Obtaining Mechanical Shock Fragility Statistics for Simple Stochastic Cushioning Design

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ABSTRACT

Proper cushioning to prevent product damage and over-packaging must consider the mechanical-shock fragility of the product. Furthermore, improved cushioning design can be achieved by performing stochastic cushioning design using mechanical-shock fragility statistics and transport hazard statistics. However, many samples are required to obtain mechanical-shock fragility statistics from standard testing comprising critical velocity change tests and critical-acceleration tests (the “conventional method”). In many cases, the required number of samples cannot be prepared. Thus, this research is designed to develop testing methods requiring half the number of samples of the conventional method. Thus far, “test method with one sample” has been developed by improving the standard testing method required two samples. Hence, we propose a new statistical method (the “proposed method”) that obtains statistics by multi-sample testing using a test method with one sample. The proposed method is one in which the shock of a single velocity change (the “test velocity change”) is given by increasing the acceleration in a step-wise fashion, and the results indicate the failure rate at the test velocity change and provide the critical-acceleration statistics. In these experiments, the critical-acceleration statistics for a test velocity change larger than the critical velocity change were equivalent to those obtained from the conventional method. The accuracy of the failure rate at test velocity changes was clarified. Moreover, examples are provided showing the results when the proposed method is applied to simple stochastic cushioning design.

KEY WORDS

mechanical-shock fragility test, cushioning design, shock test, statistical analysis, damage boundary curve

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INTRODUCTION

Products dropped during transport may be damaged, which can result in financial losses. Cushioning designs are necessary when dropping damage is likely to occur during product transport. However, the addition of excessive cushioning causes increased transportation costs and/or environmental problems. Therefore, designing proper cushioning is crucial.

As shown in Figure 1, cushioning design is performed by comparing the hazard during transport with the shock fragility of the product and compensating with the necessary cushioning. As part of the improved cushioning design, it is important to realize the lowest cost while still addressing the failure rate expected during transport. Quality cost minimization has been advocated in the field of quality assurance [1]–[3]. Optimal cushioning design results from balancing the cost of transportation accidents owing to defective packaging with the cost owing to over-packaging. The failure rate can be calculated by applying the stress-strength model [4] used in the field of strength of materials, which calculates the failure rate by statistically comparing environmental stress and product strength.

As shown in Figure 2, statistics on transport hazards and shock fragility can be applied to stochastic cushioning design for calculating the failure rate during transport. Thus, proper cushioning requires knowledge of both the hazard during transport and the shock fragility of the product.

This study involves one of these two key elements and deals with a test method for determining the shock fragility of a product. The accepted method for testing the shock fragility of a product was proposed by R. E. Newton [5], and, as a mechanical-shock fragility test of products, it is prescribed in the American Society for Testing Materials (ASTM) D3332 [6] and Japan Industrial Standards (JIS) Z 0119 [7]. To obtain a damage boundary curve (DBC), this test requires two types of destructive tests: a critical velocity change ($\Delta V_c$) test and a critical-acceleration ($A_c$) test. In addition, it is necessary to predict the shock fragility of the sample in order to determine the test-setting values of the velocity change ($\Delta V$) and acceleration. These characteristics often prevent one from performing this test.

Therefore, we have proposed a test method for obtaining the minimum necessary information for cushioning design with only one sample (hereinafter referred to as “the test method with one sample”) [8]. In this method, only the magnitude of the relationship between $\Delta V_c$ and the evaluation criteria $\Delta V$ was used to judge the necessity of the cushioning
design. Since a specific value of $\Delta V_c$ is unnecessary, a $A_c$ test and a short half-sine shock test are performed with the evaluation criteria $\Delta V$. Therefore, it is possible to test using only one sample. The details of this method will be introduced in the first and second sections in the next chapter. In addition, Kawaguchi [9] has presented a simple shock-testing machine with which this method is applied.

In a standard shock fragility test, each test is performed on each product sample. Although the shock fragility of the test sample is known, a fragility distribution for all shipped products is not known, and these variations in shock fragility can cause over-packaging or inadequate packaging. Therefore, Nakajima et al. [10] demonstrated that a probability DBC, considering not only the average value but also the distribution of the values, is necessary to predict damage during transport. However, these authors did not present a method for setting the value of the mechanical-shock fragility test for deriving the DBC. Therefore, we subsequently proposed a test method for obtaining statistics efficiently in the mechanical-shock fragility testing of a product [11] (the conventional method). However, this method still requires a large number of samples to obtain statistics in the $\Delta V_c$ test and the $A_c$ test.

