Polarization Dynamics in Nonlinear Photonic Resonators

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Polarization Dynamics in Nonlinear Photonic Resonators

by

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A Dissertation Submitted in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy in Engineering—Communications Domain

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Acknowledgments

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Dedication

To my parents, siblings, wife, sons, & daughter. Thank you for your love, sacrifice, & encouragement, I love you all.
Curriculum Vitae

The author Saif Al Graiti completed his undergraduate study in Telecommunication Engineering Technology at Najaf Technical College from 2006 to 2010. The author worked in a cellular company in the research and development department from 2010-2011, and at the University of Kufa’s Information Technology Research and Development Center from 2012 to 2013. The author received a scholarship from the Higher Committee for Education Development in Iraq to complete his Master’s degree with Thesis in Telecommunication Engineering Technology at the Rochester Institute of Technology from 2014 to 2016. During the MS degree, the author worked as a GRA in the Photonic Systems Lab under his advisor, Dr. Drew Maywar. The author entered RIT’s Engineering PhD program in 2016 and has performed his research under Dr. Maywar. The author completed his degree of Doctor of Philosophy in Engineering—Communications Domain at RIT from 2016 to 2021.

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Abstract

The global market demand for higher-bandwidth communication is increasing exponentially. Although optical networks provide high transmission speed using light to transmit signals, a bottleneck-inducing conversion is often needed to perform the processing of optical signals in the electrical domain. Such processing imposes a major barrier that would limit the high transmission speed of fiber-optic communications. This bottleneck conversion may be mitigated by extending signal-processing capabilities directly into the optical domain itself. Thus, I have studied the dynamics of optical polarization in a nonlinear photonic resonator to understand a new optical physical behavior to enhance the capabilities of optical signal processing.

I present a theoretical model and experimental investigation to study the simultaneous occurrence of two optical nonlinear processes—nonlinear polarization rotation (NPR) and dispersive optical bistability. These two optical nonlinear processes within a nonlinear photonic resonator produce an optical signal exhibiting hysteresis curves in its state of polarization (SOP). Bistable action accompanied with simultaneous NPR is a significant departure from traditional optical memory, where the optical signal only exhibits hysteresis curves in the output power. Bistable polarization rotation (BPR) term is used to refer to the new physical process of bistable action accompanied by simultaneous NPR.

I have leveraged this new physical process of the bistable polarization rotation to realize a hysteresis-shape transformation and optimization. A diversity of hysteresis shapes are demonstrated in optical power including the canonical counter-clockwise (CCW) shape (S-shape), the clockwise (CW) shape (inverted S-shape), and butterfly shapes. The control
ABSTRACT

of the shape is performed downstream of the nonlinear photonic resonator within which the bistable signal is generated. I have derived a mathematical model to study this transformation process. Critical to our model, a generalized Malus’ law of a non-ideal linear polarizer and an elliptical input polarization.

Since all hysteresis shapes originate from the same bistable signal, all shapes exhibit the same switching input powers. Moreover, the shape-control process is used to enhance the bistable switching contrast to surpass 20 dB for the CCW and CW shapes. Additionally, the new technique of hysteresis shape control enables the ability of simultaneous distribution of the bistable signal into multiple paths. In each path, the optical signal can be independently controlled to produce a hysteresis shape. For example, CCW and CW shapes can be configured in two locations using the same BPR signal.

The theoretical and experimental work reported here is carried out for the case of a Fabry-Pérot semiconductor optical amplifier as the nonlinear photonic resonator. Both the new physical process and the new control capability presented here are extendable to other nonlinear media (such as Kerr media) and other photonic resonators (such as ring and distributed feedback resonators). The dissertation outcomes detail processes and techniques to enhance the performance of all-optical combinational gates, such as photonic AND and XOR gates, as well as all-optical sequential devices, such as photonic flip-flops.
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A.1 Polarization-resolved small-signal ASE spectra for TE and TM modes of the FP-SOA.

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A.3 The process of optimizing the input optical signal SOP into only the TM mode of the FP-SOA
1. Introduction

1.1 Motivation and Background

1.1.1 All-Optical Signal Processing

The global demand for higher bandwidth and processing speed is increasing exponentially [1]–[4]. The growing trend of users for high-speed bandwidth applications such as computing, online video & games, and other Internet-based services creates a need for high-speed transmission and processing capabilities [4].

Fiber-optic communications and optical interconnects are capable of transferring signals in a high transmission speed (in terahertz (THz) range) [1], [3], [5], [6], where researchers are working on expanding capacities of optical systems with revolutionary changes in optical communications [1], [2], [6], [7]. For example, researchers are working on developing capabilities of silicon photonics systems for and high-bandwidth communications [7], such systems are light sources and optical modulators using the complementary metal oxide semiconductor (CMOS) process compatibility to fabricate photonic integrated circuits (PICs) [7].

Optical communications also require developing high-speed capacities in processing, where processing capacities are need during the transmission of optical signals. For example, the processing is needed when transfer data from fiber to fiber, chip to chip, or within chips [3], [8]. Such processing includes routing and switching, where this kind of processing is typically performed in the electrical domain, where a bottleneck-inducing conversion is
required. This conversion is known as an optical to electrical to optical (O/E/O) conversion [3].

The electronic bottleneck conversion adds a major barrier that would limit the high-speed data rate of fiber-optic communications [3], [6], [9]. In addition, other impairments associated with electrical interfaces (e.g., interference & loss) limit the efficiency of such processing [8].

Therefore, there is a need to perform high-speed processing capacities in the optical domain itself. Processing and manipulating signals in the optical domain eliminate the bottleneck-inducing O/E/O conversion [3], [6]. Also, high-speed optical signal-processing can operate at a speed of optical communications to enhance the capability and efficiency of optical networking [4].

An optical signal property can be adjusted and controlled when it is propagating through a nonlinear medium, where nonlinearities can be realized under correct conditions in nonlinear devices [3], [4], [10]. Such nonlinear devices are the semiconductor optical amplifiers (SOAs), where SOAs have been studied widely studied to perform a nonlinear optical processing [3], [10]. For example, using SOAs to achieve all-optical signal-processing functional applications like SOA optical gates and wavelength conversion [10].

Nonlinearities are realized as results of the carrier density changes in SOAs that influence the refractive index of the nonlinear medium and it applies changes in the optical signal property exiting the device [10]. Such optical signal properties are power, phase, and polarization.

Nonlinear optical processing in SOAs can be triggered with extremely low power, such as -30 dBm (1 µW) that is one of the reasons to use it for processing purpose [11]. Furthermore, A SOAs is manufactured with semiconductor materials that make the SOA capable to be integrated with photonic integrated circuit (PIC) technologies [4], [10].

With the revolutionary changes of PICs technologies, it is feasible to realize all-optical functional applications via SOAs nonlinearities [10]. Therefore, all-optical processing
capabilities in SOAs can be used to overcome the limitation of the electronic bottleneck conversion [10].

A wide variety of nonlinear processes have been studied as a mechanism to perform all-optical signal processing, such processes include nonlinear polarization rotation and dispersive optical bistability, where these two nonlinear phenomena have as been studied separately in a variety of SOAs and other nonlinear media [11]–[29].

### 1.1.2 Nonlinear Polarization Rotation

Nonlinear polarization rotation (NPR) influences an optical signal polarization, where when an input power varies into a nonlinear device, the output state of polarization changes (undergoes a nonlinear rotation). Thus, NPR manipulates an optical signal polarization. NPR has been studied to demonstrate combinational all-optical applications [27], [30]–[32], using a variety of nonlinear media [24], [27], [33]–[35]. NPR has been demonstrated in a variety of nonlinear photonic devices including traveling-wave SOAs (TW-SOAs) [24], [27], [30]–[32], [34]. NPR occurs due to one of both of the following nonlinear factors—gain/loss anisotropy (influences optical power) [27]–[30], [32] and birefringence (influences optical phase) [24], [28], [32]—due to an asymmetry of the TE and TM polarization modes in nonlinear photonic devices. Gain/loss anisotropy means that the TE and TM modes experiences their own gain/loss, while birefringence means that there is a nonlinear phase difference between the TE and TM modes.

During NPR the output state of polarization (SOP) varies in time when the input power is changing into a nonlinear optical device, where a key to obtain NPR is to launch the optical power into both the TE and TM modes. Thus, either or both the following quantities will cause a change in the output SOP as a function of input power change—the phase difference between the polarization modes (due to birefringence) and power strength difference between the polarization modes (due to gain/loss anisotropy).

Previously demonstrated examples of NPR are shown in Fig. 1.1. The Poincaré sphere
CHAPTER 1. INTRODUCTION

Figure 1.1: Examples of previously demonstrated nonlinear rotation (NPR) in the output SOPs traces on Poincaré sphere: (a) NPR in a travelling-wave SOA [27]. (b) NPR in a travelling-wave SOA [34].

was used to trace output SOPs change as a function of input power change (as shown in the black curve in Fig. 1.1 (a), and red dotted curve in Fig. 1.1(b), where each point on the Poincaré sphere represents a unique SOP [36]. The Poincaré sphere is a graphical three-dimensional tool that can be use to describes an SOP using optical Stokes parameters \( s_1, s_2, \) & \( s_3 \), where each point on the Poincaré sphere is a unique SOP [36]. Some examples of SOPs are shown in Fig 1.2. Additionally, the Poincaré sphere is a convenient way of visualizing the handedness of polarization. For example, the north pole and south pole correspond to right circular polarization (RCP) and left circular polarization (LCP), respectively for sign convention of \( e^{i\omega t} \). The region between the north pole and the equator

Figure 1.2: Poincaré sphere with examples of SOPs.
or south pole and the equator have an infinite numbers of elliptical SOPs. The equator of the sphere has an infinite number of linear SOPs. Furthermore, the Poincaré sphere describes orthogonal SOPs like linear horizontal polarization (LVP) is orthogonal to the linear vertical polarization (LVP).

1.1.3 Dispersive Optical Bistability

Another process is used to manipulate an optical signal power via dispersive optical bistability, where when an input power varies into a nonlinear device, the output power exhibits two bistable branches, where these branches are different in power (strength) and those branches occur at a different input power. Dispersive optical bistability has been studied widely as a means of achieving a range of sequential and combinational all-optical signal processing applications [37]–[45]. Dispersive optical bistability has been studied widely in variety of nonlinear photonic resonators, such as in vertical cavities [12], dual-bus ring resonators [13], photonic-crystal nanocavities [14], diffraction gratings [15], and Fabry-Pérot resonators [11], [16], [17]. The previous demonstrations of dispersive optical bistability are observed in the total output optical power [11]–[23].

Dispersive optical bistability occurs when an interferometric resonance shifts onto the optical-signal wavelength to produce the high-power branch (upward-switching). While the low-power branch occurs when the resonances shift away from the optical-signal wavelength (downward-switching).

This shift occurs due to a nonlinear refractive index that depends on the input optical power, where if an input power strength varies, the causes a change in gain profile inside the active region of a nonlinear photonic resonator. The nonlinear refractive index can occur due to an assortment of mechanisms, including thermal-optic effects [13], two-photon absorption [14], the Kerr effect [19], and the carrier dynamics characterized by the linewidth enhancement factor [12], [11], [15]–[17].

When the input power increases, the optical power increases inside the active region
and it saturates the carrier density. Therefore, the refractive index increases. Increasing the refractive index makes an interferometric resonance shift towards the optical-signal wavelength that causes the upward-switching. While when the input power decreases inside the active region, the refractive index reduces. Then this reduction causes the interferometric resonance to shift away from the optical-signal wavelength that causes the downward-switching.

Examples of previously demonstrated dispersive optical bistability are shown in Fig. 1.3, where the arrows illustrating the upward-switching and downward-switching. In this nonlinear interaction, two stable optical powers occur in the output power of the nonlinear resonator [46].

![Figure 1.3](image)

**Figure 1.3:** Examples of previously demonstrated dispersive optical bistability: (a) Reported traditional optical-power hysteresis curve in Fabry-Pérot SOA [16]. (b) Reported traditional optical-power hysteresis curve in polymer SU-8 race-track resonator [13].

There are more aspects of light than just optical power that may be useful. For example, polarization varying between the two bistable branches of the output power has not been seen in the traditional optical-power hysteresis curve.

Thus, the simultaneous occurrence of dispersive optical bistability and nonlinear polarization rotation, where two bistable branches in dispersive optical bistability have a different state of polarization, has not been studied in the previous work [11]–[29].
1.1.4 Diversity of Hysteresis Shapes

Optical bistability has been studied widely to be used for sequential and combinational all-optical signal processing applications [37]–[45], [47], [48]. Several studies have been demonstrated using a nonlinear photonic resonator that exhibited dispersive optical bistability. Such nonlinear photonic resonators include distributed feedback (DFB) semiconductor optical amplifiers (SOAs) [15], [38], [39], [49], vertical cavity SOAs (VC-SOAs) [12], [40], [44], [50], and Fabry-Perot SOAs (FP-SOAs) [11], [15]–[17], [23], [41], [43], [51]. Previous studies of dispersive optical bistability have reported a variety of hysteresis shapes, including the canonical counter-clockwise shape (S-shape) [11], [12], [15]–[17], [23], [49], [51], the clockwise shape (inverted S-shape) [12], [15], [23], [49]–[55], and butterfly shapes [15], [49], [51], [55]. Such examples of diversity of hysteresis shapes are shown in Fig. 1.4.

![Figure 1.4: Examples of diversity of hysteresis shapes](image)

However, most of these previous bistable systems are capable of producing more than one hysteresis shape only in the reflection port of the nonlinear photonic resonator, where S-shape, inverted S-shape, and butterfly shapes have been reported in the reflective bistability from DFB-SOAs, VC-SOAs, FP-SOA. The reflection port does not allow for a straightforward concatenation operation for all-optical logic gates. Furthermore, the previous demonstrations
for the S-shape and inverted S-shape shapes showed a limited switching contrast of less than 10 dB [11], [12], [16], [17], [23], [49]–[55].

Additionally, the shape control mechanism is achieved by changing the conditions of the bistable system to produce a needed hysteresis shape. Specifically, the shape control is achieved by either the SOA drive current (i.e., small-signal gain) [15], [51], [55] or the wavelength of the injected optical signal [49], such examples for changing derive current or wavelength of input optical signal are shown in Fig. 1.4 (a) and Fig. 1.4 (b) respectively.

1.2 Overview of Dissertation

1.2.1 New Physical Behavior

In this dissertation, we investigate a new concept of optical memory based on the simultaneous occurrence of these two nonlinear processes—nonlinear polarization rotation and dispersive optical bistability. We refer to the simultaneous occurrence of two nonlinear processes as bistable polarization rotation (BPR).

The new concept of optical memory is a significant departure from the common concept of optical memory. A traditional optical memory could occur when optical power exhibits a hysteresis curve, where power at point A (in the center of the blue-line) is different than the power at point B (in the center of the red-line) as illustrated in the left-most graph of Fig. 1.5. There are more aspects of light properties in addition to optical power to be investigated, such an important aspect of a light property is polarization.

In our work, a new kind of optical memory occurs when an optical signal exhibits hysteresis curves in its state of polarization (SOP), as illustrated in the right-most graph Fig. 1.5, where such hysteresis curves need to be observed in the optical Stokes parameters ($s_1$, $s_2$, & $s_3$). Having optical Stokes parameters that reveal hysteresis behavior means each point on a hysteresis curve has its own SOP.

This dissertation provides a comprehensive study to deeply understand the new concept
CHAPTER 1. INTRODUCTION

Figure 1.5: Demonstrated new concept for optical memory, whereby the state of polarization exhibits hysteresis.

of having a simultaneous occurrence of nonlinear polarization rotation and dispersive optical bistability. The outcomes of this study are leveraged to investigate a new technique to control the hysteresis shape.

1.2.2 New Optical-Control Capability

The new concept of optical memory opens up potential nonlinear optical processing capacities. By leveraging the new concept of optical memory, the dissertation investigates a simple, flexible, and robust control mechanism to manipulate a hysteresis shape. The control mechanism can be achieved by only using linear-polarization components to produce a variety of hysteresis shapes. The linear-polarization components are located downstream of the nonlinear photonic resonator as illustrated in Fig. 1.6.

In my work, the bistable system is used to generate the bistable polarization rotated (BPR) signal, then this signal enters a hysteresis shape controller (HSC) system, where the HSC system is used to produce a variety of hysteresis shapes.

The polarization controller PC is configured to change the relative SOPs on the BPR curve with respect to the pass-axis of the linear polarizer. Realizing hysteresis shape diversity is achieved by controlling the relative power passing through the polarizer between the upper and lower branches of the BPR signal.
Figure 1.6: Simple, linear polarization components are used to transform the polarization bistable signal into a variety of optical-power hysteresis shapes.

Thus, the working principle of our technique does not act on the bistable system to select a hysteresis shape; where there is no signal sent back to the bistable system. Previously hysteresis shapes are achieved by acting on the bistable system, where it is performed by either changing an injected wavelength of the optical signal or an injected drive current. The potential benefits of this new control mechanism are as follows:

- Produce of a diversity of hysteresis shapes including counter-clockwise shape (S-Shape), clockwise shape (Inverted S-shape), a downward-switching butterfly shape, and an upward-switching butterfly shape.
- Originate all hysteresis shapes from the same BPR signal, this results in having hysteresis shapes that exhibit the same switching input powers, where having the same switching power means that AND and XOR logic gates, for example, can be driven using the same-power control pulses.
- Enhance the contrast of hysteresis shapes to exceed 20 dB, specifically, for the CCW shape and the CW shape.
- Produce a diversity of hysteresis shapes occurs at a location downstream (transmission) of the nonlinear photonic resonator, where the transmission is a typical concatenation configuration for all-optical signal processing.
- Perform the control mechanism to occur outside and downstream of the nonlinear
resonator. Our technique enables the functionality of distribution and simultaneous local control of a hysteresis shape. The BPR signal is sent into two locations A and B. A variety of hysteresis shapes experimentally demonstrated simultaneously at locations A and B, whereby an HSC unit independently and simultaneously is used at each location to select a hysteresis shape. This provides the flexibility to produce a needed hysteresis shape at each location (e.g., S-shape at location A and inverted S-shape at location B) to support the operation of an optical application.

1.2.3 Methodologies: Research Design & Strategy of Inquiry

Methodologies for Mathematical Modeling

Mathematical models are developed to model bistable polarization rotation (BPR) and hysteresis shape transformation. Mathematical modeling provides clear insight into behaviors based on the logical steps of mathematical expressions, where key quantities can be defined to describe system behavior. Defining the key quantities gives the opportunity to deeply observe and evaluate the influence of these quantities [56]. The mathematical model is carried out in two phases.

