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Frequency dependent effective cell volume estimation for circular disk transducers using Lommel diffraction formulation

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ABSTRACT

In medical ultrasonic imaging the signal reflected from the tissue often has a random character to it. It is believed that the random nature of the tissue scattering microstructure is responsible for the stochastic nature of the echo signal. Chen, et. al. have proposed a signal processing scheme that is based on the statistical moments calculated on the Fourier transform of the time gated echo signal. The theory requires the knowledge of a frequency-dependent effective cell volume term. This paper describes the use of a closed form expression (Lommel diffraction formulation) for this purpose. Our simulation results suggest that reliable estimation of the cell volume is possible only when the time duration of the excitation pulse is small compared to the time gate length.

Key words: Ultrasound, Moments, Fourier Transform, Diffraction, Lommel Functions, Cell volume, Generalized K-distribution

1. INTRODUCTION

Ultrasound back-scattered signal from tissue has the character of random signals. Authors of several recent articles have attempted to model the echo signal as a random process characterized by some probability distribution function (pdf). The phenomenon is quite ubiquitous and is commonly known as speckle. The term is reserved for the pattern generated by interference between contributions from a large number of scattering centers. The phenomenon has been observed with microwave echoes from rough sea surface[2] as well as in the analysis of laser speckle in Optics[3]. In recent years there has been an increased effort to interpret speckle in medical ultrasound in terms of theory that has been developed for Optics and radar. The echo signal value at any given time is considered to be a superposition of echoes from a finite number of scatterers within a resolution cell. The cell volume for ultrasonic imaging performed in B or C scanning mode is approximately bounded by the -3 dB beam extent in the lateral direction and the time(distance) pulse width in the axial direction. A more precise definition follows later. The superposition effects have been successfully explained in other fields as a random walk in two dimensional complex plane. The resultant signal can be thought of as being a vector sum of randomly-phased components from a number of independent scatterers. The limiting case when the number of scatterers is large and their phases are uniformly distributed between 0 and 2π leads to the well known circularly Gaussian statistics on the complex plane or equivalently Rayleigh statistics for the signal amplitude [1]. Any deviation from this is generally referred to as non-Gaussian statistics. The deviations are manifestly due to the break down of the two conditions stated above, i.e. when the number of scatterers in the cell volume becomes small or their phases are not uniformly distributed or both.

The deviations due to the first condition, namely small number of scatterers can be studied via the first order statistics of the echo signal. Either the probability distribution function (PDF) or the statistical moments can be used for this purpose. Majority of researchers have considered various higher moments of the time-domain echo signals in their analysis. In the Rayleigh limit the expectation value of the second normalized intensity moment I_2 is 2[1]. Here no tissue relevant information can be gleaned from the moments. The deviation manifestly results in I_2 attaining a value greater than 2. In this situation statistical estimation of the effective number of scatterers per resolution cell volume from the moments is possible. The PDF is invariably found to obey K-distribution. Various models have been proposed and this has been the major thrust of recent research. Sleaf and Lele[4] examined the fourth normalized moment of the rf echo signal. Wear et.al. [5], Rao and Zhu [6], Narayanan et.al. [7] have used the second normalized

moment of the intensity(squared envelop of the rf) signal for the same purpose. As we will show the resolution cell volume depends on the imaging system parameters only, at least for a propagation medium that has no frequency dependent attenuation. If one can determine the resolution cell volume, then it is possible to estimate the effective scatterer number density (SND). Only few [4,6] have considered this concept in detail.

Chen et.al. [8] with a frequency domain analysis approach have developed a method to estimate frequency dependent effective scatterer number density $N_{eff}(\omega)$. In their approach the statistical moments are calculated on the Fourier transform of the time gated echo signal. Consequently there is a need to estimate “frequency dependent effective resolution cell volume” $V_{eff}(\omega)$ so that $N_{eff}(\omega)$ may be estimated from the moments. In this paper the frequency domain formalism of the diffraction from a circular aperture has been used to estimate $V_{eff}(\omega)$. Section 2 serves to define the mathematical problem and the terms that will be used in this paper. Section 3 introduces the reader to the Lommel diffraction formalism. Section 4 presents the calculation details followed by presentation and discussion of our results in section 5.