Furthermore, the design of stochastic cushioning requires statistics on transport hazards. There are various reports of transport hazards measured in Japan [12], the USA [13]–[15], China [16], and between Europe and the USA [17]. However, transportation conditions such as the region, means of transportation, size of cargo, and weight rarely coincide completely with the conditions studied in these reports. Therefore, it is difficult for packing-design engineers to overcome all of the appropriate barriers.

We propose a test method that is relatively easy to perform. As shown in Figure 3, the test method (the proposed method) is a new method designed to obtain statistics by using the test method with one sample. The proposed method enables one to obtain the minimum statistical information necessary for a simple stochastic cushioning design with half the number of samples, as compared to the conventional method [11]. In our simple stochastic cushioning design, the use of the drop test height of the transport test standard [18–20] enables stochastic cushioning design more easily than is possible with the conventional method.

![Fig. 3: Concept of the proposed method](image)

The remainder of this paper has been structured as follows: first, we introduce the test method with one sample for mechanical-shock fragility, and a multi-sample method (the proposed method) to obtain statistics for shock fragility with the test method with one sample. Next, we present experiments using the proposed method. Then, we discuss the experimental results and shows examples of applying these results to simple stochastic cushioning design. Finally, we list conclusions drawn from this research.
TESTING THE PROPOSED METHOD

In this chapter, first and second chapters introduce the test method with one sample for mechanical-shock fragility, and Section 2.3 introduces a multi-sample method (the proposed method) to obtain statistics for shock fragility with “the test method with one sample”.

The test method of a product with one sample for mechanical-shock fragility [8]

In the mechanical-shock fragility test of a product, the number of samples is reduced by limiting the obtained results to the minimum information necessary for cushioning design. Two kinds of information necessary for the cushioning design procedure are shown in Figure 4: “necessity of cushioning design” and “cushioning design acceleration.” The two steps in the procedure are described below:

Step 1: The necessity of cushioning design can be determined by whether or not a sample is damaged at a predetermined ∆V (hereinafter referred to as the “test velocity change ∆V_t”) based on the drop height of a drop test or during transport. In other words, the necessity of cushioning design is judged based on the magnitude relationship between ∆V_c and ∆V_t. Assuming that the reference drop height is h, coefficient of restitution is e, and gravitational acceleration is g, ∆V_t can be determined by Equation (1).

\[
\Delta V_t = (1 + e)\sqrt{2gh}
\]

(1)

Step 2: A cushioning design acceleration is determined by a A_c. However, if ∆V_c is larger than ∆V_t, this step is unnecessary.

Based on the cushioning design procedure, ∆V_c only needs to obtain the magnitude relationship with ∆V_t, and the A_c needs to be obtained at ∆V_t.

Therefore, sufficient information can be obtained with one sample by performing the A_c test and only a short half-sine shock (hereinafter referred to as the “∆V test”) at ∆V_t. The procedure for the test method with one sample is as follows and is diagrammed in Figure 5:

1. A_c test: Conduct an A_c test with ∆V_t. This test is conducted over a range from the minimum acceleration to the maximum acceleration accessible with the trapezoidal shock test machine. This method of increasing the acceleration (the “constant-magnification method”) is discussed in the next section.

2. ∆V test: Conduct a test that gives a short half-sine shock with ∆V_t.

On the cushioning design procedure (in figure 4), the test result is applied as follows.

Step 1: In the case of no damage, ∆V_c is larger than ∆V_t, and cushioning design is unnecessary. In the case of damage, ∆V_c is smaller than ∆V_t, and cushioning design is necessary.

Step 2: In the case of no damage, this step is unnecessary. In the case of damage, A_c is deter-
A method to increase test acceleration with constant magnification [8]

Kipp [21] suggested a fatigue effect in the determination of DBC resulting from the number of shocks. Kitazawa [22] confirmed the effects of repeated shocks on DBC in experiments. If the number of shocks is very large in the method with one sample of the last section, the fatigue effect may underestimate the $A_c$. The test method for mechanical shock fragility specified by JIS Z 0119 stipulates that the test sample should be subjected to five or six shocks or less. However, it is difficult to cover the range from minimum to maximum shock (as specified by the test machine) with six shocks, and only a rough test can be performed. Therefore, to eliminate this limitation, we have defined an index of “accumulated fatigue rate” as the standard for evaluating accumulated fatigue and have proposed a method for setting an acceleration increase magnification based on the allowable value of the accumulated fatigue rate.

