $K^{21}$ are not used. This is correct.

The singular dimension assumptions that are imposed on the solution process of these elements forces the connectivity sections of the full stiffness matrix to be neglected. By doing so, the global matrix size is reduced by a factor of four. This significantly eases the strain on the partitioning methods needed to allow a full matrix run in Microsoft Excel. The purpose of the fully coupled stiffness matrix is to relieve the need for an iterative process to define the planar interaction of forces and deflections. If this process included the off diagonal coupling between dimension, this iteration sequence would eventually converge to the exact solution. The largest resulting error from this assumption is located in the
determination of the lateral deflection analysis. Since this deflection is an order of magnitude below the rest of the system, this error was deemed acceptable given the gain in calculation efficiency.

The effective error described above, imposed by not coupling the dimensional solutions, is created when these iterations are not carried out. Basically, the restructuring of the nodal locations due to off dimensional displacement are not included. Given that this model was meant to have a course mesh density and low absolute accuracy, this error is acceptable.

Full Reddy stiffness matrix equation

\[
\begin{bmatrix}
[M] & [0] \\
[0] & [M]
\end{bmatrix}
\begin{bmatrix}
[u] \\
[v]
\end{bmatrix}
+ \begin{bmatrix}
[K^{11}] & [K^{12}] \\
[K^{21}] & [K^{22}]
\end{bmatrix}
\begin{bmatrix}
\{u\} \\
\{v\}
\end{bmatrix}
= \begin{bmatrix}
\{F^1\} \\
\{F^2\}
\end{bmatrix}
\]

Where

\[
M_{ij} = \int_{\alpha'} c \psi_i \psi_j \text{d}x \text{d}y, \text{ where } c = \rho h
\]

\[
K_{ij}^{11} = \int_{\alpha'} h \left( c_{i1} \frac{\partial \psi_i}{\partial x} \frac{\partial \psi_j}{\partial x} + c_{i6} \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_j}{\partial y} \right) \text{d}x \text{d}y
\]

\[
K_{ij}^{12} = K_{ij}^{21} = \int_{\alpha'} h \left( c_{i2} \frac{\partial \psi_i}{\partial x} \frac{\partial \psi_j}{\partial y} + c_{i6} \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_j}{\partial x} \right) \text{d}x \text{d}y
\]

\[
K_{ij}^{22} = \int_{\alpha'} h \left( c_{i6} \frac{\partial \psi_i}{\partial x} \frac{\partial \psi_j}{\partial x} + c_{i2} \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_j}{\partial y} \right) \text{d}x \text{d}y
\]

\[
F_i^1 = \int_{\alpha'} h \psi_i f_i \text{d}x \text{d}y + \oint_{\Gamma} h \psi_i t_i \text{d}s, \quad F_i^2 = \int_{\alpha'} h \psi_i f_i \text{d}x \text{d}y + \oint_{\Gamma} h \psi_i t_i \text{d}s
\]

Equation set 3.26a
Modified singular dimension analysis matrix equation

\[
\begin{bmatrix}
[M] & [0] \\
[0] & [M]
\end{bmatrix}
\begin{bmatrix}
\{\dddot{u}\} \\
\{\dddot{v}\}
\end{bmatrix}
+
\begin{bmatrix}
[K^{11}] & [0] \\
[0] & [K^{22}]
\end{bmatrix}
\begin{bmatrix}
\{u\} \\
\{v\}
\end{bmatrix}
=
\begin{bmatrix}
\{F^1\} \\
\{F^2\}
\end{bmatrix}
\]

Where the off diagonal stiffness values are zero!

for the static case, the equation is reduced to...

\[
\begin{bmatrix}
[K^{11}] \\
[K^{22}]
\end{bmatrix}
\begin{bmatrix}
\{u\}
\end{bmatrix}
=
\begin{bmatrix}
\{F^1\} \\
\{F^2\}
\end{bmatrix}
\]

Where

\[
K^{11}_y = K^{22}_y = \int h \left( \frac{\partial \psi_i}{\partial x} \frac{\partial \psi_j}{\partial x} + \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_j}{\partial y} \right) dxdy
\]

or

\[
= a \int_1^l \left( \frac{\partial \psi_i}{\partial x} \frac{\partial \psi_j}{\partial x} + \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_j}{\partial y} \right) J |\delta \xi \delta \eta|
\]

Equation set 3.26a cont.

Being that the decoupled system is now defined, the application of forces and boundary conditions must now be considered. The process direction equation is subject to a direct boundary condition as a result of the belt tension and roll deflection. The lateral direction equation, since it is decoupled, has no direct loading. Recall that the lateral deformation of the roll was set to zero. This assumption must be augmented with a superficial loading sequence to provide a more accurate portrayal of the system. Note that in future versions of the system, the lateral deflection assumption will not be activated and this augmentation will no longer be required.

The augmentation to the lateral loading is defined as an external force that is distributed to the belt element nodes as a function of position.
The distribution was derived using the MAPLE V software used to determine the stiffness matrix of the belt elements. The results of this analysis are shown in Figure 3.9

![Figure 3.9 Augmented forcing function applied to the belt element lateral solution](image)

The equations from Maple are then implemented by using the Poisson ratio as the value of $Q$ (integration constant), scaled by the Belt tension, and the deflection of the roll in the process direction. This deflection is translated by a relationship defined by the average deflection value and the modulus of the roll. It is currently known that this configuration is not valid. Again, this “patch” is used only to augment the loading and coupling process that was disabled in the decoupling process. The results for lateral loading are found to be on the right order of magnitude as the process deflection times the Poisson Ratio of the material. This result is indeed consistent with the actual system and therefore deemed appropriate as a temporary patch. This method is not endorsed by RIT nor Xerox. The next version of this software will not contain this augmentation.

3.3.4. Analysis Choices

The selection of Finite Elements to use for an analysis has a considerable impact on the results that are achieved. With this in mind, numerical calculation expense and efficiency were weighted as a restraining but secondary consideration to overall system quality in representation. That is not to say however, that the accuracy of the model has not been sacrificed in order to achieve an end. The definition of this thesis promotes that a coarse solution technique that retains a higher efficiency than traditional FEA software packages is
of value to the corporation. It should further be noted that the element type selections described here have been fully implemented into the software spreadsheet package. If the element selection does not coincide with the requirements of the user, the software must be modified accordingly. With this said, it is further recommended that the formulation used in this software package not be modified at this time. Further investigation is slated as a future project at Xerox. The complexity and interaction of cells within the format of the solution sequence has not been 100% documented. Without a benchmark as to what results should be achieved given certain parameters, the risk of misadjusting the software is present. Any user that requires an updated version of this package or has a request for a change is urged to use the contact information at the end of this thesis. For all other users, the charter version of this software has selected a cubic Hermite beam element and quadratic quad element for use in this analyzer.

The Cubic Hermite Beam Element was chosen for lack of substantial increase in calculation investment with respect to the other elements. A primary benefit to the use of these elements is the extremely high level of internodal accuracy achieved. This is reflected in the post processing of the results at a particular point of interest. Further, because a limited mesh density was used, it was hypothesized that future versions of this software might require a more accurate internodal result. Lastly, the element selection of Cubic Hermite elements utilizes only 2 physical nodes per element. The four Psi functions that comprise the Cubic Element are split between deflection and rotation degrees of freedom. Hence, not only are added nodal degrees of freedom gained, but also the cubic element is compatible with any number of elements requested. Hence, regardless of the belt configuration, using a cubic beam element ensures that the system-interlocking requirement can be met.

The quadratic belt element was chosen for much the same reasons as the cubic beam. However, in the case of the belt quad element, the numerical derivation and formulation of automating a solution sequence has a substantial impact. The Xerox Corporation presently knows that the reaction of the belt is not linear. Although first estimations of belt systems may often utilize a linear assumption, it was deemed unrealistic for this application. The reaction of the belt in the third or transverse dimension would require in itself a non-linear element to be used. The nodal configuration of linear elements does not contain any interior
nodes to deflect in this direction under loading. Hence, the choice is narrowed to 2\textsuperscript{nd} order, 3\textsuperscript{rd} order or a combination of the two. The derivation of both the quadratic and cubic elements is quite similar as shown earlier and in the Appendix. However, the interaction of the belt subsystem with the roll again seems to dominate the element selection process. The cubic element requires that there exist two internal nodes along each element section. This means that the overall number of nodes along the axis of the roll must be a multiple of 3, as opposed to 2 with the quadratic element. This may seem to be a minor difference, but when compiled with the idea of having 50\% more nodes required along the axis of the roll for the same number of elements is a clear disadvantage. Even further, it is contemplated that the highest gradient of belt deflection occurs at the edges of the belt. Recall that it was assumed that the edges of the belt, not attached to the rollers, were assumed to be fixed in the transverse, out of plane, direction. Because this is the case, and the accuracy of the system in this domain has already been disturbed. It would be foolish to implement a high order accuracy element on a section of known and appreciated error. Thus, the quadratic element was chosen for the analysis.

The transition quadratic to cubic element was shown as an example of how to circumvent the restraints of this type of modeling. Transition elements provide the opportunity to vary element accuracy within the local and global domains. However, given that this variation is non-repetitive, the procurement of a general solution varied with orientation. The definition is included for future manipulation sake, but it has not been foreseen as a necessary component for system accuracy. Given a huge increase in user input and restriction on node and degree of freedom existence on the rest of the model, its use could not be justified at this time.

3.4. Component Repeatability in Modeling

The breakdown of any mechanical system results in a variable number of components or individual systems. The assumption that is made states that the rollers of the system, regardless of material or geometrical differences, will all act as simplified beams. Further, each of these beams will have approximately the same order of response to boundary condition inputs, since they are indeed in a closed mechanical loop. The sections of belt that
connect these rollers thus must also be similar in nature and response to system inputs. Since each roller is derived in the same fashion, the output for each derivation will be of a generic format, for instance cubic. If this is the case, then the attachment of the belt sections to rollers does not depend on the actual definition of the roller itself, but on the form of the roller definition equation.

Furthermore, using a linear roll does not require a linear belt provided that the two can still be mated mathematically. Hence, the repeated use of similar elements allows for a significant decrease in the derivation and interlocking of the individual components of the system. This assumption becomes vital when the number of individual components changes between analyses. In the current engineering process, adding or subtracting a subsystem requires that an entirely new Finite Element model be created. The interlocking nature of the components used here promotes the mentality that the string of components itself is of variable length. The addition or subtraction of subsystems occurs with simple detachment and reconfiguration. As will be shown in Section 4.0, this capability allows for greater latitude in the experimentation of new photoreceptor design.
3.5. 2D vs. 3D modeling Techniques

The derivation of the belt and roll elements were completed for a singular dimension and then replicated to represent the three relative directions of interest, out-of-plane, in-plane process and in-plane lateral. Each dimensional result was calculated as a scalar and then vectorily added during post processing to get a resultant deformation tensor. The sequence of assumptions that allows for this simplification begins with a 3-dimensional space. This space is split into a set of 2-d spaces, followed by a set of 1-d spaces. The coupling effects of each dimension are approximated by calculating only the first iteration of the coupling sequence. By doing so, the speed of the program is increased while accuracy is sacrificed.

The transition from the 3 to 2 dimensional analysis domains is made, provided that the equations of motion in the system are of a generic format, and that the resulting matrix equations are of workable form. The third dimension results from the first step in an iteration methodology that extracts one dimension at a time and uses the Poisson effect as the go between for each iteration. It should be noted that for this type of analysis step to be accurate, many iterations would need to be processed. Because the accuracy of this analysis is not required to be extremely tight, a single iteration is used and the resulting error is tolerated.

As an example, consider a cantilevered beam. A 2-D cantilevered beam analysis is shown in Figure 3.10. A 3-D version of the same analysis is shown in Figure 3.11. Looking more closely at the 3-D system reveals that the system itself consists of two 2-D systems, one in the xy and another in the xz planes. This configuration step must also be accommodated by a change in boundary condition, inherently the absence of gravity in the xz plane. The theory used to derive the deflection equations of beams, including cantilevered, is based in vector geometry. The beam configuration allows for the same type of vector to be utilized in multiple directions. These relationships depend neither on the magnitude or direction of forces within a space, but to the systems upon which they act.
Consider the system shown in Fig. 3.12. The axial deflection is common between the 2-D analyses, but indeed the jump is being made to 3-DOF not 4-DOF (2 for each component). As will be explained in Section 3.6 Belt/Roll interaction, the axial deflection of the beam elements is assumed to be zero. Hence, what remains is a resolved load system, two equations and two unknowns, a common fundamental of Statics and Mechanics. It is this simplicity which this thesis draws upon to represent more difficult systems, with more complex interactions, but with the same vantage point to the third dimension. If it can be proven that an element will react similarly in two of the three dimension, or all three for that matter, then the derivation required for that element is repetitive and therefore minimized.
3.6. Belt/Roll interaction

The interaction of the belt with respect to the rolls is defined as the displacement and force transition between the quadrilateral belt elements and beam roll elements. This interaction is dominated by non linear effects in both materials as well as the contact region between them. The modeling strategy for this system is to retain accuracy while only modeling some of the parameters involved. The simplifying assumptions are as follows. The displacements of the elements in contact have a one-to-one ratio at the nodes. Further, the force transfer through these nodes is calculated as if the elements were rigid bodies, which do not absorb any of the force. The simplifying assumptions do induce error but the required output may be extracted as a result of this type of calculation step.

As previously stated, the efficiency improves when multiple dimensions of an analysis are shown to be of the same format. The accuracy of any given dimensional result is limited only in that no material non-linearity and no dimension coupling is accounted for. Provided that these parameters are acceptable, the results of the analysis will be representative of the system.

Again referring to the cantilevered beam example, the respective dimensional solution sequence, the axial deformation of the rod, was concluded to be the same in each planar
configuration by assuming that this deformation was zero throughout the system regardless of orientation. Thus, when the individual planar sets were solved, the boundary condition set by this assumption was preserved throughout the analysis.

The use of simplifying assumptions such as zero axial deformation provide compounding benefits when the system is modeled as individual subsystems and dimensions. The belt, which is under a nominal tension $T$ over its length, produces a force on the roll. In the actual system, as the belt is stretched, the Poisson effect causes the belt to shrink toward its centerline. This creates an axial component to the force translated to the roll that the belt is acting on. The decision as to how this force would be translated became 2-dimensional as a result of the zero axial deformation of the roll assumption. This is not to say, however, that the belt does not shrink in this direction. The assumption specifically states that regardless of this deflection, and thus of the force translated by this deflection to the roll, the matrix solution vector becomes a null set, $U_{z\text{roll}} = 0$, and is shown in Figure 3.13. Hence, an increase in the magnitude or change in the direction of the force translated would not have any effect on the deformation of the roll because the roll deflection along its axis was set to zero as a boundary condition. Although this relation implies an over-constraint, noting that because the dimensional analysis was split into individual analyses, the in-plane process and lateral forcing functions are no longer coupled. It is therefore critical to derive a correct forcing function for the in-plane lateral analysis. This forcing function must take into account both the Poisson effect and the deformation restrictions at the roll node locations. An example of this deflection is portrayed in Figures 3.13 and 3.14 below. Note that for simplicity, the model shown only contains 3 rollers. The methodology and solution response is the same for all systems however.
Figure 3.13. Real system before and after tension is applied to the belt (no axial def. in belt)

Figure 3.14 Modeled system which shows the axial deformation of the roll to be zero

The result of this simplification is the transferal of force from the belt in a 2-dimensional plane. Note that each roll will support an arbitrary amount of wrap, where the belt is in contact with the roll. This wrap, or wrap angle, will vary as a function of roll placement and diameter. The tensile force in the belt is assumed to act purely in the plane of the belt. In the actual system, this belt may not lie in a perfect plane, but for the purposes of the inputs to this
model, it can be assumed as such. This belt force configuration acts in a tensile fashion as shown in Fig. 3.15. Resolving these forces into a singular distributed load along the axis of the roll reveals that the magnitude and direction of this load are roll independent. The magnitude of the force, $T'$, will always follow the relation in Equation 3.27. The direction of the force will always act along the bisector of the wrap angle pointed inward with respect to the system. Utilizing this, the loading of the roll can be separated from the rest of the analysis. Because of this independence to belt definition and geometry, the loading and deflection of the roll as a function of belt tension provides an optimal starting point for the analytical analysis sequence.

$$T' = (2\cos(\theta/2)) T$$

Eq. 3.27

![Diagram of belt force configuration](image)

Figure 3.15. Resulting load configuration of the roll as a result of belt tension

Once the deflection of the roll is known, it may be resolved into the coordinate frames of the belt that is attached to it. By doing this, the belt is now supplied with a displacement boundary condition for both out-of-plane and the in-plane process directions. This connectivity completes the analysis cycle for belt/roll interaction. A detailed solution sequence can be found in the Description of the Spreadsheet Formulation, Section 4.0.
3.7. Generic roll analysis

Due to the complexity of even a simplified photoreceptor module model, it becomes increasingly efficient to have stand-alone solvers for specific subsystems within a system. One such system is the individual roll component used in this analysis. Recalling the discussion of the independence of roll loading to belt geometry, it becomes apparent that the deflection of each roll will have a significant impact on the response of the system. Not only does each roll withstand the bulk of the changing parameters in the system, i.e. position, length, diameter, material, and wrap but also these rolls provide the forcing function for the belt deflection models that are of final interest to us in this analysis.