2. MATHEMATICAL PROBLEM

Consider a medium, as shown in Fig.1, that consists of point scatterers that are randomly distributed in three dimensional space. A circular piston transducer couples a pressure wave disturbance to the medium in the form a short pulse $p(t)$. The harmonic composition of the pulse is determine by its Fourier transform. For an ultrasound transducer this is determined by the product $[T(\omega)B_0(\omega)]$ where $T(\omega)$ is the two way complex transfer function of ultrasound transducer at the angular frequency ω . $B_0(\omega)$ is a complex superposition coefficient corresponding to the frequency composition of the electrical drive signal, i.e. the spectrum of the input pulse signal. The echo time signal can be written as a superposition of echoes from all scatterers that can possibly contribute and are individually labeled by the index “i”:

$$V(t) = \int_{-\infty}^{+\infty} d\omega T(\omega) B_0(\omega) e^{+i\omega t} \sum_{i=1}^{M_{all}} \psi_i(\omega) [A(\vec{r}_i, \omega)]^2 \quad (1)$$

The integral over ω signifies that we are taking an inverse Fourier transform to obtain the time domain signal. $\psi_i(\omega)$ is the value of the angular distribution factor at a 180° scattering angle for the i th scatterer whose position is $\vec{r}_i$. The sum is over all scatterers, M_{all} in space. However in reality not all scatterers will contribute because of the finite extent of the transducer beam. The number of scatterers is limited by the spatial extent of the beam. As we will see shortly, this beam profile is also important in defining the effective resolution cell volume. If a scatterer happens to be located within the cell volume then it will contribute to the echo signal. $A(\vec{r}_i, \omega)$ here refers to the circular piston transducer beam profile at the

scatterer location and for the excitation frequency ω . We can regard $A(\vec{r}, \omega)$ as a complex one way diffraction filter. The reciprocity principle implies that the process involved in the echo signal generated by the point scatterer at $\vec{r}_i$ and phase-sensitively integrated over the face of the same transducer can be interpreted as another identical diffraction filter. That is why the diffraction filter term appears as second power in equation (2). A closed form approximate expression to calculate $A(\vec{r}_i, \omega)$ will be considered in the next section. If echo signal analysis from a region of interest at some axial distance from the transducer is desired, it is necessary to time gate the echo signal defined in equation (1). Following Chen et.al.[] the Fourier transform of the gated signal can be expressed as:

Fig.1 Resolution cell volume with time gating function

Fig.2 Geometry for diffraction integral calculations

$$V_g(\omega) = \int_{-\infty}^{+\infty} d\omega_1 T(\omega_1) B_0(\omega_1) W(\omega - \omega_1) \sum_{i=1}^{M_{all}} \psi_i(\omega_1) [A(\vec{r}_i, \omega_1)]^2 \quad (2)$$

where $W(\omega - \omega_1)$ is the Fourier transform of the gating function. For a rectangular gate, this is simply $(\tau/2\pi) \text{sinc}[(\omega - \omega')(\tau/2\pi)]$. (Assume the zero of clock of has been set in the middle of the time gate and the gate duration is τ). The integration over ω_1 represents a convolution.

The number M_{all} can be replaced by the number of scatterers M in a small volume $\Delta\Omega$ (as shown in Fig.1) that extends laterally a distance that is larger than the transducer beam and extends a distance axially to include all the scatterers contributing significantly to the time-gated echo signal segment. Eqn. (2) can be rewritten as

$$V_g(\omega) = \int_{-\infty}^{+\infty} d\omega_1 T(\omega_1) B_0(\omega_1) W(\omega - \omega_1) \sum_{i=1}^M \psi_i(\omega_1) [A(\vec{r}_i, \omega_1)]^2 \quad (3)$$