In phase A, a bistable polarization rotates (BPR) signal is modeled in a nonlinear photonic resonator. An ordinary differential equation (ODE) solver is used to find a solution for the FP-SOA nonlinear resonator rate equation, where the FP-SOA rate equation is modified to account for both polarization modes. Furthermore, Fabry-Pérot transmissivity is adapted to count for both polarization modes and anisotropic quantities. Optical Stokes parameters are used to represent a state of polarization (SOP) for an optical bistable signal exiting the FP-SOA.

In phase B, we have leveraged the mathematical model of the BPR signal with our mathematical model of a downstream hysteresis-shape controller to study the new capacity of controlling a hysteresis shape. Jones calculus is used to derive the mathematical modeling of the linear polarization complements in the downstream hysteresis shape controller, where
a generalized Malus’ law of the linear polarizer for an elliptical input polarization is derived and it includes insertion loss and a finite extinction ratio of the non-ideal polarizer.

Hakki-Paoli data analysis method is used to account for both FP-SOA polarization modes, where this method is used to calculate consistent values based on the lab conditions and the measured Fabry-Pérot resonances in the lab. Consistent values have been used in the modeling to validate the accuracy of the model and compare the modeling results with the experimental investigation.

**Methodologies for Experimental Research**

Lab data is an important part of our research because lab data is evidence that the research can be transferred into practice. Additionally, we extract consistent values based on the lab conditions to be used in the mathematical model. Also, experimental investigations help to discover new physical behaviors and perform optimization.

For each research activity, the experimental set-up is designed and laid out with the needed optical and electrical equipment. The design layout will consider setting values vs measured values, calibrating, signal synchronization, optimizing, simplicity, personal and equipment safety.

For producing an optimum BPR signal, optimization is carried out in our lab demonstrations by selecting the input optical signal conditions to reveal the highest NPR that is associated with bistable curves.

For the hysteresis shape controller (HSC), the relative passing power through a non-ideal linear polarizer between the upward branch and downward branch of the hysteresis curve is controlled to achieve the hysteresis shape transformation.

A Fabry-Pérot semiconductor optical amplifier (FP-SOA) is used as the nonlinear photonic resonator in the lab demonstrations. The FP-SOA is an active photonic device, where the material type is InGaAsP/InP. The device coating is designed to support the Fabry-Pérot resonances.
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1.2.4 Dissertation Outline

This dissertation is organized as follows:

Chapter 2 describes the theoretical model of the new physical process of bistable polarization rotation, where it takes into account: small-signal and power-induced birefringence; gain, loss, and nonlinearity anisotropy; the anisotropic impact of the Fabry-Pérot resonances on both optical power and phase; and the injection of the optical signal into the TE and TM polarization modes. Also, this chapter describes how to extract consistent values based on the lab conditions; bistable polarization rotation (BPR) is then modeled and quantified, introducing a single quantity to capture the extent of bistable polarization rotation.

Chapter 3 describes the experimental set-up and demonstration of bistable hysteresis curves for the state-of-polarization (SOP); the dependence of BPR on the input-power imbalance into polarization modes and on small-signal birefringence is investigated.

Chapter 4 describes a model that elucidates the physical process of this new hysteresis-transformation technique, where it shows how a linear polarizer can act on the polarization-rotating bistable signal to yield CCW, CW, and butterfly hysteresis shapes; crucial to modeling this transformation is a generalized Malus’ law accounting for elliptical input polarization, insertion loss, and a finite extinction ratio. Also, this chapter describes the optimization process to enhance a hysteresis shape contrast by using a polarization controller before the polarizer.

Chapter 5 describes lab demonstration of the hysteresis-transformation technique, where it describes the experimental setup. Also, it discusses the process of achieving optical bistability and polarization-mode-resolved hysteresis shapes, and the demonstrated diversity of hysteresis shapes.

Chapter 6 offers concluding remarks of the dissertation research and discusses the main contributions and future work.
2. Bistable Polarization Rotation: Theory

2.1 Introduction

Optical bistability exhibited by optical power has been widely studied in a variety of photonic devices and structures [11]–[22], [57]–[65]. One kind of optical bistability is known as dispersive optical bistability as it forms based on an interferometric resonance interaction with an optical signal. Dispersive optical bistability has been studied in a variety of photonic resonators, including vertical cavities [12], dual-bus ring resonators [13], photonic-crystal nanocavities [14], diffraction gratings [15], and Fabry-Pérot resonators [11], [16], [17]. Dispersive optical bistability in a nonlinear resonator occurs as an interferometric resonance shift onto and off an optical-signal wavelength when the input power changes into the nonlinear bistable system [19]; this shift is due to nonlinear refractive index dependence on the input optical power. The nonlinear refractive index can occur due to a variety of mechanisms, including thermal-optic effects [13], two-photon absorption [14], the Kerr effect [19], and the carrier dynamics characterized by the linewidth enhancement factor [12], [11], [15]–[17]. Demonstrations of traditional dispersive optical bistability focus on the bistable hysteresis curves of optical power [11]–[17].

Nonlinear polarization rotation (NPR) has been previously modeled in SOAs, where these models introduced factors, including anisotropic gain [27]–[29], anisotropic loss
[27], [29], and anisotropic single-pass phase [27]–[29]. An anisotropic single-pass phase occurs due to both small-signal (i.e., intrinsic) and optical-power-dependent (i.e., optically induced) birefringence [32]. NPR has been used as the basis of optical applications, such that an AOFF is build based on using two TW-SOAs coupled by a polarization beam splitter [30], [31]. However, these models only considered TW-SOA and avoided Fabry-Pérot resonator physics. Fabry-Pérot resonator physics is required to directly apply them to study simultaneous dispersive optical bistability in FP-SOAs; such Fabry-Pérot physics introduces the interferometric resonances that are an integral part of dispersive optical bistability.

Previous models of dispersive optical bistability in FP-SOAs considered optical signals injected into only a single polarization mode and avoided anisotropic resonator quantities. Anisotropic resonator quantities are required to study how the polarization modes are different in optical power strength and phase. These quantities are the key to study simultaneous NPR. Steady-state models are common for the study of optical bistability in FP-SOAs, starting with the earliest published model [21]. Earlier work considered steady-state models to study the optical bistability in FP-SOAs, where steady-state models are a simplified approach. Some important advancements follow the earliest published model [21]. These advancements considered the internal power driving bistability [11], and included the gain dynamics [22]. Additionally, dual input injections into the FP-SOA model was studied [23]; this modeling allowing for different amounts of spectral detuning between each optical-signal wavelength and a Fabry-Pérot resonance. However, this modeling approach did not introduce the anisotropic resonator quantities and did not study light propagation into the TE and TM polarization modes.

While optical bistability and NPR are both present in previous studies, its model lacks the interferometric resonance effects required to study dispersive optical bistability in nonlinear resonators. Such resonances are critical to take into account because such resonances need to study the effect of an imbalance in both the optical power and phase portions of the two polarization modes. Thus, dispersive optical bistability and nonlinear polarization
rotation processes have only been investigated separately as two isolated nonlinear optical phenomena [11]–[29].

In this chapter, we investigate the simultaneous occurrence of the two nonlinear phenomena—nonlinear polarization rotation (NPR) and dispersive optical bistability. We develop a mathematical model to study the bistable hysteresis curves of the Stokes parameters and establish the relevant scaling parameters for bistable action as described in the following sections.

2.2 Optical Signal into Fabry-Pérot SOA

The Fabry-Pérot semiconductor optical amplifier (FP-SOA) is an optoelectronic device that is used to amplify an optical signal passing through the active region of the device [10]. The radiation outcome of the SOA can result in spontaneous emission, stimulated emission, and stimulated absorption [10].

In addition to using FP-SOA as an amplifier, FP-SOA can be used to realize optical nonlinear processing [10], [11]. Optical nonlinear phenomena in SOAs including FP-SOA can be triggered with extremely low power, such as -30 dBm (1 $\mu$W) [11], where this nonlinear phenomenon is a key to operate the functionality of optical signal processing applications. An FP-SOA is depicted in Fig. 2.1, possessing an active-region width $w$ along the x-axis, thickness $d$ along the y-axis, and length $L$ along the z-axis.

![Figure 2.1 Schematic of a Fabry-Pérot SOA.](image)

An optical signal propagates down the device in the form of orthogonal TE and TM modes aligned with the x and y axes, respectively. The FP-SOA is anisotropic — the TE and TM modes will, in general, experience different changes in optical power (i.e.,
gain anisotropy and loss anisotropy) and phase accumulation (i.e, linear and nonlinear birefringence).

The electric field at the input \( z = 0 \) to the FP-SOA is given by

\[
E_{\text{in}} = E_{\text{in}}^{\text{TE}} \hat{x} + E_{\text{in}}^{\text{TM}} \hat{y},
\]

(2.1a)

\[
E_{\text{in}}^{\text{TE}} = \sqrt{P_{\text{in}}^{\text{TE}}} \exp(i\Psi_{\text{in}}^{\text{TE}}) \exp(-i\omega t),
\]

(2.1b)

\[
E_{\text{in}}^{\text{TM}} = \sqrt{P_{\text{in}}^{\text{TM}}} \exp(i\Psi_{\text{in}}^{\text{TM}}) \exp(-i\omega t),
\]

(2.1c)

where \( \omega \) is the angular frequency of the carrier wave, \( \Psi_{\text{in}} \) is the input optical phase, and \( P_{\text{in}} \) is the input optical power. The \( \text{TE} \) and \( \text{TM} \) superscripts are used to associate a quantity with a specific polarization mode.

In this model, total power is the sum of the power in each polarization mode for fully polarized assumption. Therefore, the optical power injected into each polarization mode is given by:

\[
P_{\text{in}}^{\text{TE}} = \frac{1}{2} (1 + s_{1\text{in}}) P_{\text{in}},
\]

(2.2a)

\[
P_{\text{in}}^{\text{TM}} = \frac{1}{2} (1 - s_{1\text{in}}) P_{\text{in}},
\]

(2.2b)

\[
P_{\text{in}} = P_{\text{in}}^{\text{TE}} + P_{\text{in}}^{\text{TM}},
\]

(2.2c)

where the normalized Stokes parameter \( s_{1\text{in}} \) is defined as

\[
s_{1\text{in}} = \frac{P_{\text{in}}^{\text{TE}} - P_{\text{in}}^{\text{TM}}}{P_{\text{in}}^{\text{TE}} + P_{\text{in}}^{\text{TM}}} = \frac{P_{\text{in}}^{\text{TE}} - P_{\text{in}}^{\text{TM}}}{P_{\text{in}}}. \quad (2.3)
\]

The Stokes parameter \( s_{1\text{in}} \) expresses the imbalance of the input power into each polarization mode. \( s_{1\text{in}} = 0 \) when the power is equally split between the modes, and \( s_{1\text{in}} = \pm 1 \) when the power is solely launched into the TE and TM mode, respectively.
To observe and study the bistable hysteresis behavior, the input power $P_{in}$ is varied in sinusoidal fashion with respect to time into the nonlinear photonic resonator. A sinusoidal fashion is chosen instead of a square-wave input because the input power should avoid any sudden high-power increment in input power (e.g., via using a square-wave) to unambiguously trace out the hysteresis behavior of the output power. The input power $P_{in}$ is modulated by a sinusoidal wave defined as:

$$P_{in} = \frac{P_0}{2} \left[ 1 - \cos\left(\frac{2\pi t}{T}\right) \right],$$

(2.4)

where $P_0$ is the peak input power and $T$ is the modulation period.

### 2.3 Semiconductor Optical Amplifier (SOA) Carrier Density

The carrier density $N$ is saturated by the internal optical power of each mode $P^{TE}$ and $P^{TM}$ within the SOA, and its rate equation is given by:

$$\frac{dN}{dt} = \frac{J}{qd} - \frac{N}{\tau} \frac{a^{TE} \Gamma^{TE}}{h f w d} (N - N_T) P^{TE} - \frac{a^{TM} \Gamma^{TM}}{h f w d} (N - N_T) P^{TM},$$

(2.5)

where $J$ is the current density, $q$ is the electron charge, $\tau$ is the carrier recovery time, $\Gamma$ is the optical confinement factor, $a = dg/dN$ is the differential gain with $g$ being the modal gain coefficient, $N_T$ is the carrier density at transparency, $h f$ is the photon energy, $h$ is Planck’s constant, and $f$ is the optical frequency. Both the carrier density and optical power each depend on the longitudinal coordinate $z$. In formulating Eq. (2.5), we have taken a rate equation for $N$ [18], and expanded the power-dependent term to include both polarization modes and assumed that each mode interacts with the same pool of charge carriers. We have also neglected fast carrier dynamics on the order of 1 ps and shorter, transverse carrier
diffusion, and amplified spontaneous emission (ASE) [11], [12], [15], [16], [18], [21]–[23], [28], [66]. Fast carrier dynamics dissipate because the modulation period used in our experiment (50 µs) is much longer than the intraband relaxation time (~0.05 ps) [18]. A transverse carrier diffusion term is neglected in Eq. (2.5) because the diffusion length is generally longer than the width and height of the gain region, thereby resulting in a nearly constant carrier-density profile [18]. ASE is neglected in Eq. (2.5) because its power (< -50 dBm) in the spectral region near the optical signal is over 40 dB weaker than the bistable transition powers.

The carrier rate equation (2.5) can be simplified as

\[
\frac{dN}{dt} = \frac{N_0 - N}{\tau} - N - N_T \left( \frac{1}{\eta P_{TE} + P_{TM}} \right),
\]

where \( N_0 = \frac{J \tau}{qd} \) is the small-signal carrier density and \( N_T \) is the carrier density at transparency. Normalized optical powers \( P \) in Eq. (2.6) and throughout this paper are normalized by the TM mode saturation power \( P_{sat}^{TM} \):

\[
P_{TE} = \frac{P_{TE}}{P_{sat}^{TM}}, \quad P_{TM} = \frac{P_{TM}}{P_{sat}^{TM}}.
\]

The saturation power \( P_{sat} \) of each polarization mode is defined as:

\[
P_{sat}^{TE} = \frac{U_{sat}^{TE}}{\tau} = \frac{hfwd}{\tau a^{TE} \Gamma^{TE}}, \quad (2.8a)
\]

\[
P_{sat}^{TM} = \frac{U_{sat}^{TM}}{\tau} = \frac{hfwd}{\tau a^{TM} \Gamma^{TM}}, \quad (2.8b)
\]

where \( U_{sat} \) is the saturation energy. The quantity \( \eta \) found in Eq. (2.6) is defined as the ratio of the saturation powers:

\[
\eta = \frac{P_{sat}^{TE}}{P_{sat}^{TM}} = \frac{a^{TM} \Gamma^{TM}}{a^{TE} \Gamma^{TE}}.
\]

The quantity \( \eta \) governs gain anisotropy in our model. In SOAs, where \( \Gamma^{TE} \) is typically
larger than \( \Gamma^{TM} \), gain isotropy can be achieved by engineering \( a^{TM} \) to be larger than \( a^{TE} \) [27]. Despite these efforts, \( \eta \) typically exhibits a wavelength dependence, with longer wavelengths favoring the TM gain for bulk semiconductors [67].

### 2.4 Gain Anisotropy

The TE and TM modes each experience their own modal gain coefficient \( g \), defined as

\[
g^{TE} = a^{TE} \Gamma^{TE} (N - N_T), \\
g^{TM} = a^{TM} \Gamma^{TM} (N - N_T),
\]  \hspace{1cm} (2.10a, b)

where the gain can thus be different for each mode even for a common carrier density \( N \) [27]. Anisotropy in the gain coefficient is expressed by combining Eqs. (2.9) and (2.10) to yield

\[
g^{TM} = \eta g^{TE}.
\]  \hspace{1cm} (2.11)

\( \eta \) is thus a measure of gain anisotropy.

A rate equation for each gain coefficient can be derived by differentiating Eqs. (2.10a) and (2.10b) with respect to time and then substituting in the carrier density rate equation (2.6). Doing so yields:

\[
\tau \frac{dg^{TE}}{dt} = g^{TE}_0 - g^{TE} (1 + \frac{1}{\eta} P^{TE} + P^{TM}) = \frac{\tau dg^{TM}}{\eta dt}, \hspace{1cm} (2.12a)
\]

\[
\tau \frac{dg^{TM}}{dt} = g^{TM}_0 - g^{TM} (1 + \frac{1}{\eta} P^{TE} + P^{TM}) = \eta \tau \frac{dg^{TE}}{dt}, \hspace{1cm} (2.12b)
\]
where the small-signal gain coefficients are defined as

\[ g_{0}^{TE} = a^{TE} \Gamma^{TE} (N_0 - N_T) = \frac{1}{\eta} g_{0}^{TM} , \]  

\[ g_{0}^{TM} = a^{TM} \Gamma^{TM} (N_0 - N_T) = \eta g_{0}^{TE} . \]  

Eqs. (2.12) show that an increase in optical power saturates the SOA gain. The amount of gain saturation \( g_p \) can be expressed as the drop of the gain coefficient \( g \) from its steady-state value \( g_0 \):

\[ g_{p}^{TE} = g_{0}^{TE} - g^{TE} = \frac{1}{\eta} g_{p}^{TM} , \]  

\[ g_{p}^{TM} = g_{0}^{TM} - g^{TM} = \eta g_{p}^{TE} . \]

### 2.5 Single-Pass Gain

As the optical signal travels along one pass of the SOA (from \( z = 0 \) to \( L \)), the change in its power is governed by the single-pass gain \( G \), given by:

\[ G^{TE} = \exp \left[ \int_{0}^{L} (g^{TE} - \alpha_{int}^{TE}) \, dz \right] , \]  

\[ G^{TM} = \exp \left[ \int_{0}^{L} (g^{TM} - \alpha_{int}^{TM}) \, dz \right] , \]  

where \( \alpha_{int} \) is the internal loss. In formulating Eqs. (2.15), we have expanded a polarization-independent form from Ref. [66] to account for the two polarization modes. Without loss of generality, these gain expressions can be recast in terms of average quantities over the length \( L \) of the SOA, where the average (for an arbitrary quantity \( b \)) is defined as
\[ \bar{b} = \frac{1}{L} \int_0^L b(z) dz. \] Doing so yields:

\[ G^{TE} = \exp \left[ \left( \frac{1}{\eta} \bar{g}^{TM} - \frac{1}{\gamma} \bar{\alpha}^{TE}_{\text{int}} \right) L \right], \quad (2.16a) \]

\[ G^{TM} = \exp \left[ \left( \eta \bar{g}^{TE} - \gamma \bar{\alpha}^{TM}_{\text{int}} \right) L \right], \quad (2.16b) \]

where the loss anisotropy \( \gamma \) is defined as

\[ \gamma = \frac{\bar{\alpha}^{TM}_{\text{int}}}{\bar{\alpha}^{TE}_{\text{int}}}. \quad (2.17) \]

Eqs. (2.16) show how the gain and loss anisotropy factors \( \eta \) and \( \gamma \), respectively, drive the single-pass gain \( G \) away from its isotropic state determined by \( \bar{g}^{TM} = \bar{\alpha}^{TM}_{\text{int}} \eta / \gamma (\gamma - 1) / (\eta - 1) \); this expression is found by setting Eq. (2.16a) equal to 2.16b and solving for \( \bar{g}^{TM} \).