Clearly, running the full model for a multitude of roll designs is drastically inefficient. Instead, it is proposed here that the deflection response curve of the roll be set to a desired value or shape. For instance, let's say that an engineer ran the static model and discovered that the drive roll was deflecting in such a way that the belt sections in contact with it were exceeding some predetermined deformation tolerance. A closed form beam calculation would achieve the amount of roll deformation which would cause this condition to occur, thus setting a breaking point for the subsystem of the drive roll. This analyzer is simply an automated calculation tool of closed form equations. The analyzer allows for more complex geometry than typical beams in that the cross section and wrap angle to distributed load relationships are embedding within the calculation cells. This type of information could be more efficiently processed using a generic roll analyzer as opposed to hand calculations or the full static analysis software. This generic roll analyzer is indeed included in the array of software sets presented here.

The generic roll analyzer is spreadsheet based in Microsoft Excel and generates deformation, moment and maximum stress plots for any generic roll on the basis of fundamental simply supported beam theory. Model inputs include all of those required by the parent static software set with the addition of rib definition and control. The modeling inputs of this analyzer have purposely been produced independently from the static model input section. In this manner, the generic roll analyzer can be modified, tested and optimized without impacting the solver as a whole.
For demonstration purposes, an example roll is shown in Fig. 3.16. The results for this example are shown in Fig. 3.17, 3.18 and 3.19. The spreadsheet itself is not shown here because the equations used are predominantly imbedded. However, a link to the software set is included in the Appendix section.

![Diagram of a roll with labeled parameters](image)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value (Units)</th>
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<tr>
<td>Wrangle</td>
<td>90 degrees</td>
</tr>
<tr>
<td>Wrangle</td>
<td>1.571 radians</td>
</tr>
<tr>
<td>Nominal Tension</td>
<td>.175 N/mm</td>
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<tr>
<td>Wrap adjusted</td>
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<td>Roll E</td>
<td>70 Gpa</td>
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<tr>
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<tr>
<td><strong>Maximum</strong></td>
<td>less than 0.5 mm</td>
</tr>
<tr>
<td><strong>MAX DEFLECTION</strong></td>
<td><strong>.0924 mm</strong></td>
</tr>
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Figure 3.16. Roll Design Specification / Parameters for the Generic Roll Test Case
Figure 3.17. Deflection plot for the Generic Roll Test Case

Figure 3.18. Moment plot for the Generic Roll Test Case
In conclusion, it is important to note that this analyzer can be used for any roll, regardless if the full static analyzer is employed or not. However, for general engineering design, this analyzer is not equipped to handle the dynamic issues such as torque response, Moment of Inertia, mass, and irregularities in roll dimension such as runout. It is therefore recommended that if this analyzer is used offline, that the deformation restraint, as well as other roll parameters, reflect the dynamic restraints that apply to the system.
4. Correlation

Correlating the results of an analysis provides confidence that the analysis is correct and that the system modeling selection is truly representative. The correlation of this software package does not follow the conventional correlation path. As stated in the Introduction, the purpose of this analysis was to provide an approximate order of magnitude result to photoreceptor static response. Hence, the percent error is allowed to be large given that the reason for the error, as well as the magnitude of that error remains constant. In other words, if two p/r module designs were being considered and both were modeled using MSC/NASTRAN and the solution solver developed here, there would be clear cut differences in the results. Obviously the Nastran models would be more accurate. However, if both sets of analysis in Excel deviated from their Nastran counterparts by a constant magnitude, then the comparison of the models in Excel and in NASTRAN would yield the same conclusion. Ideally, it is this relative comparison accuracy that is needed at the Xerox Corp. Once a benchmark fixture is analyzed in the solver, any other fixture may be measured and compared. It is this methodology which is currently being implemented until a full analysis on the error of this program can be completed. The future expectations and plans are discussed in Section 7.0.

4.1. The MSC/NASTRAN Model

The Nastran model itself is quite basic. The model uses beam elements for the rolls and shell elements for the belt. For simplicity, only the belt section between rollers one and two are considered. The Boundary Conditions were set to match those in the example problem set. Each roll end was restricted in the transitional degrees of freedom, except the z of one end to prevent over constraint, and the belt was attached to the rolls such that no slip could occur. MSC/Patran was used to develop the graphical interface to the Nastran data deck. Figure 4.1 shows the model as geometry while the corresponding Data Deck can be found in the Appendix under “Nastran Data Deck”.
Forces shown in yellow depict resolved belt tension force along the bisector of wrap.

Boundary Conditions are shown in blue. Far end of roller is restricted 123, near end 12.

Roll 3, 10 nodes used to create belt 23 & 101 coordinate planes.

Figure 4.1. Nastran Model of Belt12 as portrayed by MSC/PATRAN
4.2. Roll Deflection Comparison

The deflection of the rolls in the Excel solver has moderate accuracy with respect to the Nastran version. However, as shown in Figure 4.2, the general trend of the deflection as well as magnitude is well within tolerance. The deflection of the Nastran beam implies that the third order solution was not necessary. Perhaps a second order element would have been more appropriate. However, the use of cubic Hermite Elements for the beams is deemed appropriate based on the versatility of cubic elements over the quadratic elements.

![Roll 1 Solution Comparison](image_url)

**Figure 4.2. Roll 1 Deflection Comparison**
4.3. Belt Deflection Comparison

The comparison of the belt results is not as cut and dried as that of the roll. Because the Excel solver split the results of the belt analysis into three separate segments, and there is error associated with each of the solutions, the overall error is compounded. However, this error is minimal with respect to the order of magnitude of the results in most cases. For example, Figure 4.3 portrays a comparison of out-of-plane deflection. The result, due to the orientation of the nodes, does not produce a user-friendly plot in Excel. The nodal configuration is not the same across each of the rows because the serendipity element was used. The spaces are taken as zeros instead of blanks and the resulting plot is misleading. For this reason, the point-to-point errors are considered on an array only basis. Matching any point on the belt between the three charts allows the user to visualize the relative error.

The first section corresponds to the Excel static solver results. The cells highlighted in Yellow represent generic element numbers. The section results shown pertain to the belt section between rolls one and two. The upper section of the belt, highlighted in red, represents the connects to roll one while the lower section corresponds to roll 2. The magnitude of the results is consistently of micron level with the exception of the area around element 6 and 7. This error was investigated and found not to be a function of the stiffness matrix, but of the partitioning method used for the in plane and out of plane analysis. As will be discussed in Section 7.0, future plans involve a Matlab version of this solution. An advantage of this is the advanced matrix manipulation and plotting capabilities required for these types of results.

The second section of Figure 4.3 represents the same belt configuration as the Excel results, but with respect to the Nastran model. These results represent the analytical target that the static solver was trying to represent. As is evidenced by the third section, Absolute error, the static solver was within an order of magnitude everywhere across the belt section with the exception of element 6 and 7. For reference sake, the Nastran results have been shown in Figure 4.4. These results portray the vector sum of deflection in all directions.
Excel out of plane belt 12

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ABSOLUTE ERROR

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Figure 4.3 Belt out-of-plane results comparison for section 12
Figure 4.4. Nastran results for belt section 12 all deflections
5. Current /Future Plans

5.1. The Generic Dynamic Model of the belt/roll system

As was referred to in the body of this thesis, there does exist a dynamic solver with this solution package. However, the accuracy and scheme of results post processing has not been elevated to an acceptable level. This has occurred primarily in the realization that the number of inputs required for the model to run efficiently is far greater than that which was originally endeavored. The dynamics of any system brings into account a number of additional variables. With respect to Photoreceptors, these can include things such as drag, slip, wear, mass, inertia, subsystem connection, active control methods, and geometric variation among others. Because the broad spectrum of both inputs and outputs, there was significant difficulty in coordinating an automated solver which could handle all of these requests. Further, unlike the static solver, the absence of subassemblies can significantly skew the results. Without prior knowledge of these subsystems and their effect on the photoreceptor, proper generic interaction sequences were not fully developed. Lastly, with the extraction of a normal mode for each degree of freedom in the system, there exists prominent post processing requirements to achieve the desired dynamic results from the model. Although in existence, the dynamic model is not fully integrated yet. The basics of the model are described here as a foundation for future work to build on.

5.1.1. System Response Requirements

The system response requirements for the current dynamic analysis are quite basic. Provided a 10 roll system, as in the static analysis, each roll would be connected through a series of springs rather than belt sections. This significantly reduces the number of nodes in the model and allows for the absence of membrane modeling in the structure. Further, by doing this, the system has now been reduced to a 3-dimensional spring mass damper system. The damping in this case is assumed to be Reynolds damping for easier calculation. Upon which the Eigen values are extracted, the stiffness matrix can be modified to account for the effects of this damping. The results of this analysis are the eigen vectors themselves. No post processing efforts have been successful to date in extracting the prominent modeshapes
accurately from this data. Those efforts that have indeed created modeshapes have proven to be too time intensive in deciphering which modes represent system and not noise modes. This is the primary reason for the temporary abandonment of the dynamic analysis for this thesis. This type of information is of great interest to the Xerox Corporation and will be endeavored later in this year.

5.1.2. Analysis Derivation

The derivation for this analysis is quite basic in nature. To obtain relatively accurate result, 5 nodes were used to represent each roll. Considering that each node has 6 degree of freedom, and that the rigid body mode of the rotation of the belt about the module, DOF6 has been restricted, this leaves a total 250 DOF's in the system. A pictorial view of a section of the p/r system is shown in Figure 5.1.

Figure 5.1. Representation of the dynamic system formulation
With this established, the global connectivity matrices may be constructed. Although the solution sequence of the dynamic model is not split into components as the static model was, the matrices for the global connection of nodal degrees of freedom are indeed derived independently and joined within the formulation of the matrix. Shown in Figure 5.2, the mass matrix is a culmination of mass and inertia elements while the stiffness matrix is a combination of beam and spring stiffness matrices.

Basic Model

Total of 50 nodes, each with 5 degrees of freedom. (only Uz is neglected)

Start with underdamped system, assume Reynolds damping such that after diagonalization, it may be added as a coeff of K matrix

Examples

5.1.3. Spreadsheet Solution Sequence and Matrix manipulation

The matrix sequence in the spreadsheet follows a vertical path since the width of each matrix set is approximately the maximum width of the spreadsheet itself. The order of matrices in the sheet from top to bottom is as shown in Figure 5.3.
It should be noted that two forms of Eigen value extraction were set up for this modeling sequence. The first, which perpetrated the formation of the matrix hierarchy, utilized the tools solver to solve for the Phi matrix as a function of the Eigen vector matrix elements. This was done by summing the off diagonal terms of the Eigen vector matrix both vertically, and then horizontally summing these values. By setting tolerances on the maximum and minimum values that the off diagonal terms could be, the solution slowly converges toward the correct solution. Since the matrices are linked by embedded formulas such that the results in the Eigen value matrix are dependent on those in the phi matrix, the solver can be set to use the phi matrix as a set of variables in achieving a diagonal Eigen value matrix. In short, the solver is trying to diagonalize one matrix by varying the coefficients within the embedded formulas that derive that matrix. Unfortunately, the current solver is not capable of solving systems of over 200 variables. Hence, the second solver technique, using Matlab, was enacted.
5.1.4. Matlab Eigen value Extraction

By extracting the mass and stiffness matrices into Matlab, the Eigen value extraction command can be used to get the modal results. The matrices in question need to be saved in a new sheet as a space delimited text file. Once in Matlab, loading this file, “load FILENAME.txt” brings the matrices into the variable set that is active. Typing “whos” will verify this. Finally, the command, “E = eig (FILENAME)” reveals the results.

Since this type of solution involves an additional software step, and a basic knowledge of Matlab, the dynamic analysis does not meet the requirements aspired to by this thesis. As is described in Section 5.2, a full Matlab version of this solver is slated for development in 1999. Once completed, the dynamic analyzer will become an integrated part in the analysis suite.

5.2. Matlab Version

As was stated a number of times throughout this thesis, the solution set provided here represents the first of its kind, version 1.0 if you will. It is the intent of the design and implementation of this solution to expand into more powerful and comprehensive analysis domains, primarily Matlab. The mathematical modeling potential in Matlab does not restrict matrix size and manipulation as Excel does, thus freeing the solver to contain a higher mesh density and resulting resolution of accuracy. This extension will be created and verified at the Xerox Corporation over the coming years. As this software evolves, the Excel version of the solver will remain as a tutorial based solver to aid the user in navigating the analysis hierarchy.

5.3. Fully integrated static to dynamic

Also related to the spreadsheet size and mathematical limitations of a spreadsheet solution, the integration of the individual analyses shown here has become a future endeavor. This culmination into a master solver would allow less variation between solution techniques and cut down on user input time. Further, it is hypothesized that at some juncture, the two
analyses could be fully integrated creating a more accurate depiction of the system itself. Again, this type of development will occur at the Xerox Corp. over the coming years.

5.4. Higher order element usage

Along with the increased mathematical modeling capability, comes the opportunity to use higher order elements. Limited by only the initial investment of calculation and input time, these types of elements could be used to model advanced phenomena such as warping, roll runout and belt conicity. Further, it is also the aspiration of this software to allow the user to choose what types of elements are used in the solver. A Matlab solution would likely incorporate this with an "m" file for each type of element and a subroutine that calls upon whatever elements are specified. The juncture of these elements would be structured similarly to the way in which the belt and rolls are linked now. By keeping the juncture points common and of the same format, subassemblies may be linked without consideration to the numerical significance. The Force and Boundary Condition limitations, along with the Finite Element Method already provide for element type discrepancies provided that the mating side is matched correctly.

5.5. Visual Basic user interface in Tutorial

In order to ensure that the usage and modification of this software, once distributed within Xerox, certain aspects will need to be locked out while others highlighted. The current scheme of using cell highlights has no guarantee of ensuring this. Therefore, it would be advantageous to construct a visual basic program that could walk the user through the analysis from start to finish. This program is currently under construction at the Xerox Corporation.
6. Conclusions

The analysis derived and constructed over the past 18 months has been both an accomplishment and a learning experience. The correlation results showed that the Excel solver achieved an order of magnitude approximation of the FEA mode. The modeling procedure is therefore deemed appropriate and can be expanded on. This expansion includes the items discussed in the Future Plans section of this report.

Quite notably, the existence of opportunity for improvement on this foundation is bountiful. Primarily, the number of links in the spreadsheet has now exceeded well into the thousands. These links cause the solution process to slow and the accuracy of the results to diminish. Further, the need for matrix partitioning adds a number of input, calculation and post processing steps that are ultimately unnecessary. The mathematical limitations in Excel, predominantly to matrix size, causes the mesh density of the model to remain low, inherently ensuring that the accuracy of the model not go beyond an order of magnitude approximation. Factors such as shear and bending in the rolls, as well as twist all contribute to and are accounted for in the real system and commercial FEA packages. In order for this program to compete successfully, it must be able to apply these constraints and reactions to the modeled system. This also applies to the belt sections where assumptions on membrane restraint and zero slip with respect to the rolls is not representative of the real system.

Despite these shortcomings however, the model itself produces vital insight into the deflection relationship of the rolls and belt in a photoreceptor module. On a comparative basis, new and current designs can be pitted against each other to derive deflection results over the entire system. This knowledge can then influence the first design steps for a photoreceptor and help eliminate the occurrence of resource expenditure on non-productive designs.
7. Thanks

As with any true learning experience, it is not only the end result that provides the enlightenment, but the journey. The journey I have taken presented me with a number of difficult choices and obstacles. I, like many of my colleagues, had to sacrifice a great deal to achieve this end. This sacrifice effected a number of people, and those individuals who would tell you they did nothing are whom I most want to thank.

The Xerox Corporation Research and Technology Division. Orlando Lacayo and the Mechatronics group have provided me with both the time and tools by which this thesis was created. Further, through scheduling mishaps and difficulties, the Corp. has done nothing but support me in this endeavor, and I thank everyone at Xerox who helped me through this.

Dr. Kevin Kochersberger, one less thing for you to keep track of. From the first class I took from you, you had always given me confidence. When I went to you for thesis ideas, you did not hesitate to connect me with Bill Nowak at the Xerox Corporation. That alone has given me the chance to prove myself in the engineering world. I thank you for the opportunities you’ve shown me, and the advice you’ve bestowed.

Bill Nowak, here’s to finally working a paid 40 hr work week! You have allowed the progression of this thesis on limited company time and given me the raw objectives and requirements to make this thesis possible. Further, you’ve guided me along a path of development I believe will mature my skills as an engineer and culminate in a quality engineering mentality and ability. For this, and the time you’ve spent getting me to this end, I thank you.

Dr. Hany Ghoneim, for all the analytical background and theory. The FEA class in the RIT Masters program was only the beginning. The partitioning and element derivation and selection processes used largely came from our discussions back in early 1998. Nearly 18 months later, your advice has paid off, and I thank you.

Dr. Budynas, for the theory of vibration foundation which this thesis is based. The matrix manipulation and equation of motion formulations here draw heavily on the Vibrations ideals that you taught to me. Thank you.
Dr. B. Karlekar. Although the ideals of fluids and heat transfer are not among those used in the analytics of this thesis, the spreadsheet manipulation and personal drive to succeed certainly were. From our meeting in Numerical Methods some 4 years ago to your reference letter permitting my acceptance into the BSMS program, my personal commitment and motivation has increased significantly. I appreciate all the advice and musical analogies we have drawn upon. I wish you all the best and want to thank you for the confidence and standards that you have helped instill in me.

Tracy, well well well. Your little bro has finally made it. Let me just tell you that Mr. Kakhi has only been this proud of one other person in his life. I know I don’t have to go into detail, but there comes a time in a sibling’s life when they become the sole person you can trust. And to hold ourselves inside, captive to our emotions is what you overcame some time ago, as I am doing now. We are becoming the people we have aspired to in our own respects. Different as they may be, we are both passionate and determined, with a vision and strategy for change. Mom used to tell us that we are judged by our associations, those who we gravitate to, we learn from. As I come to this crossroads, head held high, ready for the next stage in my journey, I draw heavily from knowing that the number one association I’ve had, has been you. I look up to, respect and love you very much.