The assumptions made by Chen et.al. under which the derivation of their final result is valid are summarized here. (1) Spatial location of scatterer i is independent of the location of another scatterer j . (2) The number of scatterers in a given volume is assumed to be Poission distributed. (3) Both the pulse and gate durations are long enough compared to the period of the ultrasonic wave. The ratio of the fourth moment to the square of the second moment of the time-gated signal spectrum is given by:

$$\frac{\langle [V_g(\omega) V_g^*(\omega)]^2 \rangle}{\langle V_g(\omega) V_g^*(\omega) \rangle^2} = 2 + \frac{\langle \|\psi(\omega)\|^4 \rangle_s}{\langle N \rangle \langle \|\psi(\omega)\|^2 \rangle_s^2} \times \frac{\iiint_{\Delta\Omega} dr \|J(\vec{r}, \omega)\|^4}{\left\{ \iiint_{\Delta\Omega} dr \|J(\vec{r}, \omega)\|^2 \right\}^2} \quad (4)$$

where $\langle \cdot \rangle$ denotes the mean value (mathematical expectation), $\|\cdot\|$ denotes the modulus of a complex variable. $V_g^*(\omega)$ is the complex conjugate of $V_g(\omega)$. N is the actual scatterer number density in the volume $\Delta\Omega$. The term $J(\vec{r}, \omega)$ is given by

$$J(\vec{r}, \omega) = \int_{-\infty}^{+\infty} d\omega_1 T(\omega_1) B_0(\omega_1) W(\omega - \omega_1) [A(\vec{r}, \omega_1)]^2 \quad (5)$$

This is actually a complex convolution that can also be written as

$$J(\vec{r}, \omega) = \{T(\omega) B(\omega) [A(\vec{r}, \omega)]^2\} \otimes W(\omega) \quad (6)$$

We can define the effective scatterer number density $N_{eff}(\omega)$ as

$$N_{eff}(\omega) \equiv \langle N \rangle \times \frac{\langle \|\psi(\omega)\|^2 \rangle_s^2}{\langle \|\psi(\omega)\|^4 \rangle_s} \quad (7)$$

this is a parameter that combines the effect of both actual number density and the effect of the frequency-dependent differential scattering cross section $\|\psi(\omega)\|^2 \equiv \psi(\omega) \psi^*(\omega)$ at a scattering angle 180° . From Eqn.(4) when the number of scatterers N goes to infinity the moments approach a limiting value of 2. We can also define the deviation of the moment from its limiting value as

$$Y(\omega) \equiv \frac{\langle [V_g(\omega) V_g^*(\omega)]^2 \rangle}{\langle V_g(\omega) V_g^*(\omega) \rangle^2} - 2 \quad (8)$$

and the effective volume $V_{eff}(\omega)$ as

$$V_{eff}(\omega) = \frac{\left\{ \iiint_{\Delta\Omega} dr \|J(\vec{r}, \omega)\|^2 \right\}^2}{\iiint_{\Delta\Omega} dr \|J(\vec{r}, \omega)\|^4} \quad (9)$$

Then Eqn.(4) can be written in a concise form as

$$Y(\omega) = \frac{1}{N_{eff}(\omega)} \times \frac{1}{V_{eff}(\omega)} \quad (10)$$

The left hand side of Eqn.(4) can be estimated from the data, that is the Fourier transform of the time gated signal. $V_{eff}(\omega)$ must be determined so that the tissue dependent parameter $N_{eff}(\omega)$ could be calculated. The main contribution of this paper is to demonstrate the utility of Lommel diffraction formulation in calculating $V_{eff}(\omega)$, the imaging system dependent effective volume parameter. Next section provides a brief introduction those readers who are not familiar with the Lommel formulation. For further details please refer to our published papers and research monograph[9,10,11].

3. LOMMEL DIFFRACTION FORMULATION

Before we introduce Lommel diffraction formulation, we need to introduce some related mathematical functions. The following two functions U_n and V_n with two variables are called Lommel functions:

$$U_n(u, v) = \sum_{s=0}^{\infty} (-1)^s \left(\frac{u}{v}\right)^{n+2s} J_{n+2s}(v) \quad (11)$$

$$V_n(u, v) = \sum_{s=0}^{\infty} (-1)^s \left(\frac{v}{u}\right)^{n+2s} J_{n+2s}(v) \quad (12)$$

Here u and v are real variables, n is non-negative integer and $J_n(u)$ Bessel function of first kind of order n .

