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David Merritt
Rochester Institute of Technology

Slawomir Piatek
New Jersey Institute of Technology

Simon Portegies Zwart
University of Amsterdam

Marc Hemsendorf
Rutgers University

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CORE FORMATION BY A POPULATION OF MASSIVE REMNANTS

DAVID MERRITT [1], SLAWOMIR PIATEK [2], SIMON PORTEGIES ZWART [3], AND MARC HEMSENDORF [4]

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ABSTRACT

Core radii of globular clusters in the Large and Small Magellanic Clouds show an increasing trend with age. We propose that this trend is a dynamical effect resulting from the accumulation of massive stars and stellar-mass black holes at the cluster centers. The black holes are remnants of stars with initial masses exceeding $\sim 20-25 M_\odot$; as their orbits decay by dynamical friction, they heat the stellar background and create a core. Using analytical estimates and N-body experiments, we show that the sizes of the cores so produced and their growth rates are consistent with what is observed. We propose that this mechanism is responsible for the formation of cores in all globular clusters and possibly in other systems as well.

Subject headings: black hole physics — gravitation — gravitational waves — galaxies: nuclei

1. INTRODUCTION

An enduring problem is the origin of *cores*, regions near the center of a stellar or dark matter system where the density is nearly constant. Resolved cores clearly exist in some stellar systems, e. g. globular clusters (Harris 1996). In other systems, such as early-type galaxies, cores were long believed to be generic but were later shown to be artifacts of the seeing (Schweizer 1979). Nevertheless a few elliptical galaxies do exhibit bona-fide cores (Lauer et al. 2002) while many others show a central density that rises only very slowly toward the center (Merritt & Fridman 1995). Density profiles of structures that form from gravitational clustering of density perturbations in an expanding universe are believed to lack cores (Power et al. 2003), although there is evidence for dark matter cores in the rotation curves of some late-type galaxies (e.g. Jimenez, Verde & Oh (2003)).

The existence of a core is usually deemed to require a special explanation. For instance, galaxy cores may form when binary black holes eject stars via the gravitational slingshot (Ebisuzaki, Makino & Okumura 1991).

A useful sample for testing theories of core formation is the ensemble of globular clusters (GCs) around the Large and Small Magellanic Clouds (LMC/SMC). These clusters have masses similar to those of Galactic GCs, but many are much younger, with ages that range from $10^6 - 10^{10}$ yr. Furthermore ground-based (Elson, Freeman & Lauer 1989; Elson 1991, 1992) and HST (Mackey & Gilmore 2003a,b) observations reveal a clear trend of core radius with age: while young clusters ($\tau \ll 10^8$ yr) have core radii consistent with zero, clusters older than $\sim 10^9$ yr exhibit the full range of core sizes seen in Galactic GCs, 0 pc $\lesssim r_c \lesssim$ 10 pc (Figure 1). The maximum core radius observed in the LMC/SMC GCs is an increasing function of age and is given roughly by $r_c \approx 2.25 \mathrm{pc} \log_{10}\tau_{\mathrm{yr}} - 14.5$. Attempts to explain the core radius evolution in terms of stellar mass loss (Elson 1991), a primordial population of binary stars, or time-varying tidal fields (Wilkinson et al. 2003) have met with limited success. The difficulty is to find a mechanism that can produce substantial changes in the central structure of a GC on time scales as short as a few hundred Myr, while leaving the large-scale structure of the cluster intact.

In this paper, we describe a new mechanism for the formation of GC cores and their evolution with time. Massive stars and their black hole remnants sink to the center of a GC due to dynamical friction against the less massive stars. The energy transferred to the stars during this process, and during the three- and higher-N encounters between the black holes that follow, has the effect of displacing the stars and creating a core. The rate of core growth implied by this model is consistent with the observed dependence of core size on age in the LMC/SMC clusters.