Mom, I love you and could never have done this without your steadfast foundation supporting me. I have never come across a more amazing person than you. I once gave you a glass slipper as a symbol of your devotion and true impact on my life. Every day I only regret that my analogy was an understatement. I could never truly express the strength I draw from you. I’ve never asked for it, nor really understood it until now, but it has been there all along. I still have that elephant card. I read it now and then and every time I do, it means more to me than the last time. I could say so much more, but I know in my heart that you know where I am headed. Just know that I love you and I thank you for giving me all that I have. Thank God this apple didn’t fall far from the tree 😊!
8. References

APPENDIX

A1 Frame Analysis - Verification of closed form techniques

The ensuing analysis compares the results from closed form simplified structure models to that of a full blown FEA model solved using MSC/NASTRAN. The use of closed form equations to analyze the photoreceptor frame can significantly lower the user input time when calculating the reaction of the frame. The primary assumption is that the cross members of the frame act as beams while the side plates remain relatively rigid. The analysis path therefore concludes that the beams have some inherent stiffness due to material type and geometry. The dependency on the grounding configuration of these "springs" relates whether they are in parallel (cantilevered) or in series (simply supported). The analysis itself is configured in Microsoft Excel such that it is fully automated by system inputs. This automation is what makes the simplified calculation that much more efficient than the FEA model. The spreadsheet solution for the closed form analysis is shown below. Included in this spreadsheet are the model parameters, assumptions, and governing equations that differentiate the closed form model from the actual system. Also included are the results of both analyses for comparative purposes. The spreadsheet itself is shown as a series of stages. Stage 1 represents the frame analysis in general. The operator "a" signifies that the system is simply supported and "f" that the analysis is a frequency response. The absence of both symbols represents the cantilevered deflection case. The Nastran solution sequence used was sol 101 for the static deflection case and sol 103 for the dynamic case.
The frame shown in Fig A.1 was constructed from planar 2-D shell elements. This element was chosen since the thickness of the sheet metal used to construct these frames is typically much less than the in-plane dimensions. The actual dimensions for this frame are considered proprietary and thus can not be divulged in this report. However, for comparison reasons, the areas of the closed form and FEA models have been matched accordingly. The mounting of the FEA model in space represents a fundamental difference with respect to the closed form
analysis. In reality, a 3-point mount on one of the side plates is used for cantilevering the module. A 5-point mount is used for the simply supported case. The closed form model assumes that the entire side plates on either side are collapsible into a single node. This assumption effectively combines the aforementioned mounting configurations into the theoretical cantilevered and simply supported cases. This difference becomes important when the distinction as to whether the rolls are being translated from their original positions, and in what direction, is made. It is this distinction that will differentiate which of the models is sufficient, and for what mounting condition, to proceed to the static and dynamic analyses presented later in this thesis.
SUBCASE: Stage 1

COMPONENTS: L/R side plates, 3 crossbar members

ASSUMPTIONS:
1) Right side plate is ground, left out of calculations
2) Crossbars considered in parallel since each is grounded
3) Lumped mass parameter at cantilever end represents left side plate
4) Geometries of x-bars: bends are negligible, as is angled orientation of bottom bar
5) $y_{max}$ occurs at the left end, despite patran representation

GOVERNING EQ's:
1) $k$ cantilever beam
   \[ k = \frac{3EI}{L} \text{ N/mm} \]
2) $k$ eq springs in parallel
   \[ k_{eq} = \sum k_i \text{ N/mm} \]
3) I of a composite x-section
   \[ I = \sum A_i y_i^2 \text{ mm}^4 \]
4) Centroid of a x-section
   \[ y = \frac{\sum A_i y_i}{\sum A_i} \]
5) Beam deflection, distributed load + end load
   \[ y_{max} = \frac{-wL^4}{8EI} + \frac{-F L^3}{3EI} \text{ mm} \]
6) Weight $W = mg$
   \[ m = \rho V \]
   \[ W = \rho V g \text{ N} \]

MAT'L PROPERTIES:
steel
\[ \rho = 7.80 \times 10^3 \text{ Mg/m}^3 \]
\[ E = 2.07 \times 10^5 \text{ N/mm}^2 \]
\[ I = 2.5 \text{ mm} \text{ throughout} \]

SCHEMATIC:
crossbar top
crossbar middle
crossbar bottom

SOLUTION:

Centroid Calculations:
\[ y_{reference} = \text{bottom edge} \]

Top: section 1 2 3
A $\text{mm}^2$ 99.375 335.3125 291.25
$y_{bar}$ 135.5 67.6625 1.25
product 13465.31 22486.69 364.0625

Middle: section 1 2 3 4 5
A $\text{mm}^2$ 250 315.3 198.75 315.3 250
$y_{bar}$ 253.615 191.68 127.245 63.06 1.125
product 63403.75 60436.7 25289.94 19682.92 1151.25

Bottom: section 1 2 3
A $\text{mm}^2$ 115 322.5 115
$y_{bar}$ 1.25 67.6625 136.625
product 143.75 22486.31 15711.875

\[ Y_{bar} = 50.02672 \]
\[ Y_{bar} = 127.3513 \]
\[ Y_{bar} = 88.16167 \]
Overall Centroid
set zero at middle centroid
Area
Top: 7.26E+02
Middle: 1.33E+03
Bottom: 5.63E+02

Moment of Inertia Calculations:
Top:
Middle:
Bottom:

Overall Moment of Inertia:
Total Mol
Top: 2004878.963 mm^4
Middle: 11397385.21 mm^4
Bottom: 1545966.875 mm^4

Beam Spring Constant Calculations:
Top:
Middle:
Bottom:

Equivalent Spring Constant Calculation:

Composite Beam Spring Constant:

Equivalent System:

where: W = the lumped mass of the left side plate
W_{1,2} = the uniformly distributed load of each cross member
Component Weight Calculations:

\[ W = mg \]
\[ pV = 1.14 \times 10^{-9} \text{Mg/mm}^3 \]

left side plate

\[ p \quad V \quad g \quad W \]
\[ \text{Mg/mm}^3 \quad \text{mm}^3 \quad \text{m/s} \quad \text{KN} \]

7.80E-09 423294.03 9.81 3.24E-02

equivalent cross member

\[ p \quad A \quad g \quad W_{13} \]
\[ \text{Mg/mm}^3 \quad \text{mm}^2 \quad \text{m/s} \quad \text{KN/mm} \]

7.80E-09 2617.7875 9.81 2.00E-04

Beam Free End Deflection:

\[ w \quad L \quad E \quad I \quad F \quad W \quad Y_{max} \]
\[ \text{N/mm} \quad \text{mm} \quad \text{N/mm}^2 \quad \text{mm}^4 \quad \text{N} \quad \text{mm} \]

2.00E-01 498.8 207000 1.49E+07 3.24E+01 -8.69E-04

Note: area taken from surface 22 in Patran model

TOTAL WEIGHT

1.30E+01 KN

Comparison to the Patran Model:

<table>
<thead>
<tr>
<th>( Y_{max} )</th>
<th>mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analytical</td>
<td>-0.0009</td>
</tr>
<tr>
<td>Nastran</td>
<td>-0.0033</td>
</tr>
<tr>
<td>error</td>
<td>0.002</td>
</tr>
</tbody>
</table>

Reasons for the discrepancy:

1) The analytical deflection does not account for the shear factor in the beams.
2) Also, the Analytical deflection does not account for the effect of the moments caused by the actual (instead of lumped) mass
3) The distribution of deflection is approximated in Nastran due to the color representations, the actual deflection ranges 8.003 to 8.0016.

As shown in Fig A.2 and A.3, the deflection of the cantilevered case produces a combined plate and system deflection configuration. The closed form analysis assumed that the free side plate did not deflect in the z-direction. However, it is shown that the effect of crossbar plate deflection on the side plates does indeed exist. Given that the model was created using units of millimeters, the results show an out-of-plane z deflection on the order of less than 10 microns. It is therefore substantiated that this assumption is indeed valid for the cantilevered case. Further, the comparison of the two methods coincides with the expected result. The FEA model takes into account the shear factor in the planar elements.
Since the closed form solution does not do this, and the rigidity of the system is relatively high, the deflection is dominated by shear rather than bending, the Nastran solution is much greater than the closed form solution. However, it is important to recognize that the purpose of this part of the analysis is to justify the use of these systems as coarse estimates for a provided system. The difference in end deflection is undisputed. However, the magnitude of the deflection itself presents the motivation to state that although there exists a significant error between the models, this error exists within the noise floor of the analysis.

Figure A.2. MSC/PATRAN plot of the NASTRAN solution 101 results for stage 1
Figure A.3. MSC/PATRAN plot of the NASTRAN solution 101 results for stage 1
SYSTEM NATURAL FREQUENCY CALCULATIONS: Stage 1f

STATEMENT: use Lagrangian to get eq of motion, solve for system NF and compare to Patran

COMPONENTS: L/R side plates, 3 crossbar members

ASSUMPTIONS:
1) Right side plate is ground, left out of calculations
2) Crossbars considered as a composite beam
3) Lumped mass parameter at cantilever end and representing left side plate is omitted.
4) Geometries of crossbars are negligible, as is angled orientation of bottom bar.
5) End deflection is sinusoidal.
6) .226 of beam mass used to approximate cantilever geom.

GOVERNING EQ: 1) Function \( F = \dot{f}(t, y) \) where \( t \) = time, \( y \) = defl at end, \( y' \) = slope at end

SCHEMATIC:

SOLUTION: \( F = T - V \) where \( T \) = the kinetic energy, \( V \) = the potential energy of the system

\[
T = \frac{1}{2} m v^2 = \frac{1}{2} \left( 1.60 \times 9.81 \right)\\
V = \frac{1}{2} k x^2\\
F = \frac{1}{2} m \left( y' \right)^2 - \frac{1}{2} k \left( y \right)^2\\
\frac{\delta F}{\delta y} = k \left( y \right)\\
\frac{d}{dt} \left( \frac{\delta F}{\delta y} \right) = -m \left( y' \right)^2
\]

Thus:

\[
m \dddot{y} + k x \left( y \right) = 0
\]

Conclusion: There exists a discrepancy in the analytical model. It is my belief that the assumed geometry, i.e., the simple mass spring system, was not valid. Also, the weight of the crossbars is a significant factor. Thus, the formulas used for \( m \) become invalid. Use a correction factor given in the text.

Natural Frequency Calculation:

\[ \omega_n = \sqrt{\frac{k}{m}} \]

\[ \omega_n = \frac{639.7259 \text{ rad/sec}}{2.255634} \]
CONCLUSION:

There still exists some error in the model, most probably due to the rigidity of the side plates or the internal moments of the beams.

CORRECTION METHOD:

Eq for the natural frequency of a cantilever beam:

\[ w_n = \frac{3.52(EI/mL'^{1/2})}{m} \]

where \( m \) = mass per unit length

\[ k = \frac{(3EI/L)}{m} \]

\( w_n = \frac{207000}{1.74E+08} \)

\( w_n \) corrected = 2.032\( w_n \)

2.032 is the cantilever beam correction factor, (Theory of Vibrations, Thomson/Dahleh, fifth edition)

\[ w_n = 1300.098 \text{ rad/sec} \]

\( f_n = 206.917 \text{ Hz} \)

Comparison to the Patran Model:

<table>
<thead>
<tr>
<th></th>
<th>NF</th>
<th>Hz</th>
<th></th>
<th>NF</th>
<th>Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analytical</td>
<td>206.92</td>
<td>Hz</td>
<td>Nastran Mode 4</td>
<td>206.3</td>
<td>Hz</td>
</tr>
<tr>
<td>error</td>
<td>-0.299</td>
<td>%</td>
<td>error</td>
<td>9.891</td>
<td>%</td>
</tr>
</tbody>
</table>

Reasons for the discrepancy:

1) Once again, since the model is actually 3 DOF and has been condensed, the internal moments caused at the left side plate have not been fully accounted for.

2) The mass distribution of the model plays a significant role. The cross member, once condensed, is actually about 3/4 the total weight of the model. Obviously, this causes the negligible mass assumption to break down. However, the Euler beam theory did verify that the .226 mass factor for cantilever beams is valid for finding the fundamental frequency.

3) After consideration of the mode shapes, it was determined that the first few modes were in fact system noise and did not represent the deformation which would be expected. In fact this deformation was not reached until the 4th mode and a frequency of 206.3 Hz. This does not mean that the above analysis is incorrect. However, it does provide insight to the inner workings of the model. In the frequency domain, the beam inertia had to be modeled as a composite using the parallel axis theorem. Despite this, the Nastran mode inherently contains both individual plate modes and resonant stress concentrations which cause the discrepancy in the fundamental frequency.

4) The Analytical model does not account for shear and is as expected, higher than the Nastran version.

5) It is concluded that, due to the preceding conclusion, that mode 4 must be the fundamental mode and thus the natural freq. = 206 Hz.

6) The reason for showing both mode 4 and 5 lies in that the mode shapes are far from clearly cut. Each is a combination of the fundamental modes and can only be assumed to be the closest to the analytical version. Mode 4 was selected since by intuition, the model which does not contain shear has a higher k value and thus a higher freq.
The natural frequencies of the frame are more closely aligned than the deflection results for the cantilevered case. The 0.3% error found in matching the fundamental modes is outstanding. This result clearly leads to the conclusion that a multi crossbar cantilevered system does and can be reduced to a single composite 1DOF system. The Mode 4 and 5 designation in the MSC/NASTRAN results implies the existence of resonance prior to rotary one shown in Figure A.4. These modes consist primarily of plate excitation and interaction response. The fifth mode closely paralleled the fourth with the exception of increased crossbar plate activity. For comparison sake, the fifth mode was included to show that the presence of the plates in the crossbars can throw the natural frequency found off by as much
as 10%. This factor should be noted for any frame design that incorporates plate like cross members and uses the closed form solution. This solution, in effect, combines the moments of inertia of the entire cross bar geometry into a single value used for calculation purposes. Lost in this translation are the individual resonances present in the crossbar element themselves. Although this effect is thought to be minimal, a radical change in frame design may require this type of condition to be looked at in more detail.
SUBCASE: Stage 1a

COMPONENTS: L/R side plates, 3 crossbar members

ASSUMPTIONS:
1) Right and Left side plates are supported by a 4 point fixed mount, and a latch mechanism. Assume that the geometry is simply supported to account for fixture in the side walls.
2) Crossbars considered in series since none are grounded
3) The weights of the side plates are added to lessen flexure error.
4) Geometries of x-bar bends are negligible, as is angled orientation of bottom bar
5) ymax occurs in the center, which agrees with the patran representation

GOVERNING EQ's:
1) k simply supported beam
   \[ k = \frac{6F}{L^2} \] N/mm
2) k eq springs in series
   \[ k = \sum k_i \] N/mm
3) I of a composite x-section
   \[ I_x = \int r^2 \, dr \] mm^4
4) Centroid of a x-section
   \[ y = \frac{\sum A_i y_i}{\sum A_i} \] mm
5) Beams deflection, distributed load
   \[ y_{max} = \frac{3FL^4}{384EI} \] mm
6) Weight \[ W = mg \] m = \[ \rho V \] \, W = \[ \rho Vg \] N

MAT'L PROPERTIES:
steel \( \rho = 7.80 \times 10^3 \text{ Mg/m}^3 \)
\( E = 2.075 \times 10^5 \text{ N/mm}^2 \)
t = 2.5 mm throughout

SCHEMATIC:
- Crossbar top
- Crossbar middle
- Crossbar bottom

SOLUTION:
Centroid Calculations:
\[ t = \frac{2.5}{3} \]

Ybar
Top: section 1 2 3
\( A \text{ mm}^2 \quad 99.375 \quad 335.3125 \quad 291.25 \)
\( y_{bar} \quad 135.5 \quad 67.0625 \quad 1.25 \)
product \( 13465.31 \quad 22486.89 \quad 364.0625 \)

Middle: section 1 2 3 4 5
\( A \text{ mm}^2 \quad 250 \quad 315.3 \quad 198.75 \quad 315.3 \quad 350 \)
\( y_{bar} \quad 253.615 \quad 191.68 \quad 127.245 \quad 127.245 \quad 63.06 \)
product \( 63403.75 \quad 60436.7 \quad 25299.94 \quad 19682.62 \quad 281.25 \)

Bottom: section 1 2 3
\( A \text{ mm}^2 \quad 115 \quad 332.5 \quad 115 \)
\( y_{bar} \quad 1.25 \quad 67.665 \quad 136.625 \)
product \( 143.75 \quad 22485.31 \quad 15711.88 \)

Ybar = \( 50.02672 \)
Ybar = \( 127.3513 \)
Ybar = \( 68.16167 \)
Moment of Inertia Calculations:

Top: $I_t = \int y^2 \, dA = \int y^2 \, h \, dy = \sum \frac{1}{4} y_b y^3 \, dy = \frac{1}{3} \left( 39.75 \cdot y^3 + 2.5 \cdot y^3 + 116.5 \cdot y^3 \right) = 2004879 \, \text{mm}^4$

Middle: $I_m = \int y^2 \, dA = \int y^2 \, h \, dy = \sum \frac{1}{4} y_b y^3 \, dy = \frac{1}{3} \left( 2100 \cdot y^3 + 2.25 \cdot y^3 + 79.5 \cdot y^3 \right) = 11397385 \, \text{mm}^4$

Bottom: $I_b = \int y^2 \, dA = \int y^2 \, h \, dy = \sum \frac{1}{4} y_b y^3 \, dy = \frac{1}{3} \left( 246.0 \cdot y^3 + 2.5 \cdot y^3 \right) = 1545967 \, \text{mm}^4$