When $u/v > 1$, Eqn.(11) converges slowly. So we can use the following equations to numerically calculate U_n :

$$U_{2n+1}(u, v) + V_{-2n+1}(u, v) = (-1)^n \sin\left(\frac{1}{2}\left[u + \frac{v^2}{u}\right]\right) \quad (13)$$

and

$$-U_{2n}(u, v) + V_{-2n}(u, v) = (-1)^n \cos\left(\frac{1}{2}\left[u + \frac{v^2}{u}\right]\right) \quad (14)$$

For a focused or unfocused transducer excited with a monochromatic pulse, the one-way disturbance $H_1(\rho, z, \omega)$ sensed by a point receiver at an off-axis distance ρ and axial distance z can be written

$$H_1(\rho, z, \omega) = \frac{1}{2\pi} \int_{\sigma_0} f(\sigma_0) \frac{e^{-jkr}}{r} d\sigma_0 \quad (15)$$

σ_0 is the area of the aperture, ρ_0 is the off-axis distance of σ_0 and r is the distance from an elemental area on the face of the aperture to the point receiver located at axial distance z and off-axis distance ρ . k is the wave number which equals to $2\pi f/c$ (f is the frequency of ultrasound and c is speed of sound). $f(\sigma_0)$ is the

velocity distribution on the face of the aperture which is unity under assumption of spatial uniformity. This equation is called *Rayleigh-Sommerfeld diffraction integral* (Chaz, [14]).

Using Fresnel approximation and assuming the circular symmetry of a piston transducer, Eqn.(15) can be rewritten as:

$$\hat{H}_1(\rho, z, \omega) = \frac{1}{z} e^{-jk(z + \frac{\rho^2}{2z})} \int_0^a e^{-jk\frac{\rho_0^2}{2z}} J_0\left(\frac{k\rho}{z} \rho_0\right) \rho_0 d\rho_0 \quad (16)$$

where a is the radius of the transducer aperture and ρ_0 is the off-axis distance at the source plane, while ρ is the off-axis distance at the observation plane. After some manipulation this integral expression for the filter can be expressed in closed form using Lommel functions. We will give here the expressions for both the focused and unfocused case. For the unfocused case we get:

$$\hat{H}_1(\rho, z, \omega) = \frac{1}{k} e^{-j(kz + \frac{v^2}{2u} + \frac{u}{2})} [U_1(u, v) + jU_2(u, v)] \quad (17)$$

where $u = ka^2/z$ and $v = k\rho/z$.

For the focused case we get:

$$\hat{A}(\rho, z, \omega) = \frac{\varepsilon}{kz} e^{-i(kz + \frac{k\rho^2}{2\varepsilon} + \frac{k\alpha^2}{2\varepsilon})} [U_1\left(\frac{k\alpha^2}{\varepsilon}, \frac{k\rho}{z}\right) + iU_2\left(\frac{k\alpha^2}{\varepsilon}, \frac{k\rho}{z}\right)] \quad (18)$$

where $1/\varepsilon = 1/z - 1/R$, R is the focal distance of the focused transducer. We are using the notation $\hat{A}(\rho, z, \omega)$ to indicate that this is a Fresnel approximation to the exact solution to the diffraction problem that was designated by $A(\vec{r}, \omega)$. Here $|\vec{r}| = \sqrt{\rho^2 + z^2}$. It is important to point out that an exact solution to the diffraction problem, i.e. the solution to the integral defined in Eqn.(1) does not exist in closed form for the case of monochromatic excitation. The integral must be evaluated numerically. What we present here is a close form solution that can be readily calculated but it is a consequence of a fresnel approximation to the phase term. Nevertheless this approximation has validity over a large region in front of the transducer.

