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Simplified Explanation of Pneumatic Tire Behavior

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George H. Sutherland, PhD, PE
Professor, MMET/PS Department
Rochester Institute of Technology
Rochester, NY 14623

One can get some insight into how a pneumatic tire behaves when it is vertically loaded by studying a thick-skinned spherical membrane (like a junior size basketball) being compressed between two flat plates. Bicycle, motorcycle, truck, general purpose and early model automobile tires have a circular cross-section similar to a junior-size basketball. However modern automobile tires have a wide aspect ratio when unloaded (like the loaded state shown in Figure 1), have unloaded line contact with the ground across the tread width, and have relatively stiff sidewalls that contribute to the tire's load carrying capability (and in the case of run-flat tires can totally support the car's weight). Thus the analysis in this report is quantitatively representative for tires of initially round cross-section, like a typical motorcycle tire, but provides only a qualitative idea of the tire support mechanism for modern automobile tires.

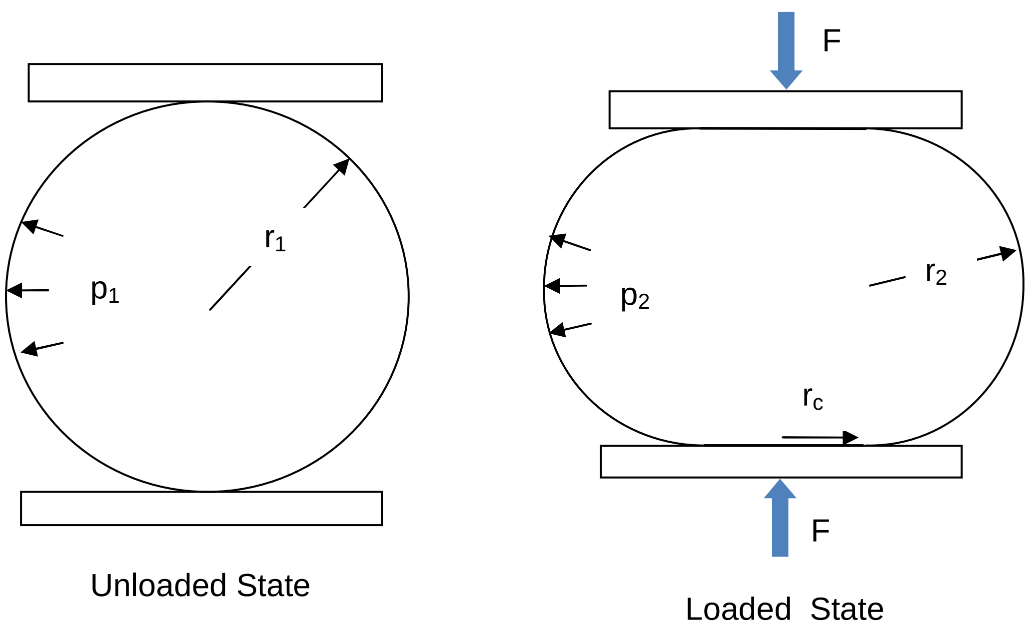

Figure 1 Compressed and Uncompressed Sphere

The sphere (basketball) starts in state 1 with a certain mass of air in it such that it has radius r_1 and internal pressure p_1. The sphere is then compressed vertically between two flat plates so it takes the shape of a cylinder of radius r_c surrounded by a ring with a half-circle cross-section of radius r_2. (It is the same shape as that of a jelly-filled doughnut.)

The sphere initially has volume

$$V_1 = (4/3)\,\pi\, r_1^3 \qquad (1)$$

and surface area

$$S_1 = 4\,\pi\, r_1^2 \qquad (2)$$

After being compressed it has volume

$$V_2 = \pi\, r_2\,(2\, r_c^2 + (4/3)\, r_2^2 + \pi\, r_2\, r_c) \qquad (3)$$

and surface area

$$S_2 = 2\,\pi\,[r_c^2 + r_2\,(2\, r_2 + \pi\, r_c)] \qquad (4)$$

As a first approximation it is reasonable to assume, since no additional air is to be added to the thick-skinned sphere, that the surface area does not change in going from state 1 to state 2. The condition $S_1 = S_2$ yields

$$0 = 2\,(r_2^2 - r_1^2) + \pi\, r_2\, r_c + r_c^2 \qquad (5)$$

In addition it can be assumed that air behaves as a perfect gas and that the temperature of the air in states 1 and 2 is the same. In this case

$$p_1\, V_1 = p_2\, V_2 \qquad (6)$$

Substituting (1) and (3) into (6) yields

$$4\, p_1\, r_1^3 = p_2\, r_2\,(6\, r_c^2 + 4 r_2^2 + 3\,\pi\, r_2\, r_c) \qquad (7)$$