Beam Spring Constant Calculations:

<table>
<thead>
<tr>
<th>E</th>
<th>I</th>
<th>L</th>
<th>L'</th>
<th>k = 48EI/L'</th>
</tr>
</thead>
<tbody>
<tr>
<td>N/mm²</td>
<td>mm³</td>
<td>mm</td>
<td>mm³</td>
<td>N/mm</td>
</tr>
<tr>
<td>Top:</td>
<td>207000</td>
<td>2004879</td>
<td>486.8</td>
<td>1.17E+08</td>
</tr>
<tr>
<td>Middle:</td>
<td>11397385</td>
<td>486.8</td>
<td>1.17E+08</td>
<td>968668.3444</td>
</tr>
<tr>
<td>Bottom:</td>
<td>1545967</td>
<td>486.8</td>
<td>1.17E+08</td>
<td>131827.9876</td>
</tr>
</tbody>
</table>

Equivalent Spring Constant Calculation:

$\frac{1}{k_{eq}} = \sum \frac{1}{k_i}$

$keq = 68990.39116 \, \text{N/mm}$

Equivalent System:

$k_{eq} = 68990.39 \, \text{N/mm}$

where: $W_{1,3}$ is the uniformly distributed load of each cross member

Component Weight Calculations:

$W = mg$

<table>
<thead>
<tr>
<th>m = \rho V</th>
<th>W = \rho Vg</th>
<th>7.8E-9 Mg/mm³</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>g</td>
<td>g</td>
</tr>
<tr>
<td>Mg/mm³</td>
<td>mm²</td>
<td>m/s</td>
</tr>
<tr>
<td>Top:</td>
<td>7.80E-09</td>
<td>725.9375</td>
</tr>
<tr>
<td>Middle:</td>
<td>7.80E-09</td>
<td>1329.35</td>
</tr>
<tr>
<td>Bottom:</td>
<td>7.80E-09</td>
<td>562.5</td>
</tr>
</tbody>
</table>

equivalent cross member

<table>
<thead>
<tr>
<th>m = \rho V</th>
<th>W = \rho Vg</th>
<th>7.8E-9 Mg/mm³</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>g</td>
<td>g</td>
</tr>
<tr>
<td>Mg/mm³</td>
<td>mm²</td>
<td>m/s</td>
</tr>
<tr>
<td>Top:</td>
<td>7.80E-09</td>
<td>2617.788</td>
</tr>
</tbody>
</table>
Beam Center Deflection:

\[
\begin{array}{cccc}
W & L & E & I \\
N/mm & mm & N/mm^2 & mm^4 \\
W_{\max} & & & \text{mm} \\
\hline
\text{Top:} & 5.55E-02 & 488.8 & 207000 & 2004879 \\
& & & -1.99E-05 \\
\text{Middle:} & 1.02E-01 & 488.8 & 207000 & 11397385 \\
& & & -6.41E-06 \\
\text{Bottom:} & 4.30E-02 & 488.8 & 207000 & 1545967 \\
& & & -2.00E-05 \\
\text{Equivalent beam} & 2.00E-01 & 488.8 & 207000 & 14948231 \\
& & & -9.62E-06 \\
\end{array}
\]

Comparison to the Patran Model:

\[
\begin{array}{ccc}
\gamma_{\text{max}} & \text{TOP} & \text{MIDDLE} \\
\text{Analytical} & -2.0E-05 & -6.4E-06 & -2.0E-05 \\
\text{Nastran error} & <.001 & <.001 & <.001 \\
\text{Nastran} & \#\text{VALUE!} & \#\text{VALUE!} & \#\text{VALUE!} \\
\end{array}
\]

Reasons for the discrepancy:

1) The analytical deflection does not account for the shear factor in the beams and is as expected lower than the actual.
2) Also, the Analytical deflection does not account for the effect of the moments caused by the actual (instead of lumped) mass geometry.
3) Nastran deflection value is maximum for the color range shown in the deformation plot.
Figure A.5. **MSC/PATRAN plot of the NASTRAN solution 101 results for stage 1a**

The deflection shown in Fig A.5 emphasizes the condition discussed for the cantilevered deflection case. In this mounting configuration, the simple supports prevent any amount of significant deflection along the side plates to occur. In fact, the deflection is dominated not only by shear, but shear only in the cross members of the system. Furthermore, this deflection is symmetrical and trails off to zero as the side plates are approached. Likewise, the closed for solution of this case shows deflections on the order of $10^{-6}$ mm., or virtually zero. It is therefore determined that the simply supported case, in the deflection domain, can be represented by the closed form solution. Further, it is deemed that the simply supported frame does not statically interact with the belt / roll system in any significant way regardless of geometry.
SUBCASE: Stage 1af

SYSTEM NATURAL FREQUENCY CALCULATIONS:

STATEMENT: use La Grange to get eq of motion, solve for system NF and compare to Patran

COMPONENTS: L/R side plates, 3 crossbar members

ASSUMPTIONS:
1) Right side plate is ground, left out of calculations
2) Crossbars considered in parallel since each is grounded
3) Lumped mass parameter at cantilever end and represents left side plate
4) Geometries of x-bars: bends are negligible, as is angled orientation of bottom bar
5) End deflection is sinusoidal

GOVERNING EQ's: 1) Function $F(t,y,y')$ where $t =$ time, $y =$ defl at end, $y'$ = slope at end
2) La Grange $\frac{\delta F}{\delta y} \frac{d}{dt} \frac{\delta F}{\delta y'} = 0$

SCHEMATIC:

SOLUTION:

LET: $F = T - V$ where $T =$ the kinetic energy, $V =$ the potential energy of the system

$T = \frac{1}{2} m v^2 = \frac{1}{2} \left( \frac{1.60 \ E - 2 \ 9.81}{9.81} \right)$

$V = \frac{1}{2} k x^2$

$F = \frac{1}{2} m \left( y'' \right)^2 - \frac{1}{2} k \left( y' \right)^2$

$\frac{\delta F}{\delta y} = k \mu (y)$

$\frac{\delta F}{\delta y'} = -m \left( y' \right)$

$\frac{d}{dt} \left( \frac{\delta F}{\delta y'} \right) = -m \ddot{y}$

Thus:

$m \dddot{y} + k \mu (y) = 0$

$\text{NF} = \sqrt{\frac{k \mu}{m}}$

Mass Calculation:

$M = Wg$

Consider only cross bar weight

$W = 9.79E+01$ N

$g = 9.81 \text{ m/s}^2$

$M = 9.98 \text{ kg}$

Natural Frequency Calculation:

$\omega_n = 356.9635 \text{ Hz}$

Conclusion:
There exists a discrepancy in the analytical model. It is my belief that the assumed geometry, i.e. the simple mass spring system, was not valid.
CORRECTION METHOD:

**Eq for the natural frequency of a simply supported beam:**

\[ \omega_n \approx \left( \frac{EI}{mL^4} \right)^{1/2} \]

where \( m \) = mass per unit length.

\[ \omega_{n, \text{corrected}} = 1.424555 \omega_n \]

5.696 is the cantilever beam correction factor, (Theory of Vibrations, Thomson/Dahleh, fifth edition)

\[ \omega_{n, \text{corrected}} = 508.514 \text{ rad/sec} \]

**Correction Factor**

\[ \omega_n = 80.93 \text{ Hz} \]

**CORRECTION METHOD 2:**

Euler Beam theory p273 (Theory of Vibrations, Thomson/Dahleh, fifth edition)

\[ (BL)^2 \]

<table>
<thead>
<tr>
<th>units correction</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.97 207000 1448231 2.04E-02 488.6 5.71E+10 508.5344 60.93576 Hz</td>
</tr>
</tbody>
</table>

**Comparison to the Patran Model:**

<table>
<thead>
<tr>
<th>NF Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analytical 80.9 Hz</td>
</tr>
<tr>
<td>Nastran 110.8 Hz</td>
</tr>
<tr>
<td>error 26.956 %</td>
</tr>
</tbody>
</table>

**Reasons for the discrepancy:**

1) Once again, the effect of the moments created by the distributed beam elements was not accounted for.

2) There must exist some interaction of the cross members since the analytical frequency is lower than the actual. This goes against intuition. Therefore, one of the beams must be dominating the system, most probably the middle one due to its weight and geometric properties.

**CORRECTION METHOD 2: REVISITED**

Euler Beam theory p273 (Theory of Vibrations, Thomson/Dahleh, fifth edition)

<table>
<thead>
<tr>
<th>units correction</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.97 207000 11397395 1.04E-02 488.6 5.71E+10 623.1256 99.17352 Hz</td>
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</tbody>
</table>

**Comparison to the Patran Model for Middle Beam Domination:**

<table>
<thead>
<tr>
<th>NF Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analytical 99.2 Hz</td>
</tr>
<tr>
<td>Nastran 110.8 Hz</td>
</tr>
<tr>
<td>error 10.493 %</td>
</tr>
</tbody>
</table>

**Result:**

1) Indeed, the Analytical fundamental frequency closes in on the Nastran value. However, it is still lower which contradicts theory. Perhaps, it is the essence of this problem, i.e., the beams are trying to act independently and the combination of them actually raises the NF.
Figure A.6. MSC/PATRAN plot of the NASTRAN solution 103 results for stage 1af
Unfortunately, the dynamic response of the simply supported frame is not as cut and dried as its static counterpart. As clearly shown in Figure A.6 and A.7, the resonance of the system produces significant side plate deflection at 110.8 Hz. However, this resonance is strictly related to the exact mounting configuration of the frame. Again referring to Figure A.6 and A.7, it becomes evident that the dominant motion is the central cross member in the z direction. The mass of this cross bar with respect to the stiffness of the side plates allows for this type of motion at its natural frequency. It is important to note that the mass and location of this crossbar is dependent on the individual frame design of each photoreceptor. Furthermore, the resonant frequency given an individual system may not be excited during operation. If this were the case, then this type of resonant behavior could be ignored. This presents the only frame condition that must be met in order to advance to the static and dynamic analyses of this thesis.
A2 FEM Element Derivations

A2.1 Belt Element Derivation 1st order

The belt transverse deflection derivation assumes that all four edges of the belt are restrained such that the element acts as a membrane under out-of-plane loading. Since only the belt and rollers are considered in this analysis, the amount or incidence of this loading directly on the belt is neglected or set to zero. However, as will be shown in the example problem, these forces may be added at any point in the system. It is therefore assumed that all input to the belt reaction shape is a function of the boundary conditions that act on the belt sections. The edges of the belt not attached to the rollers do not allow for out-of-plane deflection. It is already known through Xerox research that the outer edge of the belt contains a higher tension than the inside. This outer tension will tend to flatten the belt on the outer edges with respect to the belt section as a whole, which allows this assumption to be made. The sections of belt that do indeed contact the rollers are attached numerically by a ratio of 1. This means that there can exist no gap or slip between the belt and roll surfaces. Given this, when the roller is deflected under a load, the belt is pulled with it, transmitting through its boundary condition, the deflection pattern. By resolving this boundary condition into its components, it becomes feasible, that the belt derivation can exist using only these conditions as a forcing function.

The belt section mesh must also coincide with the roller mesh that is chosen. Since this mesh is an "n" element array along the axis of the roll, hence the belt must be compatible with the roll node configuration. This is not to say, however, that the higher order elements can not be used with the current configuration of roll mesh density. The number of belt elements may differ from that of the roll provided that the number of nodes required is the same and that the connectivity of those nodes is intact. For symmetrical calculation advantages, the process direction of the belt was matched to the lateral mesh density resulting in an arbitrary 10x10 mesh definition per belt section. The element definition is shown in Figure A.8.
10x10 mesh density = 100 elements

h,l are therefore section specific
\[ U \equiv \sum_{e=1}^{100} \hat{U}^e \]
\[ \hat{U}^e = \sum_{i=1}^{4} \psi_i U^e_i \]
\[ \psi_i = \frac{1}{4} (1 + \zeta \zeta_i)(1 + \eta \eta_i) \]

Element Equation
\[-a \nabla^2 U = f(x, y)\]
\[ R = -a \nabla^2 U - f(x, y) \neq 0 \]
\[ \int_{\Omega} \psi_i \left( -a \nabla^2 U - f(x, y) \right) d\Omega = 0 \]

\[ \int G \nabla^2 F = -\int \nabla G \cdot F d\Omega + \frac{1}{2} \int \nabla F d\Gamma \]
\[ a \int_{\Omega} \psi_i \cdot \nabla \hat{U} d\Omega - a \oint \psi_i \left( \hat{n} \cdot \nabla \hat{U} \right) d\Gamma - \int \psi_i f(x, y) d\Omega = 0 \]
\[ \sum \int_{\Omega} \left( \frac{\partial \psi_i}{\partial x} \frac{\partial \psi_j}{\partial x} + \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_j}{\partial y} \right) d\Omega U^e_j = \int \psi_i f(x, y) d\Omega + a \oint \psi_i \left( -n_x \frac{\partial \hat{U}}{\partial x} + n_y \frac{\partial \hat{U}}{\partial y} \right) d\Gamma \]

\[ [K]_{4 \times 4} U^e = F^e + b^e \]

\[ K^e_{ij} = a \int_{\Omega} \left( \frac{\partial \psi_i}{\partial x} \frac{\partial \psi_j}{\partial x} + \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_j}{\partial y} \right) d\Omega \]
\[ F^e_i = \int_{\Omega} \psi_i f(x, y) d\Omega \]
\[ b^e_i = a \oint \psi_i \left( n_x \frac{\partial \hat{U}}{\partial x} + n_y \frac{\partial \hat{U}}{\partial y} \right) d\Gamma \]

Recall that for d\Omega approx. dxdy we need the Jacobian
\[ [J] = \begin{bmatrix} \frac{\partial X}{\partial \zeta} & \frac{\partial Y}{\partial \zeta} \\ \frac{\partial X}{\partial \eta} & \frac{\partial Y}{\partial \eta} \end{bmatrix} \]

\[ dx dy = |J| d\zeta d\eta \]

also

\[ \left( \begin{array}{c} \frac{\partial \psi_i}{\partial x} \\ \frac{\partial \psi_i}{\partial y} \end{array} \right) = [J]^{-1} \left( \begin{array}{c} \frac{\partial \psi_i}{\partial \zeta} \\ \frac{\partial \psi_i}{\partial \eta} \end{array} \right) \]

\[ \frac{\partial X}{\partial \zeta} = \sum_{i=1}^{4} \frac{\partial \psi_i}{\partial \zeta} x_i \]

\[ = \sum \frac{1}{4} (\zeta_i)(1 + \eta \eta_i)x_i \]

\[ = \frac{1}{4} \left\{ -(1 - \eta)x_i + (1 - \eta)x_2 - (1 + \eta)x_3 + (1 + \eta)x_4 \right\} \]

\[ \frac{\partial X}{\partial \eta} = \sum_{i=1}^{4} \frac{\partial \psi_i}{\partial \eta} x_i \]

\[ = \sum \frac{1}{4} (\eta_i)(1 + \zeta \zeta_i)x_i \]

\[ = \frac{1}{4} \left\{ -(1 - \zeta)x_1 - (1 + \zeta)x_2 + (1 - \zeta)x_3 + (1 + \zeta)x_4 \right\} \]
\[ \frac{\partial y}{\partial \zeta} = \sum_i \frac{1}{4} \frac{\partial \psi_i}{\partial \zeta} y_i \]
\[ = \frac{1}{4} \left\{ -(1-\eta)y_1 + (1-\eta)y_2 - (1+\eta)y_3 + (1+\eta)y_4 \right\} \]

\[ \frac{\partial y}{\partial \eta} = \sum_i \frac{1}{4} \frac{\partial \psi_i}{\partial \eta} y_i \]
\[ = \frac{1}{4} \left\{ -(1-\zeta)y_1 + (1+\zeta)y_2 + (1-\zeta)y_3 + (1+\zeta)y_4 \right\} \]

For a Typical Element of \( h,1 \)
\[ \frac{\partial x}{\partial \zeta} = \frac{1}{4} \left\{ (1-\eta)\Delta x + (1+\eta)\Delta x \right\} = \frac{1}{4} \left\{ 2\Delta x \right\} = \frac{\Delta x}{2} \]

Likewise
\[ \frac{\partial x}{\partial \eta} = 0 \]
\[ \frac{\partial y}{\partial \zeta} = 0 \]
\[ \frac{\partial y}{\partial \eta} = \frac{\Delta y}{2} \]

Hence:
Solving for \( K_e \)

\[
J = \begin{bmatrix}
\frac{\Delta x}{2} & 0 \\
0 & \frac{\Delta y}{2}
\end{bmatrix} = \begin{bmatrix}
h/2 & 0 \\
0 & l/2
\end{bmatrix}
\]

\[
|J| = \frac{\Delta x \Delta y}{4} = \frac{hl}{4}
\]

\[
J^{-1} = \begin{bmatrix}
\frac{2}{\Delta x} & 0 \\
0 & \frac{2}{\Delta y}
\end{bmatrix} = \begin{bmatrix}
\frac{2}{h} & 0 \\
0 & \frac{2}{l}
\end{bmatrix}
\]

\[
[K] = \frac{1}{6} \begin{bmatrix}
4 & -1 & -1 & -2 \\
-1 & 4 & -2 & -1 \\
-1 & -2 & 4 & -1 \\
-2 & -1 & -1 & 4
\end{bmatrix}
\]
A2.2 3\textsuperscript{rd} order

The cubic quadrilateral elements provide an even more in depth and accurate look at the deflection of the belt under loading, however, the computational requirement has now increased to a 12 shape function per element calculation. The derivation is as follows.