The accumulated fatigue rate is defined as shown in Figure 6, and Equation (2) shows the accumulated fatigue rate resulting from damage owing to the $n$th shock. The sum of the fatigue values up to the $n$th shock is defined as $S_n$ and is expressed by Equation (3). The following assumptions are used in defining the accumulated fatigue rate: $A_i$ is the input acceleration of the $i$th shock, and the trapezoidal shock is approximated as a rectangular shock. When the $A_c$ is obtained, the response magnification is doubled, the response acceleration of the fragile part is $2A_i$, and the fatigue value of the fragile part is $(2A_i)^\alpha$.

Figure 7 shows the accumulated fatigue rate resulting from increasing the acceleration by a constant-interval (“constant-interval method”). The initial acceleration is set at 100 m/s$^2$, and the increment is 100 m/s$^2$. The acceleration factor $\alpha$ is 6. In this case, the accumulated fatigue rate increases as the number of shocks increases, and it is difficult to distinguish from the fatigue failure. Therefore, one must limit the number of shocks to suppress the effects of accumulated fatigue.
Figure 8 shows the accumulated fatigue rate observed in the constant-magnification method. The initial acceleration is set 100 m/s², the magnification factor is 1.3, and α is 6. In this case, even if the number of shocks increases, the accumulated fatigue rate does not increase; rather, it converges to a constant value. In addition, when the magnification factor or the α changes, the accumulated fatigue rate converges to a different value. Therefore, it is unnecessary to limit the number of shocks.

This converged value in the constant-magnification method is defined as an “allowable accumulated fatigue rate” (hereinafter, \( \Delta_{\text{fatigue}} \)). The value \( \Delta_{\text{fatigue}} \) can be expressed as shown in Equation (4), which results from Equation (2). Assuming that the acceleration increase magnification factor is given by \( r \), then \( A_i \) is expressed by Equation (5), and Equation (6) is obtained from Equations (4) and (5). The calculated \( r \) is represented by α and \( \Delta_{\text{fatigue}} \), and used for the acceleration increase magnification factor of the \( A_c \) test in the last section. α is determined for each product, and \( \Delta_{\text{fatigue}} \) is set as the fatigue tolerance of the test. However, because it is difficult to determine α and \( \Delta_{\text{fatigue}} \), in this paper, the method for determining \( r \) is described in the Appendix.

\[
\Delta_{\text{fatigue}} = \lim_{n \to \infty} \frac{S_{n-2}}{(2A_n)^\alpha + (2A_{n-1})^\alpha}
\]

(4)

\[
A_i = r^{i-1} \times A_1
\]

(5)

\[
r = \left( \frac{1 + \Delta_{\text{fatigue}}}{\Delta_{\text{fatigue}}} \right)^\frac{1}{\alpha}
\]

(6)

**Multi-sample method for obtaining statistics with “the test method with one sample”**

In this section, we propose a multi-sample method to obtain statistics for shock fragility with the “the test method with one sample.”

In the test method with one sample, the constant-magnification method is selected since \( A_c \) is unknown. In the multi-sample method, enhancement of statistical accuracy is realized by the constant-interval method of narrowing down the range of the test by setting the acceleration of the next sample using the data for the samples studied up to that point [11].

The constant-magnification method and the constant-interval method both have advantages and disadvantages, as shown in Figure 9. To take advantage of these two acceleration-increasing methods, we apply them in the proposed method as follows: the constant-magnification method is applied when the number of non-censored data is less than three, and the constant-interval method is applied when that number is three or more.
Figure 10 shows a flowchart of the test procedure and Figure 11 shows the recording procedure.

$\Delta V_t$ and a failure rate criterion are determined by the drop test standard or the transportation environment survey. For the first to the third samples, test acceleration values in the $A_c$ test are determined by the constant-magnification method using a machine specification and a predetermined minimum increase magnification (in this research, 1.23 times; see Appendix). The pulse width of the half-sine shock in the $\Delta V$ test is set to a sufficiently short time (in this research, it is set to 2.2 ms).

Fig. 9: Applying two kinds of acceleration increase methods to the proposed method: (a) Constant magnification method, (b) Constant interval method using statistics.