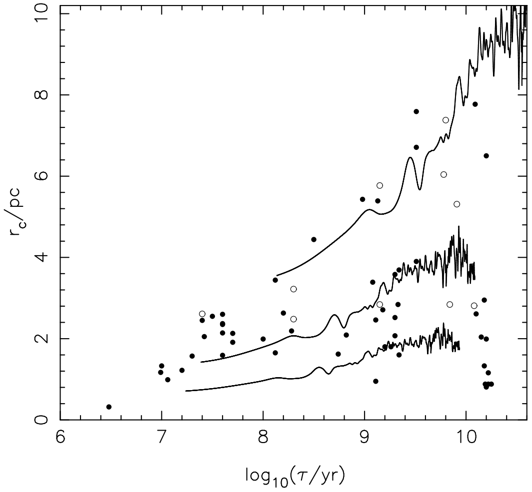

FIG. 1.— Core radius versus age for LMC and SMC GCs from the samples of Mackey & Gilmore (2003a,b). Lines show core radius evolution from the N-body simulations with initial cusp slope $\gamma = 1$ and three different scalings to physical units; see text for details.

2. CORE FORMATION TIMESCALES

Consider a gravitationally bound stellar system in which most of the mass is in the form of stars of mass m, but which also contains a subpopulation of more massive objects with masses m_{BH}. The orbits of the more massive objects decay due to dynamical friction. Assume that the stellar density profile is initially a power-law in radius, $\rho(r) = K(r/a)^{-\gamma}$, $K = (3-\gamma)M/4\pi a^3$ with M the total stellar mass and a the

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[1] Department of Physics, Rochester Institute of Technology, Rochester, NY 14623

[2] Department of Physics, New Jersey Institute of Technology, Newark, NJ 07102

[3] Astronomical Institute "Anton Pannekoek" and Institute for Computer Science, University of Amsterdam, Kruislaan 403, Amsterdam, The Netherlands

[4] Department of Physics and Astronomy, Rutgers University, New Brunswick, NJ 08903

density scale length; the expression for K assumes that the density follows a Dehnen (1993) law outside of the stellar cusp, i.e. $\rho \sim r^{-4}$ at large r. The effective radius (the radius containing $1/2$ of the mass in projection) is related to a via $R_e/a \approx (1.8, 1.3, 1.0)$ for $\gamma = (1, 1.5, 2)$.

Due to the high central concentration of the mass, the orbits of the massive particles will rapidly circularize as they receive nearly-impulsive velocity changes near pericenter. Once circular, orbits shrink at a rate that can be computed by equating the torque from dynamical friction with the rate of change of orbital angular momentum. We adopt the usual approximation (Spitzer 1987) in which the frictional force is produced by stars with velocities less than the orbital velocity of the massive object. The rate of change of the orbital radius, assuming a fixed and isotropic stellar background, is then

$$\frac{dr}{dt} = -2\frac{(3-\gamma)}{4-\gamma}\sqrt{\frac{GM}{a}}\frac{m_{BH}}{M}\ln\Lambda\left(\frac{r}{a}\right)^{\gamma/2-2}F(\gamma), \tag{1}$$

$$F(\gamma) = \frac{2^{\beta}}{\sqrt{2\pi}}\frac{\Gamma(\beta)}{\Gamma(\beta-3/2)}(2-\gamma)^{-\gamma/(2-\gamma)} \times \int_0^1 dy\, y^{1/2}\left(y+\frac{2}{2-\gamma}\right)^{-\beta},$$

where $\beta = (6-\gamma)/2(2-\gamma)$ and $\ln\Lambda$ is the Coulomb logarithm, roughly equal to 6.6 (Spinnato, Fellhauer & Portegies Zwart 2003). For $\gamma = (1.0, 1.5, 2.0)$, $F = (0.193, 0.302, 0.427)$. Equation (1) implies that the massive object comes to rest at the center of the stellar system in a time

$$\Delta t \approx 0.2\sqrt{\frac{a^3}{GM}}\frac{M}{m_{BH}}\left(\frac{r_i}{a}\right)^{(6-\gamma)/2} \tag{2}$$

with r_i the initial orbital radius; the leading coefficient depends weakly on γ. Equation (2) can be written

$$\Delta t \approx 3\times 10^9 \mathrm{yr}\, a_{10}^{3/2} M_5^{1/2} m_{BH,10}^{-1}\left(\frac{r_i}{a}\right)^{(6-\gamma)/2} \tag{3}$$

with a_{10} the density scale length in units of 10 pc (e.g. Figure 1 of van den Bergh (1991)), $M_5 = M/10^5 M_\odot$, and $m_{BH,10} = m_{BH}/10M_\odot$, the approximate masses of black hole remnants of stars with initial masses exceeding $\sim 20-25M_\odot$ (Maeder 1992; Portegies Zwart, Verbunt & Ergma 1997). This time is of the same order as the time ($\sim 10^9$ yr) over which core expansion is observed to take place (Mackey & Gilmore (2003a,b), Fig. 1).