Since the single-pass gain Eqs. (2.16) depend on spatially averaged quantities, similarly averaged rate equations are required. Averaging Eqs. (2.12) in this way yields

\[ \tau \frac{d\bar{g}^{TE}}{dt} = \bar{g}^{TE}_0 - \bar{g}^{TE} (1 + \frac{1}{\eta} \bar{P}^{TE} + \bar{P}^{TM}) = \eta \frac{d\bar{g}^{TM}}{dt}, \quad (2.18a) \]

\[ \tau \frac{d\bar{g}^{TM}}{dt} = \bar{g}^{TM}_0 - \bar{g}^{TM} (1 + \frac{1}{\eta} \bar{P}^{TE} + \bar{P}^{TM}) = \eta \frac{d\bar{g}^{TE}}{dt}, \quad (2.18b) \]

where we have invoked the mean-field approximation to replace the average of the product of the gain coefficient and power with the product of the average power and average gain coefficient [18].
2.6 Birefringence

Similar to the modal gain coefficient, the modal refractive index $n$ depends on the carrier density and is polarization dependent [28]:

\[
\begin{align*}
n^{TE} &= n^{TE}_T + \Gamma^{TE} \frac{dn^{TE}}{dT}(N - N_T), \\
n^{TM} &= n^{TM}_T + \Gamma^{TM} \frac{dn^{TM}}{dT}(N - N_T),
\end{align*}
\]  

(2.19a)

(2.19b)

where $n_T$ is the modal index at transparency. A change in the refractive index is related to a change in gain by the linewidth enhancement factor $\alpha$ [11], which is defined for the two polarization modes as:

\[
\begin{align*}
\alpha^{TE} &= -\frac{4\pi}{\lambda} \frac{dn^{TE}}{dN} / a^{TE}, \\
\alpha^{TM} &= -\frac{4\pi}{\lambda} \frac{dn^{TM}}{dN} / a^{TM},
\end{align*}
\]  

(2.20)

where $\lambda$ is the free-space wavelength of the optical signal.

Using Eqs. (2.10), (2.19), and (2.20), the modal refractive indices can be written in terms of the gain coefficients as:

\[
\begin{align*}
n^{TE} &= n^{TE}_T - \frac{\lambda}{4\pi} \alpha^{TE} g^{TE} = n^{TE}_T - \frac{\lambda}{\epsilon \eta} \frac{4\pi}{\lambda} \alpha^{TM} g^{TM}, \\
n^{TM} &= n^{TM}_T - \frac{\lambda}{4\pi} \alpha^{TM} g^{TM} = n^{TM}_T - \epsilon \eta \frac{\lambda}{4\pi} \alpha^{TE} g^{TE},
\end{align*}
\]  

(2.21a)

(2.21b)

where the nonlinearity anisotropy $\epsilon$ is defined as:

\[
\epsilon = \frac{\alpha^{TM}}{\alpha^{TE}}.
\]  

(2.22)

Recent measurements show this quantity can be as small as 0.68 [34]. Eqs. (2.21) show that gain-dependent birefringence occurs only when the product of anisotropies $\epsilon \eta$ differs from unity.
2.7 Single-Pass Phase

As the optical signal travels along one pass of an SOA, the change in its phase is expressed by the single-pass phase shift $\phi$. This quantity is defined for each polarization mode as follows:

$$
\phi_{TE} = \int_0^L \frac{2\pi n_{TE}}{\lambda} \, dz, \quad \phi_{TM} = \int_0^L \frac{2\pi n_{TM}}{\lambda} \, dz. \quad (2.23)
$$

Leveraging the modal-index Eqs. (2.21), the single-pass phase expressions can be rewritten in terms of the averaged gain coefficients as:

$$
\phi_{TE} = \phi_{TE}^T - \frac{1}{2} \alpha_{TE} \frac{g_{TE}}{g} L = \phi_{TE}^T - \frac{1}{2} \varepsilon \eta \alpha_{TM} \frac{g_{TM}}{g} L \quad (2.24a)
$$

$$
\phi_{TM} = \phi_{TM}^T - \frac{1}{2} \alpha_{TM} \frac{g_{TM}}{g} L = \phi_{TM}^T - \frac{1}{2} \varepsilon \eta \alpha_{TE} \frac{g_{TE}}{g} L, \quad (2.24b)
$$

where the phase terms at transparency are given by:

$$
\phi_{TE}^T = \frac{2\pi n_{TE}^T}{\lambda} L, \quad \phi_{TM}^T = \frac{2\pi n_{TM}^T}{\lambda} L. \quad (2.25)
$$

As with the gain coefficient, the single-pass phase shift $\phi$ can be thought of as the sum of a small-signal portion $\phi_0$ (an intrinsic birefringence) and a power-dependent portion $\phi_p$:

$$
\phi_{TE} = \phi_{TE}^0 + \phi_{TE}^p, \quad \phi_{TM} = \phi_{TM}^0 + \phi_{TM}^p. \quad (2.26)
$$

The small-signal phase shifts are defined as:

$$
\phi_{TE}^0 = \phi_{TE}^T - \frac{1}{2} \alpha_{TE} \frac{g_{TE}^0}{g_0} L = \phi_{TE}^T - \frac{1}{2} \varepsilon \eta \alpha_{TM} \frac{g_{TM}^0}{g_0} L, \quad (2.27a)
$$

$$
\phi_{TM}^0 = \phi_{TM}^T - \frac{1}{2} \alpha_{TM} \frac{g_{TM}^0}{g_0} L = \phi_{TM}^T - \frac{1}{2} \varepsilon \eta \alpha_{TE} \frac{g_{TE}^0}{g_0} L, \quad (2.27b)
$$
and the power-induced phase shifts are defined as:

\[ \phi_{pE}^T = \frac{1}{2} \alpha_{pE} T^E \alpha_{TE} T^E g_{pE} T^E L = \frac{1}{2} \epsilon \eta \alpha_{TE} T^E g_{pE} T^E L, \]  

(2.28a)

\[ \phi_{pM}^T = \frac{1}{2} \alpha_{pM} T^M \alpha_{TE} T^E g_{pM} T^M L = \frac{1}{2} \epsilon \eta \alpha_{TE} T^E g_{pM} T^E L. \]  

(2.28b)

The single-pass phase-shift difference \( \Delta \phi \) between the two polarization modes is an expression of the birefringence of the SOA, and is defined as:

\[ \Delta \phi = \phi_{TM} - \phi_{TE} = \Delta \phi_0 + \Delta \phi_p, \]  

(2.29)

where the small-signal birefringence \( \Delta \phi_0 \) is given by

\[
\Delta \phi_0 = \phi_{0M}^T - \phi_{0E}^T = \Delta \phi_T - \frac{\alpha_{TE} T^E g_{0E} T^E L}{2} (\eta \epsilon - 1),
\]  

(2.30a)

\[
= \Delta \phi_T - \frac{\alpha_{TM} T^M g_{0M} T^M L}{2} \left( 1 - \frac{1}{\eta \epsilon} \right),
\]  

(2.30b)

where the small-signal birefringence at transparency is given by:

\[ \Delta \phi_T = \phi_{TM}^T - \phi_{TE}^T, \]  

(2.31)

and the power-induced birefringence \( \Delta \phi_p \) is given by:

\[ \Delta \phi_p = \phi_{pM}^T - \phi_{pE}^T = \frac{\alpha_{TE} T^E g_{pE} T^E L}{2} (\eta \epsilon - 1). \]  

(2.32a)

\[ = \frac{\alpha_{TM} T^M g_{pM} T^M L}{2} \left( 1 - \frac{1}{\eta \epsilon} \right). \]  

(2.32b)

The product \( \epsilon \eta \) of gain anisotropy and nonlinearity anisotropy plays an important role. According to Eq. (2.30), this product must differ from unity in order for the small-signal birefringence \( \Delta \phi_0 \) to depend on the small-signal gain \( g_0 \). According to Eq. (2.32), the
product must differ from unity in order for any amount of power-induced birefringence $\Delta \phi_p$ to occur.

## 2.8 Fabry-Pérot Transmittivity & Output Power

The total output power $S_0$ is given by

$$
S_0 = P_{out}^{TE} + P_{out}^{TM} + P_{un},
$$

(2.33)

where $P_{un}$ is the power of unpolarized light. We ignore ASE in our model and take $P_{un} = 0$ such that the total output power is the sum of the power of the polarization modes.

As the optical signal traverses the FP-SOA, we assume that the resonator responds instantaneously to changes in gain and modal index. This adiabatic approximation is valid for single-pass transit times much shorter than the SOA recovery time $\tau$ and the modulation period $T$ [18]. Under this assumption, the output optical power $P_{out}$ of each polarization mode is related to the input power $P_{in}$ by a steady-state transmittivity $T$:

$$
P_{out}^{TE} = P_{in}^{TE} T^{TE},
$$

(2.34a)

$$
P_{out}^{TM} = P_{in}^{TM} T^{TM},
$$

(2.34b)

where the transmittivities are given by:

$$
T^{TE} = \frac{(1 - R_1)(1 - R_2)G^{TE}}{(1 - G^{TE}R)^2 + 4G^{TE}R \sin^2(\phi^{TM} - \Delta\phi_0 - \Delta\phi_p)},
$$

(2.35a)

$$
T^{TM} = \frac{(1 - R_1)(1 - R_2)G^{TM}}{(1 - G^{TM}R)^2 + 4G^{TM}R \sin^2(\phi^{TM})},
$$

(2.35b)

In these expressions, $R_1$ and $R_2$ are the reflectivity of the first and second facet, respectively, and $R = \sqrt{R_1R_2}$. These expressions are based on an isotropic FP-SOA model [11], adapted
here to include gain anisotropy and birefringence.

The Fabry-Pérot resonances expressed by these transmittivities are in general different for the TE and TM modes, and this difference significantly impacts the output SOP. The two sets of resonances are aligned when the birefringence $\Delta \phi = \Delta \phi_0 + \Delta \phi_p = 0$, an example of which is shown in Fig. 2.2. Both the small-signal birefringence $\Delta \phi_0$ and power-induced birefringence $\Delta \phi_p$ may independently misalign the two sets of resonances.

![Figure 2.2 Simulated small-signal transmittivity spectra for the TE and TM modes, indicating the small-signal birefringence $\Delta \phi_0$, optical signal $S$, and TM-mode single-pass phase $\phi_{TM}^0$. Other parameter values are defined in Table 4.1.](image)

Dispersive optical bistability in a nonlinear resonator occurs as an interferometric resonances shifts onto (high-power hysteresis branch) and off of (low-power branch) the optical signal. Bistability in an *anisotropic* nonlinear resonator can be accompanied by polarization rotation because the product $\varepsilon \eta$ causes the TE and TM resonances to shift by a different amount [via Eqs. (2.32)], and because the balance of $\eta$ and $\gamma$ set the relative strength of TE and TM resonances [via Eqs. (2.16)].
2.9 Internal Optical Power

The average internal optical powers that drive gain saturation in Eqs. (2.18), and therefore drive dispersive optical bistability, are related to the output powers by:

\[ P_{TE}^{T} = H_{TE}^{T} P_{out}^{TE}, \]  
(2.36a)

\[ P_{TM}^{T} = H_{TM}^{T} P_{out}^{TE}, \]  
(2.36b)

where the \( H \) factors are given by:

\[ H_{TE}^{T} = \frac{(R_{2}G_{TE}^{T} + 1)(G_{TE}^{T} - 1)}{(1 - R_{2})G_{TE}^{T}(\xi_{TM}L - \alpha_{TM}^{T}L)}, \]  
(2.37a)

\[ H_{TM}^{T} = \frac{(R_{2}G_{TM}^{T} + 1)(G_{TM}^{T} - 1)}{(1 - R_{2})G_{TM}^{T}(\xi_{TM}L - \alpha_{TM}^{T}L)}. \]  
(2.37b)

These steady-state equations have been adapted from a bistable isotropic FP-SOA model (Ref. [11]) to include gain and internal-loss anisotropy.

2.10 Phase Transfer Function & Output Phase

The output phase \( \Psi_{out} \) of each polarization mode is related to the input phase \( \Psi_{in} \) by a phase transfer function \( \Phi_{T} \):

\[ \psi_{out}^{TE} = \Psi_{in}^{TE} + \Phi_{TE}, \]  
(2.38a)

\[ \psi_{out}^{TM} = \Psi_{in}^{TM} + \Phi_{TM}, \]  
(2.38b)
where the phase transfer functions are given by

\[
\Phi^{TE} = \phi^{TE} + \arctan(u) = \phi^{TM} - \Delta\phi + \arctan(u),
\]

\[
\Phi^{TM} = \phi^{TM} + \arctan(v) = \phi^{TE} + \Delta\phi + \arctan(v).
\]

(2.39a)

(2.39b)

Whereas the single-pass phase terms are based on modal index and gain, the \(u\) and \(v\) quantities are due to interference effects within the Fabry-Pérot cavity and are given by:

\[
u = \frac{RG^{TM} \sin(2\phi^{TM})}{1 - RG^{TM} \cos(2\phi^{TM})}.
\]

(2.40a)

\[
v = \frac{RG^{TM} \sin(2\phi^{TM})}{1 - RG^{TM} \cos(2\phi^{TM})}.
\]

(2.40b)

A similar expression has been used for an isotropic FP-SOA [11]; we have adapted it here to include anisotropy.

The output SOP is partially determined by the output phase difference \(\Delta\Psi_{out}\), defined as:

\[
\Delta\Psi_{out} = \Psi_{out}^{TM} - \Psi_{out}^{TE},
\]

(2.41a)

\[
\Delta\Psi_{out} = \Delta\Psi_{in} + \Delta\phi_0 + \Delta\phi_p + \arctan \left[\frac{v - u}{1 + uv}\right],
\]

(2.41b)

where the input phase difference \(\Delta\Psi_{in} = \Psi_{in}^{TM} - \Psi_{in}^{TE}\), and Eq. (2.41b) was obtained by combining Eqs. (2.38) and (2.39).

The output phase difference from an FP-SOA depends not only on single-pass phase difference \(\Delta\phi\), but also on the Fabry-Pérot structure via the \(u\) and \(v\) terms. These structural terms are yet another way in which NPR through an FP-SOA differs from NPR through a TW-SOA.
CHAPTER 2. BISTABLE POLARIZATION ROTATION: THEORY

2.11 Output State of Polarization (SOP)

The output SOP is determined by the relative phase difference $\Delta \Psi_{out}$ and the relative strength of the power of each polarization mode. The Stokes parameters provide an unambiguous description of the SOP [68]:

\begin{align}
S_1 &= P_{out}^{TE} - P_{out}^{TM} \\
S_2 &= 2\sqrt{P_{out}^{TE}P_{out}^{TM}} \cos (\Delta \Psi_{out}), \\
S_3 &= -2\sqrt{P_{out}^{TE}P_{out}^{TM}} \sin (\Delta \Psi_{out}).
\end{align}

The negative sign in front of the third Stokes parameter is required to represent right-handed-circular polarization by $S_3 = 1$ for the sign convention of the electric field shown in Eq. (2.1) [68]; doing so matches the $S_3$ convention of the polarimeter used in the lab results below.

The values of the Stokes parameters can be conveniently forced to fall between $\pm 1$ by normalizing them by polarized power $P_{pol} = P^{TE} + P^{TM}$; this normalization also places the tip of the Stokes vector at the surface of a unity Poincaré sphere. Using Eqs. (2.2), (2.34), and (2.42), the Stokes parameters become:

\begin{align}
s_1 &= \frac{S_1}{P_{pol}} = \frac{(1 + s_{1in})T^{TE} - (1 - s_{1in})T^{TM}}{(1 + s_{1in})T^{TE} + (1 - s_{1in})T^{TM}}, \\
s_2 &= \frac{S_2}{P_{pol}} = 2\sqrt{(1 + s_{1in})(1 - s_{1in})}T^{TE}T^{TM} \cos (\Delta \Psi_{out}), \\
s_3 &= \frac{S_3}{P_{pol}} = -2\sqrt{(1 + s_{1in})(1 - s_{1in})}T^{TE}T^{TM} \sin (\Delta \Psi_{out}).
\end{align}

These expressions show that launching the optical signal solely into the TE mode ($s_{1in} = 1$) results in fully TE-polarized output light $(s_1, s_2, s_3) = (1, 0, 0)$ irrespective of the relative strength of the transmittivities $T^{TE}$ and $T^{TM}$ or the amount of phase difference.
\[ \Delta \Psi_{out} \]. Similarly, launching the optical signal solely into the TM mode \((s_{1in} = -1)\) results in fully-TM-polarized output light \((s_1, s_2, s_3) = (-1, 0, 0)\). Launching the optical signal into both polarization modes \((s_{1in} \neq \pm 1)\) leads to polarization rotation due to birefringence or anisotropy in the transmittivity. This polarization rotation can occur even during dispersive optical bistability, as shown in the following section.

### 2.12 Executing the Mathematical Model

BPR can be modeled by applying a numerical ordinary differential equation (ODE) solver to one of the two nonlinear equations, either Eq. (2.18a) or (2.18b) for \( g^{TE} \) or \( g^{TM} \), respectively. Although either equation may be used, we choose the latter because the TM-mode lasing threshold is less than that of the TE mode (as discussed below). The equations can be further simplified by introducing the normalized time \( t' = t / \tau \), where \( \tau \) is the SOA recovery time. Doing so converts Eq. (2.18b) to:

\[
\frac{d g^{TM}}{dt'} = \frac{g^{TM}}{g^{TM}_0} - g^{TM}(1 + \frac{1}{\eta} P^{TE} + P^{TM}),
\]

(2.44)

where the right-hand side is the derivative function used in the ODE solver. The internal powers \( P^{TE} \) and \( P^{TM} \) can be expressed in terms of the input power \( P_{in} \) by applying Eqs. (2.2), (2.4), (2.34) and (2.36), resulting in:

\[
P^{TE} = H^{TE} T^{TE} \frac{1}{2} (1 + s_{1in}) P_{in},
\]

(2.45a)

\[
P^{TM} = H^{TM} T^{TM} \frac{1}{2} (1 - s_{1in}) P_{in},
\]

(2.45b)

\[
P_{in} = \frac{P_0}{2} (1 - \cos(2\pi t' / T')),
\]

(2.45c)

where the normalized modulation period \( T' = T / \tau \).

