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Some Issue in Statistical Process Control

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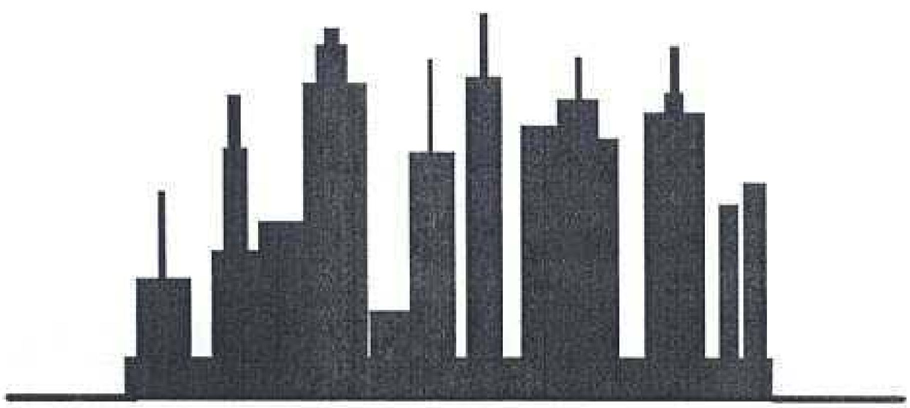

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PREFACE

Welcome to the thirtieth annual conference of the Northeast Decision Sciences Institute. The 2001 meeting is being held at the Westin Convention Center Pittsburgh. The Program includes 38 sessions of competitively judged papers and 6 special sessions in the form of symposia, panels, tutorials and workshops. Authors of the papers and special session participants include members of academia as well as individuals from both the private and public sectors.

Without question, the quality of the meeting depends on the quality of the papers and special session proposals which were submitted, the subsequent review process and the final selection process. Many individuals committed significant amounts of time reviewing papers and providing constructive feedback to the authors. All of the reviewers did an outstanding job. All submitted papers were competitively double-blind reviewed by at least two reviewers.

The *Proceedings* has several features designed to aid conference participants locate papers of special interest to them. The papers in the *Proceedings* itself are organized alphabetically within their respective tracks. Two other indices appear in the *Proceedings*. The first index is located in the front of the book and alphabetically lists each paper or special session by title within each track. The second index is an author index located in the back of the book.

We extend special thanks to each of you who have volunteered your time in organizing and preparing this year's meeting. We understand the multitude of demands placed on your time and appreciate the personal commitment that each of you has made to the Northeast Decision Sciences Institute.

We encourage you to attend as many sessions as possible. We hope this year's meetings will provide you with much information and that it will serve as a forum for meeting colleagues with similar interests in research and teaching.

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SOME ISSUES IN STATISTICAL PROCESS CONTROL

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ABSTRACT

The objective of this paper is to discuss some of the issues in statistical process control (SPC), along with some methods to deal with them, so that the SPC users would get the most benefit out of their quality control efforts. An alternative approach to process control, i.e., process adjustment, which might be preferred over process control in some situations, will also be briefly discussed

INTRODUCTION

SPC is an integral part of an effective quality management system for focusing on controlling and reducing the variation in the process. As with other tools and methodologies, SPC also evolves as the process conditions and the environment changes. Within the past decade several issues emerged in the SPC arena. In this paper we will discuss some of those issues. Recognizing these issues is important, because not dealing with them properly could cause harm to the process through, for example, either getting unnecessary out-of-control signals or not getting one fast enough when the process conditions have changed. This in turn would reduce the credibility of SPC techniques among the users. The issues that will be discussed in this paper are:

- Existence of Autocorrelation
- Short Run Production
- Multiple (and potentially correlated) number of quality variables
- Changes in distribution parameters other than mean and variance

In addition to the above issues, the process adjustment (regulation) approach, which may be preferred over process control (monitoring) for some processes, will be briefly discussed.