To estimate the effect of the massive remnants on the stellar density profile, consider the evolution of an ensemble of massive particles in a stellar system with initial density profile $\rho \sim r^{-2}$. The energy released as one particle spirals in from radius r_i to r_f is $2m_{BH}\sigma^2\ln(r_i/r_f)$, with σ the 1D stellar velocity dispersion. Decay will halt when the massive particles form a self-gravitating system of radius $\sim GM_{BH}/\sigma^2$ with $M_{BH} = \sum m_{BH}$. Equating the energy released during infall with the energy of the stellar matter initially within r_c, the "core radius," gives

$$r_c \approx \frac{2GM_{BH}}{\sigma^2}\ln\left(\frac{r_i\sigma^2}{GM_{BH}}\right). \tag{4}$$

Most of the massive particles that deposit their energy within r_c will come from radii $r_i \approx$ a few $\times\, r_c$, implying $r_c \approx$ several $\times\, GM_{BH}/\sigma^2$ and a displaced stellar mass of $\sim$ several $\times\, M_{BH}$. If $M_{BH} \approx 10^{-2}M$ (Portegies Zwart & McMillan 2000), then $r_c/a \approx$ several $\times\, 2M_{BH}/M$ and the core radius is roughly 10% of the effective radius.

Evolution continues as the massive particles form binaries and begin to engage in three-body interactions with other massive particles. These superelastic encounters will eventually eject most or all of the massive particles from the cluster. Assume that this ejection occurs via the cumulative effect of many encounters, such that almost all of the binding energy so released can find its way into the stellar system as the particle spirals back into the core. The energy released by a single binary in shrinking to a separation such that its orbital velocity equals the escape velocity from the core is $\sim m_{BH}\sigma^2\ln(4M_{BH}/M)$. If all of the massive particles find themselves in such binaries before their final ejection and if most of their energy is deposited near the center of the stellar system, the additional core mass will be

$$M_c \approx M_{BH}\ln\left(\frac{M}{M_{BH}}\right) \tag{5}$$

e.g. $\sim 5M_{BH}$ for $M/M_{BH} = 100$, similar to the mass displaced by the initial infall. The additional mass displacement takes place over a much longer time scale however and additional processes (e.g. core collapse) may compete with it.

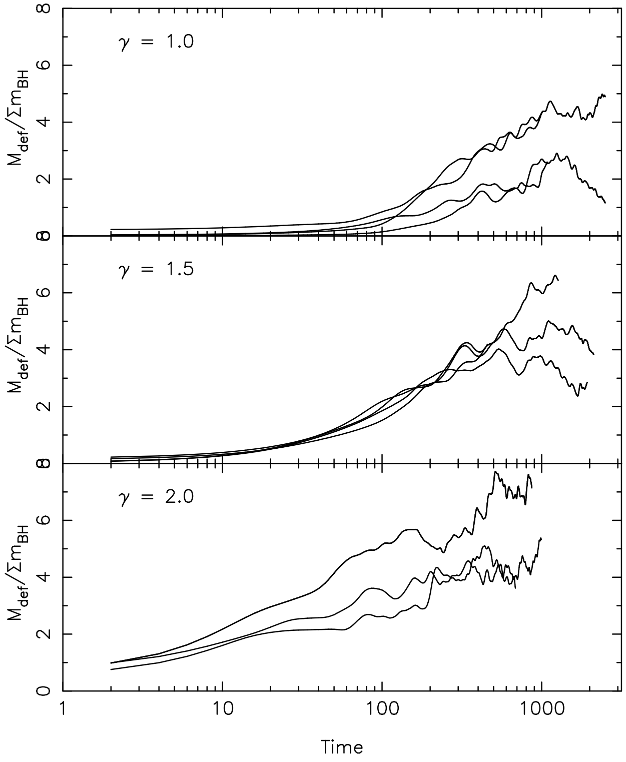

FIG. 2.— Evolution of the mass deficit, from the N-body experiments with $N_{BH} = 10$. Data were smoothed with cubic splines.