![Diagram of Individual Cubic Belt Element definition](image)

**Notes:**
- \( h \) is constant throughout photoreceptor
- \( l \) is constant for each belt section
- \( \zeta \) is the local GWR coord. Axis
- \( \eta \) is the local GWR coord. Axis
- Denotes a node location

\[
\frac{\partial x}{\partial \zeta} = \sum_{i=1}^{12} \frac{\partial \psi_i}{\partial \zeta} x_i
\]

\[
= \frac{1}{32} \begin{bmatrix}
10x_1 - 27x_2 + 27x_3 - 10x_4 - 9x_5 + 9x_6 - 9x_7 + 9x_8 + 10x_9 - 27x_{10} + 27x_{11} - 10x_{12} + \\
\eta(-10x_1 + 27x_2 - 27x_3 + 10x_4 + 27x_5 - 27x_6 - 27x_7 + 27x_8 + 10x_9 - 27x_{10} + 27x_{11} - 10x_{12}) + \\
\eta^2(-9x_1 + 9x_4 + 9x_5 - 9x_6 + 9x_7 - 9x_9 + 9x_{10}) + \\
\xi^3(9x_1 - 9x_4 - 27x_5 + 27x_6 + 27x_7 - 27x_8 - 9x_9 + 9x_{12}) + \\
\eta(18x_1 - 18x_2 - 18x_3 + 18x_4 + 18x_5 - 18x_{10} - 18x_{11} + 18x_{12}) + \\
\xi^2(-27x_1 + 81x_2 - 81x_3 + 27x_4 - 27x_5 + 81x_{10} - 81x_{11} + 27x_{12}) + \\
\eta(27x_1 - 81x_2 + 81x_3 - 27x_4 - 27x_5 + 81x_{10} - 81x_{11} + 27x_{12}) \\
\end{bmatrix}
\]

\[
= \frac{1}{32} \begin{bmatrix}
10x_1 - 27x_2 + 27x_3 - 10x_4 - 9x_5 + 9x_6 - 9x_7 + 9x_8 + 10x_9 - 27x_{10} + 27x_{11} - 10x_{12} + \\
\eta(-10x_1 + 27x_2 - 27x_3 + 10x_4 + 27x_5 - 27x_6 - 27x_7 + 27x_8 + 10x_9 - 27x_{10} + 27x_{11} - 10x_{12}) + \\
\eta^2(-9x_1 + 9x_4 + 9x_5 - 9x_6 + 9x_7 - 9x_9 + 9x_{10}) + \\
\xi^3(9x_1 - 9x_4 - 27x_5 + 27x_6 + 27x_7 - 27x_8 - 9x_9 + 9x_{12}) + \\
\eta(18x_1 - 18x_2 - 18x_3 + 18x_4 + 18x_5 - 18x_{10} - 18x_{11} + 18x_{12}) + \\
\xi^2(-27x_1 + 81x_2 - 81x_3 + 27x_4 - 27x_5 + 81x_{10} - 81x_{11} + 27x_{12}) + \\
\eta(27x_1 - 81x_2 + 81x_3 - 27x_4 - 27x_5 + 81x_{10} - 81x_{11} + 27x_{12}) \\
\end{bmatrix}
\]

\[
= \frac{1}{32} \begin{bmatrix}
10x_1 - 27x_2 + 27x_3 - 10x_4 - 9x_5 + 9x_6 - 9x_7 + 9x_8 + 10x_9 - 27x_{10} + 27x_{11} - 10x_{12} + \\
\eta(-10x_1 + 27x_2 - 27x_3 + 10x_4 + 27x_5 - 27x_6 - 27x_7 + 27x_8 + 10x_9 - 27x_{10} + 27x_{11} - 10x_{12}) + \\
\eta^2(-9x_1 + 9x_4 + 9x_5 - 9x_6 + 9x_7 - 9x_9 + 9x_{10}) + \\
\xi^3(9x_1 - 9x_4 - 27x_5 + 27x_6 + 27x_7 - 27x_8 - 9x_9 + 9x_{12}) + \\
\eta(18x_1 - 18x_2 - 18x_3 + 18x_4 + 18x_5 - 18x_{10} - 18x_{11} + 18x_{12}) + \\
\xi^2(-27x_1 + 81x_2 - 81x_3 + 27x_4 - 27x_5 + 81x_{10} - 81x_{11} + 27x_{12}) + \\
\eta(27x_1 - 81x_2 + 81x_3 - 27x_4 - 27x_5 + 81x_{10} - 81x_{11} + 27x_{12}) \\
\end{bmatrix}
\]

\[
= \frac{3h}{2} = \frac{\Delta x}{2}
\]
\[
\frac{\partial x}{\partial \eta} = \sum_{i=1}^{12} \frac{\partial \psi_i}{\partial \eta} x_i \\
\begin{bmatrix}
10x_1 - 9x_3 + 10x_4 - 27x_5 - 27x_6 + 27x_7 + 27x_8 - 10x_9 + 9x_{10} + 9x_{11} - 10x_{12} + \\
\zeta(-10x_1 + 27x_2 - 27x_3 + 10x_4 + 27x_5 - 27x_6 - 27x_7 + 27x_8 + 10x_9 - 27x_{10} + 27x_{11} - 10x_{12}) + \\
\zeta^2(-9x_1 + 9x_4 + 9x_5 - 9x_6 + 9x_7 - 9x_8 - 9x_9 + 9x_{12}) + \\
\zeta^3(9x_1 - 27x_2 + 27x_3 - 9x_4 + 27x_{10} - 27x_{11} + 9x_{12}) + \\
\eta(18x_1 + 18x_4 - 18x_5 - 18x_6 - 18x_7 - 18x_8 + 18x_9 + 18x_{12}) + \\
\eta^2(-27x_1 - 27x_4 + 81x_5 + 81x_6 - 81x_7 - 81x_8 + 27x_9 + 27x_{12}) + \\
\eta^3(-18x_1 + 18x_4 + 18x_5 - 18x_6 + 18x_7 - 18x_8 - 18x_9 + 18x_{12}) + \\
\eta^2(27x_1 - 27x_4 - 81x_5 + 81x_6 + 81x_7 - 81x_8 - 27x_9 + 27x_{12}) \\
\end{bmatrix} = \frac{1}{32} [0] = 0
\]

\[
\frac{\partial y}{\partial \eta} = \sum_{i=1}^{12} \frac{\partial \psi_i}{\partial \eta} y_i \\
\begin{bmatrix}
10y_1 - 9y_2 + 10y_4 - 27y_5 - 27y_6 + 27y_7 + 27y_8 - 10y_9 + 9y_{10} + 9y_{11} - 10y_{12} + \\
\zeta(-10y_1 + 27y_2 - 27y_3 + 10y_4 + 27y_5 - 27y_6 - 27y_7 + 27y_8 + 10y_9 - 27y_{10} + 27y_{11} - 10y_{12}) + \\
\zeta^2(-9y_1 + 9y_4 + 9y_5 - 9y_6 + 9y_7 - 9y_8 - 9y_9 + 9y_{12}) + \\
\zeta^3(9y_1 - 27y_2 + 27y_3 - 9y_4 + 27y_{10} - 27y_{11} + 9y_{12}) + \\
\eta(18y_1 + 18y_4 - 18y_5 - 18y_6 - 18y_7 - 18y_8 + 18y_9 + 18y_{12}) + \\
\eta^2(-27y_1 - 27y_4 + 81y_5 + 81y_6 - 81y_7 - 81y_8 + 27y_9 + 27y_{12}) + \\
\eta^3(-18y_1 + 18y_4 + 18y_5 - 18y_6 + 18y_7 - 18y_8 - 18y_9 + 18y_{12}) + \\
\eta^2(27y_1 - 27y_4 - 81y_5 + 81y_6 + 81y_7 - 81y_8 - 27y_9 + 27y_{12}) \\
\end{bmatrix} = \frac{1}{32} [48l] \\
\frac{3l}{2} = \frac{\Delta y}{2}
\]
\[ [J] = \begin{bmatrix} \frac{\Delta x}{2} & 0 \\ 0 & \frac{\Delta y}{2} \end{bmatrix} = \frac{3}{2} \begin{bmatrix} h & 0 \\ 0 & l \end{bmatrix} \]

\[ |J| = \frac{\Delta x \Delta y}{4} = \frac{9}{4} hl \]

\[ [J]^{-1} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \frac{1}{h} \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{l} \end{bmatrix} \]

\[ \left( \frac{\partial \psi_i}{\partial x}, \frac{\partial \psi_i}{\partial y} \right) = [J]^{-1} \left( \frac{\partial \psi_i}{\partial \zeta}, \frac{\partial \psi_i}{\partial \eta} \right) \]

\[ \frac{\partial \psi_i}{\partial x} = \frac{2}{3h} \frac{\partial \psi_i}{\partial \zeta} \\
\frac{\partial \psi_i}{\partial y} = \frac{2}{3l} \frac{\partial \psi_i}{\partial \eta} \]

So:

\[ K_0 = a \int \left\{ \frac{\partial \psi_i}{\partial x} \frac{\partial \psi_j}{\partial x} + \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_j}{\partial y} \right\} [J] \delta \zeta \delta \eta \]

\[ = a \int \left\{ \left( \frac{2}{3h} \right)^2 \frac{\partial \psi_i}{\partial \zeta} \frac{\partial \psi_j}{\partial \zeta} + \left( \frac{2}{3l} \right)^2 \frac{\partial \psi_i}{\partial \eta} \frac{\partial \psi_j}{\partial \eta} \right\} \left( \frac{9}{4} hl \right) \delta \zeta \delta \eta \]

\[ = a \int \left\{ \left( \frac{1}{h} \right)^2 \frac{\partial \psi_i}{\partial \zeta} \frac{\partial \psi_j}{\partial \zeta} + \left( \frac{1}{l} \right)^2 \frac{\partial \psi_i}{\partial \eta} \frac{\partial \psi_j}{\partial \eta} \right\} (hl) \delta \zeta \delta \eta \]

Once again, the mathematical derivation of the stiffness matrix is more efficient using MAPLE V. The results of that analysis, as well as the notation are of the same format as the serendipity elements. Because of the redundancy inherent in their derivation, this information is not included in this report. However, the Partial derivatives are as follows.

MAPLE V input: Psi function derivatives
\[ \psi_1 = \frac{1}{32} (1 - \zeta)(1 - \eta) \left[ -10 + 9 (\zeta^2 + \eta^2) \right] \]
\[ = \frac{1}{32} \left[ 1 - \zeta - \eta + \eta \zeta \left[ -10 + 9 \zeta^2 + 9 \eta^2 \right] \right] \]
\[ = \frac{1}{32} \left[ -10 + 9 \zeta^2 + 9 \eta^2 + 10 \zeta - 9 \zeta^3 - 9 \eta^2 \zeta + 10 \eta - 9 \zeta^2 \eta - 9 \eta^3 - 10 \eta \zeta + 9 \eta \zeta^3 + 9 \zeta \eta^3 \right] \]
\[ \frac{\partial \psi_1}{\partial \zeta} = \frac{1}{32} \left[ 18 \zeta + 10 - 27 \zeta^2 - 9 \eta^2 - 18 \eta \zeta - 10 \eta + 27 \eta \zeta^2 + 9 \eta^3 \right] \]
\[ \frac{\partial \psi_1}{\partial \eta} = \frac{1}{32} \left[ 18 \eta - 18 \eta \zeta + 10 - 9 \zeta^2 - 27 \eta^2 - 10 \zeta + 9 \zeta^3 + 27 \zeta \eta^2 \right] \]

\[ \psi_2 = \frac{9}{32} (1 - \eta)(1 - \zeta^2)(1 - \zeta) \]
\[ = \frac{9}{32} (1 - \zeta - \eta + \eta \zeta^2)(1 - \zeta) \]
\[ = \frac{9}{32} (1 - 3 \zeta - \eta + 3 \eta \zeta - \zeta^2 + 3 \zeta^3 + \eta \zeta^2 - 3 \eta \zeta^3) \]
\[ \frac{\partial \psi_2}{\partial \zeta} = \frac{9}{32} (-3 - 3 \eta - 2 \zeta + 9 \zeta^2 + 2 \eta \zeta - 9 \eta \zeta^2) \]
\[ \frac{\partial \psi_2}{\partial \eta} = \frac{9}{32} (-1 - 3 \zeta + \zeta^2 - 3 \zeta^3) \]

\[ \psi_3 = \frac{9}{32} (1 - \eta)(1 - \zeta^2)(1 + \zeta) \]
\[ = \frac{9}{32} (1 - \zeta - \eta + \eta \zeta^2)(1 + \zeta) \]
\[ = \frac{9}{32} (1 + 3 \zeta - \eta - 3 \eta \zeta - \zeta^2 - 3 \zeta^3 + \eta \zeta^2 + 3 \eta \zeta^3) \]
\[ \frac{\partial \psi_3}{\partial \zeta} = \frac{9}{32} (3 - 3 \eta - 2 \zeta - 9 \zeta^2 + 2 \eta \zeta + 9 \eta \zeta^2) \]
\[ \frac{\partial \psi_3}{\partial \eta} = \frac{9}{32} (-1 - 3 \zeta + \zeta^2 + 3 \zeta^3) \]
\[ \psi_4 = \frac{1}{32} (1+\zeta)(1-\eta)[-10+9(\zeta^2+\eta^2)] \]
\[ = \frac{1}{32} (1+\zeta-\eta-\eta\zeta)[-10+9\zeta^2+9\eta^2] \]
\[ = \frac{1}{32} [-10+9\zeta^2+9\eta^2-10\zeta+9\zeta^3+9\eta^2\zeta +10\eta-9\eta\zeta^2-9\eta^3+10\eta\zeta-9\eta\zeta^3-9\eta^3\zeta] \]
\[ \frac{\partial \psi_4}{\partial \zeta} = \frac{1}{32} [18\zeta-10+27\zeta^2+9\eta^2-18\eta\zeta+10\eta-27\eta\zeta^2-9\eta^3] \]
\[ \frac{\partial \psi_4}{\partial \eta} = \frac{1}{32} [18\eta+18\eta\zeta+10-9\zeta^2-27\eta^2+10\zeta-9\zeta^3-27\eta^2\zeta] \]
\[ \psi_5 = \frac{9}{32} (1-\zeta)(1-\eta^2)(1-\eta) \]
\[ = \frac{9}{32} (1-\zeta-\eta^2+\eta^2\zeta)(1-\eta) \]
\[ = \frac{9}{32} (1-3\eta-\zeta+3\eta\zeta-\eta^2+3\eta^3+\eta^2\zeta-3\eta^3\zeta) \]
\[ \frac{\partial \psi_5}{\partial \zeta} = \frac{9}{32} (-1+3\eta+\eta^2-3\eta^3) \]
\[ \frac{\partial \psi_5}{\partial \eta} = \frac{9}{32} (-3+3\zeta-2\eta+9\eta^2+2\eta\zeta-9\eta^2\zeta) \]
\[ \psi_6 = \frac{9}{32} (1+\zeta)(1-\eta^2)(1-\eta) \]
\[ = \frac{9}{32} (1+\zeta-\eta^2-\eta^2\zeta)(1-\eta) \]
\[ = \frac{9}{32} (1-3\eta+\zeta-3\eta\zeta-\eta^2+3\eta^3-\eta^2\zeta+3\eta^3\zeta) \]
\[ \frac{\partial \psi_6}{\partial \zeta} = \frac{9}{32} (1-3\eta-\eta^2+3\eta^3) \]
\[ \frac{\partial \psi_6}{\partial \eta} = \frac{9}{32} (-3-3\zeta-2\eta+9\eta^2-2\eta\zeta+9\eta^2\zeta) \]
\[ \psi_\eta = \frac{9}{32} (1 - \zeta)(1 - \eta^2)(1 + 3\eta) \]
\[ = \frac{9}{32} (1 - \zeta - \eta^2 + \eta^2 \zeta)(1 + 3\eta) \]
\[ = \frac{9}{32} (1 + 3\eta - \zeta - 3\eta \zeta - \eta^2 - 3\eta^3 + \eta^2 \zeta + 3\eta^3 \zeta) \]
\[ \frac{\partial \psi_\eta}{\partial \zeta} = \frac{9}{32} (-1 - 3\eta + \eta^2 + 3\eta^3) \]
\[ \frac{\partial \psi_\eta}{\partial \eta} = \frac{9}{32} (3 - 3\zeta - 2\eta - 9\eta^2 + 2\eta \zeta + 9\eta^2 \zeta) \]
\[ \psi_\zeta = \frac{9}{32} (1 + \zeta)(1 - \eta^2)(1 + 3\eta) \]
\[ = \frac{9}{32} (1 + \zeta - \eta^2 - \eta^2 \zeta)(1 + 3\eta) \]
\[ = \frac{9}{32} (1 + 3\eta + \zeta + 3\eta \zeta - \eta^2 - 3\eta^3 - \eta^2 \zeta - 3\eta^3 \zeta) \]
\[ \frac{\partial \psi_\zeta}{\partial \zeta} = \frac{9}{32} (1 + 3\eta - \eta^2 - 3\eta^3) \]
\[ \frac{\partial \psi_\zeta}{\partial \eta} = \frac{9}{32} (3 + 3\zeta - 2\eta - 9\eta^2 - 2\eta \zeta - 9\eta^2 \zeta) \]
\[ \psi_\eta = \frac{1}{32} (1 - \zeta)(1 + \eta)[-10 + 9(\zeta^2 + \eta^2)] \]
\[ = \frac{1}{32} [-10 + 9\zeta^2 + 9\eta^2] \]
\[ = \frac{1}{32} [-10 + 9\zeta^2 + 9\eta^2 + 10\zeta - 9\zeta^3 - 9\eta^2 \zeta - 10\eta + 9\zeta^2 \eta + 9\eta^3 + 10\eta \zeta - 9\eta \zeta^3 - 9\eta \eta^3] \]
\[ \frac{\partial \psi_\eta}{\partial \zeta} = \frac{1}{32} [18\zeta + 10 - 27\zeta^2 - 9\eta^2 + 18\eta \zeta + 10\eta - 27\eta \zeta^2 - 9\eta^3] \]
\[ \frac{\partial \psi_\eta}{\partial \eta} = \frac{1}{32} [18\eta - 18\eta \zeta - 10 + 9\zeta^2 + 27\eta^2 + 10\zeta - 9\zeta^3 - 27\eta \eta^2] \]
\[ \psi_{10} = \frac{9}{32} (1 + \eta)(1 - \zeta^2)(1 - 3\zeta) \]
\[ = \frac{9}{32} (1 + \eta - \zeta^2 - \eta\zeta^2)(1 - 3\zeta) \]
\[ = \frac{9}{32} (1 - 3\zeta + \eta - 3\eta\zeta - \zeta^2 + 3\zeta^3 - \eta\zeta^2 + 3\eta\zeta^3) \]
\[ \frac{\partial \psi_{10}}{\partial \zeta} = \frac{9}{32} (-3 - 3\eta - 2\zeta + 9\zeta^2 - 2\eta\zeta + 9\eta\zeta^2) \]
\[ \frac{\partial \psi_{10}}{\partial \eta} = \frac{9}{32} (1 - 3\zeta - \zeta^2 + 3\zeta^3) \]
\[ \psi_{11} = \frac{9}{32} (1 + \eta)(1 - \zeta^2)(1 + 3\zeta) \]
\[ = \frac{9}{32} (1 + \eta - \zeta^2 - \eta\zeta^2)(1 + 3\zeta) \]
\[ = \frac{9}{32} (1 + 3\zeta + \eta + 3\eta\zeta - \zeta^2 - 3\zeta^3 - \eta\zeta^2 - 3\eta\zeta^3) \]
\[ \frac{\partial \psi_{11}}{\partial \zeta} = \frac{9}{32} (3 + 3\eta - 2\zeta - 9\zeta^2 - 2\eta\zeta - 9\eta\zeta^2) \]
\[ \frac{\partial \psi_{11}}{\partial \eta} = \frac{9}{32} (1 + 3\zeta - \zeta^2 - 3\zeta^3) \]
\[ \psi_{12} = \frac{1}{32} (1 + \zeta)(1 + \eta)[-10 + 9(\zeta^2 + \eta^2)] \]
\[ = \frac{1}{32} [1 + \zeta + \eta + \eta\zeta][-10 + 9\zeta^2 + 9\eta^2] \]
\[ = \frac{1}{32} [-10 + 9\zeta^2 + 9\eta^2 - 10\zeta + 9\zeta^3 + 9\eta^2\zeta - 10\eta + 9\zeta^2\eta + 9\eta^3 - 10\eta\zeta + 9\eta\zeta^3 + 9\zeta\eta^3] \]
\[ \frac{\partial \psi_{12}}{\partial \zeta} = \frac{1}{32} [18\zeta - 10 + 27\zeta^2 + 9\eta^2 + 18\eta\zeta - 10\eta + 27\eta\zeta^2 + 9\eta^3] \]
\[ \frac{\partial \psi_{12}}{\partial \eta} = \frac{1}{32} [18\eta + 18\eta\zeta - 10 + 9\zeta^2 + 27\eta^2 - 10\zeta + 9\zeta^3 + 27\zeta\eta^2] \]
A2.3 2\textsuperscript{nd}/3\textsuperscript{rd} order transition

