The Persistence of Ecological-Economic Systems: Alternate Measures and their Properties¹

by

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Abstract

A number of ecologists and some economists have commented on the significance of persistence as a stability concept for jointly determined ecological-economic systems. This notwithstanding, the economics and the regional science literatures have paid scant attention to the problem of computing an ecological-economic system's persistence. As such, this paper has two objectives. First, we derive three measures of the persistence of a jointly determined ecological-economic system whose temporal behavior is governed by the presence of thresholds. Next, we compare and contrast these three measures. Finally, we discuss their theoretical properties and their practical uses.

Keywords: Ecological-Economic System, Persistence, Stability, Threshold, Uncertainty

JEL Classification: Q20, D81

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1. Introduction

There now exists a sizeable literature in economics and in regional science on the stability of economic and regional systems. In particular, in the context of economic systems, researchers such as Jordan (1988), Mitra *et al.* (1991), and Balasko and Royer (1996) have analyzed the many meanings of stability. Similarly, in the regional science literature, Damania (1993) has analyzed the stability of a collusive spatial oligopoly, Lanaspa and Sanz (2001) have studied the stability properties of Paul Krugman's core-periphery model, and Zeng (2002) has provided a theoretical analysis of the stability of a migration model. Despite this great interest in studying the stability of economic and regional systems, economists and regional scientists have paid scant attention to the problem of computing the stability of dynamic and stochastic *ecological-economic* systems that are jointly determined.³ This is surprising not only because the study of the stability of ecological-economic systems is mathematically challenging, but also because in recent times, a number of researchers such as Dasgupta (1996), Perrings (1998), and Batabyal (1999) have commented on the need for studying ecological-economic systems thoroughly.

Ecologists have used the word *stability* to mean at least five different things (Pimm, 1991, p. 13; Krebs, 1994, p. 544). Of these five different things, economists have focused their attention

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This means that the dynamics of the joint system reflect the nature of the connections between each of the two sub-systems, i.e., the ecological and the economic sub-systems. Consequently, as Perrings (1996) has noted, shocks to the joint system generate a set of ecological and a set of economic effects, and these effects are interlinked. The extent to which these interlinked effects affect the stability of the joint system depends on the degree to which the two sub-systems are "connected." Highly connected systems co-evolve. Consequently, as compared to loosely connected ecological-economic systems, the stability of highly connected ecological-economic systems is more likely to be affected by ecological and/or economic shocks.

almost exclusively on *resilience*. However, as noted by Pimm (1991, pp. 13-15), Krebs (1994, pp. 544-545), Batabyal (1999), and Keeling (2000), for certain kinds of research questions, the apposite stability concept is *persistence*, where persistence refers to “how long a variable lasts before it is changed to a new value” (Pimm, 1991, p. 14).⁴ The reader should note that the use of the word “variable” in this definition is deliberate. Because ecologists have focused on alternate concepts of stability at distinct levels of ecological organization, it is possible to substitute the word “variable” with another word such as “ecosystem.” To this end, suppose that a researcher is interested in determining how long a particular community will retain its current species composition. Next, suppose that this researcher—*a la* Belovsky (1987) and Goodman (1987)—would like to ascertain how long a particular population lasts before becoming extinct. In both these cases, this researcher will want to know the persistence of, respectively, the concerned community and the population.

Although persistence is the appropriate stability concept for a number of research questions, with three exceptions, economists and regional scientists have paid no attention to the task of constructing measures of persistence. Recently, in two complementary papers, Batabyal and Beladi (1999) and Batabyal (2000a) have provided a closed-form expression for the persistence of a jointly determined ecological-economic system in a dynamic and stochastic setting. However, this expression depends on the assumptions that (i) persistence depends only on the keystone species⁵ of an ecological-economic system, that (ii) there are no interaction effects between the keystone species of an ecological-economic system, and that (iii) there is perfect substitutability between these

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An implication of this definition is that persistence is measured in time units.