4. EFFECTIVE CELL VOLUME CALCULATIONS

We present in this section a procedure to calculate $V_{eff}(\omega)$ using Eqn.(9) in conjunction with Eqn.(6). Furthermore we have also incorporated the approximate diffraction filter given by Eqn.(18) in place of $A(\vec{r}, \omega)$ in Eqn.(6). The calculations were performed for a focused circular disk transducer with a diameter of 12 mm and radius of curvature or focal length of 5 cm. The frequency response of the transducer, $T(\omega)$ was assumed to be a Gaussian function centered at $f = \omega/2\pi = 3.5$ MHz and a -20dB amplitude bandwidth of 4 MHz. The calculations were carried out initially for a time gate centered at the focal distance. Ultrasound speed of propagation was assumed to be 1500 m/s. The frequency domain convolution operation embodied in Eqn.(6) was performed via a multiplication in the time domain as shown below:

$$J(\vec{r}, \omega) = FFT\{ \{IFFT\{T(\omega)B_0(\omega)[\hat{A}(\rho, z, \omega)]^2\} \} w(t) \} \quad (19)$$

The functions in the frequency domain were sampled from -18 MHz to +18 MHz at 8192 points for the purpose of taking the inverse discrete Fourier transform (IFFT). $w(t)$ is the time gate function that was multiplied in the time domain before taking the FFT to get the frequency domain function $J(\vec{r}, \omega) = J(\rho, z, \omega)$. In order to study the variations of the cell volume with excitation signal bandwidth the time domain transducer excitation signal was modeled as a Hanning weighted window multiplied by the sinusoid at $f_0 = 3.5$ MHz:

$$b(t) = 0.5[1 - \cos(\frac{2\pi t}{T})]\cos(2\pi f_0 t) \quad (20)$$

Thus the $B_0(\omega)$ term in Eqn.(6) can be calculated as $\text{FFT}\{b(t)\}$. It is possible to show that the center frequency of $B_0(\omega)$ is f_0 and the bandwidth can be calculated as $BW = 1977.53T(\text{MHz})$ where T is in unit of μs .

The three dimensional integrations defined in Eqn.(9) can now be performed numerically. Note that the radial symmetry of the diffraction filter can be exploited here as we replace $d\vec{r}$ by $[2\pi\rho d\rho dz]$

$$V_{\text{eff}} = \frac{2\pi \left\{ \sum_{z=z_1}^{z_2} \left[\sum_{\rho'=0}^{\rho_0} \|J(z', \rho', \omega)\|^2 \rho' \Delta\rho' \right] \Delta z' \right\}^2}{\sum_{z=z_1}^{z_2} \left[\sum_{\rho'=0}^{\rho_0} \|J(z', \rho', \omega)\|^4 \rho' \Delta\rho' \right] \Delta z'} \quad (21)$$

We can verify that the above equation has the dimensions of volume. The step size $\Delta\rho$ was set at 0.1mm and ρ_0 was 2.5mm such that the side lobes even at the lowest frequency(2.5 MHz) were included in the integration.

5. RESULTS AND DISCUSSION

It is not possible to visualize the effective cell volume $V_{\text{eff}}(\omega)$ in three dimensions as something bounded by some surface. Nevertheless this volume has some relationship to aninsonification volume that one could visualize as an approximate cylinder which is bounded by the time gate in the axial direction and the -6dB beam profile value in the lateral or off-axial direction. Thus we may expect that larger the frequency the smaller would be $V_{\text{eff}}(\omega)$ because the ultrasound beam profile becomes narrower as frequency increases. Fig.3 is a plot of $V_{\text{eff}}(\omega)$ as a function of frequency, for several different input pulse bandwidths. The time gate was rectangular in shape and was $10\mu\text{s}$ wide for all the curves in this figure. When the bandwidth of the excitation pulse is large (greater than 1 MHz) the behavior of $V_{\text{eff}}(\omega)$ is understandable, the decreasing trend with frequency is obvious. However this behavior seems to be asymptotically approached as the bandwidth becomes large. In fact at the other extreme by the time the bandwidth becomes very small (for example 0.2 MHz) the behavior changes significantly. The curve dips down at low and high frequencies as well as in the neighbourhood of the center frequency $f_0 = 3.5$ MHz. The reason is not clear to us at the present time but based on some observed facts we can form a hypothesis. At the bandwidth where there are significant deviations ($\approx 0.2\text{MHz}$) the time domain pulse width has become comparable to the time gate length. Thus it appears that the formula begins to underestimate the cell volume. In other words the formula provides reasonable values for the cell volume when the pulse time width of the excitation is very small compared to the time gate length. There will always be some finite partial volume that is located near the two axial distances that correspond to the two