![Diagram of a rectangular element with labels for x, y, h, l, n, \(\eta\), \(\zeta\), and a note on orientation]

Notes:
- \(h\) is constant throughout photoreceptor
- \(l\) is constant for each belt section
- \(\zeta\) is the local GWR coord. Axis
- \(\eta\) is the locl GWR coord. Axis
- \(\bullet\) Denotes a node location

ORIENTATION IS NOW AN ISSUE! MUST DERIVE EACH ORIENTATION SEPARATELY

Figure A.11. Individual Quad-Cubic Transition Belt Element definition

The derivation of transitional elements resides in an incomplete form. The achievement of a stiffness matrix for this type of element was achieved for the orientation shown in Figure A.11. However, the use of these elements provides that the orientation of the element is also variable. This would require 3 more derivations for the use of this one type of element. However valid, the efficiency of this practice did not coincide with the ideals of this project. I suspect that future modeling attempts will include transitional elements, which is why they are mentioned here. At the request of the user, the research and derivation of this element is available.
A3.1 Overview and Organization

The static analyzer for photoreceptor systems is currently based in Microsoft Excel. The use of Multiple worksheets has allowed an organizational configuration that aids the user interface quality and reduces the risk of user error. A total of 24 worksheets are currently available in the software package. This translates into a 10 roll photoreceptor configuration, noting that any number of these rolls can be used to model backerbars or other belt influences throughout the system provided that the influence is beam like in nature. An example of such a system is shown in Fig. A3.1 below.

Figure A3.1. Example 10 Roll system (cross sectional view)
If the number of rolls in the system needs to be modified, certain restrictions apply. The roll that is unwanted in the system should be moved inline with the resulting belt section of the desired system, as shown in Figure A3.2. By doing this, the belt force which is transmitted to the roll approaches zero, i.e. a wrap angle of approximately zero. However, it should be recognized that in turn, the boundary condition for the belt as it comes into contact with the roll is also negligible. It can not be foreseen by this report as to the impact of such an influence on any one particular analysis system. It is therefore recommended that if the number of rolls needs to be decreased, that these rolls are all taken out of the same section of belt and that this is done with consideration that the resulting belt section accuracy may lose its transverse deflection accuracy in the vicinity of the roll.

If the number of rolls requested is higher than that which is provided, there exists two courses of action which are appropriate. First and foremost, the software package can be manipulated by adding a component set to the link configuration. It is not recommended that the user do this independently, given the dependent nature of the cell calculation format on these links. The other possibility is to induce restraints that mimic a roll on the required section of belt using force loading or boundary conditions. By utilizing the generic roll analyzer, the deflection curve along the axis of the roll can be created. Discretizing this curve into the node locations of the static model will produce the sufficient boundary condition to “fool” the model into thinking that the roll was in the system. Further, it was hypothesized that no more than 10 actual rollers would be present in any given photoreceptor module. This promotes the activity of increasing the occurrence of backer bars into the belt path. This type of restraint can also be easily implemented by using boundary conditions on the belt surface.
A Zero Wrap condition results in the Perpendicular Component of the Tension Force $T$ to equal Zero. The result is a roll deflection of zero and a corresponding belt deflection contribution due to this deflection to also be zero. The resulting belt deflection will strictly be a result of the surrounding belt quad element interactions with the exception of the transverse direction. Since the roll nodes are not allowed to deflect in the axial deflection, the reaction of the belt along unused rolls has not been evaluated.

Figure A3.2. Example configuration which eliminates the effect of a given roller

The organization of the static analysis is quite basic. Of the 24 worksheets included in the system, only two require user input, the input/output and Geom., worksheets. Described later in this section, these sheets control the rest of the analysis much like any control room retains links to its satellite subsystems. The system output for the analysis is placed completely on the input/output page with the exception of in depth types of subsystem results such as surface deflections by node in their actual orientation. There exists two non-integrated worksheets which describe the derivation and mathematical formulation of the rest of the analyzer. These “model” worksheets occur for both the roll and the belt elements. The rest of the worksheets are provided for each subsystem in the analysis. This configuration allows easy access to more in depth results or the manipulation of a particular subassembly for any reason.

In the interest of future formulation changes, all cells that are involved in a mathematical calculation step, or at least the matrix it resides in, is labeled with an appropriate name. In this way, when viewing the formulas used for any calculation in the spreadsheet, it is clearly evident what is being done and to what values. If for some reason, the notation used does not reveal the required information, or a more in depth look at the
mathematical steps is desired, the user may use the INSERT/NAMES function to find the location of the named cell. Simply scroll through the list until the desired cell is highlighted. At the bottom of the window appears the sheet and cell location of the desired information. This type of link correlation is quite useful in that when a change is made to a cell that does not result in a consistent formulation, the cell displays the error message of \textit{VALUE}. This type of error illumination makes the process of matrix manipulation and the upgrading of future versions of this software occur with a reduced risk of missing a connection between cells. A complete list of spreadsheet variables is also included in the Nomenclature section of this report. With this in mind, let us now proceed to the analyzer.

A3.2 Solution Sequence overview

The solution sequence of the solver was required to circumvent calculation limitations in Excel and is therefore not tradition in format. The analysis path is segmented and rerouted to mimic a circular reference but without the automation link which would make the system unsolvable. Because of this organizational structure, Figure A3.3 has been included as a brief summary of what is occurring and the order by which the calculations are being made.

![Diagram](image)

\textbf{Figure A3.3. Brief Analysis Methodology Overview}
Figure A3.4. Input/Output Overview page selector and guide map
1) Input Section for Global Roll Placement

2) Resolved Tensile Force which acts on each roll

Typical P/R module geometry

Input deck

<table>
<thead>
<tr>
<th>Roll #</th>
<th>NAME</th>
<th>x-coord</th>
<th>y-coord</th>
<th>direction</th>
<th>x</th>
<th>y</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>drive</td>
<td>0</td>
<td>0</td>
<td>-54.35</td>
<td>-0.231</td>
<td>0.027</td>
<td>0.230</td>
</tr>
<tr>
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<td></td>
<td>32</td>
<td>-2.6</td>
<td>-132.58</td>
<td>-0.334</td>
<td>-0.273</td>
<td>0.192</td>
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<tr>
<td>3</td>
<td>tension</td>
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<td>-175.02</td>
<td>0.314</td>
<td>0.193</td>
<td>0.248</td>
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<td>-37.94</td>
<td>0.350</td>
<td>0.350</td>
<td>0.308</td>
</tr>
<tr>
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<td></td>
<td>104</td>
<td>574</td>
<td>-91.08</td>
<td>0.006</td>
<td>-0.006</td>
<td>0.000</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>18</td>
<td>-1065</td>
<td>-196.12</td>
<td>-0.275</td>
<td>-0.063</td>
<td>0.267</td>
</tr>
<tr>
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<td>-1129.8</td>
<td>-55.52</td>
<td>0.119</td>
<td>-0.092</td>
<td>0.076</td>
</tr>
<tr>
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<td>81.6</td>
<td>1073</td>
<td>22.62</td>
<td>0.109</td>
<td>-0.058</td>
<td>0.053</td>
</tr>
<tr>
<td>9</td>
<td>backer</td>
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<td>-531</td>
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<td>0.340</td>
<td>0.300</td>
<td>0.160</td>
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<tr>
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<td></td>
<td>-90.5</td>
<td>-580.4</td>
<td>-11.98</td>
<td>0.335</td>
<td>0.230</td>
<td>0.185</td>
</tr>
</tbody>
</table>

number of elements per belt section/ belt width

# of elem = 5
n = 44

belt section angles as a function of roll placement

<table>
<thead>
<tr>
<th>belt</th>
<th>transformation</th>
<th>location</th>
<th>length l</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>radians</td>
<td>degree</td>
<td>cm</td>
</tr>
<tr>
<td>1-2</td>
<td>-0.03</td>
<td>1.82</td>
<td>16.41</td>
</tr>
<tr>
<td>2-3</td>
<td>-1.45</td>
<td>93.30</td>
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<tr>
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<td>-1.61</td>
<td>86.74</td>
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<td>1.59</td>
<td>90.85</td>
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<td>14.62</td>
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</tr>
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<td>9-10</td>
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<td>89.46</td>
</tr>
<tr>
<td>10-1</td>
<td>1.28</td>
<td>72.14</td>
<td>55.33</td>
</tr>
</tbody>
</table>

TOTAL LENGTH 4403.39 mm

3) Example setup of a p/r module

4) Example Coordinate frame and Belt angle input derivations

Figure A3.5. Model Geometry Input Worksheet
A3.2.1 Input/Output Worksheet

The Input/Output (IO) Worksheet consists of the most import cells in the program. The IO sheet acts as the control center and results viewing apparatus for the user. As shown in Fig. A3.4, the set up of the spreadsheet is a two tier multifunctional design that is blocked into segments for easy recognition and modification. The gray background shown in the upper section designates the Input set of spreadsheet cells while the brown lower section designates the Output.

Again referring to Fig. A3.4, it becomes evident that five main segments construct the input section for the program. These five sections are named and numbered as follows; 1) Roll & Belt information, 2) Key, 3) Model Parameters (global), 4) Roll input forces, and 5) Belt boundary conditions.

The output section of the IO worksheet is also divided into 5 main sections as follows; 6) Roll node Deflection, 7) Roll Node Deflection Revisited, 8) Belt Node Deflection Out-of-plane, 9) Belt Node Deflection in-plane process (reg.), and 10) Belt Node Deflection in-plane lateral.

A3.2.1.1 Roll & Belt information

The R&B window designates the starting point for any analysis. The types of information found in this section include both organizational and program specific inputs such as roll dimensions and material properties.
Figure A3.6. Roll and Belt Information Window

A3.2.1.2. Key and navigational Aids

The Key, shown in Figure A3.7 provides the most important navigational information of any of the sections. Throughout the derivation and programming stages of this analysis, there arose a number of issues, implementations and suggestions which were considered, modified and included into the final spreadsheet form. Because of Xerox proprietary constraints as well as keeping an adequate focus on the task at hand, a number of these inclusions were not fully integrated into the first version of this software. Furthermore, depending on how the information is organized, i.e. where best to put active cells, those which calculate and deliver results to other cells, versus viewing cells, which simply show the user what is happening before during and after the analysis run, the user could get confused as to which cells to start with and which ones not to modify at all. For this reason, the following color indicator scheme was constructed. The only required user inputs on the IO page appear or are outlined in **RED** highlight. Red was chosen as it is an attention seeker and should stand out as a “don’t forget about me” type of reminder. All red input cells occur only on the IO page and furthermore, only in the Input section of the IO page. With the exception of the Model Geometry Worksheet, as will be discussed later in Section A3.2.2,
these red sections are the only user inputs required to run the analysis successfully. The GREEN highlighted cells represent purely organizational cells. These cells are typically not even numerical in nature but provide information as to roll names, function, global parameters, or analysis results in non analysis based units. Changing these cells will have absolutely no effect on the inner workings of the model. Finally, those cells outlined or highlighted in BLUE represent future additions to the software. Most notably in this arena is the roll end initial deflection parameter found in the Roll & Belt information window. This type of information is only partially integrated into the analysis schedule. For the initial version, these types of inputs were deemed as secondary concerns. In other words, common assumption practice would typically cause the inputs of these types of cells to become negligible. By utilizing these types of assumptions, the matrix manipulation process can change significantly, primarily by decreasing the number of active degrees of freedom and lowering the calculation time and complexity. Furthermore, the post processing of these results is also based on simplifying assumptions. In order to integrate this type of input, the matrix solvers for each roll would need to be manipulated, a step that would undoubtedly occur in future versions of this software. However, given the mathematical and matrix limitations in the Excel program, it was deemed as a valid assumption to leave these types of cells “out of the loop” so to speak. It is advisable therefore to not modify these cells in any way. Their existence is simply to allot space for future consideration, primarily in the way of allowing cantilevered photoreceptors to be modeled automatically. The naming and partial integration of these cells could effect the parent solver if deleted or modified in any way. If you see blue, do not modify these cells!

Figure A3.7. Key and Navigational Aids
A3.2.1.3. Model Parameters (global)

As is evident in Figure A3.9 below, there are only a few global parameters that are mentioned in the IO page. Of these seven inputs, only four of them are active. The belt tension “a” constitutes the amount of force per unit length that is applied to the belt throughout the system. In the real system, this value can vary along the width of the belt as well as along the belt length. However, it is assumed here, that when the integration of the force that the belt exerts on the roll over its entire axis is taken, the resulting magnitude is approximately the same. With this in mind, the value for “a” should represent the nominal or averaged tension value in the system. Current work at Xerox includes the adjustment of this nominal assumption. As this work evolves, so to will the treatment of tension in this model. The capital T highlighted in green just below the “a” represents total force throughout the system. As described by the Key, this information does not constitute any relativity nor influence on the model. The value for T is a simple notification of the system loading. A common use for this type of information resides in the matching of reaction forces with respect to FEA models to ensure model comparability and as a model to model comparison of overall loading. The Belt length window is linked to the Geometric worksheet and portrays the resulting length of the combined belt sections. This value will be lower than the actual system due to the assumption that the belt sections connect to a beam element of point diameter. A pictorial view of this assumption shows that although the individual belt sections are not effected, since the assumption remains that the belt can not slide across the roller, the overall length of the belt will be shortened by the summation of the circumferential wrap distances of the full set of rollers. This loss of belt length is shown in Figure A3.8.
The width parameter must be defined by the user and constitutes the width of the belt. In this analysis, it is assumed that the belt stretches the entire length of the rollers. Derived from the geometric specifications of the model, these windows serve as a reminder as to what model is being worked on and to ensure that the geometric inputs were entered correctly. On the right side of the global input window set, there exists the belt stiffness and relative deflection information. "Nu", the symbol for the Poisson effect or ratio is used in calculating the lateral deflection of the belt quadrilateral elements under process loading. This is required since the derivation of the belt element was completed for a singular dimension and then replicated to represent the 3 relative directions of interest, out-of-plane, in-plane process and in-plane lateral. By modifying the value of Nu, the amount of hourglassing of the belt sections can be modified. However, this value should be inherent to the belt material used and should be entered as such. Unusually high values of Nu may cause a breakdown in the
connection of the belt to the rollers due to the zero lateral deflection of the roller assumption made in the roll deflection analysis subset. Due to the ambiguity of specific models, a range of nu that will cause this can not be validated. However, by viewing the belt section deflection results, the relative lateral deflection should be approximately a multiple of Nu off from the process deflection. If it is not, then the value of nu may be incorrect. Lastly, the belt material property of stiffness must be included. In this version of the analyzer, \( k_{\text{reg}} \) is used in place of \( k_{\text{process}} \) to indicate that the belt stiffness properties within the plane of the belt are independent of direction. This value should also coincide with the material of choice for the belt. The \( k_{\text{lat}} \) stiffness value is included as an organizational reminder and will update automatically if \( k_{\text{reg}} \) is modified.