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“A role may be occupied by a single species, and the presence of that role may be critical to the community. Such important species are called *keystone species* because their activities determine community structure” (Krebs, 1994, p. 554, emphasis in original).

keystone species in the discharge of ecological functions. In addition to this, the provided measure of persistence does not directly exploit the fact that ecological-economic systems are jointly determined. Batabyal (2000b) has also studied the problem of computing an ecological-economic system's persistence. Although this paper discards the "persistence depends only on the keystone species" and the "no interaction effects" assumptions, it maintains the "perfect substitutability" assumption. Moreover, this last paper provides an upper bound and not a closed-form expression for the persistence of an ecological-economic system.

We now comment on two issues that are related to this paper's focus on the calculation of the persistence of jointly determined ecological-economic systems. First, given that ecological-economic systems evolve over time, what kinds of dynamic systems are we studying in this paper? Second, what is the relationship between existing studies of persistence in the ecology literature and our interest in constructing measures of the persistence of an ecological-economic system?

The answer to the first question is that we are analyzing the set of all dynamic ecological-economic systems whose temporal behavior is subject to *threshold* effects. We emphasize that this set is indeed a large one. For instance, Stylinski and Allen (1999) have noted that when subjected to exogenous economic (construction, landfill operations) and environmental (fire) shocks, the successional patterns of shrub communities in the semi-arid regions of the southwestern United States are subject to threshold effects. Wu *et al.* (2000) have pointed out that the temporal abundance of the steelhead trout in Oregon is determined by the presence of thresholds in the quality of the water in the relevant fisheries. More generally, the work of Homer-Dixon (1991) and that of Perrings (1999) tells us that the temporal behavior of *most* ecological-economic systems is governed by the presence of thresholds. Dynamic ecological-economic systems that are subject to threshold effects

are also stochastic in nature. Indeed, as May (1973, p. 109) has rightly noted, “real environments are uncertain, stochastic...and...[the] parameters which characterize natural biological systems all, to a greater or lesser degree, exhibit random fluctuations.” Although, strictly speaking, this statement refers to “natural biological systems,” the quoted sentence applies equally well to ecological-economic systems. Consequently, the reader should note that the systems that comprise the subject matter of this paper are, broadly speaking, the kinds of systems studied in chapter 5 of May (1973).

Although there exists a sizeable literature in ecology on persistence and related concepts,⁶ most of this literature has focused on the study of the persistence of *deterministic* systems. Commenting on this state of affairs, Ives (1995, p. 217, emphasis added) has noted that in order “[t]o apply generally to ecological communities, stability needs to be defined for *stochastic* systems in which environmental perturbations are continuous and equilibrium [population] densities are never achieved.” More to the point, Keeling (2000, p. 269, emphasis added) has pointed out that once “the essential stochasticity of a system has been acknowledged, the *persistence* of populations becomes an important consideration.”

This review of the extant literature yields two principal conclusions. First, except for a very small number of papers that have studied the computation of persistence in somewhat restrictive settings, the economics and the regional science literatures have paid no attention to the problem of constructing measures of the persistence of an ecological-economic system. Second, while persistence has been studied at some depth in ecology, most of this analysis has been conducted for deterministic systems. Given this state of affairs, our paper has two objectives. First, we construct three new

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For more on this see Pimm (1991), Krebs (1994), Akcakaya and Baur (1996), Stelter *et al.* (1997), and Robertson (2000).

measures of the persistence of dynamic and stochastic ecological-economic systems whose temporal behavior is governed by the presence of thresholds. *Inter alia*, these measures exploit the fact that ecological-economic systems are jointly determined (see footnote 3). Put differently, our measures of persistence place great emphasis on the effects that detrimental shocks have on the functioning of an ecological-economic system. Second, we compare and contrast these three measures and then we discuss their theoretical properties and their practical uses.

The rest of this paper is organized as follows: Section 2.1 constructs a measure of persistence that concentrates on the number of *shocks* before an ecological-economic system, in the words of Pimm (1991, p. 14) and Krebs (1994, p. 544), changes to a new value. Section 2.2 constructs a measure of persistence—based on the *continuous* exponential distribution—that concentrates on the *cumulative damage* that an ecological-economic system is able to withstand before it is changed to a new value. Section 2.3's measure of persistence is also based on the total damage that can be withstood by an ecological-economic system. However, this time we use the *discrete* geometric distribution. In section 3 we compare and contrast these three measures and then we discuss their theoretical properties and their practical uses. Section 4 concludes and offers suggestions for future research.