The overall vertical reaction force of the sphere’s contact surface on the plate must be equal to the load F. If a free body diagram is made of this contact area (Figure 2), it is clear that the tension, T_2, in the thick membrane contributes little to the vertical load equilibrium condition due to its almost horizontal angle. The contact force is thus given by

$$F = p_2 \pi r_c^2 \tag{8}$$

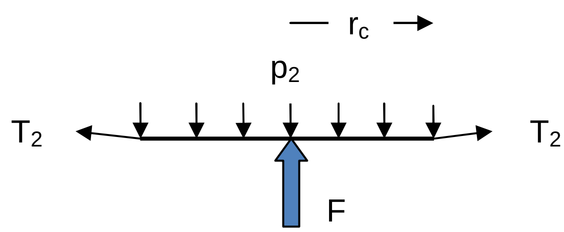

Figure 2 Free Body Diagram of Sphere/Plate Contact

Equations (5), (7) and (8) represent 3 equations in 3 unknowns p_2, r_2 and r_c, assuming p_1, r_1 and F are given. Equation (8) is solved for p_2, which can then be substituted into equation (7) yielding

$$4 \pi p_1 r_1^3 r_c^2 = F r_2 (6 r_c^2 + 4 r_2^2 + 3 \pi r_2 r_c) \tag{9}$$

Equation (5) is quadratic in both r_2 and r_c while equation (9) is a cubic in r_2 and a quadratic in r_c. Both equations have some mixed terms in r_c and r_2. Obtaining an explicit equation for either r_2 or r_c appears difficult and would likely be of 6th order if it could be developed. Thus a direct algebraic solution for r_2 or r_c does not seem possible. However a numerical state 2 solution for a given set of state 1 parameters and a force F can easily be obtained using standard computer software like Excel.

For example, for the situation where F = 1000 lbf, r_1 = 4.5 inches, and p_1 = 36 psi, the Solver routine in Excel yields

r_2 = 2.41 inches

r_c = 2.79 inches

and p_2 = 40.86 psi

Also V_1 = 381.70 in^3 and V_2 = 336.09 in^3.

The 1000 pound load is carried by deforming the sphere, increasing its internal pressure by 13.5% and reducing its internal volume by 11.9%.

The sphere example leads to some understanding as to what happens in a pneumatic tire when it is inflated to 36 psi while not installed and then placed in service with a vertical loading of 1000 pounds. With a pneumatic tire the assumption of S1 = S2 is quite accurate. Of course the overall geometry of a pneumatic tire is more complex than a sphere. However if only the cross-section of the inside of the unloaded tire (where the tire contacts the road) is considered and approximated by a circle, then there is a direct comparison to the sphere cross-section geometry. When the tire is loaded, the tire contacting the road surface will flatten out in an oval (but not necessarily perfectly circular) pattern. Where the sphere model differ substantially from an actual tire is that the upper load in the tire is transmitted to the wheel rim which contacts the tire (at the tire bead) a ways down on the cross-section from the top of the cross-section (whereas the upper flat plate contacts the sphere at the very top of the cross-section). Thus there is no symmetric flattening of the real tire cross-section at its top, but rather there is a broader area of load transfer from the tire to the rim.

With the above-mentioned caveats the sphere model provides a reasonable approximation to pneumatic tire behavior in the area of contact with the road. The tire outside of the road contact region can be considered to remain geometrically unchanged, and just serves as a reservoir of air that increases the overall volume of air to be compressed as the contact patch area flattens out and reduces the volume inside of the tire. In other words consider an imaginary sphere inside of the tire at the road contact point. Most of the volume of the inside of the tire is outside of the imaginary sphere. Only the volume inside of the sphere is reduced by ΔV when the tire is loaded. However when equation (6) is applied, the whole inside volume of the tire (4797 in^3) must be considered. Thus while a ΔV of 45.61 in^3 will lead to a pressure increase of 4.86 psi when only the sphere (381.70 in^3) is considered, when the entire tire volume is included, the pressure increase needed to support 1000 pounds is only about 0.35 psi. In other words, the pressure must increase within a pneumatic tire when it is loaded, but the increase is barely noticeable with a typical tire gage. What is more obvious is the deformation of the tire (ΔV) that occurs where the tire contacts the road surface, with the height of the tire being reduced by about 4 inches (2 ($r_1 - r_2$)) in the model. In a real tire, as the tire deforms more of the load is spread into the tire carcass (effectively increasing the system stiffness) so that the actual tire deflection will not be as much as predicted by the simple sphere model.

In summary, the primary mechanism by which a pneumatic tire supports its load is by the volume inside the tire being reduced due to deformation in the area of contact with the road surface. The road surface contact area of the deformed tire times the tire internal air pressure equals the vertical vehicle load on the tire. The tire air pressure increases slightly when the tire is loaded. The rolling radius of the tire is reduced by the local deformation of the tire at the road surface.