3. *N*-BODY SIMULATIONS

We used N-body simulations to test the core formation mechanism described above. Initial conditions were designed to represent GCs in which 1% of the total mass is initially in the form of massive objects, either stars or their black hole remnants. Integrations were carried out using NBODY6++, a high-precision, parallel, fourth-order direct force integrator which implements coordinate regularization for close encounters (Spurzem 1999). Particles had one of two masses, representing either black holes (m_{BH}) or stars (m). The number of particles representing black holes was $N_{BH} = (4, 10, 20)$ and the ratio of m_{BH} to m ranged from 10 to 25. Most of the N-body experiments used $N = 10^4$ particles. We concentrate here on the results obtained with $N_{BH} = 10$ and $m_{BH}/m = 10$; results obtained with other values of N_{BH} were consistent. All particles were initially distributed according to Dehnen's (1993) density law, $\rho(r) = ((3-\gamma)M/4\pi a^3)/\xi^\gamma/(1+\xi)^{4-\gamma}, \xi = r/a$. The logarithmic slope of the central density cusp is specified via the parameter γ. Initial velocities were generated assuming isotropy; positions and velocities of the massive particles were distributed in the same way as the stars, so that the initial conditions represented a cluster in which the massive objects had not yet begun to segregate spatially with respect to the lighter stars.

The 1% mass fraction in massive particles was based on a Scalo (1986) mass distribution with lower and upper mass limits of $0.1 M_\odot$ and $100 M_\odot$ respectively. With such a mass function, about 0.071% of the stars are more massive than $20 M_\odot$ and 0.045% are more massive than $25 M_\odot$. A star cluster containing $N_\star$ stars thus produces $\sim 6\times10^{-4} N_\star$ black holes. Known Galactic black holes have masses m_{BH} between $6 M_\odot$ and $18 M_\odot$ (Timmes, Woosley, & Weaver 1996). Adopting an average black hole mass of $10 M_\odot$ then results in a total black hole mass of $\sim 6\times10^{-3} M$.

The decision to use just two mass groups – clearly an idealization of the true situation – was made for two reasons. First, the interpretation of the N-body results is greatly simplified in such a model. Second, it is not clear what a better choice for the initial spectrum of masses would be. The distribution of black hole masses produced by stars with $m \gtrsim 25 M_\odot$ is uncertain (Timmes, Woosley, & Weaver 1996); some Galactic black holes may have masses as low as $\sim 3 M_\odot$ (White & van Paradijs 1996), close to the maximum probable masses of neutron stars. However this may be a selection effect: low-mass binary systems tend to be selected due to their longer lifetimes. The remnant mass range between $\sim 1 M_\odot$ and $\sim 3 M_\odot$ is occupied by neutron stars, but there is evidence that neutron stars receive larger kicks at birth than black holes (Lyne & Lorimer 1994) and may be ejected. (Portegies Zwart & McMillan (2000) estimate that $\sim 10\%$ of black holes are ejected from GCs by formation kicks.) In summary, the initial spectrum of masses in a GC shortly after its formation is poorly known. Future studies will attempt to include a realistic treatment of stellar physics, primordial binaries, star formation, and other processes that affect the initial mass spectrum and the initial spatial distribution of different mass groups.

If the core radius is defined as the radius at which the projected density falls to $1/2$ of its central value, Dehnen models with $\gamma \geq 1$ have $r_c = 0$. Any core that appears in these models must therefore be a result of dynamical evolution.

Henceforth we adopt units in which $G = a = M = 1$. The corresponding unit of time is

$$[T] = \left[\frac{GM}{a^3}\right]^{-1/2} = 1.44\times10^6 \text{ yr } M_5^{-1/2} a_{10}^{3/2} \qquad (6)$$

where M_5 is the cluster mass in units of $10^5 M_\odot$ and a_{10} is the cluster scale length in units of 10 pc. The effective radius R_e, defined as the radius containing $1/2$ of the light particles seen in projection, is $R_e \approx (1.8, 1.3, 1.0)$ in model units ($a = 1$) for $\gamma = (1, 1.5, 2)$. The time scaling of equation (6) is not correct for processes whose rates depend on the masses of individual stars or black holes, since our models have fewer stars than real GCs. The most important of these processes for our purposes are black hole-star interactions, which are responsible for the orbital decay of the black holes and the growth of the core. This decay occurs in our simulations at a rate that is $\sim N_\bullet/N_{BH}$ times faster than implied by the scaling of equation (6), with $N_\bullet$ the true number of black holes in a GC. Assuming a Scalo initial mass function as above, this factor may be written $\sim 6.0(N_\star/10^5)/(N_{BH}/10)$ with $N_\star$ the true number of stars in a GC.