2. The Construction of Measures of Persistence

2.1. The Number of Shocks and Persistence

Consider a stylized dynamic and stochastic ecological-economic system that consists of an arbitrary but finite number of species. Economic activities such as fishing, grazing, and hiking, and environmental events such as droughts, fires, and winter freezes result in shocks *over time* to this

ecological-economic system. With each shock, our ecological-economic system's characteristics⁷ are modified over time, for the most part, incrementally. In other words, this is *not* a static system. However, as indicated in section 1, this ecological-economic system's temporal behavior is governed by the presence of thresholds. Let us denote this threshold by $k \in \mathbb{N}$. We suppose that our ecological-economic system can withstand k shocks before it is changed to a different value. Put differently, our ecological-economic system is persistent until it is hit by the k th shock. In this connection, the reader should note that different ecological-economic systems will typically have different thresholds. Therefore, a given number of shocks will generally not have the same effect on ecological-economic systems with different threshold levels.

We now need to model the arrival process for these shocks. The simplest stochastic arrival process that has been used extensively in the natural resource and environmental economics and the ecology literatures is the Poisson process.⁸ Consequently, in the rest of this paper, we suppose that the shocks arrive over time in accordance with a Poisson process with rate λ .⁹ We now wish to compute the persistence of our ecological-economic system. Mathematically, we wish to acquire information about the lifetime T of this system until it is changed to a different value. Note that our

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By characteristics we mean things like the number and the abundance of species.

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For a more detailed corroboration of this claim, see Pielou (1969, 1977), Uhler and Bradley (1970), Arrow and Chang (1980), and Mangel (1985).

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As we have already noted in footnote 8, in both the natural resource and environmental economics and the ecology literatures, there are many precedents for using the Poisson process. This is a key reason for using this stochastic process in our paper. The Poisson process is defined to be a counting process that possesses both independent and stationary increments (Ross, 2000, p. 258). It is possible to give up the assumption of stationary increments and model the arrival of shocks with a non-homogeneous Poisson process. However, if we give up the independent increments assumption, then we would not be working with a Poisson process. In this connection, the reader should note that if one were to eschew the use of the Poisson process, then one could model the shock arrival process with a more general counting process such as a renewal process. For more on these matters, see Taylor and Karlin (1998, pp. 419-471) and Ross (2000, pp. 363-425).

ecological-economic system will change to a new value when the k th shock hits this system. Put differently, this ecological-economic system will change to a new value when the number of shocks equals the threshold value k . Also, note that because T is a random variable, the persistence of our ecological-economic system is given by the expected value of T or $E[T]$.

Let us first compute the density function of T . Because the shocks occur in accordance with a Poisson process with rate $\ddot{e}$, by Proposition 5.1 in Ross (2000, p. 262), we know that the times between successive shocks are exponentially distributed with mean $1/\ddot{e}$. Moreover, from Taylor and Karlin (1998, p. 276) and from Ross (2000, p. 248) we know that the sum of these exponentially distributed times has a gamma distribution with parameters k and $\ddot{e}$. Using these two pieces of information, we can write the density function of our ecological-economic system's lifetime T as

$$f_k(t) = \frac{\ddot{e}^k t^{k-1} \exp(-\ddot{e}t)}{\tilde{\Gamma}(k)}, \quad (1)$$

where $\tilde{\Gamma}(k)$ is the gamma function and in our case $\tilde{\Gamma}(k) = (k-1)!$. Now from Table 2.2 in Ross (2000, p. 67), the persistence of our ecological-economic system is

$$Persistence = E[T] = \frac{k}{\ddot{e}}. \quad (2)$$

Equation (2) provides us with a simple measure of an ecological-economic system's persistence that exploits the fact that the temporal behavior of this system is governed by the presence of the threshold k . This equation tells us that persistence is directly proportional to the threshold value k , and the constant of proportionality is the mean time between successive shocks ($1/\ddot{e}$).