Fig. 10: Flowchart of proposed method
EXPERIMENTAL

The experiments were performed using the conventional method and the proposed method. Recall that the conventional method is a method of obtaining each result with $\Delta V_c$ tests and $A_c$ tests.

Experimental samples

DVD players were used as experimental samples. The occurrence of damage was adjudged with the statement, “the DVD tray cannot be opened and closed.” Figure 12 depicts the method for supporting the DVD player and for simulating the receiving surface of the cushioning material. This method is the same as that used in [8] and [11].

Table 1. Parameters used to derive the test-setting values for the $\Delta V_c$ test [11]

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum number of test shocks</td>
<td>6</td>
</tr>
<tr>
<td>Minimum $\Delta V$</td>
<td>2 m/s</td>
</tr>
<tr>
<td>Maximum $\Delta V$</td>
<td>7 m/s</td>
</tr>
<tr>
<td>Minimum increase value</td>
<td>0.2 m/s</td>
</tr>
<tr>
<td>Minimum increase ratio</td>
<td>1.05x</td>
</tr>
</tbody>
</table>
Experimental procedure for the proposed method

In the proposed method, the $\Delta V_t$ values selected were 3.0, 4.0, and 5.5 m/s, and they were applied to 10 samples each. Three patterns–small, near, and large–were selected with reference to the average of $\Delta V_c$ in the conventional method discussed in Section experimental results from the conventional method. A test-support program for calculating the accelerations in the proposed method (Section multi-sample method for obtaining statistics with “the test method with one sample”) was created and used. Table 3 shows the input parameters used as the testing machine specifications. The acceleration increase magnification was set to 1.23, using the example in the Appendix.

Table 2. Parameters used to derive the test-setting values for the $A_c$ test.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum number of test shocks</td>
<td>6</td>
</tr>
<tr>
<td>Minimum acceleration</td>
<td>98 m/s²</td>
</tr>
<tr>
<td>Maximum acceleration</td>
<td>980 m/s²</td>
</tr>
<tr>
<td>Minimum increase value</td>
<td>49 m/s²</td>
</tr>
<tr>
<td>Minimum increase rate</td>
<td>1.05x</td>
</tr>
</tbody>
</table>

Experimental results

Experimental results from the conventional method

Figure 13 shows the results of the $\Delta V_c$ test [11]. Figure 14 shows the results of the $A_c$ test, and Table 3 shows the statistics for $\Delta V_c$ and $A_c$.
Experimental results for the proposed method

Figure 15 shows the results at $\Delta V_t=3.0$ m/s, Figure 16 shows the results at $\Delta V_t=4.0$ m/s, and Figure 17 shows the results at $\Delta V_t=5.5$ m/s. Table 5 shows the statistics for failure rate and $A_c$.

At $\Delta V_t=3.0$ m/s (Figure 15), the failure rate was 10% and the $A_c$ statistics could not be calculated. This was because $\Delta V_t$ was smaller than the $\Delta V_c$ and all the data were right-censored data.

At $\Delta V_t=4.0$ m/s (Figure 16), the failure rate was 40%, and, again, the $A_c$ statistics could not be calculated. This was because there were only two data samples for which the $A_c$ was obtained. The remaining eight samples were right-censored data. Six samples were not damaged, and two samples were damaged, not by a trapezoidal shock but by a half-sine shock.

At $\Delta V_t=5.5$ m/s (Figure 17), the failure rate was 100%, and the $A_c$ statistics averaged 888 m/s$^2$ with a standard deviation of 78.1 m/s$^2$ and a coefficient of variation of 9.8%.
Table 6. Comparison of the $A_c$ statistics for the conventional method and for the proposed method at $\Delta V_t=5.5$ m/s

<table>
<thead>
<tr>
<th></th>
<th>Conventional method</th>
<th>Proposed method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
<td>901 m/s²</td>
<td>888 m/s²</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>88.0 m/s²</td>
<td>78.1 m/s²</td>
</tr>
<tr>
<td>Coefficient of Variation</td>
<td>9.8%</td>
<td>8.8%</td>
</tr>
</tbody>
</table>

These results show that, while the conventional method uses a total of 20 samples in two types of tests, the proposed method at $\Delta V_t=5.5$ m/s used only 10 samples and obtained $A_c$ statistics with the same accuracy.