The transmittivities and \( H \) factors are updated each computational step with new values.
of $g^{TM}$ using Eqs. (2.11), (2.16), (2.17), (2.27b), (2.30b), (2.32b), (2.35), and (2.37). Once $g^{TM}$ is solved for all normalized time $t'$, its array of values is then used to calculate other time-dependent quantities — most notably, the output powers, phases, and Stokes parameters — without an ODE solver. All optical powers are normalized by the saturation power of the TM mode $P_{sat}^{TM}$ as defined early in this chapter. The $P_{sat}^{TM}$ is chosen because TM mode is the dominant mode in the FP-SOA. In this chapter the normalized input power $P_{in}$ stops at 3 because the peak power 2.45$c$ is chosen to be 3 to be consistent with lab conditions. The normalized input power can be bigger than one which indicates that the input power strength is higher than the saturation power $P_{sat}^{TM}$. In chapter 4, the normalized input power $P_{in}$ stops at 1 because the peak power 2.45$c$ is chosen to be 1 (to be consistent with lab conditions). When the normalized input power equals 1 it means $P_{in} = P_{in} / P_{sat}^{TM} = 1$, which indicates the input power is the same as the saturation power $P_{sat}^{TM}$.

Table 2.1

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>normalized modulation period</td>
<td>$T'$</td>
<td>$10^5$</td>
</tr>
<tr>
<td>input-power imbalance</td>
<td>$s_{1in}$</td>
<td>0</td>
</tr>
<tr>
<td>input phase</td>
<td>$\Psi_{in}^{TM}$</td>
<td>0</td>
</tr>
<tr>
<td>input phase difference</td>
<td>$\Delta \Psi_{in}$</td>
<td>$\pi$</td>
</tr>
<tr>
<td>small-signal, single-pass phase</td>
<td>$\phi_0^{TM}$</td>
<td>-0.63$\pi$</td>
</tr>
<tr>
<td>small-signal birefringence</td>
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</tr>
<tr>
<td>FP facet reflectivities</td>
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<tr>
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<tr>
<td>gain anisotropy</td>
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<tr>
<td>normalized averaged loss</td>
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<td>loss anisotropy</td>
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</tr>
<tr>
<td>nonlinearity anisotropy</td>
<td>$\epsilon$</td>
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</tr>
</tbody>
</table>

Quantity values used in simulations shown in Figs. 2.2–2.4 are listed in Table 4.1. Values for $T'$, $s_{1in}$, $\Psi_{in}^{TM}$, $\Delta \Psi_{in}$, $\phi_0^{TM}$, and $\Delta \phi_0$ are extracted based on the lab the conditions discussed in the following section. Note that the normalized modulation period satisfies the mathematical-model condition $T' \gg 1$. 

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The values for $\bar{g}^{TM}_0L$, $\eta$, $\alpha^{TM}_{int}L$, $\gamma$, and $R$ are selected to be consistent with the lasing threshold and strength of Fabry-Pérot resonances as measured in our experiments. The strength of resonances as characterized by $x$, the ratio of the peak and the valley powers of a measured ASE spectrum, is used in the Hakki-Paoli equations [69], applied here to both TE and TM ASE spectra:

$$G^{TE}R = \frac{\sqrt{x^{TE} - 1}}{\sqrt{x^{TE} + 1}}$$

$$G^{TM}R = \frac{\sqrt{x^{TM} - 1}}{\sqrt{x^{TM} + 1}}, \quad (2.46)$$

where the reflectivity $R = \sqrt{R_1R_2}$. We use these expressions to determine the $GR$ product based on the polarization-resolved experimental ASE spectra shown in Fig. 3.2(b).

The $GR$ product can be expanded using Eqs. (2.16) to yield:

$$\ln(G^{TE}R) = g^{TE}_0L - \alpha^{TE}_{int}L + \ln(R), \quad (2.47a)$$

$$\ln(G^{TM}R) = g^{TM}_0L - \alpha^{TM}_{int}L + \ln(R). \quad (2.47b)$$

Further simplification can be made by introducing expressions for the lasing threshold of a Fabry-Pérot laser:

$$\bar{g}^{TM}_{th}L = \alpha^{TM}_{int}L - \ln(R). \quad (2.48)$$

Here, we focus on the TM mode because it exhibits the lowest lasing threshold in our experiments. Eqs. (2.46), (2.47b), and (2.48) combine to yield:

$$\bar{g}^{TM}_0L = \frac{p}{p - 1} \ln \left\{ \frac{\sqrt{x^{TM} - 1}}{\sqrt{x^{TM} + 1}} \right\}, \quad (2.49a)$$

$$\alpha^{TM}_{int}L = \frac{1}{p - 1} \ln \left\{ \frac{\sqrt{x^{TM} - 1}}{\sqrt{x^{TM} + 1}} \right\} + \ln R, \quad (2.49b)$$

where $p = \bar{g}^{TM}_0L/\bar{g}^{TM}_{th}L$; a value of $p = 0.91$ is chosen to be consistent with the ratio of the experimentally measured drive current and lasing-threshold current. Eqs. (2.49) map a
measurement of the polarization-resolved ASE spectrum to a value of $g_{0}^{TM}L$ and $\alpha_{int}^{TM}L$ for a given value of $p$ and $R$.

The values of gain anisotropy $\eta$ and loss anisotropy $\gamma$ are chosen to satisfy:

$$\frac{1}{\eta}g_{0}^{TM}L - \frac{1}{\gamma}\alpha_{int}^{TM}L = \ln \left\{ \frac{\sqrt{x_{TE}} - 1}{\sqrt{x_{TE}} + 1} \right\} - \ln R,$$

where this equation is found by combining Eqs. (2.11), (2.17), (2.46), and (2.47a). A loss anisotropy value $\gamma < 1$ is chosen to correspond to our laboratory device for which $g_{th}^{TM} < g_{th}^{TE}$.

Finally, the values of the linewidth enhancement factor $\alpha^{TM}$ and nonlinear anisotropy $\varepsilon$ were selected to make the simulated hysteresis curves match the shape of the experimentally measured curves. The selection of $\varepsilon$ to be less than unity is consistent with published data [34].

### 2.13 Hysteresis Curves of the Stokes Parameters

BPR hysteresis curves are modeled as shown in Fig. 2.3 for the output optical power of the FP-SOA. The traditional total output power $S_{0}$ reveals a hysteresis curve as shown in Fig. 2.3(a).

The forming of this hysteresis curve can be traced as the input power $P_{in}$ raises, the lower branch experiences upward switching. The input power falling causes downward switching from the upper breach curve. The modeled dispersive optical bistability $S_{0}$ is similar to previous presented traditional hysteresis curves [11]–[17], [19], [41].

The traditional hysteresis curve in Fig. 2.3(a) curve shows only power and there is no way to tell if there is a simultaneous occurrence of nonlinear polarization rotation because the traditional hysteresis curve shows only power portion of the optical signal. Stokes parameters show both the power and phase portion of the optical signal. Consequently, BPR is evidently simulated in Fig. 2.3(b). In the Stokes vector, the SOP evolution is represented,
where any change in the Stokes parameters means a change in the SOP because Stokes parameters are an accurate representation. These hysteresis curves in SOP are not the traditional optical bistability in a nonlinear resonator.

Figure 2.3 Simulation of BPR: (a) Hysteresis curve of optical power, the traditional notion of dispersive optical bistability; (b) Hysteresis curves of the Stokes parameters, revealing BPR. Simulation parameter values are given in Table 4.1.

Stokes parameter $s_1$ reveals a hysteresis curve that indicates differences in transmissivity $T$ of each polarization mode due to anisotropy, where anisotropy changes the single-pass gain $G$ and birefringence that fuels each transmissivity $T$ of polarization modes. The $s_1$ hysteresis curve in drops down to a lower value ($\approx -0.66$) at the upward-switching input power as shown in Fig. 2.3(b). The TM transmittivity peak is stronger than that of the TE transmittivity, which causes this drop ($\approx -0.66$). The TM resonance strength is shown in Fig. 2.2.

The Stokes parameters $s_2$ and $s_3$ vary due to anisotropy in the transmissivity $T$ and the output phase difference $\Delta \Psi_{out}$ as per Eqs. (2.43b) & (2.43c). The sign of $s_2$ and $s_3$ is determined by the $\cos(\Delta \Psi_{out})$ and $-\sin(\Delta \Psi_{out})$ functions, respectively. Both Stokes
parameters are negative because $\Delta \Psi_{out}$ varies from $\pi$ to $0.8\pi$.

### 2.14 Degree of Bistable Polarization Rotation

The degree of bistable polarization rotation quantifies the amount of NPR between the two stable branches in the hysteresis curves in Fig. 2.4. A degree of BPR ($D$-$BPR$) is defined as:

$$D$-$BPR = \frac{\theta}{\pi} = \frac{\arccos\left(s_{r1}s_{f1} + s_{r2}s_{f2} + s_{r3}s_{f3}\right)}{\pi}. \quad (2.51)$$

The quantity $\theta$ is the angular separation on the Poincaré sphere of two arrays of three Stokes vectors $(s_{r1}, s_{r2}, s_{r3})$ and $(s_{f1}, s_{f2}, s_{f3})$, corresponding to the rising $r$ and falling $f$ portions of the input-optical power, respectively. Normalizing by $\pi$, the maximum angular separation possible on the Poincaré sphere, scales the $D$-$BPR$ from 0 (or 0%), indicating no BPR, to 1 (or 100%), indicating BPR spanning orthogonal SOPs.

![Figure 2.4](image)

**Figure 2.4** The degree of BPR ($D$-$BPR$) as a function of input power for the Stokes parameters shown in Fig. 2.3(b). The center value $C = 19.4\%$ (indicated by the marker).

The $D$-$BPR$ for the Stokes parameters of Fig. 2.3 is shown in Fig. 2.4 as a function of input power. At low and high input powers, $D$-$BPR = 0$, which indicates that a single, stable SOP exists. For the range of input powers bounded by the upward and downward switching thresholds, the non-zero $D$-$BPR$ indicates BPR. The $D$-$BPR$ quantity places the focus on the extent of rotation during bistability, instead of on the specific bistable SOP values; in
Fig. 2.4, it reaches a center value \( C = 19.4\% \). The center value is used as a single-value representation of the D-BPR curve; it is also a useful value to be chosen as an operation point for some applications of bistability, such as an AOFF [41], where the holding beam is optimally chosen to operate away from both the rising and falling switching transitions points. The negative slope is dominated by the the product of the rising \( r \) and falling \( f \) portions of the \( s_3 \) Stokes parameter.

### 2.15 Conclusion

We have mathematically modeled the simultaneous occurrence of dispersive hysteresis curve and nonlinear polarization rotation (NPR). The model provides deep insight into this physical process of having a bistable curve that undergoes NPR, where the bistable curve is modeled in the optical Stokes parameters. The modeling is carried out based on using consistent values that match the lab conditions. The model and methods reported here are applicable to investigate BPR in other structures and other nonlinear media.
3. Bistable Polarization Rotation: Lab Demonstration

3.1 Introduction

The simultaneous occurrence of the two nonlinear phenomena—nonlinear polarization rotation (NPR) and dispersive optical bistability has only been investigated separately as two isolated nonlinear optical phenomena [11]–[17], [24]–[26]. In this chapter, we reported the experimental demonstration of the simultaneous occurrence of these two nonlinear phenomena, where bistable hysteresis curves are evidently demonstrated in the optical Stokes parameters. The input optical power imbalance is optimized to be launched into the TE and TM modes of the nonlinear resonator, where we used Fabry-Pérot SOA to produce the bistable hysteresis curve. Furthermore, the input-signal wavelength is selected to optimize the effect of the birefringence between the TE and TM modes. As a result, an optimized polarization rotation of 30% orthogonality is demonstrated in the lab.

3.2 Experimental Set-Up

BPR is demonstrated in the laboratory with the experimental set-up shown in Fig. 3.2. The optical signal from a tunable laser is modulated by a Mach-Zehnder modulator (MZM) driven by a radio-frequency (RF) signal generator whose modulation period $T = 50 \mu s$ is
CHAPTER 3. BISTABLE POLARIZATION ROTATION: LAB DEMONSTRATION

Figure 3.1 Experimental set-up to generate BPR signal: PC = polarization controller; SG = RF signal generator; MZM = Mach-Zehnder modulator; OTF = optical tunable filter; PLR = polarimeter; PD = photodiode; PBS = polarization beam splitter; OSW = optical switch; OSA = optical spectrum analyzer;

much longer than the SOA recovery time of $\tau \sim 200 - 1000$ ps. Polarization controller PC1 is used to align the optical signal to the TE mode of the MZM, wherein it experiences an insertion loss of 3.8 dB. A 3-dB optical splitter then separates the optical signal into two paths — a reference path that provides a measurement of the input signal and a path that injects light into FP-SOA. The injection current $I = 62$ mA of the FP-SOA is 96% of the lasing threshold $I_{th} = 64.5$ mA; this configuration corresponds to the $g_{TM} = 0.91g_{TM}^{th}$ used in simulations.

An optical tunable filter (OTF) is used after the FP-SOA to eliminate the dominate optical noises that are exiting the FP-SOA due to the spontaneous process, where a random photons are generated as a consequences of the amplification process [10].

Before turning on the optical signal, a preliminary configuration is performed as follows. The ASE spectrum of the FP-SOA is sent along the output-measurement path, through polarimeter PLR-A, and ultimately into the optical spectrum analyzer (OSA). The polarization controller PC4 is used to adjust the polarization of the FP-SOA TM-mode ASE so that it aligns with, and thus solely exits, one of the two orthogonal output ports of the polarization
beam splitter (PBS); the OSA is used as the monitor during this optimization process because its high sensitivity can detect the weak ASE power (<50 dBm). This TM-mode alignment process concurrently aligns the orthogonal TE-mode ASE to the other output port of the PBS. The optical switch is used to select a specific polarization mode for injection into the OSA. The small-signal birefringence $\Delta \phi_0$ is determined by observing the spectral difference $\Delta \lambda$ between the peaks of the TE and TM ASE spectral responses, still without the optical signal [70], [71]. Specifically, $\Delta \phi_0 \approx \Delta \lambda \left( FSR_\phi / FSR_\lambda \right)$, where $FSR_\phi = \pi$ is the free-spectral range (FSR) of the resonances in terms of phase and $FSR_\lambda$ is the FSR of the resonances in terms of wavelength. $FSR_\lambda = 0.18$ nm for the data shown in Fig. 3.2, but $\Delta \phi_0 = 0$ because $\Delta \lambda = 0$.

![Figure 3.2](image-url)

**Figure 3.2** Polarization-resolved small-signal ASE spectra with a small-signal optical-signal injected into the TM mode, indicating the small-signal birefringence $\Delta \phi_0$ and TE-mode single-pass phase $\phi_{0,TE}^T$. 

The laser is then turned on at low power and without RF modulation. TM alignment into the FP-SOA is achieved by varying the polarization controller PC2 until the signal power passes solely through the TM port of the PBS; an example of this state is shown in the TM ASE spectrum of Fig. 3.2. The small-signal phase shift $\phi_{0,TM}^T$ is then set by tuning the laser wavelength. The small-signal regime is verified by checking that the ASE resonances are unaffected by the addition of the optical signal. For the spectral region shown in Fig. 3.2(b), $\Delta \phi_0 = 0$ and $\phi_{0,TM}^T = -0.63 \pi$. 

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3.3 Hystereses & Degree of BPR

The traditional total output optical power $S_0$ is shown in Fig. 3.3(a) for a free-space wavelength of 1591.16 nm, laser-output power of 8 dBm, $s_{1in} = 0$, $\phi_0^{TM} = -0.63\pi$, and $\Delta\phi_0 = 0$. The bistable hysteresis is measured in two diagnostics—oscilloscope (shown along the left vertical axis) and polarimeter (shown along the right vertical axis). The polarimeter reveals the bistable hysteresis with several artifacts, where there are slower switching transitions and a "knee" occurring after the downward-switching input power. However, polarimeter

\[ \text{Figure 3.3 Experimental demonstration of BPR for } s_{1in} = 0, \phi_0^{TM} = -0.63\pi, \text{ and } \Delta\phi_0 = 0: \text{ (a) Bistable hysteresis curves of the total optical power as measured by the oscilloscope (left axis) and the polarimeter (right axis); the polarimeter faithfully reveals the bistable region, albeit with slower switching transitions and a "knee" artifact after the downward transition; (b) Bistable hysteresis curves of the three Stokes parameters; the slow transitions and "knee" are artifacts persist here and for the } D\text{-BPR}; \text{ (c) The degree of BPR reaches } C = 16.5\% \text{ at the center of the hysteresis (indicated by the marker).} \]
evidently capable of measuring bistable hysteresis as in Fig. 3.3(a). Thus, the polarimeter is used to measure the BPR in the Stokes parameters exhibit clear hysteresis with the same artifacts shown in the total optical power.

The hysteresis curves in the Stokes parameters obviously indicate a polarization rotation, where the polarization separation can be measured based on the angular separation between the lower branches (rising vector) and higher branches (falling vector) of the Stokes parameters according to Eq. (2.51) produces the degree of BPR shown in Fig. 3.3(c). The center values $C = 16.5\%$ is achieved between the two vectors.

### 3.4 Dependence on Input-Power Imbalance

The dependence on input optical power imbalance is investigated in this section, shifting more power into the TE modes show an increase in the D-BPR. An input $s_{1in} = 0$ was chosen for the initial BPR demonstration in Fig. 3.3 because of its symmetry — an equal amount of optical power is injected into each polarization mode. Introducing an imbalance between the power injected into the two modes is achieved by varying polarization controller PC2, where changing PC2 impacts the amount of BPR realized, as shown in Fig. 3.4 for a variety of $s_{1in}$ while maintaining $\phi^T_0 = -0.63\pi$, $\Delta\phi_0 = 0$, and the other experimental conditions used for Fig. 3.3. As $s_{1in}$ increases from $-0.2$, Fig. 3.3(a) reveals that the switching input powers of the hysteresis curves increase. Increasing $s_{1in}$ in this manner shifts more power into the TE mode. Since the onset of bistability is dominated by the stronger TM resonance, these higher input powers are needed to overcome the reduction in TM power due to larger $s_{1in}$.

The associated variation in the degree of BPR is shown in Fig. 3.4(b). Note that the $D$-$BPR$ increases as input power is transferred to the TE mode (by increasing $s_{1in}$). Thus, as the optical power hysteresis curve is getting "worse" (i.e., higher switching powers and lower switching contrast), the BPR is getting "better" (i.e., higher $D$-$BPR$). The range
Figure 3.4  Dependence of BPR on the input power imbalance $s_{1in}$ for $\phi_0^{TM} = -0.63\pi$ and $\Delta\phi_0 = 0$: (a) As $s_{1in}$ increases, the optical-power hysteresis curves shift to higher switching input powers; (b) As $s_{1in}$ increases, the $D$-BPR generally increases before closing down and leaving only artifacts of the polarimeter at $s_{1in} = 0.8$. An optimal $s_{1in} = 0.4$ provides wide $D$-BPR region with a center value $C = 24\%$ (indicated by the marker).

of input powers over which BPR occurs begins to close down at $s_{1in} = 0.6$, leaving only the slow transitions of the polarimeter at $s_{1in} = 0.8$. Setting $s_{1in} = 0.4$ provides a wide $D$-BPR region and high center value of $C = 24\%$. This value of $s_{1in}$ corresponds to about 2.3 times more power being injected into the TE mode than the TM mode; BPR is thus favored by preferential power injection into the mode that exhibits the weaker interferometric resonances.