![Image of Belt Tension Input and Belt Length/Width](image)

**Figure A3.9. Global parameters List and Location**

A3.2.1.4. Roll input Forces

The input forces on the roll contain a significant mix of user input and concurrent analysis calculation. Although the cells are highlighted in red, the user must be aware of the fact that modifying these cells has a significant impact on the results of the analysis. When the model geometry worksheet is completed and successfully entered into the analysis, and coupled to the tension value \( "a" \), an initial loading configuration is calculated and placed in these cells. This calculation represents the base or default roll loading configuration described in Section 3.6. However, for the sake of versatility and increased user control over the model, these cells have been detoured out of the calculation loop, paused for user input, and then reentered into the database. Any modification to these cells should be done by double clicking on the cell itself and modifying, by adding to, the formula. This type of editing will not overwrite the existing link to the belt/roll interaction derivation and will allow any addition or subtraction of force in any direction at any node.

The cell organization is such that each roll is divided into 40 possible loading vectors. The premise of the Finite Element Methodology indicates that each node has a degree of
freedom set of 3 translational and 3 rotational directions for a total of 6 as shown in Figure A3.10. Typically, DOF notation coincides with the coordinate frame used to construct the nodes upon which the degrees of freedom will act. In the case of the rollers, only 4 DOF are considered. As previously indicated, the axial translation of the roll nodes is assumed to be zero throughout the model. Further, the rotation of the roll about its axis is restricted. The reasoning for this type of assumption is simple. In the actual system, the belt revolves around the module achieving the tasks that the machine was design to do. By doing so, the rotational movement of the rollers is initiated. However, these motions, as well as the multitude of dynamic inputs throughout the system such as drag, slip and wear are not influential to the static analysis. In fact, recalling the basic analysis assumptions reveals that all integrated subsystems were assumed to have no static effect on this analysis. Thus, in order to simplify the calculations by eliminating rigid body mode singularities, and to restrict an otherwise free movement of the belt, the rotational degree of freedom (DOF 6) of the roll nodes is assumed to be zero.

Figure A3.10. Degree of Freedom (DOF) orientation definition and notation
As indicated on the far left column of Figure A3.11, the four remaining degrees of freedom are split into two categories, translational, d, and rotational, M types of input. The numeric component of the locators signifies the relative node number along the axis of the roll, with node 1 and 11 being the ends. It can also be seen in this window that the assumption of zero roll end deflection is indeed present in the analysis. d1, M1, d11, and M11 are all set to zero and are not effected by belt tension variation. It should also be noted that the existence of belt tension does not reveal any rotational input on the rolls themselves at this time. This type of interaction can be manually entered at the discretion of the user and will undoubtedly coincide with the next release of this analyzer.

The column identifiers include a two-tier configuration with the top tier consisting of the roll number and the bottom tier signifying the sub orientation direction of the force. The Roll number indication speaks for itself. The x, y orientation divider requires some clarification. Referring to Figure A3.10, the x and y signifiers relate to the DOF numbers 1, 4 and 2, 5 respectively. Hence, when the d row signifier is matched with the x column signifier, DOF 1 is enabled. In coordinate frame verbage, this would refer to translation along the x-axis. Likewise, the d row y column would enable the DOF number 2 or y axial translation. In the subcase of rotation, M, the x and y signifiers represent the axis to which the rotation is about. The x column would enable DOF 4 and the y column would enable DOF 5 of the respective node. The right hand rule is utilized in all cases to determine the positive sense of the deflection.

![Figure A3.11. Roll Input Forces Window](image-url)
A3.2.1.5. Belt Boundary Conditions

The Initial Conditions for the Belt Deflection analysis is dependent on the results of the roll deflection analysis. Recalling that the derivation of this analysis was segmented by dimension and solved in parallel results in a split input screen in the Belt Initial Conditions Window of the IO worksheet. However, the Full Belt Initial Conditions set is not shown in Figure A3.12 due to repeatability.

The first 4 sections of this window, the belt section number, belt organizational inputs, and belt element width and length are not repeated for the other dimensions due to their consistent nature. Reviewing these cells briefly reveals that the type of belt quadrilateral element that is being used is shown in the upper left hand corner. This input is not linked to the program but does serve as a reminder of the proposed accuracy that the model will achieve. Below it are the belt section identifiers. For simplicity sake, the belt sections were named by the rolls which enclose them. The first number of the belt section name constitutes the roll at the belt sections clockwise (cw) end while the second signifies the roll at the counter clockwise (ccw) end. However, a set of cells directly adjacent to these names is provided such that the individual user may specify their own names for the section if they deem it appropriate. There is no numerical significance to the belt section naming cells. However, the embedded formulas used for the calculation process also work on this type of belt and roll naming system. Hence, while viewing a particular formula for its according with an individuals requirements, it would seem strategic to retain the names given for continuity and comparison purposes. The material properties of the belt are not linked by way of this window in any fashion.

These cells were originally set up to allow for individul belt section modeling techniques which would allow for different belt properties at different sites within the photoreceptor module. Primarily, the use of smart materials was the driving force of this inclination, but once again, the relative use in today’s modeling efforts is minimal, so the effect of changing these cells is negligible.

The belt element width is defined by the nodal spacing of the rolls. This section of the Belt Boundary Conditions Window is a bit misleading. As shown in the belt element derivation diagrams Figures 3.7 and 3.8, the spacing between nodes in the lateral direction is
always defined as h. However, depending on the order of the belt element that is in use, the number of nodes that the element encompasses will change. For instance, the belt element in use here is a $2^{nd}$ order serendipity quad, which uses an Element width of $2h$. The element width multiplier shown is the nodal spacing $h$, not the element width $2h$. In order to achieve the true element width, the nodal spacing must be multiplied by the order of the element in use. The choice as to show the nodal spacing instead of the true element width was made with the dimensional derivational split methodology enabled. In order to achieve a truly nondimensionalized belt derivation, the nodal spacing becomes the defining factor. This is also representative of the overall accuracy of the mesh itself, since the mesh density is a direct relative to the nodal spacing.

The bulk of the information provided in the Belt B.C. window consists of the roll analysis output. This section, as already stated, is divided into three subsections, one for each dimension of the model. The dimension of significance is shown in the bottom of the segmented section respectively, e.g. Out-of-plane Deflection. The matrix of values within this section is arranged by the belt connectivity orientation with respect to the rolls. Nodes 2-10 represent the non roll end nodes of the roller to the counter clockwise side of the respective belt section while nodes 87-95 do the same for the roll to the clockwise side.

It should be noted that the third dimension, lateral deformation of the belt, is outlined in blue. Recalling that this signifies a gap in the interface between the current software and the spreadsheet solution user interaction protocol, it is important to review the specifics of this window. Consider that the deflection of the belt is based on the boundary conditions set by the roller to which it is attached. Further consider that this arrangement can be quantified regardless of orientation or dimension. Now applying this theory to the out-of-plane and in-plane process directions, the solution falls out rather readily because the equations involved are statically determinant. However, a static indeterminance arises when the lateral deflection of the rollers is set to zero. Therefore, the lateral deflection of the belt must be enacted in some other way, but also restricting the belt from moving laterally with respect to the roll at the roll locations themselves. This is achieved in the individual belt worksheets as part of the lateral deformation analysis and will be explained as such. However, the interaction of the cells shown in this window with respect to the lateral deflection were developed with future considerations in mind. Eventually, the belt may be allowed to "walk"
across the belt in this lateral direction. For this, these cells have been designated to account for the needed Boundary Conditions. For this version of the software, and for any analysis thereof, these cells should not be modified from their present state.

![Figure A3.12 Belt Boundary Condition Window](image)

A3.2.2. Model Geometry Worksheet

The Model Geometry Worksheet is the place where the system orientation and architecture is defined. As shown in Figure A3.5, the worksheet itself is quite basic, consisting of an example orientation for a 10 roll photoreceptor and three sets of data which pertain to it. The first set of data, shown in Figure A3.13 below, must be supplied by the user of the program. The 10 roll definitions are organized in tabular format with a NAME column for organizational purposes. The user, if so inclined, can link the name function here with that of the IO worksheet with one being the master and the other the dependent. Without customer feedback as of the first release of this software, the link was left open to facilitate less confusion as to the number cells which are linked internally. The coordinates given in this part of the worksheet are relative to the GLOBAL coordinate frame, which is centered at the drive roll for all cases. The drive roll coordinates are still considered a variable in the analysis to allow for iterative analysis in which the drive roll may be in motion with respect
to the system. This feature may also be used to investigate the effects of tolerance buildup that results in a poorly aligned drive mechanism.

<table>
<thead>
<tr>
<th>Roll #</th>
<th>NAME</th>
<th>x-coord</th>
<th>y-coord</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>drive</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>82</td>
<td>-2.6</td>
</tr>
<tr>
<td>3</td>
<td>tension</td>
<td>106</td>
<td>-207</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>119</td>
<td>-435</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>104</td>
<td>574</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>18</td>
<td>-1065</td>
</tr>
<tr>
<td>7</td>
<td>steering</td>
<td>-17.7</td>
<td>-1128.8</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>-61.6</td>
<td>-1073</td>
</tr>
<tr>
<td>9</td>
<td>backer</td>
<td>-122.5</td>
<td>-581</td>
</tr>
<tr>
<td>10</td>
<td>backer</td>
<td>-90.5</td>
<td>-280.4</td>
</tr>
</tbody>
</table>

Figure A3.13. Inputs of the Roll locations relative to the Global Coord. Frame

The second subset of the Geometric Worksheet is the calculation of the angles and bisector information for the system defined by the coordinates given in Figure A3.13. This subset, shown in Figure A3.14, consists of 4 columns. The first column is a calculation step that utilizes the trigonometric relationship between rolls to calculate the direction of a vector that bisects the wrap angle of the roll. The calculation assumes that the rolls are point masses and does not account for differences in roll diameter. This type of assumption will likely become one of the first to be revisited in future versions of this software. However, given the relatively small impact this will have on the overall loading of the roll, the simplification stands in the current version. The second column utilizes the angular information derived in the first column and resolves the vectorily loaded tension "a" into a singular resolved force that acts along the bisector of the wrap angle. It is the magnitude of this force that is shown in each cell. The last two columns, labeled x and y correspond to a second transformation of
the force loading on the roll. Instead of using the actual bisector of the wrap angle, the global x and y coordinates are used to resolve the force into a workable matrix format. This format will be shown in the roll derivation worksheet section.

<table>
<thead>
<tr>
<th>direction degrees</th>
<th>magn</th>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-54.85</td>
<td>-0.231</td>
<td>0.029</td>
<td>-0.230</td>
</tr>
<tr>
<td>-132.56</td>
<td>-0.334</td>
<td>-0.273</td>
<td>0.192</td>
</tr>
<tr>
<td>-175.02</td>
<td>0.314</td>
<td>0.193</td>
<td>0.248</td>
</tr>
<tr>
<td>-87.94</td>
<td>0.350</td>
<td>0.350</td>
<td>0.008</td>
</tr>
<tr>
<td>-91.08</td>
<td>0.005</td>
<td>-0.005</td>
<td>0.000</td>
</tr>
<tr>
<td>-196.12</td>
<td>-0.275</td>
<td>-0.063</td>
<td>0.267</td>
</tr>
<tr>
<td>-85.52</td>
<td>0.119</td>
<td>-0.092</td>
<td>0.076</td>
</tr>
<tr>
<td>22.62</td>
<td>0.109</td>
<td>-0.088</td>
<td>-0.065</td>
</tr>
<tr>
<td>0.49</td>
<td>0.340</td>
<td>0.300</td>
<td>0.160</td>
</tr>
<tr>
<td>-11.98</td>
<td>0.335</td>
<td>0.280</td>
<td>0.185</td>
</tr>
</tbody>
</table>

Figure A3.14. Roll Loading resolution sequence and visual orientation

The final window in the Geometrical worksheet calculates the general belt element parameters and overall length of the belt in use. As shown in Figure A3.15, the difference in overall belt length to the actual system is a good indicator of how much the assumption of point masses for roller centers is effecting your analysis. At the top of the window there exists two input cells, called "# of elements" and "h". The number of elements used will depend on the order of the analysis used. For this version of the analysis, 2nd order serendipity belt elements are used. This type of element utilizes a width of 2h along the axis of each roll. Given that the nominal number of roll nodes was 11, resulting in 10 roll elements of h width each, it becomes apparent that a serendipity analysis would utilize half as many or 5 belt elements for the same total width. This cell should not be modified unless serendipity elements are no longer being used. The width definition h is defined by dividing the system width, which was supplied on the IO worksheet, by the number of roll elements.
along the axis. This number of roll elements will remain constant for this analysis and is equal to ten.

Figure A3.15. Belt Element definition and Length calculation

A3.3. Solver Methods

As specified in the beginning of this section, there exists two worksheets whose only purpose is to serve as a theoretical example of the working process, one for the roll elements and one for the belt elements. Indeed, both of these worksheets exist, but merely convey the theoretical implementation of the Finite Element method to the roll and belt components. In order to gain a working knowledge of the real mathematical manipulation that occurs, both the theoretical pages and the first mathematical pages within the program will be shown and discussed.
A3.3.1. Roll Model Worksheet

The Roll Model Worksheet defines the process by which the theory of finite elements is combined with the mathematical interpretation of the system and selection of element types. Recall that in Section 3.3, the beam element chosen was cubic Hermite element. This element, shown again for reinforcement in Figure A3.16, spans only two nodes. However, its four Psi functions take into account the concurrent slopes at the beam joining points. The four Psi functions, along with their first and second derivatives are shown in Equation sequence A3.1.

*Global Coord. X
Local Coord. x

\[ U_1^e = U_1 \]
\[ U_2^e = U_1 \quad \text{Coincident Elements} \]
\[ U_3^e = U_1 \quad \text{Must Share Slopes} \]
\[ U_4^e = U_2 \]

Figure A3.16. The Cubic Hermite Beam Element Revisited

\[
\text{derivative} \quad 0 \quad 1 \quad 2 \\
Y_1 \quad 1 - 3(x/h)^2 + 2(x/h)^3 \quad (6/h^2)(-x + x^2/h) \quad (6/h^2)(-1 + 2x/h) \\
Y_2 \quad x - 2(x^2/h) + x(x/h)^2 \quad 1 - 4x/h + 3(x/h)^2 \quad (2/h)(-2 + 3x/h) \\
Y_3 \quad 3(x/h)^2 - 2(x/h)^3 \quad (6/h^2)(x - x^2/h) \quad (6/h^2)(1 - 2x/h) \\
Y_4 \quad -x(x/h) + x(x/h)^2 \quad -2x/h + 3(x/h)^2 \quad (2/h)(-1 + 3x/h)
\]

\[
[K^e] = \frac{EI}{h^3} \begin{bmatrix} 12 & 6h & -12 & 6h \\ 6h & 4h^2 & -6h & 2h^2 \\ -12 & -6h & 12 & -6h \\ 6h & 2h^2 & -6h & 4h^2 \end{bmatrix}
\]

Equation set A3.1
Figure A3.17. Derivation of Beam Equations of Motion in Matrix Format
Once the Local form of the equation is compliant, it must then be implemented over the entire system. In this case, each roll will have 10 elements along its axis. The Global formulation of the solution matrix, as well as post processing and transformation technique definition is shown in Figure A3.18.

Figure A3.18. Global Solution Sequence for Beam Elements in Analytical Matrix form
The Statically Determinant form of the Matrix solution can now be applied to individual Rolls in the system. The Transformation and Post Processing tools will be used to augment this process as well. An example of this is depicted in the next section, Section A3.4.2, for Roll 1.

A3.3.2 Roll 1. an example of the mathematical derivation for roll elements

The example Roll number is arbitrary. Each of the roll definition and analysis pages has the same format but with different inputs from the IO page. To distinguish these types of inputs, certain highlight structures have been implemented. A background highlight of YELLOW indicates that the cell is Roll Specific. Take note that this does not say anything about format or solution sequence, just numerical values due to roll geometry and material properties. Those fonts highlighted in RED are used to single out cells that are direct references to the IO page. These cells are not integrated into the analysis and thus, changing them will have no effect on the solver. This note is to reinforce that all user inputs must be entered on either the IO or GEOM worksheets.
The next section of the Roll worksheet consists of the implementation of Global X and Y Boundary Conditions and Forcing Functions. These inputs are linked directly to the IO worksheet. Recall that the Input Roll Forces segment, subset number 4 of the Input section, consists of the global forces in a resolved X and Y format. Since the stiffness matrix is independent of direction, it is only depicted once on the worksheet. However, the solution
sequence in its mathematical form is the same for both directions, just with different references as to the inputs on the right side of the equation. This segment of the Roll Analysis worksheet is shown in Figure A3.20 below.