Finally, equation (2) shows that when the frequency of the shocks increases (δ increases), *ceteris paribus*, the persistence of our ecological-economic system decreases.

One question that arises now is the following: When does it make more sense to use a measure of persistence that is based on the number of shocks rather than on the cumulative damage resulting from these shocks? To answer this question, note that shocks, at least the kind that we have in mind, are typically the result of abiotic factors. Further, it seems to us that biotic factors are more likely to have a cumulative impact on a particular system. Consequently, it seems more useful to us to employ a measure of persistence that is based on the number of shocks when the system under study is controlled largely by abiotic factors. On the other hand, it is more effectual to use measures of persistence based on the cumulative damage caused by shocks when both abiotic and biotic factors are responsible for the shocks under consideration. Many researchers—see Tikkanen *et al.* (1994) and Muotka and Virtanen (1995)—have suggested that benthic stream communities are primarily controlled by abiotic factors. Therefore, if one is interested in determining the persistence of such communities, then it makes sense to use a measure of persistence that is based on the number of shocks.

Conceptualizing persistence as a function of the number of shocks until a threshold is reached is one way, but clearly not the only way, to think of this concept. Shocks resulting from abiotic and biotic factors typically result in damage to an ecological-economic system and hence we can also think of persistence in terms of the *cumulative damage* caused by these shocks. In other words, persistence can also be viewed in terms of the aggregate damage that an ecological-economic system is able to withstand before it is changed to a new value. We now develop this approach in the next two sections of this paper.

2.2. The Cumulative Damage from Shocks and Persistence: The Continuous Case

As in section 2.1, we suppose that shocks affect our ecological-economic system over time in accordance with a Poisson process with rate μ . Once again, we remind the reader that with each shock our ecological-economic system's attributes typically *change*. However, our point is that because this system will change to a new value only when a certain threshold is reached, the individual shocks themselves cause some but not sufficient change. Viewed differently, the system under study is *not* static but dynamic in the sense that it is changing its attributes—for the most part marginally—over time.

Let $N(t)$ be the total number of shocks that have hit our system by time t and let D_i be the non-negative dollar damage caused by the i th shock. These damages $D_1, D_2, D_3, \dots$ are exponentially distributed random variables and they are distributed in conformity with the density function $g_D(d) = \mu \exp(-\mu d)$, where $d \geq 0$. The monetary damage from individual shocks accumulates additively and hence $X(t) = \sum_{i=1}^{i=N(t)} D_i$ denotes the aggregate damage sustained by our ecological-economic system by time t . Now, consistent with the discussion in sections 1 and 2.1, we suppose that our ecological-economic system is persistent as long as the cumulative damage is less than some threshold value $\hat{X}$. In other words, when the total monetary damage exceeds the threshold $\hat{X}$, our ecological-economic system will change to a new value.¹⁰ We now compute persistence or $E[T]$, the expectation of the lifetime T of this system until it is changed to a different value.

First note that $T > t$ if and only if $X(t) < \hat{X}$. Then, using equation 6.4 in Taylor and Karlin (1998,

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In the time period between any two shocks, particularly if this time period is relatively long, it is possible that the ecological-economic system under study will recover to some extent. Although we do not explicitly model this recovery, such a possibility is certainly accounted for in our modeling framework. Indeed, if such a recovery does take place in the time period between, say, D_2 and D_3 , then the impact of D_3 on the system will be less than what it would have been in the absence of such a recovery. Nevertheless, it is important to note that our procedure for deriving the two measures of persistence does *not* depend on whether a recovery does or doesn't take place.