3.5 Dependence on Small-Signal Birefringence

The small-signal birefringence $\Delta\phi_0$ is optimized to reveal a 30% D-BPR as shown in Fig. 3.5. Using the optimized value $s_{1in} = 0.4$ from Section 3.4, $\phi_0^{TM} = -0.63\pi$, and the other experimental conditions used for Fig. 3.3. The value of $\Delta\phi_0$ is selected by observing the overlap of TE and TM ASE spectra at different spectral regions. For example, a change of $\Delta\phi_0$ by $0.4\pi$ requires a spectral translation of 13.6 nm, which is about a span of 75 resonances for a free-spectral range of 0.18 nm. After a spectral region is selected, the
laser is tuned to provide \( \phi_0^{TM} = -0.63\pi \) with a nearby resonance. The largest center value \( C = 30\% \), occurs for \( \Delta \phi_0 = -0.2\pi \).

![Graph](image)

**Figure 3.5** Dependence of BPR on the small-signal birefringence \( \Delta \phi_0 \) for \( s_{1in} = 0.4 \) and \( \phi_0^{TM} = -0.63\pi \): (a) The hysteresis curves generally shift to smaller input powers with a decrease in \( \Delta \phi_0 \); (b) The largest center value \( C = 30\% \) (indicated by the marker) of \( D\text{-BPR} \) is realized at \( \Delta \phi_0 = -0.2\pi \).

### 3.6 Conclusion

We have experimentally demonstrated the new physical process of having a bistable optical signal that undergoes nonlinear polarization rotation. This means each point on the bistable curve is unique in its state of polarization. This behavior broadens the notion of traditional optical bistability. The input power imbalance and the selection of the input optical signal wavelength are optimized to measure a 30\% NPR rotation between the two stable branches of the hysteresis curves.
4. Hysteresis-Shape Transformation: Theory

4.1 Introduction

Optical bistability exhibited by optical power has been widely studied in a variety of photonic devices and structures [11]–[22], [57]–[65]. Hysteresis curves are applied to several of all-optical signal processing gates and applications including optical flip-flops [37]–[41], gate and packet switches [47], [48], parity checkers [72], AND gates [43]–[45], and square-wave optical clocks [73].

With an eye towards integration, many studies of optical hysteresis curves have been demonstrated in compact, monolithically integrateable photonic devices including semiconductor lasers [16], distributed feedback (DFB) semiconductor optical amplifiers (SOAs) [15], [38], [39], [49], vertical cavity SOAs (VC-SOAs) [12], [40], [44], [50], and Fabry-Pérot SOAs (FP-SOAs) [11], [15]–[17], [23], [41], [43], [51].

A well-known kind of optical bistability is dispersive optical bistability that works based on an interferometric resonance interaction with an input optical signal [46]. This bistability has been studied in different structures of nonlinear resonators [11]–[17].

In the exploration of optical hysteresis, a diversity of hysteresis shapes have been reported, including the canonical counter-clockwise shape (S-shape) [11], [12], [15]–[17], [23], [49], [51], [57], [58], [60], [62], [63], the clockwise shape (inverted S-shape) [12],
CHAPTER 4. HYSTERESIS-SHAPE TRANSFORMATION: THEORY

[15], [23], [49]–[55], [59], and butterfly shapes [15], [49], [51], [55].

Optical bistable systems are often capable of achieving more than one hysteresis shape. The shape transformation requires a sort of mechanism to transform from one shape to another. This transformation ability is useful as it would allow a single nonlinear component to operate for different applications. Both the CCW and CW hysteresis shapes have been demonstrated for dual-optical-signal injection into an FP-SOA, with the shape, transformation is achieved by tuning wavelength of the non-dominant signal, [23].

The optical signal wavelength (initial phase detuning) of the bistable signal itself can be used to select CCW, CW, or butterfly hysteresis on the reflection from chirped-grating DFB-SOAs [49]. These shapes can also be achieved by tuning the SOA drive current (i.e., small-signal gain) in a DFB-SOA [15], VC-SOA [55], and FP-SOA [51].

Each hysteresis-shape transformation technique listed above are achieved by acting directly on the bistable system, either by altering the wavelength of the optical signal entering the system, or altering a drive current of the system. Such techniques have limitations. For example, the different hysteresis shapes exhibit different switching powers. Also, only one type of hysteresis shape can be sent at a time (which limits distribution). Additionally, the contrast ratio of the previously demonstrated work using a variety of nonlinear devices is limited to be less than 10 dB [11], [12], [16], [17], [23], [49]–[55].

A hysteresis-transformation technique has been investigated using an FP-SOA that realizes CCW, CW, and butterfly shapes, where all shapes exhibit the same switching powers [74]. Our technique does not act on the bistable system, but rather control the bistable signal itself in the downstream of the nonlinear resonator, allowing for the distribution and local selection of the hysteresis shape at each destination.

Moreover, a key to this new transformation technique is that it acts on the BPR signal whose bistable nature includes a change in its state of polarization (SOP) as discussed in chapters 2 and 3. The hysteresis-shape transformation technique acts on the bistable SOP by means of a linear polarizer and waveplates.
An essential feature capability of the new transformation technique is the ability to optimize the *contrast* of the hysteresis shapes, produces CCW and CW shapes having contrasts exceeding 20 dB [74]. This is a substantial improvement of the contrast of prior work using dispersive optical bistability [11], [12], [16], [17], [23], [49]–[55]. Examples for some approximates contrast values as follows: 3 dB contrast for CCW shape in DFB-SOA [49], 8.5 dB for CCW shape and -5 dB for the CW shape in VC-SOA [12], 3 dB for CCW shape and -2.5 dB for CW shape in laser diode amplifiers [51], 6 dB for the CCW shape and -1.5 dB for the CW shape in FP-SOA [23]. The FP-SOA is reasoned to have a theoretically contrast limit to be less than 10 dB [43]. The contrast is bounded because it is not possible to obtain a zero output power in one of the bistable branches to achieve high contrast. The forming mechanism of a dispersive bistable hysteresis curve relies on interferometric resonance interaction with an input optical signal that exhibits a given output power (non-zero power) based on the resonance profile (e.g. strength). In this chapter, a model is provided that describes the physical process of this new hysteresis-transformation technique, where it shows how a linear polarizer can act on the polarization-rotating bistable signal to yield CCW, CW, and butterfly hysteresis shapes. Also, this chapter reports a generalized Malus’ law accounting for elliptical input polarization.

### 4.2 Bistable Polarization-Rotating Signal

This section presents executive summary for our recent model for the bistable polarization-rotating signal (BPR) [46], an essential ingredient for our optical-power hysteresis-shape transformation technique. The model presented here is for the specific case of the simultaneous occurrence of dispersive optical bistability and nonlinear polarization rotation within a Fabry-Pérot semiconductor optical amplifier (FP-SOA). Other means of generating a bistable polarization-rotating signal would also be valid for our transformation technique.
4.2.1 Jones Vector Representation for an Optical Signal

An optical signal can be decomposed into components pointing in the x and y directions:

\[
E(z,t) = E^x \hat{x} + E^y \hat{y},
\]

(4.1a)

\[
E^x(z,t) = \sqrt{P^x} \exp(i\Psi^x) \exp(i(\omega t - \beta^x z)),
\]

(4.1b)

\[
E^y(z,t) = \sqrt{P^y} \exp(i\Psi^y) \exp(i(\omega t - \beta^y z)),
\]

(4.1c)

where \(\hat{x}\) and \(\hat{y}\) are the unit vectors of the transverse plane, \(E^x\) and \(E^y\) are the electric fields associated with the \(\hat{x}\) and \(\hat{y}\) unit vectors, \(P^x\) and \(P^y\) are the corresponding optical powers, \(\Psi^x\) and \(\Psi^y\) are corresponding optical phases, \(\beta^x\) and \(\beta^y\) are the corresponding wavenumbers, and \(\omega\) is the angular frequency of the carrier wave.

The total optical power \(P\) of the optical signal is given by:

\[
P = P^x + P^y.
\]

(4.2)

The state of polarization (SOP) of the optical signal can be understood in term of two quantities that are derived from the electric-field expressions, the phase difference \(\delta\), defined as:

\[
\delta = \Psi^y - \Psi^x,
\]

(4.3)

and field-distribution angle \(\gamma\) (also called the auxiliary angle), which relates the amplitudes \(\sqrt{P^x}\) and \(\sqrt{P^y}\) to the amplitude associated with the total power \(\sqrt{P}\) as follows:

\[
\sqrt{P^x} = \sqrt{P} \cos(\gamma),
\]

(4.4a)

\[
\sqrt{P^y} = \sqrt{P} \sin(\gamma).
\]

(4.4b)

Using these derived quantities, the SOP is concisely written in the Jones vector notation as
follows:

$$J(t) = \begin{bmatrix} \sqrt{P_x} \\ \sqrt{P_y} \exp(i \delta) \end{bmatrix} = \sqrt{P_T} \begin{bmatrix} \cos(\gamma) \\ \sin(\gamma) \exp(i \delta) \end{bmatrix}. \quad (4.5)$$

The above Jones vector is a generalized form for an elliptical state of polarization (SOP) that is required in our study, but other specific cases of SOPs (e.g., circular SOP) can be obtained for a given values of \( \gamma \) and \( \delta \).

### 4.2.2 Simultaneous Dispersive Optical Bistability & Nonlinear Polarization Rotation

Here, we introduce a FP-SOA as the nonlinear photonic resonator. The schematic of the nonlinear photonic resonator is shown in Fig. 4.1, where the transverse electric (TE) mode is aligned \( \hat{x} \) axis and the transverse magnetic (TM) mode is aligned with the \( \hat{y} \) axis of the nonlinear photonic resonator.

![Figure 4.1](image)

**Figure 4.1** Schematic of the nonlinear photonic resonator producing a bistable polarization-rotating signal. The input signal is shown as a function of time with a green rising slope and a black falling slope. The output signal is shown as an input-output hysteresis curve; the SOP on the rising (green) is different than the SOP on the falling (black). The z-axis locations \( k = \{a, b\} \) are at the input and output of the resonator, respectively.

BPR signal is simulated by applying a numerical ordinary differential equation (ODE) solver to the TM nonlinear equation as discussed in chapter 2. The parameter values were used as shown in Figs. 4.2 and 2.3 are obtaining based on calculating consistent parameters based on the lab conditions in chapter 5 using Hakki-Paoli method [69]. The Hakki-Paoli is
applied to both TE and TM ASE spectra that are measured in the laboratory as discussed in chapter 2 [46]. The consistent values are shown in Table 4.1:

Table 4.1 Quantity values for consistent values based on the lab conditions that are used to model the BPR signal in Figs. 4.2 and 4.3

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>normalized modulation period</td>
<td>$T'$</td>
<td>$10^3$</td>
</tr>
<tr>
<td>amplitude-distribution angle</td>
<td>$\gamma_a$</td>
<td>26.57°</td>
</tr>
<tr>
<td>input phase</td>
<td>$\Psi_{in}^{TM}$</td>
<td>0</td>
</tr>
<tr>
<td>input phase difference</td>
<td>$\Delta\Psi_{in}$</td>
<td>$\pi$</td>
</tr>
<tr>
<td>small-signal, single-pass phase</td>
<td>$\phi_0^{TM}$</td>
<td>-0.603$\pi$</td>
</tr>
<tr>
<td>small-signal birefringence</td>
<td>$\Delta\phi_0$</td>
<td>-0.298$\pi$</td>
</tr>
<tr>
<td>FP facet reflectivities</td>
<td>$R_1, R_2$</td>
<td>0.6</td>
</tr>
<tr>
<td>normalized small-signal gain coefficient</td>
<td>$\overline{g}_0^{TM}$</td>
<td>2.547</td>
</tr>
<tr>
<td>gain anisotropy</td>
<td>$\eta$</td>
<td>1.05</td>
</tr>
<tr>
<td>normalized averaged loss</td>
<td>$\overline{\alpha}_{int}^{TM}$</td>
<td>2.288</td>
</tr>
<tr>
<td>loss anisotropy</td>
<td>$\Gamma$</td>
<td>0.58</td>
</tr>
<tr>
<td>linewidth enhancement factor</td>
<td>$\alpha_H^{TM}$</td>
<td>10.38</td>
</tr>
<tr>
<td>nonlinearity anisotropy</td>
<td>$\varepsilon$</td>
<td>0.865</td>
</tr>
</tbody>
</table>

4.2.3 Bistable Total Output Power

After executing the BPR mode using the consistent values as reported in Section 4.2.2, the simulated BPR signal $P_b$ is shown in Fig. 4.2. The total output power $P_b$ represents the sum of the output optical powers in the polarization modes given by Eq. (2.34). When the input power increase into the nonlinear resonator, the interferometric resonance shifts onto the optical signal wavelength to produce upward-switching (green-line). Then, when the input power decreases, the interferometric resonances shifts away from the optical-signal wavelength to produce the he downward-switching (black-line) [46].

The hysteresis curve in BPR signal $P_b$ is characterized at the center of the hysteresis loop by a contrast $C$ between the center the rising-input-power branch $P^R$ and the center of the falling-input-power branch $P^F$, given by:

$$C = \frac{P^F}{P^R},$$

(4.6)
This particular definition could result in a positive or a negative contrast $C$ in the dB scale for CCW and inverted CW hysteresis curves, respectively. The sign of the contrast $C_{dB}$ depends on the power strength between the rising-input-power branch $P_k^R$ and the falling-input-power branch $P_k^F$. For example, if $P_k^R$ is stronger than $P_k^F$ in the hysteresis region, the contrast $C_{dB}$ is negative for those with inverted CW hysteresis curves. This definition of contrast simplifies the analysis of the transformation of the hysteresis shape.

For the bistable polarization-rotating (BPR) signal shown in Fig. 4.2, the contrast $C_b = 3.25$ dB. This poor contrast is not unusual for bistable photonic systems governed by dispersive bistability. The center of the raising branch exhibits a relatively high-power of $P_b^R = 0.027$ due to the fact of having an FP resonance shifting toward the optical signal wavelength when the input power is increasing, thus it is impossible to obtain a zero-power value at the center of the raising branch to reveal a high-contrast.

The limited NPR affects the strength of the $P_b^F$ due to a maximum degree of rotation of D-BPR= 30% that is experimentally measured in a nonlinear resonator (FP-SOA) [46], and the simulated degree of rotation is D-BPR= 31.59% to be consistent with the capability of the FP-SOA that is used in the laboratory.

Figure 4.2 Optical hysteresis curve of the total power exiting the nonlinear resonator at location $k = b$ for input $\gamma_0 = 26.57^\circ$, $\phi_0^{TM} = -0.603\pi$, and $\Delta\phi_0 = -0.298\pi$. The contrast hysteresis curves such as this, whose contrast $C_b$ at the center of the hysteresis between the rising (green) input-power branch and the falling (black) input-power branch is only 3.25 dB. This hysteresis shape can be transformed into both high-contrast CCW-shaped, CW-shaped, and butterfly-shaped hysteresis curves.
4.2.4 Bistable SOP Curve

As the total optical power traces out the hysteresis shape shown in Fig. 4.2, the optical signal simultaneously undergoes nonlinear polarization rotation (NPR), as reported in previous chapters in terms of Stokes parameters and the Poincare sphere [46]. The output optical signal (BPR signal) exhibits a different SOP for each point on the bistable curve due to asymmetric the TE and TM transmittivities given by Eq. (2.35) that causes nonlinear polarization rotation.

The corresponding SOP for the output optical signal, shown in Fig. 2.3, can be represented in terms of amplitude distribution angle $\gamma_b$ as in Fig. 2.3(a). $\gamma_b$ varies in time, thus each point on this curve shows a unique amount of output power distribution in each polarization mode based on Eq. 4.4.

![Figure 4.3](image)

**Figure 4.3** Bistable polarization rotation as evident in the (a) amplitude-distribution angle $\gamma_b$ and (b) the phase detuning $\delta_b$ of the signal exiting the nonlinear resonator at location $k = b$. This change in SOP accompanies the change in optical power shown in Fig. 4.2 and is the basis for controlling the optical hysteresis shape.

The phase difference $\delta_b$ between the TM and TE modes is shown in Fig. 2.3(b). Both $\gamma_b$ and $\delta_b$ change in time, where this change is evidence that the output dispersive optical bistability undergoes a simultaneous NPR and each point on the output total power is different in polarization.

The center of the hysteresis curves occurs at a normalized input power of 0.66. On the rising-input-power branch, this input power corresponds to $\gamma_b^R = 40.31^\circ$ and $\delta_b^R = 0.50\pi$. On the falling-input-power branch, this input power corresponds to $\gamma_b^F = 66.03^\circ$ and $\delta_b^F = 0.64\pi$. 

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These values of $\gamma_b$ and $\delta_b$ will be used throughout the paper as an example of output optical signal. These are the rising and falling center points of the hysteresis.

The output state of polarization exhibits hysteresis as shown in the example of the BPR signal in Figs. 4.2 is the heart of enabling the functionality of optical-hysteresis shape transformation as discussed in the following sections.

### 4.3 Optical-Power Hysteresis-Shape Transformation via Linear Polarizer

The shape, contrast, and power of the optical-power hysteresis curve can be altered by passing the BPR signal through a linear polarizer as shown in Fig. 4.4. The BPR signal enters the linear polarizer in the downstream, the output hysteresis curve shape, contrast, and power at location $d$ depends on the relation between each SOP on the BPR signal and the orientation angle $\theta$ of the polarizer.

#### 4.3.1 Generalized Malus’ Law for Elliptical SOP Through a Linear Polarizer

A linear polarizer allows passage of optical power align to pass-axis that is oriented at an angle $\theta$ from the x-axis [68] as shown in Fig. 4.4. The positional index $k$ takes on the values of $a$, $b$, $c$, and $d$ to signify the input and signals of various components as identified in Fig. 4.4. The SOP at location $z = k$ can be expressed as a Jones vector.

The output Jones vector from a non-ideal linear polarizer can be founded by the following matrix product of the input SOP into the polarizer $J_c(t)$ and the non-ideal polarizer Jones matrix $M_\theta$ whose pass-axis is oriented at angle $\theta$:

$$J_d(t) = M_\theta J_c(t).$$  \hfill (4.7)
Figure 4.4 Schematic of using a linear polarizer to transform the hysteresis shape of a bistable polarization-rotating signal. The z-axis positions $k = \{b, c\}$ are at the output of the resonator and the input to the polarizer, respectively, and are co-located for this case. The location $k = d$ is at the output of the polarizer.