SOME ISSUES

i. Existence of autocorrelation:

Autocorrelation, r_k, is a measure of linear association between the data points of the same variable that are k time periods apart. Autocorrelation values change between -1 and $+1$. The r_k values near -1 imply strong negative autocorrelation and those near $+1$imply strong positive autocorrelation. An autocorrelation value of 0 or near 0 indicates no association.

The existence of autocorrelation, which is mainly due to the increasing use of high technology in processes, is an emerging problem for many SPC users. This is especially true in those continuous processes where an automated process control mechanism is in place, and those in which parts are produced very close to each other using high-tech machines. If there is a positive autocorrelation, the regular statistical process control charts (i.e., Shewhart charts) would underestimate the width of the control limits. This, in turn, will cause more unnecessary "out-of-control" signals. The presence of negative autocorrelation, on the other hand, causes the overestimation of the width of the control limits. This would cause the control chart to be less sensitive to process changes. Thus the presence of autocorrelation should be checked as part of an SPC analysis.

When Shewhart introduced X-bar and R charts, autocorrelation was hardly a concern. There was little, if any, automatic process control and manufacturing was not highly automated; both of which contribute to autocorrelation. That's why the Shewhart model for process, which is under statistical control, is simply:

$$X_t = \mu_x + \varepsilon_t \qquad (1)$$

In this model the average value of the statistics being monitored (μ_x) is assumed to be constant. The question that is tested is whether the observations at time t (X_t) is equal to the average value with some error term (ε_t). The assumption is that error terms are uncorrelated and follow normal distribution with mean 0 and constant variance σ^2. This is the correct model as long as there is no autocorrelation or any other non-random pattern in the data. However, today the trend is just the opposite. According to Alwan [2], for example, a conservative estimate of the rate of occurrence of autocorrelation in SPC data is at least 70%. Thus the potential existence of autocorrelation should be considered in SPC analysis.

Autocorrelation could exist due to some removable causes. If these causes can be found and removed, then autocorrelation would be eliminated. However, if the autocorrelation is part of the process, i.e., the causes cannot be removed, then conventional SPC approaches should be revised to take this into account. In other words, the model in equation (1) is no longer the valid model for the process. Holmes & Gordon [11], Montgomery & Mastrangelo [20], Alwan and Roberts [1], Alwan [2], Maragah [18] and others proposed some ways to deal with the autocorrelation. Holmes and Gordon [11] and others have suggested that the form of autocorrelation, i.e., first order, second order, etc., should be determined and modeled. Then the deviation (i.e., the difference between the actual and the model value of the statistic being monitored) from this model should be monitored on the proper control chart(s). Under this approach the question and the model used to test the question is different than the Shewhart model, i.e., the statistic being monitored is no longer constant but changes in time with the fitted model and the question being tested is whether the deviations from the model are significant over time. In other words, if the deviation is not significant, this implies that the autocorrelation model fitted to the process is still the one explaining the behavior of autocorrelation existed in the process. If the deviation is significant, this means the autocorrelation structure has changed near point (t) so that the cause(s) for this change should be investigated and if necessary the new model should be introduced. See the above references also to deal with the autocorrelated data in SPC.

ii. Short Run production:

Griffith [10] describes short-run conditions as follows:

i. Not enough parts in a single production run to achieve/maintain control limits on the process

ii. The process cycles so quickly that even large size production runs are over before data can be gathered.

iii. Many different parts are made for many different customers (in small lot sizes).

The increasing emphasis on decreasing manufacturing setup times has led to shorter production runs (e.g., Just-in-Time systems). These shorter production runs have led in turn to increased awareness of the necessity to modify SPC practices to fit the situation. For several studies that address Short Run SPC see references [6, 7, 8, 9, 10, 15, 21]. There are two basic approaches to short-run SPC: one is to use coded data, e.g., deviation from the target; the other one is to standardize the coded data when the variances of the different product lines are significantly different. The first approach is implemented using:

$$\text{Coded data} = X_A - \text{Target}_A \qquad (2)$$

where X_A is current value for part A and Target_A is the desired value for part A. Instead of individual and Target values, subgroup averages and historical averages for parts can also be used if the data is abundant.