p. 319), we have

$$E[T] = \int_0^{\infty} \text{Prob}\{T > t\} dt = \int_0^{\infty} \left[\sum_{n=0}^{\infty} \frac{(\ddot{e}t)^n \exp(-\ddot{e}t) G^{(n)}(\hat{X})}{n!} \right] dt, \quad (3)$$

where $G^{(n)}(\hat{X})$ is the convolution of the n damage random variables $D_1, \dots, D_n$. Mathematically, we have $G^{(n)}(\hat{X}) = \text{Prob}\{D_1 + D_2 + D_3 + \dots + D_n \leq \hat{X}\}$. Interchanging integration and summation, we get

$$E[T] = \frac{1}{\ddot{e}} \sum_{n=0}^{\infty} G^{(n)}(\hat{X}). \quad (4)$$

Because the D_i 's are distributed in accordance with the exponential density function $g_D(d) = \mu \exp(-\mu d)$, equation (4) can be simplified further. In particular, we use the fact that the sum of these exponentially distributed damage random variables has a gamma distribution (see Ross (2000, p. 248)) to obtain $\sum_{n=0}^{\infty} G^{(n)}(\hat{X}) = \sum_{n=0}^{\infty} \sum_{k=n}^{\infty} \{(\mu \hat{X})^k \exp(-\mu \hat{X})\} / k!$. Simplifying the right hand side (RHS) of this last equality, we get $\sum_{n=0}^{\infty} G^{(n)}(\hat{X}) = 1 + \mu \hat{X}$. This equality and equation (4) together allow us to conclude that

$$\text{Persistence} = E[T] = \frac{1 + \mu \hat{X}}{\ddot{e}}. \quad (5)$$

Equation (5) gives us a damage based measure of the persistence of an ecological-economic system. This measure is based in part on the threshold $\hat{X}$ and this threshold captures the idea that there is a cumulative dollar damage that our ecological-economic system is able to withstand before it is changed to a new value. Equation (5) tells us that even when one adopts a cumulative damage

based view of persistence, the persistence measure is still directly proportional to the threshold value $\hat{X}$ of the total monetary damage that our ecological-economic system is able to withstand before it is changed to a new value. Moreover, comparing equations (2) and (5), we see that these two measures of persistence are qualitatively similar. In particular, both measures are ratios in which the denominator is the rate $\bar{\epsilon}$ of the Poisson arrival process of, respectively, the shocks, and the damages resulting from these shocks. Finally, equations (2) and (5) tell us that when the frequency of the shocks and the damage from these shocks rises, i.e., when $\bar{\epsilon}$ rises, *ceteris paribus*, the persistence of our ecological-economic system falls.

In our analysis thus far, we have made considerable use of the exponential distribution to model the times between successive shocks and the damage originating from these shocks. Although this has enabled us to obtain a closed-form measure of an ecological-economic system's persistence, it should be noted that the exponential distribution is a *continuous* distribution with a constant hazard (or failure) rate.¹¹ One can ask what happens to our methodology when the damage from the shocks to our ecological-economic system is more appropriately modeled with a *discrete* distribution. We now show how the methodology of this paper can be used to construct a measure of persistence in this last case. As indicated above, the exponential distribution has a constant hazard rate. Consequently, to be consistent in our comparison of the persistence measures in the continuous and in the discrete cases, we need to model the D_i 's with a *discrete* distribution that also has a constant hazard rate. The geometric distribution satisfies this requirement and hence this is the distribution that we use in the next section.

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For more on this concept see Taylor and Karlin (1998, p. 36) and Ross (2000, p. 246).

2.3. The Cumulative Damage from Shocks and Persistence: The Discrete Case

Consistent with the analysis in sections 2.1 and 2.2, suppose that shocks damage our ecological-economic system over time in conformity with a Poisson process with rate $\ddot{e}$. For the reasons given in section 2.2, we remind the reader that our ecological-economic system is *not* a static system. Let $N(t)$ be the total number of shocks by time t and let D_i be the non-negative dollar damage caused by the i th shock. Now, these damage random variables $D_1, D_2, D_3, \dots$ are geometrically distributed with $\text{Prob}\{D=k\} = p(1-p)^k$, where $k=0,1,2,\dots$, and p is the parameter of the geometric distribution. The monetary damage from individual shocks accumulates additively and therefore $X(t) = \sum_{i=1}^{i=N(t)} D_i$ denotes the cumulative damage sustained by our ecological-economic system up to time t . As in sections 2.1 and 2.2, we suppose that our ecological-economic system is persistent as long as the total damage is less than some threshold value $\hat{X}$. This means that when the aggregate monetary damage exceeds the threshold $\hat{X}$, our ecological-economic system will change to a new value. Let us now compute persistence or $E[T]$, the expectation of the lifetime T of this system until it is changed to another value.