The Jones matrix $M_\theta$ of a non-ideal linear polarizer is given by [36]:

$$M_\theta = R(-\theta) M_0 R(\theta)$$

(4.8a)

$$= \begin{bmatrix} t_p \cos^2(\theta) + t_b \sin^2(\theta) & (t_p - t_b) \sin(\theta) \cos(\theta) \\ (t_p - t_b) \sin(\theta) \cos(\theta) & t_p \sin^2(\theta) + t_b \cos^2(\theta) \end{bmatrix},$$

(4.8b)

where $R$ is the rotation matrix

$$R(\theta) = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix},$$

(4.9)

and $M_0$ is the Jones matrix for a non-ideal linear polarizer, whose pass-axis aligned to $\hat{x}$:

$$M_0 = \begin{bmatrix} t_p & 0 \\ 0 & t_b \end{bmatrix}.$$  

(4.10)

The quantities $t_p$ and $t_b$ are the complex-valued transmission coefficients for the pass-axis.
and block-axis, respectively:

\[ t_p = \sqrt{T_p} \exp(i\phi_p), \quad (4.11a) \]

\[ t_b = \sqrt{T_b} \exp(i\phi_b), \quad (4.11b) \]

where \( \phi_p \) and \( \phi_b \) are the change in phase of the optical field along each axis, and are assumed to be equal for this work. The quantities \( T_p \) and \( T_b \) are the transmittivity of pass-axis and block-axis, respectively.

The generalized Malus’ law output power passing through a non-ideal linear polarizer can be determined based on the product of the output Jones vector \( J_d(t) \) and conjugate transpose of the output Jones vector \( J_d(t)^\dagger \) at location \( k = d \), as follows:

\[ P_d = J_d(t)^\dagger J_d(t), \quad (4.12) \]

\[
P_d = [T_p \cos^2(\theta) + T_b \sin^2(\theta)] P_c^T \]
\[ + [T_p \sin^2(\theta) + T_b \cos^2(\theta)] P_c^M \]
\[ + 2[T_p - T_b] \sqrt{P_c^T P_c^M} \sin(\theta) \cos(\theta) \cos(\delta_c). \quad (4.13) \]

The transmittivity \( T = P_d^T / P_c^T \) of the non-deal linear polarizer is found by dividing both sides by the total input power \( P_c^T \):

\[
T = [T_p \cos^2(\theta) + T_b \sin^2(\theta)] \cos^2(\gamma) 
+ [T_p \sin^2(\theta) + T_b \cos^2(\theta)] \sin^2(\gamma) 
+ 2[T_p - T_b] \cos(\theta) \sin(\gamma) \sin(\gamma) \cos(\delta_m). \quad (4.14)
\]

Instead of using the transmittivity of the block axes \( T_b \), commercial polarizers are
commonly specified in terms of their extinction ratio \[68\], defined as:

\[
ER = \frac{T_p}{T_b},
\]

(4.15a)

\[
ER_{dB} = 10\log_{10}(ER) = L_{dB} - IL_{dB}.
\]

(4.15b)

where the insertion loss \(IL_{dB}\) and block-axis loss \(L_{dB}\) are defined as:

\[
IL_{dB} = -10\log_{10}(T_p), \quad L_{dB} = -10\log_{10}(T_b).
\]

(4.16)

The transmittivity of the non-ideal linear polarizer can be expressed in terms of \(IL\) and \(ER\) by factoring out the pass-axis transmittivity \(T_p = 10^{-IL_{dB}/10}\) from all terms in Eq.(4.14) and using Eq. (4.15a):

\[
T = T_p \left\{ \cos^2(\theta) + \left(\frac{1}{ER}\right) \sin^2(\theta) \right\} \cos^2(\gamma)
\]

\[
+ \left[ \sin^2(\theta) + \left(\frac{1}{ER}\right) \cos^2(\theta) \right] \sin^2(\gamma)
\]

\[
+ 2\left[ 1 - \left(\frac{1}{ER}\right) \right] \cos(\theta) \sin(\theta) \cos(\gamma) \sin(\gamma) \cos(\delta_m) \right\}.
\]

(4.17)

Eqs. 4.13, 4.14, and 4.17 are three versions of a generalized Malus’ law that is valid for elliptical input SOPs and includes the practical aspects of insertion loss and extinction ratio. These are crucial aspects to include for the transformation technique modeled here. The reduction of this generalized Malus’ law to more familiar forms is given in the following section.

### 4.3.2 Reduction of Generalized Malus’ Law

The generalized Malus’ law given, in three forms, as Eqs. 4.13, 4.14, and 4.17 is appropriate for elliptical input SOPs and for non-ideal polarizers having insertion loss and a finite extinction ratio. This generalized law is reduced here for several more common cases.
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For a linear polarizer with its pass-axis aligned to \( \hat{x} \) (that is, \( \theta = 0 \)) [68], Eq. (4.14) and Eq. (4.17) become, respectively:

\[
T = T_p \cos^2(\gamma) + T_b \sin^2(\gamma) = (T_p - T_b) \cos^2(\gamma) + T_b, \quad (4.18)
\]

\[
T = T_p [1 - (1/ER) \cos^2(\gamma) + 1/ER]. \quad (4.19)
\]

For this case, the pass-axis transmittivity is independent of the phase detuning \( \delta \).

For an ideal polarizer, for which \( IL = 0 \) dB and \( ER = \infty \), Eq. (4.17) reduces to:

\[
T = \cos^2(\theta) \cos^2(\gamma) + \sin^2(\theta) \sin^2(\gamma) + 2 \cos(\theta) \sin(\theta) \cos(\gamma) \sin(\gamma) \cos(\delta). \quad (4.20)
\]

This expression is appropriate for studying the passage of elliptical (general) polarization through an ideal linear polarizer.

For the case of linear SOPs passing through the polarizer, the phase detuning \( \delta = p\pi \), where \( p \) is an integer. The ideal-polarizer transmittivity Eq. (4.20) then simplifies to:

\[
T^\pm = (\cos(\theta) \cos(\gamma) \pm \sin(\theta) \sin(\gamma))^2, \quad (4.21)
\]

where the \( \pm \) signs corresponds to even and odd values of \( p \), respectively. Specifying the sign leads to

\[
T^+ = \cos^2(\gamma - \theta), \quad (4.22a)
\]

\[
T^- = \cos^2(\gamma + \theta). \quad (4.22b)
\]

Eqs. (4.22) can be written in terms of polarization ellipse angle \( \alpha \) based on Eq. (2.20), where
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\[ \gamma = \alpha \] for even \( p \) and \( \gamma = -\alpha \) for odd \( p \):

\[ T = \cos^2(\alpha - \theta) \] \hspace{1cm} (4.23)

Eq. (4.23) is the canonical Malus’ law for linear SOP passing through an ideal polarizer orientated at \( \theta \) [36]. Maximum transmittivity occurs when \( \alpha = \theta \).

### 4.3.3 Hysteresis Behavior of the Polarizer Transmittivity

As the optical signal traces out the \( \gamma_b \) and \( \delta_b \) values shown in Fig. 2.3, the transmittivity \( T \) traces out its own hysteresis curve, as governed by Eq. 4.17 and shown in Fig. 4.5 for the specific orientation angle \( \theta = 98.3^\circ \).

![Figure 4.5](image_url)

**Figure 4.5** (a) The transmittivity of the polarizer itself exhibits a bistable hysteresis curve due its dependence on the incoming SOP. For an orientation angle \( \theta = 98.3^\circ \), IL\( _{dB} \) = 0.55 dB, and ER\( _{dB} \) = 26 dB, the transmittivity contrast \( C_T = 3.09 \) dB. (b) The output power from the polarizer is the product of the hysteresis curves of this transmittivity the input power shown in Fig. 4.2. The contrast has improved to \( C_d = 6.34 \) dB.

The transmittivity hysteresis curve is characterized by a transmittivity contrast \( C_T \) that quantifies the separation between bistable states. This contrast is defined in terms of the rising-input-power branch \( T^R \) and the fall-input-power branch \( T^F \):

\[ C_T = T^F / T^R. \] \hspace{1cm} (4.24)

The example shown in Fig. 4.5(a), \( C_T = 3.09 \) dB. The contrast definition in Eq. (4.24) allows
the $C_T$ to be negative for other orientation angles of the polarizer, which is be useful for understanding the control of hysteresis shape.

The total power $P_d$ exiting the polarizer is simply the product of the total input power $P_b$ and the transmittivity $T$, both of which have their own hysteresis shapes:

$$P_d = P_c T. \quad (4.25)$$

This expression is the essence of the shape control elucidated in this paper; whereas $P_b$ will be the same for all examples shown, the transmittivity $T$ will be selected to alter the shape of the total output power $P_d$.

The power $P_d$ exiting the polarizer has a level set by the product of $P_b$ and $T$. This applies point-by-point along the hysteresis curve; for the center-point powers $P^R$ and $P^F$, the power levels are:

$$P^R_d = P^R_c T^R, \quad P^F_d = P^F_c T^F. \quad (4.26)$$

The contrast after the polarizer is found by the ratio of these powers, and simplifies to the product of the linear contrast values:

$$C_d = \frac{P^F_d}{P^R_d} = \frac{P^F_c T^F}{P^R_c T^R} = C_c C_T. \quad (4.27)$$

The change in the contrast is easily found by comparing Fig. 4.2 and Fig. 4.5, where $C_d = 6.34$ dB is the sum of the initial contrast $C_b = C_c = 3.25$ dB and the transmittivity contrast $C_T = 3.09$ dB. Positive values of all contrasts in this example indicates that the falling-input-power branch is stronger than the rising branch.

The altered transmittivity hysteresis curve will itself alter the hysteresis curve of the total output power $P_d$. Therefore, the optical power level of the lower branch of $P^R_d$ is reduced to be around 0.375% from the original power level $P^R_b$ in Fig. 4.2 due to the transmittivity value at $T^R = 0.375$. Additionally, the optical power level of the upper branch of $P^F_d$ has
been reduced to be around 76.4% from the original power level $P^F_b$ in Fig. 4.2 due to the transmittivity value at $T^F = 0.764$.

The specific orientation angle $\theta = 98.3^\circ$ found based on Fig. 4.6, where the largest contrast $C_T$ occurs at of $98.3^\circ$, at which value the $T^R$ falls to its minimum value of 0.375. However, the contrast is limited because the input SOPs at the $P^R_b$ and $P^F_b$ are elliptical SOPs, these SOPs alters transmittivity hysteresis curve based on Eq. (4.14) for elliptically polarized light.

### 4.3.4 Hysteresis-Shape Transformation

For the example BPR signal shown in Fig. 2.3, the full range of transmittivity contrast $C_T$ is achievable, where by using the rising-branch center-point values $\gamma^R_b$ and $\delta^R_b$ and the falling-branch center-point values $\gamma^F_b$ and $\delta^F_b$ with Eq. 4.14 gives the full range of orientation angle $\theta$ as shown in Fig. 4.6.

Figure 4.6 Hysteresis curves of the polarizer transmittivity and total output power for IL = 0.55 dB and ER = 26 dB. (a) The transmittivity of the polarizer itself exhibits a bistable hysteresis curve due its dependence on the incoming SOP (shown in Fig. 3). For an orientation angle $\theta = 98.3^\circ$, the transmittivity contrast $C_T = 3.09$ dB. (b) The output power from the polarizer is the product of this nonlinear transmittivity and the power shown in Fig. 2. The contrast has improved to $C_d = 6.34$ dB.

The strength difference between the transmissivity of the center points of the rising-branch center-point $T^R$ and falling-branch center-point $T^F$ are varying for a given value of
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The transmissivity of the rising-branch center-point \( T^R \) and falling-branch center-point \( T^F \) alters the shape of the output power, where for the example shown in Fig. 4.6(b), a diversity of hysteresis curve can be obtain for a given \( \theta \) based on the strength difference between the \( P^R \) and \( P^F \). When \( P^R \) is smaller than \( P^F \), this provides a traditional CCW hysteresis shape. A CW hysteresis curve is obtained when \( P^R \) is bigger than \( P^F \) while a butterfly hysteresis curve when \( P^F \) and \( P^R \) have the same power (as shown in crossing points).

**Figure 4.7** Transformation of the output-power hysteresis shape based on the polarizer-orientation angle \( \theta \), showing a (b) CW shape, (d) downward-switching butterfly shape, and (f) upward-switching butterfly shape for \( \theta = 12.7^\circ, 35.97^\circ, \) and \( 166.82^\circ \), respectively. The corresponding transmittivity \( T \) hysteresis curves are shown in parts (a), (c), and (e). \( IL_{dB} = 0.55 \) dB and \( ER_{dB} = 26 \) dB.

The contrast of the transmissivity \( C_T \) and the output power \( C_d \) are shown in Fig. 4.6(c) and (d) respectively. The positive contrast values are corresponding to the CCW shape, and it is limited to be less than 7 dB a and the negative contrast is corresponding to the CW shape to be less than 4 dB shown in Fig. 4.6(d).

Using the determined orientation angles \( \theta \) from Fig. 4.6, the corresponding hysteresis
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shapes are shown in Fig. 4.7. For $\theta = 12.7^\circ$ results in CW hysteresis shape, where the transmittivity is shown in Fig. 4.7(a) with a contrast of $C_T = -6.39$ and the total output power with a limited contrast of $C_d = -3.14$ as shown in Fig. 4.7(b). For $\theta = 35.97^\circ$ results in downward-switching butterfly shape hysteresis shape as shown in Fig. 4.7(c) and (d) for transmittivity and the total output power, respectively. For $\theta = 166.88^\circ$ results in upward-switching butterfly shape hysteresis shape as shown in Fig. 4.7(e) and (f) for transmittivity and the total output power, respectively. In both of these butterfly shapes, the center crossing point between the upward and downward switching in butterfly shape exhibits the same power $P_d$.

Although using only a non-ideal linear polarizer exhibits a diversity of hysteresis shapes for the example BPR signal shown in Fig. 2.3, the polarizer alone is not sufficient to enhance the contrast of the CCW and CW shapes, where the hysteresis contrast $|C_d|$ of the CCW and CW are less than 7 dB. Additionally, each hysteresis shape requires a rotation angle of the polarizer to transform the optical-power hysteresis curve into CCW, CW, and butterfly shapes.

4.4 Optimized Hysteresis-Shape Transformation

The contrast $|C_d|$ can be optimized to exceed 20 dB for CCW and CW shapes by using a polarization controller before the polarizer as depicted in Fig. 4.8. This controller, shown here as consisting of three waveplates in series, alters the SOP of the BPR signal from the nonlinear photonic resonator for optimal passage through the linear polarizer. Additionally, the transformation of the optical-power hysteresis curve into CCW, CW, and butterfly shapes can be achieved without changing the rotation of a linear polarizer. Optimization of the hysteresis shape is carried out by only adjusting the waveplate-orientation angles for a fixed polarizer-orientation angle $\theta$. 
Figure 4.8 Schematic of using a polarization controller to optimize the transformation of the hysteresis shape through a linear polarizer. The z-axis positions \( k = \{b, c\} \) are at the output of the resonator and the input to the polarizer, respectively, and occur at either end of the polarization controller.

4.4.1 Polarization Controller

We consider a Lefevre’s three-loop fiber-optic polarization controller [36], comprised of a quarter-wave plate \( Q_1 \), a half-wave plate \( H \), and another quarter-wave plate \( Q_2 \) in series as shown in Fig. 4.8. These waveplates alter the phase components of the electric field with the assumption that no loss is introduced. We use the standard Jones-matrix model of this polarization controller, included here for completeness.

The polarization controller varies the input SOP \( J_b(t) \) from the nonlinear photonic resonator to a new SOP \( J_c(t) \) for injection into the linear polarizer as described by

\[
J_c(t) = Q_2 \ H \ Q_1 \ J_b(t).
\]

The Jones matrix \( W_\phi \) for a wave plate that is aligned with respect to the x-axis is given by:

\[
W_\phi = \begin{bmatrix}
\exp(-i\frac{\phi}{2}) & 0 \\
0 & \exp(i\frac{\phi}{2})
\end{bmatrix}.
\]
The Jones matrix for the half waveplate \( H \) oriented at an angle \( \sigma_2 \) is given by [68]:

\[
H = R(-\sigma_2) \, W_\pi \, R(\sigma_2), \quad (4.30a)
\]

\[
H = \begin{bmatrix}
\cos(2\sigma_2) & \sin(2\sigma_2) \\
\sin(2\sigma_2) & -\cos(2\sigma_2)
\end{bmatrix}, \quad (4.30b)
\]

The Jones matrix \( H \) of a half wave plate (HWP) is obtained when \( \phi \) equals \( \pi \) in Eq. (4.29).

The Jones matrix \( Q_u \) for each quarter-wave plate \( u = \{1, 2\} \) is obtained when \( \phi = \frac{\pi}{2} \) in Eq. (4.29), where \( Q \) is oriented at angle \( \sigma_u \) with respect to the x direction: [68]:

\[
Q_u = R(-\sigma_u) \, W_{\pi/2} \, R(\sigma_u), \quad (4.31a)
\]

\[
Q_u = \begin{bmatrix}
\cos^2(\sigma_u) - i \sin^2(\sigma_u) & a_{12} \\
\sin^2(\sigma_u) - i \cos^2(\sigma_u) & a_{21}
\end{bmatrix}, \quad (4.31b)
\]

where \( a_{12} = a_{21} = \cos(\sigma_u) \sin(\sigma_u) + i \cos(\sigma_u) \sin(\sigma_u) \).

Eq. 4.28 produces a new amplitude-distribution angle \( \gamma_c \) and phase difference \( \delta_c \) at \( k = c \) as depicted in Fig. 4.8 that are fed into the generalized Malus’ Law Eq. 4.14 to perform the transformation of the hysteresis shapes.

### 4.4.2 Optimization Process

The contrast of the CCW hysteresis shape can be greatly enhanced by using a polarization controller before the linear polarizer. An optimization technique is modeled here that is deliberately similar to the steps taken in the laboratory setting — the waveplate orientation angles are varied one at a time and signal exiting the polarizer is viewed. The optimization process is depicted in the flowchart in Fig. 4.9.

The first step is to set initial conditions for the waveplate orientation angles and the
Set initial orientation angle for waveplates ($\sigma_1, \sigma_2, \sigma_3$) and polarizer ($\theta$) and polarizer orientation angle; optimization is illustrated using $\sigma_1 = 0^\circ$, $\sigma_2 = 15^\circ$, $\sigma_3 = 30^\circ$, and $\theta = 45^\circ$ and for the example BPR signal shown in Fig. 2.3. The polarizer $ER_{dB} = 26$ dB, and $IL_{dB} = 0.55$ dB are chosen to match laboratory conditions. The output-power hysteresis curve shape at location $k = d$ is observed to be a CW hysteresis shape with a contrast of $C_d = -2.9$ dB, as shown in the upper left corner of Fig. 4.9. Next, the desired hysteresis shape is decided; the CCW shape is selected for the example in Fig. 4.9.