As discussed above, we expect the stellar mass displaced by the massive particles to scale roughly with M_{BH}. One way to illustrate this is via the *mass deficit*, defined as in Milosavljević et al. (2002): it is the mass difference between the initial stellar density $\rho(r,0)$ and the density at time t, integrated from the origin out to the radius at which $\rho(r,t)$ first exceeds $\rho(r,0)$. The mass deficit is a measure of the core mass. Figure 2 shows $M_{\rm def}(t)/M_{BH}$ for the N-body experiments. The density center was computed via the Casertano-Hut (1985) algorithm. The black holes displace a mass in stars of order $2-8$ times their own mass; the larger values correspond to the larger values of γ although there is considerable scatter from experiment to experiment for a given γ. The results for $\gamma = 2$ are consistent with the analytic arguments presented above, which implied a core mass of a few times M_{BH} after a time in model units of $\sim 0.2M/m_{BH} \approx 20$ (cf. equation 2) followed by a slower displacement of a similar mass as the black hole particles engage in three-body interactions (cf. equation 5).

Some of the N-body simulations show a decrease in the core radius after $t \approx 10^3$ (Figure 2, 3). By this time, the majority of the black holes have been ejected. Our simulated clusters are then effectively reduced to equal-mass systems, which take about 15 half mass relaxation times to experience core collapse (Spitzer 1987). The two-body relaxation time is roughly $T_R \approx 0.2 T_D N/\ln N \approx 200$ in our N-body models, with T_D the crossing time. It is therefore not surprising that, once the black holes are ejected, the cluster core shrinks again on a time scale of $\sim 10^3$ time units.

About one-tenth of the known globular clusters in the Galaxy have vanishingly small cores and are inferred to be in a state of core collapse (Harris 1996). We note here the large number (~ 10) of GCs in Figure 1 with small or zero core radii; this may indicate that a much larger fraction ($\sim 80\%$) of the LMC globular clusters are on their way to core collapse. We predict that these clusters have lost most or all of their black holes, while the $\sim$ two old clusters in Figure 1 with substantial cores still contain a few stellar mass black holes in their cores. We find no indication from their structural parameters that these two clusters differ systematically from the other clusters in Figure 1.

Figure 3 shows the evolution of the core radii in these simulations. Computation of r_c was based on its standard definition as the projected radius at which the surface density falls to $1/2$ of its central value. Projected densities were computed via a kernel estimator (Merritt & Tremblay 1994; Merritt 2004). To reduce the noise, values of r_c from all experiments with the same γ and with $N_{BH} = 10$ were averaged together. Figure 3 shows that core sizes increase roughly as the logarithm of the time, consistent with the time dependence of the upper envelope of Figure 1, and reach values at the end of the simulations of $\sim 10\%$ of the half-mass radius.

Based on equation (6) and the discussion following, the conversion factor from model time units to physical time units is approximately

$$8.9\times10^6 \text{ yr } M_5^{-1/2} a_{10}^{3/2} N_{\star,5} \qquad (7)$$

with $N_{\star,5}$ the number of stars in the GC in units of 10^5. This scaling was used to plot three curves in Figure 1: with $M_5 = N_{\star,5} = 2$, $a_{10} = 0.5$ (bottom), $M_5 = N_{\star,5} = 0.5$, $a_{10} = 1$ (middle), and $M_5 = N_{\star,5} = 1$, $a_{10} = 2.5$ (top). The curves in Figure 1 were taken from the experiments with $\gamma = 1$; the experiments with $\gamma = 1.5$ and 2 give similar results (note that R_e/a varies by a factor ~ 2 from $\gamma = 1$ to $\gamma = 2$, hence r_c/R_e varies less than r_c/a in Figure 3). The logarithmic time dependence of the upper envelope of the r_c distribution is well reproduced, and with appropriate (and reasonable) scaling, points below the envelope can also be matched. As noted above, the smaller core radii that begin to appear in SMC/LMC clusters with $\tau \gtrsim 10^9$ yr are plausibly due to evolution toward core collapse in these clusters, as seen also in some of the simulations.