Modifying the section 2.2 analysis to account for the discrete nature of the damage process, we get an equation corresponding to equation (4). That equation is

$$E[T] = \frac{1}{\ddot{e}} \sum_{n=0}^{n=\infty} G^{(n)}(\hat{X}-1) \quad (6)$$

and $G^{(n)}(\cdot)$ is as in section 2.2. Using equation 3.6 in Taylor and Karlin (1998, p. 26) first and then using equation 6.21 in Taylor and Karlin (1998, p. 55), we can simplify the RHS of equation (6). This simplification gives

$$E[T] = \frac{1}{\ddot{e}} [1 + \{ \sum_{k=0}^{\hat{X}-1} p(1-p)^k (1-p)^{-k-1} \}]. \quad (7)$$

Note that the expression inside the curly brackets on the RHS of equation (7) reduces to $\hat{X}p/(1-p)$.

Using this result and equation (7), the persistence of our ecological-economic system is

$$Persistence = E[T] = \frac{1}{\ddot{e}} [1 + \frac{\hat{X}p}{1-p}]. \quad (8)$$

Equation (8) gives us a second cumulative damage based measure of the persistence of an ecological-economic system. Allowing for the discrete nature of the damage process, this last measure is also based primarily on the threshold dollar damage that can be sustained by our ecological-economic system before it is changed to a new value. Before concluding this section, we reiterate that relative to persistence measures that are purely shock based, our standpoint is that it is more useful to use the measures of persistence based on the cumulative damage caused by shocks when both abiotic and biotic factors are responsible for the shocks under consideration. We now compare and contrast the theoretical properties of these three persistence measures. Then, we comment on the practical uses of these three measures.

3. Discussion

3.1. Theoretical Properties of the Three Persistence Measures

Before we proceed to our comparative exercise, it is worth repeating a point that we have made before and that point is this: the ecological-economic systems studied in sections 2.1-2.3 are *not* static systems. Although we have not modeled these systems explicitly with differential equations, this does not mean that these systems are static. The properties of the ecological-economic systems

we have studied in this paper are affected by the shocks that hit these systems *over time*. These temporally distributed shocks *do* alter the properties of the systems being studied. Consequently, these systems are *changing over time* and it is in this sense that they are dynamic. It is just that they are not changing enough to turn into a new value until a certain threshold is reached.

Table 1 contains a complete description of this paper's three measures of persistence and

insert Table 1 about here

their theoretical properties. Reading horizontally, the second row of the Table tells us that in every instance, persistence is given by the product of two terms. Moreover, the first term in this product is always the same and it equals the reciprocal of the rate δ of the Poisson shock arrival process. Inspection of Table 1 shows that the differences between the three measures come from the second term in the square brackets.

Suppose the rate δ of the Poisson shock arrival process increases. Then, *ceteris paribus*, in any given time period our ecological-economic system will be subjected to a higher number of these damaging shocks. In this circumstance we would expect this system's persistence to decrease. The third row of Table 1 provides evidence to support this intuition. In all three cases, there is a negative effect of a rise in the rate δ on persistence.

A similar intuitive result holds as far as the impact of the threshold on our ecological-economic system's persistence is concerned. If the threshold k or $\hat{X}$ goes up then this means that, *ceteris paribus*, our ecological-economic system will persist in its current state for a longer time period before changing to a new value. In other words, we expect a rise in either k or $\hat{X}$ to have a positive impact on persistence. The fourth row of Table 1 tells us that this is indeed the case.

Finally, with regard to the measures derived in sections 2.2 and 2.3, the last row of Table 1

demonstrates the impact of a rise in the parameters of the two damage distribution functions on the persistence of our ecological-economic system. We see that irrespective of whether the damage distribution is continuous or discrete, the impact of a rise in either μ or p is the same on system persistence. Specifically, when either μ or p go up, our ecological-economic system's persistence also goes up.