**DISCUSSION**

**Accuracy of $A_c$ statistics**

We compared the $A_c$ statistics for the conventional method with those for the proposed method using $\Delta V_t=5.5$ m/s. As shown in Table 6, the average, standard deviation, and coefficient of variation did not show large differences between the two methods. Figure 18 has the failure rate based on the $A_c$ statistics from the proposed method at 5.5 m/s superimposed on the probabilistic DBC of the conventional method. (Originally, DBC was written as a curve; however, for simplicity, it was written here as a perpendicular DBC.) Figure 18 presents failure rates of 10%, 50%, and 90% in the normal distribution, and the accelerations for failure rates of 10%, 50%, and 90% were well matched in the two methods.

**Fig. 17: Results of $\Delta V_t=5.5$ mls.**

**Fig. 18: Comparison of $A_c$ failure rates of the conventional method and proposed method at $\Delta V_t=5.5$ mls**

**Accuracy of the failure rate determined for each $\Delta V$**

A simple stochastic cushioning design requires a failure rate at a $\Delta V$ corresponding to a reference drop height in order to determine the need for the cushioning. To evaluate the failure rate at each $\Delta V$ obtained with the proposed method, the results were compared with those from the conventional method.

The $\Delta V_c$ statistics obtained from the conventional method were used to calculate the failure rates at $\Delta V=3.0, 4.0,$ and 5.5 m/s. These calculated
results and the results from the proposed method are shown in Table 7. In the proposed method, the failure rate is determined in 10% step increments because there were 10 samples. When both results were compared, differences in the number of damaged samples were < one sample (<10% difference) for the 3.0 m/s and 5.5 m/s trials. Conversely, at 4.0 m/s, the difference was 19%, which corresponded to more than one damaged sample. Thus, the difference between the conventional method and the proposed method became particularly large at around the average ∆Vₜ (failure rate around 50%), and ∆Vₜ and the evaluation failure rate should be chosen in recognition of this observation.

Examples of how to apply the results obtained by the proposed method to stochastic cushioning design

This section applies the proposed method to stochastic cushioning design. The method shown in this section constitutes a very simple example and utilizes the conditions specified in Figure 19. The ∆Vₜ values applied were ∆Vₜ=3.0, 4.0, and 5.5 m/s, as used in the proposed method in the last chapter.

![Fig. 19: Example application of the proposed method of stochastic cushioning design](image)

### Table 7: Comparison of the failure rate of each ∆V in the conventional method and the proposed method.

<table>
<thead>
<tr>
<th>∆V</th>
<th>Failure rate in conventional method</th>
<th>Failure rate in proposed method</th>
<th>Difference of failure rate of two methods</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.0 m/s</td>
<td>0.08%</td>
<td>10%</td>
<td>9.92%</td>
</tr>
<tr>
<td>4.0 m/s</td>
<td>59%</td>
<td>40%</td>
<td>19%</td>
</tr>
<tr>
<td>5.5 m/s</td>
<td>100%</td>
<td>100%</td>
<td>0%</td>
</tr>
</tbody>
</table>

### Table 8: Examples of applying the results obtained by the proposed method to stochastic cushioning design.

<table>
<thead>
<tr>
<th>∆Vₜ</th>
<th>3.0 m/s</th>
<th>4.0 m/s</th>
<th>5.5 m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Necessity of cushioning design</td>
<td>unnecessary</td>
<td>necessary</td>
<td>necessary</td>
</tr>
<tr>
<td>The cushioning design acceleration</td>
<td>unnecessary</td>
<td>703 m/s²</td>
<td>787 m/s²</td>
</tr>
<tr>
<td>Cushioning design using experiment results</td>
<td>possible</td>
<td>partially possible</td>
<td>possible</td>
</tr>
</tbody>
</table>

**Fig. 19: Example application of the proposed method of stochastic cushioning design**

**Results for ∆Vₜ = 3.0 m/s**

When ∆Vₜ=3.0 m/s, the failure rate was 10% and cushioning design was unnecessary. Therefore, there was no problem if the Aₜ statistics could not be obtained.

**Results for ∆Vₜ = 4.0 m/s**

When ∆Vₜ= 4.0 m/s, the failure rate was 40% and cushioning design was necessary. However, the cushioning design acceleration could not be set because the Aₜ statistics could not be calculated.
This was because there were few uncensored data since the two samples were right-censored data owing to the location of a $A_c$ between the maximum accelerations of the trapezoidal shock and the half-sine shock. This problem could be avoided by using a machine providing a larger maximum acceleration for trapezoidal shocks or by performing all tests with half-sine shocks.