The optimization process then enters the phase of varying the angle of each waveplate,
one waveplate at a time, for multiple iterations until the desired shape, power, and contrast is obtained. Overall, all-hysteresis shapes can be achieved with less than \( j = 20 \) iterations of changing the waveplates orientation angles, a similar number of iterations found in the laboratory process of manually changing the waveplates.

Fig. 4.9 shows the data for the first iteration, \( j = 1 \). First, the orientation angle \( \sigma_1 \) of the waveplate \( Q_1 \) is varied over its 180° and the center point of the rising-branch \( P^R \) (green-line) and of the falling-branch \( P^F \) (black-line) are observed. The CCW shape occurs when falling branch point is higher than the rising-branch point; this occurs in the figure inset for \( \sigma_1 \) orientation angles less than 40° and greater than 130°. The value of \( \sigma_1 = 18° \) is chosen because it gives the desired shape with high power in falling-branch \( P^F \) and low rising-branch center point value that yields a relatively large contrast of 7.8 dB.

The orientation angle \( \sigma_2 \) of the \( H \) waveplate is then varied in the same manner as discussed for \( \sigma_1 \). The \( j = 1 \) data shown in the figure inset reveals that the hysteresis curve mostly remains in a CCW shape over the range of \( \sigma_2 \), and increases contrast when the rising-branch point drops to low power. \( \sigma_2 = 85.55° \) provides a contrast of \( C_d = 11.1 \) dB as shown in Fig. 4.9. The orientation angle \( \sigma_3 \) of the \( Q_2 \) waveplate is then varied in the same manner as discussed for \( \sigma_1 \) and \( \sigma_2 \). \( \sigma_3 = 35.2° \) yields a contrast of \( C_d = 12.2 \) dB as shown in Fig. 4.9.

After adjusting each waveplate, the shape, contrast, and power are checked for their desired states. The power is optimized by ensuring the highest in falling-branch \( P^F \) with the lowest power in the rising-branch \( P^R \). At the end of the illustrated \( j = 1 \) iteration, the CCW is achieved, but the contrast \( C_d = 12.2 \) dB is still low and the power of the falling-branch \( P^F \) is also still low; this case is shown near the bottom left of Fig. ???. Since the desired state has not been achieved, a second \( j = 2 \) iteration of varying the waveplate orientation angles is enacted. To obtain the final shape shown in the bottom inset of Fig. 4.9, 15 iterations where required.

In the final shape, a CCW is achieved with high contrast and the highest power in the
falling-branch $P_d^F$. Overall, the high contrast CCW shape is achieved by optimizing the SOP at the lower-branch of the bistable curve to be blocked by the polarizer using the polarization controller PC, and at the same time changing the PC is also optimizing to allow SOP at the upper-branch to pass through the polarizer with the highest power possible.

### 4.4.3 High-Contrast CCW Hysteresis Shape

The CCW hysteresis shape with contrast $C_d = 28.4$ dB is shown in Fig. 4.10(d), where the optimized waveplates angles are $\sigma_1 = 41.15^\circ$, $\sigma_2 = 10.5^\circ$, and $\sigma_3 = 149^\circ$. This hysteresis shape is the product of the initial hysteresis shape shown in Fig. 2.3 and the transmittivity shape shown in Fig. 4.10(c); at the center of the hysteresis, $C_b + C_T = C_d$.

The corresponding amplitude-distribution angle $\gamma$ and phase shift $\delta$ at location $k = c$ are shown in Fig. 4.10(a) and (b), respectively. Each exhibit a hysteresis curve. Note that the center point of the lower branch has $\delta_R^c = \pi$ and $\gamma_R^c = 45.0^\circ$; corresponding to linear

![Figure 4.10](image-url)
polarization with an angle perpendicular to the polarizer orientation angle $\theta$. Thus, the center point of the lower branch is thus optimally blocked by the polarizer; indeed, the transmittivity $T^R_c = 0.003$ at the center of the lower switching branch and can be seen to be very small in Fig. 4.10(c).

As the SOP varies away from $SOP_R$ along the hysteresis curve, the optical increasingly passes through the polarizer. At the center point of the falling branch, $\gamma^F_c = 39.82^\circ$ which corresponds to a nearly linear SOP, and corresponds to a high transmittivity $T^F_c = 0.85$. The contrast of the transmittivity $C_T = 25.15$ dB. The high-contrast S-shape hysteresis curve is shown in Fig. 4.10(d) based on Eq. (4.25).

### 4.4.4 Impact of ER and IL on the Transmittivity Contrast & Power

The transmittivity $T$ hysteresis curve in Fig. 4.10 and throughout the paper use the specific values of $ER_{dB} = 26$ dB and $IL_{dB} = 0.55$ dB, selected to match laboratory components. The transmittivity curves depend significantly on extinction ratio and insertion loss, as illustrated in Fig. 4.11(a) for the case of a CCW shape.

![Figure 4.11](image)

**Figure 4.11** Impact of ER and IL on the (a) transmittivity contrast $C_T$ and (b) upper-branch transmittivity $T^R$. The orientation angles are $\sigma_1 = 41.15^\circ$, $\sigma_2 = 10.5^\circ$, $\sigma_3 = 149^\circ$ for the waveplates, and $\theta = 45^\circ$ for the polarizer.

The extinction ratio ER comes into play especially for low-transmittivity values and prevents the transmittivity from reaching the value of zero. For the case illustrated in
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Fig. 4.11(a), as the ER decreases to 20 dB, the contrast lowers to 19.473 dB for $IL_{dB} = 0.55$ dB. The insertion loss has a lesser effect on the transmittivity contrast $C_T$. Fig. 4.11(b) shows three cases of insertion loss, with slight separation at lower ER values. For example, for the case of having $ER = 26$ dB, increasing the IL from $IL = 0.1$ dB to $IL = 1.55$ dB, the contrast is reduced by 0.66 dB.

In addition to high contrast, it is also important to achieve a high transmittivity of the upper branch $T^F$. The variation of $T^F$ is illustrated in Fig. 4.11(b). There is minimal dependence on ER but a noticeable dependence on IL. The $T^F$ curve basically scales with the value of IL, where when IL increases the transmittivity $T^F$ drops.

### 4.4.5 High-Contrast CW Hysteresis Shape

The optimization process for forming the CW hysteresis shape follows the general steps discussed in Section 4.4.2 and shown in Fig. 4.9, but seeks the rising branch point to be higher than the falling branch point. The waveplate orientation angles are optimized until a CW shape is formed with a high transmittivity contrast $|C_T| = 24.28$ dB for $\theta = 45^\circ$, this yields $\sigma_1 = 142.8^\circ$, $\sigma_2 = 1.7^\circ$, and $\sigma_3 = 173.4^\circ$.

These waveplates orientations yield a field-amplitude distribution angle $\gamma_c^F = 45.0^\circ$ and phase difference $\delta_c^F = \pi$ at the center of the falling branch as shown in Fig. 4.12(a). Thus, the SOP of on the falling branch is linearly polarized and optimally blocked by the polarizer whose orientation angle $\theta = 45^\circ$. The transmittivity is 0.0024 at the center of the upper branch to block SOP$_c^F$. At the center of the lower branch, the SOP has rotated enough to yield $T_c^R = 0.632$, corresponding to a transmittivity as shown in Fig. 4.12(c). The high-contrast CW shape with contrast of $C_d = 21.03$ dB is shown in Fig. 4.12(d).

### 4.4.6 Butterfly Hysteresis Shapes

The polarization controller can also be used to yield the downward-switching and upward-switching butterfly hysteresis shapes. As seen in Fig. 4.6 for in the case of rotating polarizer
Figure 4.12 A CW hysteresis shape exhibiting a high contrast $C_d = -21.03$. The (a) amplitude-distribution angle $\gamma$ and (b) output phase difference $\delta$ exciting the polarization controller produce (c) a transmittivity that transforms the initial CCW shape to the high-contrast CW shape. (d) The transmitted power through the polarizer. Configuration: $\sigma_1 = 142.8^\circ$, $\sigma_2 = 1.7^\circ$, $\sigma_3 = 173.4^\circ$, $\theta = 45^\circ$, $ER_{dB} = 26$ dB, and $IL_{dB} = 0.55$ dB.

orientation angle, butterfly hysteresis occur during the transition between CW and CCW hysteresis shapes. Butterfly shapes are remarkable in that there is an input power within the hysteresis region that corresponds to the same output power. Butterfly shapes can be identified by varying the orientation angles and tracking the center point of the rising and falling branches, as seen in the right-hand-side insets of Fig. 4.9; the variation of $\sigma_1$, for example, reveals 4 such cross points of the center points.

Examples of achieving both downward-switching and upward-switching butterfly shapes by means of adjusting the waveplate orientation angles are shown in 4.13(b) and (d), respectively. For the cases shown, the downward-switching shape is achieved using $\sigma_1 = 170.1^\circ$, $\sigma_2 = 0^\circ$, and $\sigma_3 = 179^\circ$, and the upward-switching shape is achieved using $\sigma_1 = 119^\circ$, $\sigma_2 = 2^\circ$, and $\sigma_3 = 6.5^\circ$. The corresponding transmittivity curves are shown in part (a) and (c). Note that the transmittivity contrast $T_c$ for each case is the correct magnitude and sign to balance out the contrast of total optical power exiting the nonlinear resonator $C_b$. In this manner, the power of the center point is forced to be the same on each hysteresis
Figure 4.13 (b) Downward-switching and (d) upward-switching butterfly hysteresis shapes achieved by adjusting the waveplate orientation angles. Parts (a) and (c) show the corresponding transmittivity hysteresis curve; the contrast $T_c$ for the each case exactly balances out the the contrast $C_b$, forcing the center point on each branch to have the same output power. Configurations: $\theta = 45^\circ$, $ER_{dB} = 26$ dB, and $IL_{dB} = 0.55$ dB. For downward switching, $\sigma_1 = 170.1^\circ$, $\sigma_2 = 0^\circ$, $\sigma_3 = 179^\circ$. For upward switching, $\sigma_1 = 119^\circ$, $\sigma_2 = 2^\circ$, $\sigma_3 = 6.5^\circ$.

branch, and a butterfly hysteresis shape is formed.

4.5 Conclusion

We have mathematically modeled a new technique of hysteresis-shape transformation by using a non-ideal linear polarizer to transform the optical-power hysteresis shape for bistable systems whose signal SOP varies along the hysteresis curve.

The mathematical model provides a deep understanding of how a non-ideal linear polarizer is capable of changing the hysteresis shape. A key component of this model is a generalized Malus’s law, applicable for elliptical SOPs and the practical consideration of the ER and IL of the linear polarizer. It is shown that the transmittivity of the linear polarizer itself exhibits a hysteresis curve and that the point-by-point product of this transmittivity curve and total optical power of the BPR signal produces the output power hysteresis curve with its myriad shapes.
For the CW and CCW hysteresis shapes, high contrast is achieved by using the polarization controller to align the center-point SOP on one of the two branches to the block-axis of the polarizer. A contrast of 20 dB is easy to achieve. For the butterfly hysteresis shapes, the contrast of the center-point transmittivities exactly balances the contrast of the optical power exiting the nonlinear resonator; consequently, the center-point optical powers are the same for each bistable branch.
5. Hysteresis-Shape Transformation: Lab Demonstration

5.1 Introduction

Optical bistability has been studied widely as a means of achieving a range of sequential and combinational all-optical signal processing applications, such as optical flip-flops [37]–[41], gate and packet switches [47], [48], parity checkers [72], and AND gates [43]–[45]. In this chapter, we experimentally demonstrate a new control mechanism to produce a diversity of bistable hysteresis shapes. As depicted in Fig. 5.1, this new technique controls the hysteresis shape downstream of the nonlinear photonic resonator that generates the bistable signal; this shape control is performed without any signal sent upstream to the bistable system. The basic functional blocks are as follows: the input optical-signal generator produces the optical signal that is injected into the nonlinear resonator, which in turn generates the bistable optical (BPR) signal. After a fiber distance of $L$, the optical signal encounters the downstream hysteresis-shape controller wherein the optical signal is controlled to produce a variety of hysteresis shapes. As will be shown, the CCW shape, the CW shape, and butterfly shapes are all selectable without changing any condition of the bistable system (such as drive current) or input optical signal (such as wavelength).

Several features demonstrated here are particularly useful for all-optical signal processing applications. First, all hysteresis shapes exhibit the same switching input powers, whereas
previous techniques of realizing different hysteresis shapes required different switching input powers for different shapes [15], [23], [49], [51], [55]. This means that AND and XOR gates, for example, can be driven using the same-power control pulses. Second, the CCW and CW shapes are produced with switching contrasts at or exceeding 20 dB, which are substantially higher contrast values than shown by previous demonstrations (less than 10 dB) [11], [12], [16], [17], [23], [49]–[55]. Third, shape diversity is available using the transmission port of the nonlinear photonic resonator, unlike most previous geometries based on dispersive optical bistability that exhibit shape diversity only in the reflection port [12], [15], [49], [51], [55]; the transmission port may lend itself to a more straightforward concatenation of photonic-logic gates.

5.2 Experimental Set-Up

The experimental setup is conducted in the laboratory as shown in Fig. 5.2. There are main four stages experimental set-up—Input optical-signal generator, A BPR signal generation, producing a diversity of hysteresis shapes, and diagnostics.

Input optical-signal generator is designed to produce input optical signal with a sinusoidal fashion, where a tunable laser is used to generate a continuous wave optical signal. Then a
Figure 5.2 (a) Experimental set-up of downstream hysteresis-shape control using a Fabry-Pérot semiconductor optical amplifier (FP-SOA) as the nonlinear resonator: TLS = tunable laser source; PC = polarization controller; SG = RF signal generator; MZM = Mach-Zehnder modulator; TE = transverse electric; TM = transverse magnetic; PLZR = polarizer; EDFA = erbium-doped fiber amplifier; OTF = optical tunable filter; OSA = optical spectrum analyzer; PLR = polarimeter; PD = photodiode.

Mach-Zehnder modulator (MZM) is used to modulate the continuous wave optical signal with a signal generator (SG) driven in a sinusoidal fashion at 0.5 MHz. To optimize the output power through the MZM, polarization controller PC1 is used to align input TE SOP into the MZM to experiences a 4 dB insertion loss.

To generate the bistable signal (BPR), the FP-SOA is driven with 62.5 mA driving current (equivalent to 97% of the lasing threshold). The MZM output optical signal is injected into the FP-SOA, where polarization controller PC2 is used to optimize the input SOP to be linearly polarized with 33% of the optical power injected into the transverse-magnetic (TM) mode of the FP-SOA, where the input optical signal power is -7.8 dBm at peak.

To generate diverse hysteresis shapes, the bistable optical signal (BPR) is injected into the downstream hysteresis the controller (HSC), where the HSC is located after a fiber distance of (L = 9 m). The DSC passive system consistent from two optical components—a polarization control PC4 followed by a linear polarizer (PLZR) in series. The mechanism of producing diverse hysteresis shapes will be discussed in detail in the following results section.

In the diagnostics stage, the 50/50 optical splitter located at the upstream location of the FP-SOA is used to send the optical signals through the monitor port to be measured.
The output signals are boosted with an Erbium-doped fiber amplifier (EDFA) that provides 16.2-dB gain for the optical-signal wavelength of 1607.09 nm used in our demonstrations.

As consequences of the amplification process, optical noises are generated in the EDFA [10]. Thus, an optical tunable filter (OTF) with 1.2-nm-bandwidth following the EDFA is centered on the signal wavelength to remove out-of-signal optical noise, named as amplified spontaneous emission (ASE). Both input and output temporal powers are measured using 23-GHz photodiodes and a 1-GHz oscilloscope.

5.3 Polarization-Mode-Resolved Hysteresis Curves

This section focuses on discussing the obtained bistable hysteresis behavior at the output of the FP-SOA. The dispersive bistable curve forms based on an interferometric resonance shift onto and off the optical signal when the input optical power changes into the nonlinear resonator. To achieve that, the input optical signal wavelength is tuned to the long-wavelength side of a Fabry-Pérot resonance, as shown in Fig. 5.3(a); four Fabry-Pérot resonances in the figure are labeled with "R" and are evident due to the ASE generated by the SOA. Additionally, the input signal is modulated to exhibit a sinusoidal fashion to change the input power strength into the FP-SOA from rising input power to falling input power.

Thus, when the input power increases, the optical signal saturates the carrier density and thus increases the refractive index. Consequently, the interferometric resonance shifts towards the optical-signal wavelength, which leads to a general increase in output power (rising input power). As the resonance snaps onto the signal wavelength, the output power switches to its higher branch. The signal remains on the upper branch of the hysteresis curve until the input power is decreased (falling input power). As the input power falls, the optical signal allows the carrier density to recover and thus decreases the refractive index. The resonance then shifts away from the optical signal wavelength, and as it snaps off of the optical signal, the output power switches to its lower hysteresis branch.
By temporarily removing the downstream hysteresis-shape controller components from the experimental set-up, the spectral and temporal data are measured directly after the FP-SOA as shown in Figs. 5.3(a) and 3(b), respectively.

Since there are two sets of interferometric resonances, one set belonging to the TE polarization mode of the nonlinear resonator and one set belonging to the TM polarization mode, where these two exhibit different strengths and different spectral locations with respect to each other resonances due to Birefringence and gain anisotropy effects. By launching the optical signal into both polarization modes, its orthogonal components experience different variations in optical power and phase due to the anisotropic resonances, yielding
the demonstrated polarization rotation during dispersive optical bistability [75].

To investigate the unique bistable curve in each polarization mode of the nonlinear resonator, the downstream polarization controller PC4, polarizer, and OSA is first used to identify the FP-SOA polarization modes. By rotating PC4, it is readily found that the FP-SOA ASE can be polarization-resolved into two unique sets of resonances, one set for each FP-SOA polarization mode. In this manner, the TM spectrum is identified as shown in Fig. 5.3(c) with four of the resonances labeled with $R_{TM}$. This figure also shows the unmodulated optical signal sent through the FP-SOA with PC2 set to provide a linear SOP with 33% of its power injected into the TM mode. In all cases shown in Sections 5.3 and 5.4, the small-signal optical signal wavelength is detuned 0.112 nm to the long-wavelength side of a TM resonance. The detuning from a $TE$ resonance is different because of birefringence, as discussed below.