In the case of all the persistence measures of this paper, we modeled the shocks from economic activities and those from environmental events *jointly*. From an ecological-economic system manager's perspective, it is sometimes but not always possible to separate the economic shocks from the environmental ones. So, how do we modify our modeling approach when it is possible to separate the economic and the environmental shocks? We now answer this question. Suppose the economic shocks occur in accordance with a Poisson process with rate $\acute{a}$. Further, suppose that the environmental shocks arise in conformity with a Poisson process with rate $\hat{a}$. Then, the *total* number of shocks hitting our ecological-economic system is the sum of the economic and the environmental shocks. Now, as long as the economic and the environmental shock processes are independent, we can invoke Theorem 1.1 in Taylor and Karlin (1998, p. 268) and conclude that the *total* shock process is also Poisson distributed with rate $\tilde{a} = \acute{a} + \hat{a}$. So, even though we could work with two independent Poisson processes with non-identical rates, we don't have to. In particular, despite the fact that we have two independent Poisson processes at work here, using the above theorem, we can still derive measures of persistence based on the single rate $\tilde{a}$.

One question that arises in practical settings relates to the estimation of the unknown Poisson parameter $\ddot{e}$ or $\tilde{a} = \acute{a} + \hat{a}$. Depending on the limitations imposed by data availability concerning the actual observance of the relevant shocks by an ecological-economic system manager, one can use

alternate techniques of estimation. Available techniques include the method of control charts (see Yacout and Chang (1996)) and Bayesian methods (see Antelman *et al.* (1997)). When the underlying Poisson process has a spatial dimension to it, one can also use the techniques delineated in Kutoyants (1998) to estimate the relevant parameter.¹² We now proceed to comment on the practical uses of the three persistence measures of this paper.

3.2. Practical Uses of the Three Persistence Measures

The measures of persistence that we have derived in this paper have a number of practical applications. In what follows, we discuss five such applications. First, Belovsky (1987), Goodman (1987), and Kinnaird and O'Brien (1991) have studied a population extinction model in which the average persistence time is computed as a function of certain specific demographic parameters. In this model, the likelihood of persistence depends only on these demographic parameters and not on the kinds of extrinsic factors that we have studied in this paper. However, as Krebs (1994, p. 406-408) has noted, these extrinsic factors are important. Consequently, along the lines of this demographic extinction model, we can estimate parameters such as the Poisson arrival rate $\bar{\epsilon}$ ¹³ and construct—for any given value of the threshold (k or $\hat{X}$)—a similar model for an ecological-economic system that tells us the average time (the persistence) until this system changes its state.

Second, in the context of range management, Friedel (1991) has rightly pointed out that environmental change can be discontinuous, with thresholds between alternate states. Therefore, in order to get an accurate idea of the condition of a range, it is necessary to quantitatively account for

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For more on spatial Poisson processes see Taylor and Karlin (1998, pp. 311-317).

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See section 3.1 for alternate ways of doing this.

the existence of these thresholds. The persistence measures of this paper provide range managers with a method for doing this. In other words, once a manager has estimated the appropriate parameters of the model, (s)he can use the work of Friedel (1991) and others to ascertain appropriate thresholds. With this information, range managers should be able to construct measures of the persistence of a specific range condition.

Third, an interesting practical question concerns the maintenance of biological diversity in particular fragmented landscapes like coastal southern California. Suarez and Case (2002) have shown that in order to shed light on this question, it is useful to study the persistence of specific species such as the coastal horned lizard following the occurrence of shocks that lead to fragmentation. In the case studied by Suarez and Case (2002), these shocks arise primarily from urbanization and species invasions. Consequently, in this kind of setting, quantitative information about the relevant shocks can be used to compute measures of persistence and then these persistence measures can be used to aid the task of maintaining biological diversity.