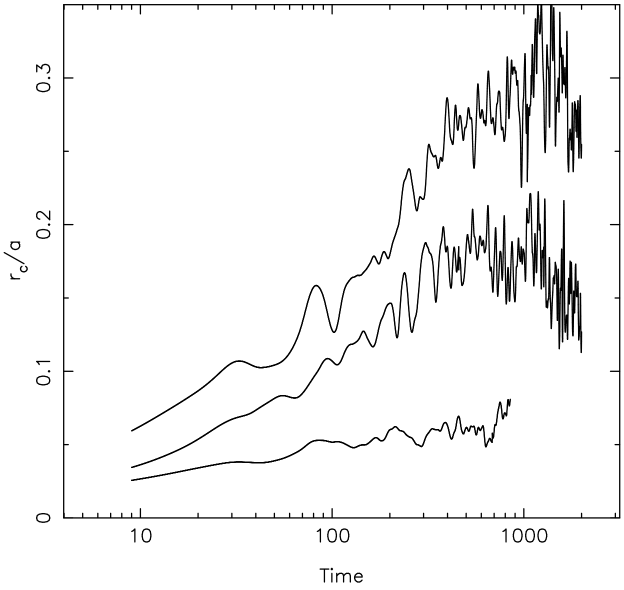

FIG. 3.— Evolution of the core radius, defined as the radius at which the projected density falls to one-half of its central value. Each curve is the average of the various experiments at the specified value of γ, with $\gamma = 0.5$ (top), $\gamma = 1$ (middle) and $\gamma = 2$ (bottom). Vertical axis is in units such that the Dehnen-model scale length $a = 1$; see text for conversion factors from a to R_e.

4. DISCUSSION

The core formation mechanism proposed here could begin to act even before the most massive stars had evolved into black holes. Evolution times for $20M_\odot$ stars are ~ 8 Myr (Schaller et al. 1992). The earliest phases of core formation, $\tau \lesssim 10^7$ yr, would therefore be driven by the accumulation of massive stars rather than by their remnants. Figure 1 shows possible evidence of core growth on time scales $\lesssim 20$ Myr in a few clusters. In this context it is interesting to mention the so-called "young dense star clusters." These clusters have ages $\lesssim 10$ Myr, sizes ~ 1 pc, and contain $\lesssim 10^5$ stars. Well-known examples are NGC2070 (Brandl et al. 1996), NGC 3603 (Vrba 2000), and Westerlund 1 (Brandl et al. 1999). All of these young clusters have small but distinct cores. The young cluster R136 in 30 Doradus ($\tau \approx 5$ Myr) shows clear evidence of mass segregation among the brightest stars (Brandl et al. 1996).

The mechanism described here is similar to core formation by a binary supermassive black hole in a galactic nucleus via the gravitational slingshot (Quinlan 1996). The latter process produces cores with masses $\sim$ a few times the binary mass, assuming that the binary separation decays all the way to the point that coalescence by gravitational wave emission can ensue (Merritt 2003). If the decay stalls at a larger separation, the displaced mass will be smaller. It is currently uncertain how often the decay would stall (Milosavljević & Merritt 2003). Core formation by a *population* of massive remnants also displaces a mass that is a few times the total mass in black holes (Figure 2), and because the smaller black holes are freer to move about, there is less prospect of stalling due to a local depletion of stars. In galactic nuclei, the imprints left on the stellar distribution by the clustering of stellar-mass black holes were probably long ago erased by the growth of the supermassive black hole, by the formation and decay of binary supermassive black holes during galaxy mergers, and by star formation.

A speculative application of these results is to cores formed at the centers of dark matter halos by the clustering of Population III remnants in the early universe (Volonteri, Haardt, & Madau 2003). The latter are believed to contain at least one-half the mass of their stellar progenitors when $m \gtrsim 250M_\odot$ (Fryer, Woosley, & Heger 2001), and the cosmological density of remnants may be similar to that of the supermassive black holes presently observed at the centers of galaxies (Madau & Rees 2001). It follows that the Population III remnants could create cores of appreciable size, *if* a number of them can accumulate in a single halo at a given time, and if the time for their orbits to decay is shorter than the time between halo mergers. Both propositions will require further investigation.

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