Figure A3.20. Example Roll Analysis Derivation Part 2
The final segment of the Roll analysis page is concerned with the transformation of the Global roll results into a workable local form for the belts which attach to it. The clockwise directional configuration is used to circumvent roll/belt numbering confusion. The output of this segment is displayed as the fifth section in the Input worksheet as the Boundary Conditions for the Belt Element analyses. The third segment is shown in Figure A3.21 below.
Figure A3.21. Example Roll Analysis Derivation Part 3
A3.3.3. Belt Model Worksheet

The Belt Model Worksheet is not as controlled as the Roll Model Worksheet for a number of reasons. Primarily, recognizing that although the derivations and analytical basis for the three dimensions analyzed is the same, the boundary conditions and loading configurations can vary greatly. Because of this, most of the belt derivation is taken into account on the roll element worksheets instead of the model. This is not to say that the element worksheets differ in any way from each other; they do not, but the construction of a model becomes ambiguous given the complexity and specificity of the analysis itself. The Belt Model worksheet thus contains a review of the belt element derivation for serendipity elements and the considerations of transforming a local elemental solution into a global one. As stated in the worksheet itself, the stiffness matrix $K_{beltq}$, which stands for Stiffness matrix of the quadratic belt element, is not linked to the analyzer. The MAPLE V output shown in Section 3.2 depicted the in depth derivation of the values within this matrix as a function of element width and length. Because of this in depth look at the element derivation, it will not be repeated here. Figures A3.22 and A3.23 show the Belt Model in its entirety.
The Galerkin Finite Element Method derivation of Belt Solutions of a Matrix format.

The Jacobian is required for the translation from $x, y$ to $n, \zeta$ coordinates.

Solution of the Jacobian in Generic Format.

Figure A3.22. The Belt Model Worksheet Part 1
K Matrix Solution Methods

Individual K matrix elements are derived using the MAPLE V output shown in Section 6.2

B and F Matrix Considerations

Derivation of B.C. set for belt exterior nodes. This is the Premise for using the residual of roll deformation as the primary source of belt transformation.

Figure A3.23. The Belt Model Worksheet Part 2
A3.3.4. Belt 12, an example of the mathematical derivation for belt elements

As opposed to the Belt Model Worksheet, the individual belt component sheets are quite in depth. In fact, they span the entire width of the Excel Spreadsheet and approximately 450 of its rows. For this reason, the depiction of the sheet itself will be quite difficult. Instead of viewing a zoomed out view, labeling and then discussing the individual windows, the following flow chart has been constructed. The relative placement of the segments is the same and their individual contributions will still be discussed. The figure numbers of these segments are also included to aid worksheet navigation.

![Flowchart of Belt Component Worksheet Navigation](image)

Figure A3.24. Belt Component Worksheet Navigational Flowchart
The primary segment of the Belt element analysis screen is shown in Figure A3.25. This screen is responsible for designating element and nodal numbers as well as compiling the local stiffness matrix from the MAPLE V output. The assumptions that appear on this screen differentiate the in-plane and out-of-plane analyses. Since the out-of-plane analysis was based on a membrane that was fixed about its edges, the belt edge nodes, highlighted in blue, can be extracted from the global equation set. Since the in-plane analysis does not require this assumption, the degrees of freedom of the edge nodes remain variables, thus remaining in the global stiffness matrix. In both sets of analysis however, the roll interaction nodes are extracted. This occurs since the in-plane process and out-of-plane roll node deflections were derived in the roll element analyses. Posted on the IO worksheet as section 5, these values cause the stiffness matrix to contain a residual, which is then moved to the right side of the equation and combined with any user-added boundary conditions and/or forcing functions. In the case of the in-plane lateral deflection analysis, the roll node deformations were assumed to be zero, thus also extracting these equations from the global sequence. The Roll nodes are differentiated by a violet highlight in the worksheet.

![Figure A3.25. Belt Segment and stiffness matrix definition](image-url)
Outer Rim of the Matrix is lined with nodal numbers, highlighted for location reinforcement and to aid in the reduction process.

Color-coded interior matrix elements show the relationship of the elements in a row and between rows of the belt section.

Outline of the in-plane stiffness reduced matrix can be seen here.

Figure A3.26. **Unmodified Global Stiffness Matrix**

In this matrix, the outlined area represents the out-of-plane reduced stiffness matrix.

Similar attributes as the unmodified Global stiffness matrix with the exception of the absence of belt edge nodes.

Figure A3.27. **Reduced Global Stiffness Matrix for out-of-plane analysis**
In order to successfully solve the matrix equations shown in the past few figures, the stiffness matrix must be inverted and multiplied by the boundary conditions. However, the solution restraints of Excel limit the invertable matrix size to 52x52. This number is actually published as 54x54 but some iterative sequencing in formula modification proved otherwise. Microsoft has confirmed that their publication was false. However, in either case, the matrix here is much too large to be solved as it is. Hence, the ideal of matrix partitioning comes into play. Matrix partitioning is the segmentation of a matrix into submatrices. Each submatrix is then treated as an element in the parent matrix. Having fewer elements, the parent may now be solved via any appropriate means. The submatrices are then solved using established subroutines. The process is shown in Figure A3.28. The end result is the circumvention of calculation limitations and the capability of solving much larger matrices. The arguments used and shown in the spreadsheet are completely linked into this sequence. Modifying these matrices will undoubtedly disrupt the program should be avoided. The inclusion of these matrices was decided based on the simplification of embedded formulas, due to an increased number of steps, and the possible need for this information in the future.
Matrix A – To be partitioned

\[
\begin{bmatrix}
A_{11} & A_{12} \\
(p \times p) & (p \times q)
\end{bmatrix}
\begin{bmatrix}
A_{21} & A_{22} \\
(q \times p) & (q \times q)
\end{bmatrix}
\]

The partition divides the parent matrix, A, into 4 elements or submatrices, A_{11}, A_{12}, A_{21}, A_{22} of order n = p+q

Matrix B – The inverse of Matrix A

\[
\begin{bmatrix}
B_{11} & B_{12} \\
(p \times p) & (p \times q)
\end{bmatrix}
\begin{bmatrix}
B_{21} & B_{22} \\
(q \times p) & (q \times q)
\end{bmatrix}
\]

The inverse matrix, B, has elements derived as the following:

\[
\begin{align*}
B_{11} &= A_{11}^{-1} + (A_{11}^{-1}A_{12})\xi^{-1}(A_{21}A_{11}^{-1}) \\
B_{12} &= -(A_{11}^{-1}A_{12})\xi^{-1} \\
B_{21} &= \xi^{-1}(A_{21}A_{11}^{-1}) \\
B_{22} &= \xi^{-1}
\end{align*}
\]

Where \(\xi = A_{22}A_{21}(A_{11}^{-1}A_{12})\)

For less repetitive calculation and spreadsheet consolidation, the following sub calculation steps have been established. Also shown are the resulting inverted matrix element formulas.

\[
\begin{align*}
\text{Arg1} &= A_{11}^{-1}A_{12} \\
\text{Arg2} &= \text{Arg1} \cdot \xi^{-1} \\
\text{Arg3} &= \text{Arg2} \cdot A_{11}^{-1} \\
B_{11} &= A_{11}^{-1} - \text{Arg2} \cdot \text{Arg3} \\
B_{12} &= -(\text{Arg1} \cdot \xi^{-1}) \\
B_{21} &= \xi^{-1}(\text{Arg3}) \\
B_{22} &= \xi^{-1}
\end{align*}
\]

Figure A3.28. Analytical Methodology for taking the Inverse of a Matrix using Partitions
Figure A3.29 depicts the final pre-invertable stiffness matrix for the out-of-plane deflection analysis. Although the actual partition size is arbitrary, the partition numbering should be noted. The colors designate the partition boundaries and the resulting elements are numbers A11, A12, A21 and A22 as they would be in theoretical Matrix mathematics.

![Matrix Partition subset for out-of-plane analysis Part 1](image)

The internal mathematics for the inverse derivation of a partitioned matrix are displayed in both the worksheet itself and in Figure A3.30. Note that due to the spreadsheet layout, the left half of the A11 inverse matrix has been cut off in the Figure.
Figure A3.30. Matrix Partition subset for out-of-plane analysis Part 2

Figure A3.31. Final out-of-plane solution sequence in Matrix Format
The partition methodology for the in-plane analysis was to use square matrices as the minor components of the parent matrix. The experience of using this method in the out-of-plane analysis provided confidence that the matrix dimensions in the arguments could be arbitrary. Figure A3.33 portrays the divisions and highlights the respective sections. The arguments of the partitioning analysis step are shown in Figure A3.34 & A3.35.
Figure A3.33. **Matrix partition subset for in-plane analysis Part 1**

Figure A3.34. **Matrix partition subset for in-plane analysis Part 2**
Figure A3.35. **Matrix partition subset for in-plane analysis Part 3**

Figure A3.36. **Final in-plane solution sequence in Matrix format**
The solution results for the in-plane analysis are restricted to the configuration shown in Figure A3.37. The color scheme and orientation matches that in the belt section definition and description. Further, the yellow cells, which designate the element centers, have been modified to portray a numerical average of the cells that surround it. These cells were not part of the analysis and are only present for plotting purposes. The surface plots for these sections were not shown due to the skewing problems encountered with non-consistent array sizes. The Future plans section indicates methods of improvement for this post-processing problem.

A3.4. System Outputs

A3.4.1. Input/Output Worksheet Revisited

The Input/Output worksheet is revisited here in order to describe the output configuration highlighted in the brown border shown in Figure A3.4. This section is split into 5 sections, four of which are currently active. The results shown on this page are restricted to numerical output only. The individual surface and deflection plots can be found on the component worksheets. Navigating these worksheets was reviewed in Section A.3.4.
A3.4.1.1. Roll Node Deflection

The Roll Deflection Results Window contains a high density of information pertaining to the roll analysis. In fact, it is the only place that the user is required to navigate for roll results of any kind. Given that this is the case, the organizational structure of this information must be sound and clear. Referring to Figure A3.38, a combination of color and matrix correlation techniques becomes the evident means for achieving this end. Along the very top of this window there exists an indicator as to what information can be found in the window. The current configuration, as just stated, only pertains to one set of roll output results. However future versions of this analysis may incorporate alternate versions of this data for which the nameplate will then become an issue. Directly beneath this window description, there exists a set of columns that is partitioned in sets of two each. Each partition represents and individual roll that is named accordingly along the top. Within each partition there exists two columns of differing color, black and blue. The black data results are the global deflections of the roll resolved into the local coordinate frame that is oriented to coincide with the belt section to the roll’s clockwise side. The blue data results are of the
same form but with respect to the counter clockwise belt local coordinate system. The individual rows of each column are differentiated by the bolded deflection markers in the column to the far left. These markers use a 3 character designation of $dlx$ for example. The $d$ represents that the result is a deflection, the $I$ corresponds to the nodal numbering sequence, again with 1 and 11 being the ends, and the $x$ pertains to the direction of displacement. An $x$ displacement in this analysis refers to an out-of-plane deflection while a $y$ displacement refers to an in-plane process deflection. This is noted to the left of the displacement markers as a reminder during analysis result viewing.

The post processing section of the window indicates the residual forces on the endpoints of each roll as a function of input loading. These forces have been resolved into shear and moment reactions in both local coordinate frames described above.

A3.4.1.2. Roll Node Deflection Revisited

The revisited Roll Deflection window is not currently in operation in the analysis. However, the window itself may be used for a number of different tasks. The user may see a need to modify the roller output in some manner, whether this be a scaling to alternate units, by some temperature variation or other unknown or unforeseen circumstance. For this reason, an additional Roll output window was constructed to enable this type of modification to occur at low risk. Other uses for this type of window include alternate coordinate frame results such as the global system coordinate frame. Even further, these results can be transferred to any ambiguous or arbitrary coordinate frame of the users choosing without entering the existing analysis code. This allows for increased variability in the agility of the program to meet the needs of the customer. For clarification purposes, the entire window has been highlighted in yellow to signify caution. By doing so, the user should realize that the window itself is not part of the actual analysis despite being in the output section. As shown in Figure A3.39 below, the current format of the window is identical to that of the enabled Roll deflection result window.
A3.4.1.3. Belt Node Deflection out-of-plane

The deflection matrix of the belt nodes is split by dimension. Each dimensional solution matrix takes the same form due to the continuity in belt derivation and modeling techniques. The location of the various nodes can be visualized in Figure 3.7 from Section 3.2 that describes the nodal layout for serendipidal elements. Each deflection is in millimeters and the columns alternate font color for easier definition of results. The top row is the only significant display change between dimensions as in it signifies the dimension that the results being viewed pertain to.
<table>
<thead>
<tr>
<th>U</th>
<th>Belt 1,2</th>
<th>Belt 2,3</th>
<th>Belt 3,4</th>
<th>Belt 4,5</th>
<th>Belt 5,6</th>
<th>Belt 6,7</th>
<th>Belt 7,8</th>
<th>Belt 8,9</th>
<th>Belt 9,10</th>
<th>Belt 10,11</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>9.0E-05</td>
<td>9.0E-04</td>
<td>1.0E+00</td>
<td>1.0E+00</td>
<td>1.0E+00</td>
<td>1.0E+00</td>
<td>1.0E+00</td>
<td>1.0E+00</td>
<td>1.0E+00</td>
<td>1.0E+00</td>
</tr>
<tr>
<td>14</td>
<td>1.0E+00</td>
<td>1.0E+00</td>
<td>1.0E+00</td>
<td>1.0E+00</td>
<td>1.0E+00</td>
<td>1.0E+00</td>
<td>1.0E+00</td>
<td>1.0E+00</td>
<td>1.0E+00</td>
<td>1.0E+00</td>
</tr>
<tr>
<td>15</td>
<td>9.0E-05</td>
<td>9.0E-04</td>
<td>1.0E+00</td>
<td>1.0E+00</td>
<td>1.0E+00</td>
<td>1.0E+00</td>
<td>1.0E+00</td>
<td>1.0E+00</td>
<td>1.0E+00</td>
<td>1.0E+00</td>
</tr>
<tr>
<td>16</td>
<td>9.0E-05</td>
<td>9.0E-04</td>
<td>1.0E+00</td>
<td>1.0E+00</td>
<td>1.0E+00</td>
<td>1.0E+00</td>
<td>1.0E+00</td>
<td>1.0E+00</td>
<td>1.0E+00</td>
<td>1.0E+00</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

Figure A3.40. Out-of-plane Belt Node Deflection Results Matrix
A3.4.1.4. Belt Node Deflection in-plane process (reg.)

The Belt Section In-plane deflection matrix is shown if Figure A3.41. Once again, the term process has been replaced with registration in order to signify its dominance with respect to the lateral solution sequence. The results of this section are derived directly from the roll deflections in the plane of the belt. Inherently, these deflections are very small. However, the relative deflections of the rolls may vary greatly producing a significantly different effect on the belt sections that connect to it.

![Figure A3.41. In-plane Process Direction Belt Node Deflection Results Matrix](image-url)
A3.4.1.5. Belt Node Deflection in-plane lateral

The lateral derivation of belt node deflection is a direct result of the process belt node deflection and the stiffness of the roll to which the belt is attached. The Poisson effect enables the interior nodes of the belt to deflect while the roll connecting nodes do not. However, the stiffness associated with this deflection must be integrated with the overall deflection of the belt section. In other words, the interior nodes must still be constrained by the rolls in which they are attached, or the model would become numerically unstable and the lateral deflections of the nodes would increase exponentially.

Figure A3.42. In-plane Lateral Direction Belt Node Deflection Results Matrix
A4 Nastran Data deck

$ NASTRAN input file created by the MSC MSC/NASTRAN input file
$ translator ( MSC/PATRAN Version 8.0 ) on April 24, 1999 at
$ 09:03:57.
ASSIGN OUTPUT2 = 'beltl2corr.op2', UNIT = 12
$ Direct Text Input for File Management Section
$ Linear Static Analysis, Database
SOL 101
TIME 600
$ Direct Text Input for Executive Control
CEND
SEALL = ALL
SUPER = ALL
TITLE = MSC/NASTRAN job created on 23-Apr-99 at 19:17:47
ECHO = NONE
MAXLINES = 999999999
$ Direct Text Input for Global Case Control Data
SUBCASE 1
$ Subcase name : thesiscorr
   SUBTITLE=thesiscorr
   SPC = 2
   LOAD = 2
   DISPLACEMENT(SORT1,REAL)=ALL
   SPCFORCES(SORT1,REAL)=ALL
   STRESS(SORT1,REAL,VONMISES,BILIN)=ALL
BEGIN BULK
PARAM POST -1
PARAM PATVER 3.
PARAM AUTOSPC YES
PARAM INREL 0
PARAM ALTRED NO
PARAM COUPMASS -1
PARAM K6ROT 0.
PARAM WTMASS 1.
PARAM,NOCOMPS,-1
PARAM PRTMAXIM YES
$ Direct Text Input for Bulk Data
$ Elements and Element Properties for region : belt12
$ Pset: "beltl2" will be imported as: "pshell.1"

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A

+ A 19.875 18.

$ Pset: "roll1" will be imported as: "pbarl.2"

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CBAR  19 3  39  42  1.  1.  0.
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$ Material Record : belt
$ Description of Material : Date: 23-Apr-99 Time: 19:06:54
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  C
+    C 1.  5
$ Displacement Constraints of Load Set : ground
SPC1  1  123  2  4
$ Displacement Constraints of Load Set : ground2
SPC1  3  12  1  3
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FORCE  1  14  4  1.6  -1.  0.  0.  0.
FORCE  1  16  4  1.6  -1.  0.  0.  0.
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FORCE  3  12  1  1.6  1.  0.  0.
FORCE  3  14  1  1.6  1.  0.  0.
FORCE  3  16  1  1.6  1.  0.  0.

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FORCE  4  27  1  1.6  -1.  0.  0.
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FORCE  4  39  1  1.6  -1.  0.  0.

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