The second approach is implemented using the standardized coded data (SCD), which is defined as:

$$\text{SCD} = \frac{X_A - \text{Target}_A}{\sigma_A} \qquad (3)$$

where σ_A is the standard deviation of part A. The same adjustment, which was mentioned above, can be done when subgroup averages are used to get the standardized coded data.

The coded data or the standardized coded data, which may come from various product lines, are then monitored on a single chart. In the short run environment, not only is there variation from setup to setup of the same part, but also variation introduced by the setup of different parts with different specifications on the same work station.

iii. Multiple (and potentially correlated) numbers of quality variables:

In some processes there may be more than one quality characteristic (variable) being monitored. Monitoring these characteristics separately requires building and maintaining many control charts (i.e., one chart per characteristics). This is not practical for the SPC users. If there are separate control charts for the characteristics and if an out-of-control signal from any of the chart triggers the conclusion that the process is out of control, then the alpha (type I error) will be different than what we planned for each individual chart. The problem will get worse if the characteristics in question are also correlated. For example, in chemical and process industries many process (input) variables are correlated due to nature of the processes. If this is the case, using separate control charts for each quality characteristics could lead to erroneous results, i.e., getting out-of-control signals when actually the process has not changed, or getting no signal when the process indeed has changed. The solution to all these problems is to use multivariate analysis techniques. In the multivariate situation, the first thing to do is to check for correlation among the variables using the correlation matrix. If correlations exist, then a multivariate process control technique should be used for more accurate results. T^2 Control charts and principal component analysis are common tools for this purpose. This statistic (i.e., T^2 statistic), which was first introduced by Hotelling [16], reduces the data on multiple variables into one statistic by taking into account the correlation structure between them as shown below:

$$T^2 = \left(X - \overline{X}\right) S^{-1} \left(X - \overline{X}\right) \qquad (4)$$

where X is the (p x 1) vector representing an observation of the p variables, $\overline{X}$ is the (p x 1) vector of the averages of the p variables, S is the sample variance-covariance matrix and S^{-1} is the inverse of S. T^2 control chart monitors this statistic. See, for example, references [13, 14, 17, 19] for this chart. Additional tests are required to find the cause of the out-of-control signal on the T^2 chart [14].

iv. Changes in distribution parameters other than mean and variance:

Almost all the process control charts are designed to monitor one parameter, e.g., mean, variance, etc. However, sometimes the change in the underlying distribution of the process may take place in other areas, such as skewness, kurtosis, etc. Though it may be rare, if it happens we would like to detect the changes as quickly as possible. An SPC tool, Chi-Square chart, to detect changes in shape parameters, other than mean and variance was discussed by Holmes and Mergen [12]. This chart checks all the shape parameters of the distribution rather than just checking the mean and/or the standard deviation. This chart requires large number of data, thus it works in the processes where data is abundant and frequently collected.

v. Process adjustment (regulation) approach:

As Box [4] argued that the objective of process monitoring is to bring the process to a basic state of statistical control without continual adjustment. This implies that the process parameter being monitored stays stable with random variation around it. Though this model works for many processes it may not work for all the processes. Some processes, due to their nature or despite the best effort to keep them stable, do not show a stable performance but they wander off their targets. These processes require continual adjustment to keep the process as close as possible to the target. Box makes an analogy equating process monitoring with statistical significance testing and process adjustment with statistical estimation. Thus if the process shows these unavoidable deviations from its target, you may be better off estimating the level of deviation and making an adjustment in advance to keep the process close to its target. See, for example, also references [3] and [5] on this topic.

CONCLUSION

In this paper we discussed some of the current issues that are facing the SPC users. The objective was to alert the user about these issues and make them consider proper models to deal with them. This way the errors that we experience in application of SPC techniques will be minimized and quality will be improved further.

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