The hysteresis shown in Fig. 5.3(d) is the TM-resolved portion of the hysteresis curve of total power shown in Fig. 5.3(b). Like the latter, it exhibits the canonical CCW shape, indicating that the TM resonance shifts onto the optical-signal wavelength in the manner discussed in the first paragraph of this section. However, its switching contrast of 5.1 dB is noticeably better than that of the initial hysteresis contrast of 2.0 dB shown in Fig. 5.3(b).

The TE spectrum is found by rotating PC4 to reveal the other set of FP-SOA spectra, highlighted in Fig. 5.3(e) using four resonances labeled with $R_{TE}$; Fig. 5.3(e) also shows the unmodulated optical signal sent through the FP-SOA with PC2 set to provide a linear SOP with 67% of its power injected into the TE mode. The high sensitivity of the OSA is essential for finding the weak resonance peaks of the TE spectrum. The different ASE power levels of the TM and TE resonances shown in Figs. 5.3(c) and 5.3(e) indicate gain anisotropy; specifically, the TM-mode gain is stronger than that of the TE mode [75]. Moreover, their relative displacement along the wavelength axis indicates birefringence; specifically, the spectral difference between peaks of the TE and TM resonances is 0.054 nm; (which corresponds to a 30% offset in terms of the 0.18-nm free-spectral range). For
this amount of birefringence, the small-signal optical signal wavelength is detuned 0.058
nm to the long-wavelength side of a TE resonance.

The hysteresis curve shown in Fig. 5.3(f) is the TE-resolved portion of the hysteresis
curve shown in Fig. 5.3(b). It exhibits a CW shape, an inverted form as compared to the
TM-resolved hysteresis. The inverted nature of this shape indicates that a TE resonance
shifts away from the optical-signal wavelength as the TM resonance shifts onto the signal;
this shifting-away process ultimately drives the optical power of the TE signal downward.
Thus, the TM resonance is the dominant contribution to the realization of dispersive optical
bistability.

The difference in behavior between the TE- and TM-resolved hysteresis curves clearly
demonstrates that NPR occurs during optical bistability. Indeed, the difference of the
optical power in the TE and TM modes, which varies significantly upon switching between
hysteresis branches, defines the $s_1$ Stokes parameter; thus, the SOP is shown to vary in a
bistable way.

### 5.4 Hysteresis-Shape Diversity

The simultaneous occurrence of nonlinear polarization rotation and dispersive optical bista-
ilarity as shown in Figs. 5.3(d) and 5.3(f) is leveraged in this section to realize a variety of
hysteresis shapes. Hysteresis-shape transformation is experimentally investigated, where
diversity of hysteresis curves have demonstrated to produce a counter-clockwise shape
(S-Shape), a clockwise shape (Inverted S-shape), a downward-switching butterfly shape,
and an upward-switching butterfly shape as shown in Fig. 5.4. The downstream hysteresis
shape controller (HSC) is used to form all these diverse hysteresis shapes from the same
BPR signal that is produced from the nonlinear resonator. In addition to using the new HSC
control mechanism to produce diverse hysteresis shapes, the HSC is used to enhance the
switching contrast of S-shape and inverted S-shape to be 20 dB and 21 dB respectively.
The functionality of the downstream shape controller is enabled based on having bistable branches with different SOPs due to NPR in the BPR signal exiting the nonlinear resonator\[46\]. The first common hysteresis shape is the hysteresis S-shape shape as shown in Fig. 5.4(a). The formation of high contrast CCW shape is traversed first by upward switching followed by downward switching, where the contrast is improved by using PC4 to orient the SOP in the middle of the lower branch of the hysteresis curve exiting the FP-SOA to be blocked by the linear polarizer.

The inverted S-shape is traversed first by downward switching followed by upward switching as shown in Fig. 5.4(b). The contrast is enhanced by using PC4 to orient the SOP in the middle of the upper branch of the hysteresis curve exiting the FP-SOA to be blocked by the linear polarizer.

**Figure 5.4** Demonstration of Hysteresis-Shape Control via the downstream shape controller depicted in Figs. 1 and 2. All shapes occur for the same bistable action occurring within the nonlinear resonator. The (a) high-contrast (20-dB) counter-clockwise shape is obtained by blocking the SOP at the center of the bottom bistable branch. The (b) high-contrast (21-dB) clockwise shape is obtained by blocking the SOP at the center of the upper bistable branch. The (c) downward-switching and (d) upward-switching butterfly shapes are obtained by blocking the two branches in a manner that matches the optical power near the middle of the input switching-power range.
The butterfly shapes are demonstrated in Fig. 5.4(c) and Fig. 5.4(d), the crossing point in the center of the switching region has the same output power for both upper and lower stable branches. There is a downward switching formation of the butterfly shape in Fig. 5.4(c) is traversed first by downward switching and followed again by downward switching. Additionally, there is an upward switching formation in Fig. 5.4(d) is traversed first by upward switching and followed again by upward switching.

The formation of each shape in Fig. 5.4(c) and (d). The crossing point is achieved to exhibit the same power for both branches by leveraging the different SOP exhibited by the two stable branches of the BPR signal shown in Fig. 5.3(b). Specifically, PC4 is oriented to block the SOP of the upper branch more than the SOP of the lower branch. where the center point SOP got the same passing power through the polarizer. This forming mechanism of a diversity of hysteresis shapes is simple and only requires two linear polarization components downstream of the nonlinear resonator (bistable system) without acting on the bistable system. Each path can have its own hysteresis shape controller system, then each path can require a hysteresis shape (e.g. S-shape or inverted S-shape) independently from the other path. This could be useful to operate different all-optical logic gates from the same BPR signal.

5.5 Conclusion

We have experimentally demonstrated diverse hysteresis shapes including S-shape (CCW), inverted S-shape (CW), and butterfly shapes using the shape selection technique downstream of the nonlinear resonator (transmission).

Since all-shapes are originated from the same upstream BPR signal, all shapes exhibit the same same switching input powers. It is important to emphasize that the new control mechanism of forming diverse hysteresis shapes uses only linear polarization components and it works downstream of the nonlinear resonator. The functionality of the DSC is
enabled because the bistable branches of the bistable signal have different SOPs due to NPR. Moreover, the shape selection technique is used to enhance the bistable switching contrast to 20 and 21 dB for the S-shape and inverted S-shape, respectively.

This new control mechanism is demonstrated using FP-SOA as a nonlinear photonic resonator and it is applicable to study another nonlinear photonic resonator. The new control mechanism provides potential features to be used to enhance the performance of future combinational and sequential all-optical signal processing applications.
6. Concluding Remarks

6.1 Summary of Main Contributions

6.1.1 New Physical Behavior

We have mathematically modeled and experimentally demonstrated a new kind of optical memory behavior that expands the traditional well-known optical memory concept. The common concept of optical memory produced by a nonlinear photonic resonator is based on a hysteresis curve of optical power, where the bistable branches are different in optical power. In our work, an optical state of polarization (SOP) exhibits a hysteresis curve. The bistable branches are different in polarization, where each point on the bistable curve has its own SOP. This bistable action relies on the new physical process of having a simultaneous occurrence of two nonlinear phenomena—dispersive optical bistability and nonlinear polarization rotation. We refer to the simultaneous occurrence of these nonlinear processes as bistable polarization rotation (BPR). Previously, these nonlinear processes have only been investigated separately as two isolated nonlinear optical phenomena [11]–[29].

I measure the BPR signal using the optical Stokes parameters. An orthogonality of 30% is measured in the bistable hysteresis, it achieved based on optimizing two aspects of the input optical power conditions. First, the input optical power imbalance into the polarization modes of the nonlinear resonator is optimized to obtain the highest effect of gain and loss anisotropy. Second, input optical signal wavelength (spectral selection) is optimized to obtain the highest effect of the small-signal birefringence between the polarization modes of
CHAPTER 6. CONCLUDING REMARKS

the nonlinear resonator.

The presented mathematical model provides a deeper understanding of lab results. The modeling results are obtained by using consistent values from the lab conditions. The ability to generate a bistable polarization-rotating (BPR) signal opens up new capabilities for optical signal processing applications.

6.1.2 New Optical Control Capability

We have demonstrated a simple and flexible technique to perform hysteresis-shape transformation. Until now, control mechanisms of the hysteresis shape have only been demonstrated by acting on the bistable system itself, where it has been performed by changing either an injected wavelength of the optical signal or an injected drive current. As such, acting on the bistable system means that previous techniques of realizing different hysteresis shapes required different switching input powers for different shapes [15], [23], [49], [51], [55]. Another issue with previous work is that the demonstrated contrast is less than 10 dB [11], [12], [16], [17], [23], [49]–[55]. Furthermore, most previous dispersive optical bistability that exhibits shape diversity is only demonstrated in the reflection port [12], [15], [49], [51], [55]; the transmission port may lend itself to a more straightforward concatenation of photonic-logic gates.

In our work, the control mechanism is achieved downstream of the nonlinear resonator by only using linear-polarization components with no signal sent back into the bistable system. A diversity of hysteresis shapes includes a high-contrast CCW shape, a high-contrast inverted CW shape, and two butterfly shapes. We provide a mathematical model for understanding the hysteresis-shape transformation mechanism by using a linear polarizer. Critical to our model is the generalized Malus’s law, applicable for input elliptical SOPs and the practical consideration of the imperfections of the linear polarizer (ER and IL).

There are several features of this hysteresis-shape selection mechanism that promise to enhance the performance of combinational and sequential optical-signal processing:
• All hysteresis shapes exhibit the same switching input powers, allowing optical-
logic gates based on these hysteresis curves to all be driven using the same optical
control-signal power (i.e., shape-agnostic control power).
• All hysteresis shapes are formed with optimum contrast, where the contrast is en-
hanced for the CCW shape and CW shape hysteresis curves exceeding 20 dB; such
contrasts are at least 10 dB larger than previously demonstrated contrasts from pho-
tonic resonators.
• All-hysteresis shapes are created using the transmission port of the photonic resonator;
the transmission port lends itself to the concatenation of photonic logic gates.
• The shape-selection mechanism occurs downstream of the nonlinear photonic res-
onator and without sending a control signal to the photonic resonator; this allows the
ability of distribution and simultaneous local control of the optical hysteresis shape.

6.2 Future Work

The experiments are carried out by using a Fabry-Perot semiconductor optical amplifier
(FP-SOA) as the nonlinear resonator. However, the FP-SOA used in our experiments was not
designed for the purpose of generating a bistable polarization-rotating signal with optimum
NPR. It may be possible to realize a larger BPR using a device specifically designed for
this purpose. In future work, the reported mathematical model could be used to design the
device to exhibits larger orthogonality beyond the measured 30%. This design would be
used to study the impact of device scaling parameters such as the anisotropic gain, loss, and
nonlinearity

Furthermore, the presented model and experimental methods in our work are applicable
to investigate the BPR in other structures. In fact, the model is readily applicable to SOA-
embedded dual-bus ring resonators by substituting the Fabry-Pérot reflectively $R \to 1 - e'$,
where $e'$ is a power-coupling coefficient and the length $L$ found throughout the model
is interpreted as half the ring circumference. Moreover, the model and methods can be extended to investigate BPR in other nonlinear media (such as Kerr media) and other photonic resonators, such as photonic-crystal and vertical cavities.

Furthermore, although our laboratory investigation uses a Fabry-Pérot cavity as the nonlinear resonator, the downstream hysteresis-shape transformation can be investigated using other resonator structures such as ring resonators, vertical cavities, and distributed feedback structures. Likewise, although our investigation used the linewidth enhancement factor of an SOA as the nonlinearity, other nonlinear effects, such as the Kerr nonlinearity, can be used to realize shape diversity.

Our demonstrations provide potential features for all-optical signal processing gates that could be key devices for future optical computing [76], [77]. Our work of leveraging the BPR signal to realize a new hysteresis-shape transformation technique enables the functionally of having distribution and simultaneous local of the hysteresis shapes.

Such hysteresis shapes are the canonical counter-clockwise (CCW) shape, the "inverted" clockwise (CW) shape, and butterfly shapes. The different switching behavior exhibited by the CCW and CW hysteresis shapes supports the functionality of different all-optical gates & applications. The CCW shape supports, for example, optical square-wave clocks [73], optical packet switches [48], AND gates [44], [45], and the $Q$ output of all-optical flop-flops [18], [37], [38], [41]. The CW shape supports, for example, XOR gates [78] and the inverted $\overline{Q}$ output of all-optical flip-flops [40]. Applications of the butterfly shapes have yet to be demonstrated.

The BPR and hysteresis-shape transformation technique can serve to enhance the performance of sequential all-optical gates & applications (requires an optical memory process), including optical flip-flops [18], [37], [38], [40], [41], optical-gate switches [47], packet switches [48], and square-wave optical clocks [73]. The digital response of bistable hysteresis curves is also applicable to combinational gates & applications such as parity checkers [72], AND gates [44], [45], and adders [79].
Recently, I have experimentally demonstrated that a BPR signal can be simultaneously sent into two locations A and B, where each location has its own local hysteresis-shape transformation unit [80]. Thus, at each location, there is a simultaneous and independent local control to produce the required hysteresis shape for given all-optical gates and applications. The ability of local distribution and local control of hysteresis shapes in multiple locations could be used to support functionality of all-optical counter [76], [77], where an optical counter requires an optical memory flip-flop gate and a combinational optical AND gate.

Another example of an optical device is an optical half adders. The performance of this device could be potentially enhanced by using our recent demonstration of local hysteresis-shape transformation [80]. The optical half adder requires an optical XOR gate and an optical AND gate to operate as shown in Fig.6.1(a). Input A and input B are the input optical signals (pulses). The XOR gate performs the binary sum between these input signals while the AND gate performs the carry of the binary addition. The truth table of the half adder is shown in Fig. 6.2(b). For example the operation concept of an optical AND gate can be based on the interaction between a nonlinear resonance the the optical signal wavelength as discussed in [45].

![Figure 6.1: All-optical half adder (a) Half-adder diagram. (b) Truth table.](image)

The series or cascaded all-optical gates could be potentially used to support the operation of an optical circuit for future optical computing. Our research provides potential features to support optical computing the using simultaneous local control of hysteresis shapes in...
multiple locations to support the operation of several optical logic gates using a single nonlinear medium.

With an eye towards integration on photonic interacted circuits (PICs), semiconductor optical amplifiers (SOAs) are attractive nonlinear devices for all-optical signal processing gates and applications [77]. We have used FP-SOA (InGaAsP gain region) as the nonlinear photonic resonator in our work, where FP-SOA is compatible to be integrated with advanced photonic integrated circuits (PICs). Our presented models and methods can be extended to realize nonlinear optical applications to be realized on a photonic integrated circuit chip, where both the nonlinear resonator (bistable system) and the hysteresis shape controller could be integrated to support the functionality of all-optical signal processing applications.
Bibliography


BIBLIOGRAPHY


A. Appendices: Laboratory Procedures

A.1 Procedure to Optimize, Measure and Calibrate an Input State of Polarization (SOP) into a Photonic Nonlinear Resonator

To generate the BPR signal the input state of polarization should be injected into both of the polarization modes of the nonlinear resonator. The following procedures discuss how to optimize, measure, and calibrate the input SOP into the FP-SOA as the photonic nonlinear resonator.

- Turn on only the FP-SOA (without turning on the optical signal).
- Use the polarization controller after the FP-SOA to align the FP-SOA TM-mode ASE to solely exits, one of the two orthogonal output ports of the polarization beam splitter (PBS). The TM mode is chosen because it the dominant stronger mode than the TE mode.
- To achieve that, after changing the optical spectrum analyzer (OSA) setting for continuous sweep mode, this setting provides real-time monitoring while changing the PC in the front of the PBS, then precisely adjust the PC until only the TM mode is passed through the PBS; the OSA is used as the monitor during this optimization process because its high sensitivity can detect the weak ASE power (<50 dBm).
- To sanity-check the process and make sure only the TM mode is passed through one of two orthogonal ports of the PBS; TM-mode alignment process is also helped to
align the other orthogonal TE-mode ASE to the other output port of the PBS, thus it is important to check the other output of the other port in the OSA and verify that only the TE-mode ASE exiting the other port, an example is shown for TM and TE ASE spectrum in Fig. A.1.

![Polarization-resolved small-signal ASE spectra for TE and TM modes of the FP-SOA.](image)

**Figure A.1** Polarization-resolved small-signal ASE spectra for TE and TM modes of the FP-SOA.

- Then, turn on the input optical signal with no RF, so it is only continuous wave (cw) from the laser.
- Use the polarization controller PC in the front of FP-SOA to optimize and the input SOP to be only injected in to the FP-SOA TM mode.
- Set the OSA on the continuous sweep mode again and adjust the PC in front the FP-SOA until input SOP of the optical signal is only injected into the FP-SOA TM mode, so the input optical signal power is not aligned with in the TE mode as shown in Fig. A.2.
- Precisely adjust the PC in front the FP-SOA unit all input optical power is solely injected into the TM mode.
- Finally, use a polarization controller PC and optical polarimeter in the input signal measurements path (as shown in the schematics of the lab set-up in Fig. 3.2 and Fig. 5.2, after completing the optimizing the of the input SOP of the optical signal to be only injected into the FP-SOA TM mode, the polarization controller PC in front of the polarimeter is adjusted so that the polarimeter displays $s_{1in} = -1$. This is the only role of the this PC in front the polarimeter, then it is fixed and never changed.
• Then PC in-front of the FP-SOA is optimized and adjusted to inject a linear SOP in to both of the polarization modes with a preferred power imbalance that can be measured in real-time in the polarimeter displays. For example, in chapter 2 BPR signal is optimized with 33/67 split was found to yield the highest NPR between the low and high stable branches of the hysteresis curve [75].

### A.2 Procedure to Configure the Scope for Real-Time Monitoring of a Hysteresis Shape

To optimize the hysteresis shape in real-time in the scope display. The following procedures are performed:

• After generating the BPR signal to be injected into the hysteresis shape controller (HSC) to produce a diversity of hysteresis shapes. First, I have worked on fixing the delay between the input and output channels in the scope, the synchronization process can be achieved by adjusting the delay option in the scope and also by adding a fiber optical cable as a delay line in one the paths of the input path or the output path. For the example of the measured data as shown in the top window in Fig. A.3, the synchronization is preformed where both valleys of the input optical signal (yellow line) and the output bistbale signal (blue line) are aligned.
• Set the scope to plot both the input and output on the same window, where the x is the input channel (input optical signal) and the y is the output channel (output bistable signal). This can be achieved by push the Acquire mode on the scope front panel, then in the waveform display window turn on the XY display option. After completing the scope setting a hysteresis shape can be seen in real-time in the scope as shown in for CW shape Fig. A.3

![Figure A.3](image)

**Figure A.3** The process of optimizing the input optical signal SOP into only the TM mode of the FP-SOA

• The influence of changing the polarization controller PC in front of the polarizer in HSC system can be observed in real-time in the scope window, this allow to optimize each shape by viewing it on the scope while precisely changing the waveplates of the PC in the HSC system.