Even without $A_c$ statistics, the cushioning design acceleration can still be obtained from the results. The average of the maximum non-damage acceleration and the damage acceleration was taken as the damage expectation acceleration, the order number at the damage expectation acceleration was determined using the average method [23], and the cumulative failure rate was determined using an approximate equation (Equation (7)) for the median-rank method [24].

For example, since the order number in sample 3 was 1, the cumulative failure rate at 703 m/s$^2$ was $(1-0.3)/(10+0.4) = 6.7\%$. Similarly, in sample 2, the order number was 2, and the cumulative failure rate at 706 m/s$^2$ was $(2-0.3)/(10+0.4) = 16.3\%$. The failure rate of $<10\%$ could be obtained by setting 703 m/s$^2$ as the cushioning design acceleration value.

Thus, there were cases in which the cushioning design acceleration could be set without using statistics. However, this method had some drawbacks: it was not possible to set the cushioning design acceleration with an arbitrary failure rate nor below the minimum cumulative failure rate (in this case, 6.7\% or less).

**Results for $\Delta V_t = 5.5$ m/s**

When $\Delta V_t=5.5$ m/s, the failure rate was 100\% and cushioning design was necessary. Using the $A_c$ statistics, an $A_c$ with a cumulative failure rate of 10\% was obtained at 787 m/s$^2$. Therefore, cushioning design was possible by using 787 m/s$^2$ as a cushioning design acceleration value.

**CONCLUSIONS**

For effective cushioning design, it is necessary to base the design on the statistics from mechanical-shock fragility tests on the product. However, the conventional method requires obtaining two sets of statistics with two types of destructive tests, and it requires a large number of samples. In this research, by improving “the test method with one sample” testing procedure, we have developed a method to obtain mechanical-shock fragility statistics that enables simple stochastic cushioning design using half the samples of the conventional method. In the proposed method, the $A_c$ statistics and failure rates at $\Delta V_t$ are obtained.

Experiments were conducted comparing the conventional method and the proposed method, and statistics were obtained for each method. When performed with $\Delta V_t= 5.5$ m/s, a value larger than the $\Delta V_c$ value, $A_c$ statistics very similar to those of the conventional method were obtained with half of the samples needed in the conventional method. As the failure rate around 50\% at the $\Delta V_t$ had a large error, it was necessary to set the $\Delta V_t$ to a value that avoids failure rates near 50\% when determining the necessity of the cushioning design. Moreover, examples in which the proposed method was applied to simple stochastic cushioning design were provided. It is expected that the proposed method will help avoid excessive or insufficient cushioning by systematizing simple stochastic cushioning design.

**Appendix**: Method for setting the acceleration increase magnification.
To determine the acceleration increase magnification, it is necessary to determine $\alpha$ and $\Delta_{\text{fatigue}}$; however, it is difficult to determine $\Delta_{\text{fatigue}}$. Therefore, we consider a method to determine the acceleration increase magnification by determining $\Delta_{\text{fatigue}}$ using the accumulated fatigue rate of six shocks in the typical constant-interval method. We consider an example in which the first acceleration is $98 \text{ m/s}^2$, and an acceleration increase of $49 \text{ m/s}^2$ is used for setting acceleration values with the typical constant-interval method. Figure A1 shows the calculated results for the accumulated fatigue rate resulting from six shocks (Equation (2)), and this accumulated fatigue rate is used as $\Delta_{\text{fatigue}}$. The acceleration increase magnification is calculated for each $\alpha$ by Equation (6), as shown in Figure A2. The $\alpha$ has a different value for each product, thus the standard makes use of representative values. For example, 5 to 8 is used in MIL-STD-810G [25], and 3 to 9 is used in JIS E 4031 [26]. In this study $\alpha = 3$ was used, giving an acceleration increase magnification of 1.23.

Nomenclature
- $A_c$: critical-acceleration
- $A_i$: input acceleration of the $i$th shock
- ASTM: American Society for Testing Materials
- DBC: damage boundary curve
- $e$: the coefficient of restitution
- $g$: gravitational acceleration
- $h$: reference drop height
- JIS: Japan Industrial Standards
- $r$: acceleration increase magnification factor
- $S_n$: summation of fatigue values
- $\alpha$: acceleration factor
- $\Delta_{\text{fatigue}}$: allowable accumulated fatigue rate
- $\Delta V$: velocity change
- $\Delta V_c$: critical velocity change
- $\Delta V_t$: test velocity change

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