Fourth, Labbe and Fausch (2000) have shown that although the management of threatened species has traditionally emphasized the maintenance of individual populations in habitats that are controlled largely by local environmental shocks, landscape level shocks arising from dispersal and habitat patch mosaic structure can also be salient. Consequently, if one is interested in the longevity of the threatened Arkansas darter, then one can use information about the relevant local environmental and landscape level shocks to evaluate the factors that influence darter persistence and thereby facilitate the construction of persistence measures for this fish.

Fifth, in the context of fisheries management, Wu *et al.* (2000) have noted that a salient problem with some contemporary programs is that they ignore threshold effects. Given this state of

affairs, the persistence measures of this paper can be used to facilitate the design of fish conservation programs that recognize the salience of thresholds. By using these measures in this way, fishery managers will be able to better comprehend the significance of cumulative effects in the relationship between water quality and fish abundance.

In addition to the applications that we have already discussed, the concept of persistence is useful in many other management settings as well. For instance, Marshall *et al.* (1994) have pointed out that information about the persistence of clofentezine residues can be used to manage resistance in the European red mite. Similarly, knowledge about the persistence of small populations such as the Acorn woodpecker can be used to develop management strategies designed to preserve small populations that are faced with high levels of environmental shocks (Stacey and Taper (1992)). As noted in Cook *et al.* (1996), information about the persistence of shrubby weeds can be used to design improved weed management strategies. Finally, it should certainly not go unsaid that epidemiologists—see Keeling (2000)—are very interested in using the concept of persistence to comprehend and manage the spread of diseases.

4. Conclusions

For certain types of research questions concerning ecological-economic systems (see section 1), the appropriate stability concept that requires attention is persistence. In this paper, we provided three *new* measures of the persistence of jointly determined ecological-economic systems whose temporal behavior is governed by the existence of thresholds. In contrast with the previous literature on this subject, our analysis makes *no* assumptions about either the dependence of persistence on only the keystone species of an ecological-economic system or about the perfect substitutability between the keystone species of an ecological-economic system in the performance of ecological functions.

We used the fact that ecological-economic systems are jointly determined to derive our shock and cumulative damage based measures of persistence.

The analysis in this paper can be extended in a number of directions. In what follows, we suggest two possible extensions. Our derivation of the three measures of persistence was based on an examination of shocks that are injurious to an ecological-economic system. For the most part, this is where we believe the emphasis ought to be. However, sometimes ecological-economic systems are affected by positive shocks. As such, it would be useful to consider cases in which the effect of the injurious shocks is mitigated by the occasional appearance of positive shocks. Second, this paper's analysis was based on the use of certain common (see footnotes 8 and 9) but nonetheless specific distribution functions. Consequently, it would be useful to determine whether closed-form measures of persistence can be constructed when the analysis is conducted with general continuous and/or discrete distribution functions. Theoretical studies that incorporate these aspects of the problem into the analysis will provide richer and more realistic characterizations of an ecological-economic system's persistence.

Table 1

Three Measures of Persistence and their Properties

Persistence and Impact of Three Criteria on Persistence	Total Number of Shocks Based Measure	Cumulative Damage Based Measure: Continuous Case	Cumulative Damage Based Measure: Discrete Case
$Persistence \equiv E[T]$	$\frac{1}{\ddot{e}}[k]$	$\frac{1}{\ddot{e}}[1 + \mu \hat{X}]$	$\frac{1}{\ddot{e}}[1 + \frac{p\hat{X}}{1-p}]$
Impact of Shock Frequency $\ddot{e}$	$\ddot{e} \uparrow \Rightarrow Persistence \downarrow$	$\ddot{e} \uparrow \Rightarrow Persistence \downarrow$	$\ddot{e} \uparrow \Rightarrow Persistence \downarrow$
Impact of Thresholds k and $\hat{X}$	$k \uparrow \Rightarrow Persistence \uparrow$	$\hat{X} \uparrow \Rightarrow Persistence \uparrow$	$\hat{X} \uparrow \Rightarrow Persistence \uparrow$
Impact of Damage Distribution Parameters μ and p	n/a	$\mu \uparrow \Rightarrow Persistence \uparrow$	$p \uparrow \Rightarrow Persistence \uparrow$

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