Fig. 3 Effective volume comparison for different excitation pulse bandwidth (center frequency $f_0=3.5\text{MHz}$, time gate: $10\mu\text{s}$ rectangle)

Fig.4 Effective volume at $f=3.5\text{MHz}$ changes with different excitation pulse bandwidth

Fig.5 Effective volume for different gate widths and different weighting function for the gate

Fig.6 Effective volume at different gate center location z_0

Effective volume value at 3.5Mhz for different time gate center location (time gate=10 μ s rectangle, input pulse BW=1.0Mhz)

Fig.7 Effective volume at 3.5MHz : variations in the neighborhood of the focal point

edges of the time gate, such that the contribution of echo signal from scatterers within this partial volume will be truncated some what due to the gating process. This is partial volume extends on either side of the gate by a distance that corresponds to the pulse width. It is therefore reasonable to expect that a fractional error due to this partial volume effect will be small when the pulse width is small compared to the gate length. It is expected to become large as the pulse width increases or the excitation bandwidth decreases. Fig.4 shows how the effective volume monotonically increases with bandwidth for calculations done at the center frequency of 3.5 MHz. But above 1.4 MHz bandwidth the value asymptotically approaches 24 mm^3 .

In Fig.5 illustrates the effect of changing gate width from 10 to $5 \mu\text{s}$ while keeping the weighting function $g(t)$ as a rectangular function (denoted as rect in the figure). The reduction in the cell volume at all frequencies is evident but the reduction is not exactly by a factor of 2 but slightly larger. The explanation probably lies in the same partial volume effect discussed earlier. At the smaller gate width the effective volume must have been slightly underestimated. The middle curve is for the case where a $10 \mu\text{s}$ wide Hanning weighting function $g(t)$ is applied. This in addition to reducing the cell volume should help in reducing the partial volume error.

Fig.6 shows how the frequency dependence for $V_{\text{eff}}(\omega)$ changes as we change the center location of the rectangular gate to different depths. The middle curve is for $z_0 = \text{focal distance} = 5 \text{ cm}$. There are two curves at $z_0 = 5 \text{ cm} - .75 \text{ cm}$ and $5 \text{ cm} - 1.5 \text{ cm}$ that correspond to regions of interest (ROI) that are closer to the transducer, in other words to the left of the focal point. The other two curves are for ROI that are located to the right of the focal point. The differences that we observe are mainly due to the differences in the behavior of the diffraction filter $\hat{A}(\rho, z, \omega)$ at different axial distances. As a general observation the frequency dependence gets stronger as we increase the axial distance moving it through the focal point. Fig.7 shows the variation of the cell volume with axial distance for the monochromatic frequency 3.5 MHz.

6. CONCLUSIONS

In this paper we have considered the task of estimating the frequency dependent effective cell volume using the beam profile information derived from the Lommel diffraction formulation. The formula for effective cell volume has been taken from the theory developed by Chen et.al. [1]. Our approach should be valid even in the near field of a flat disk circular transducer or in a region that is far away from the focal zone of a focused circular disk transducer. This is because the Lommel formulation amounts to a Fresnel approximation to the diffraction integral and the Fresnel approximation is believed to be valid over a much larger spatial region in front of the transducer than for example the Fraunhofer approximation that is often used in literature. The computational advantage here is that the diffraction filter can be calculated from a closed form expression for each frequency. Our simulation results indicate an interesting interplay between the time gate width and the excited pulse bandwidth. Sufficient care must be exercised to ensure that the excited pulse bandwidth must be large or equivalently the pulse time width must be small compared to the time gate length. Otherwise the calculated frequency dependence of the cell volume appears to be unreliable.

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