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Polychromatic MTF of a Pinhole Camera

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POLYCHROMATIC MTF OF PINHOLE CAMERA

by
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ABSTRACT

Polychromatic MTFs based on the work of Hopkins and Sayanagi are developed for a pinhole camera using standard 5500°K daylight and a black and white, and a color reversal film. On and off-axis MTF were developed for the radial and tangential cases. Conclusions are drawn about the depth of focus and "best" focal position. A monochromatic depth of field study was done, showing both the need to focus and the infinite character of the pinhole camera.

INTRODUCTION

An interest in the pinhole as an imaging device is shown in the literature. Newman and Rible have reported using an array of pinholes to replace a step and repeat camera for making integrated circuits.[1] Gallas, Gilbert, and Hitterdal used a pinhole to replace an automatic focusing system when making a simulator for space docking exercises.[2] J. H. Waddell in his article, "The Neglected Pinhole," gives other uses of the pinhole camera.[3]

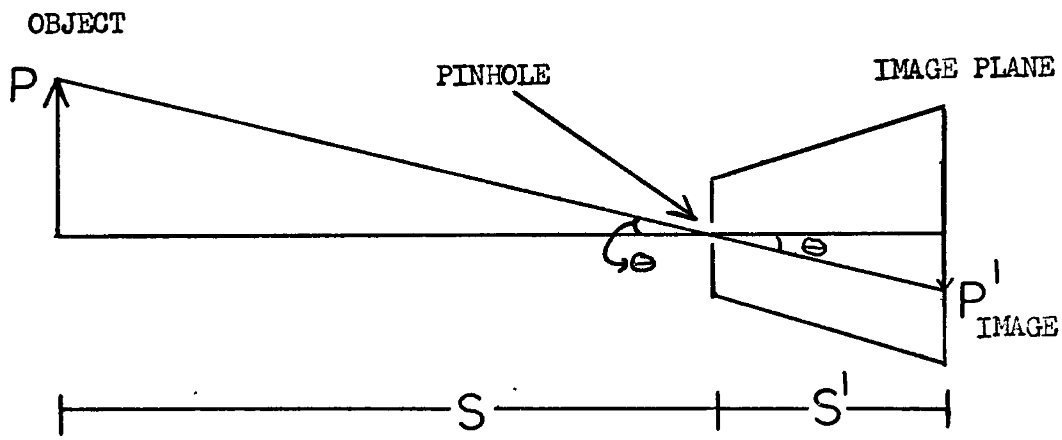


Figure One. Diagram of a Pinhole Camera

Figure One shows a diagram of a simple pinhole camera imaging an object. Experimental evidence and theoritical considerations show that if the pinhole is too large or too small no image or a very fuzzy one is all that is obtained. The major problem of the pinhole camera then is optimization: What is the relationship between pinhole diameter, d, and the distance s' that produces the best image. Figure Two is a pinhole picture taken under optimum conditions.

There are many papers in the literature that discuss the problem of optimizing a pinhole camera. Sayanagi records the oldest reference as being in the tenth century.[4] Petzval and Rayliegh wrote papers in the nineteenth century.[5,6] Recently, Selwyn, Sayanagi, Young, and Swing and Rooney have writ-

FIGURE TWO, PICTURE, 2×, d=.6mm s=162mm

ten excellent papers on the optics of the pinhole.[7,8,9,10] Others, notably Hopkins, and Steel, have written articles that have a direct bearing on understanding the imaging characteristics of pinhole camers.[11,12]

In finding this optimum relation most papers started with the object at infinity.[13] When s is equal to infinity s' is called the focal length of the pinhole, f, as in other lenses. Petzval used the criterion of pinhole diameter that produced the smallest image of a point source as his optimizing criterion. Rayleigh based his on Fraunhofer diffraction in the presence of defocus. Selwyn used the maximizing of the central intensity of the point spread function. Young used physical reasoning from geometrical and physical optics. Sayanagi used the central intensity of the point spread function as a two dimensional figure of merit and the central intensity of the line spread function as a one dimensional figure of merit. Sayanagi has shown that his, Petzval's, Rayleigh's, and Selwyn's optimization relatioship can be put into the form $d^2=kyf$. Where k is a constant depending on the criterion taken for optimization and y is the wavelength.[14]

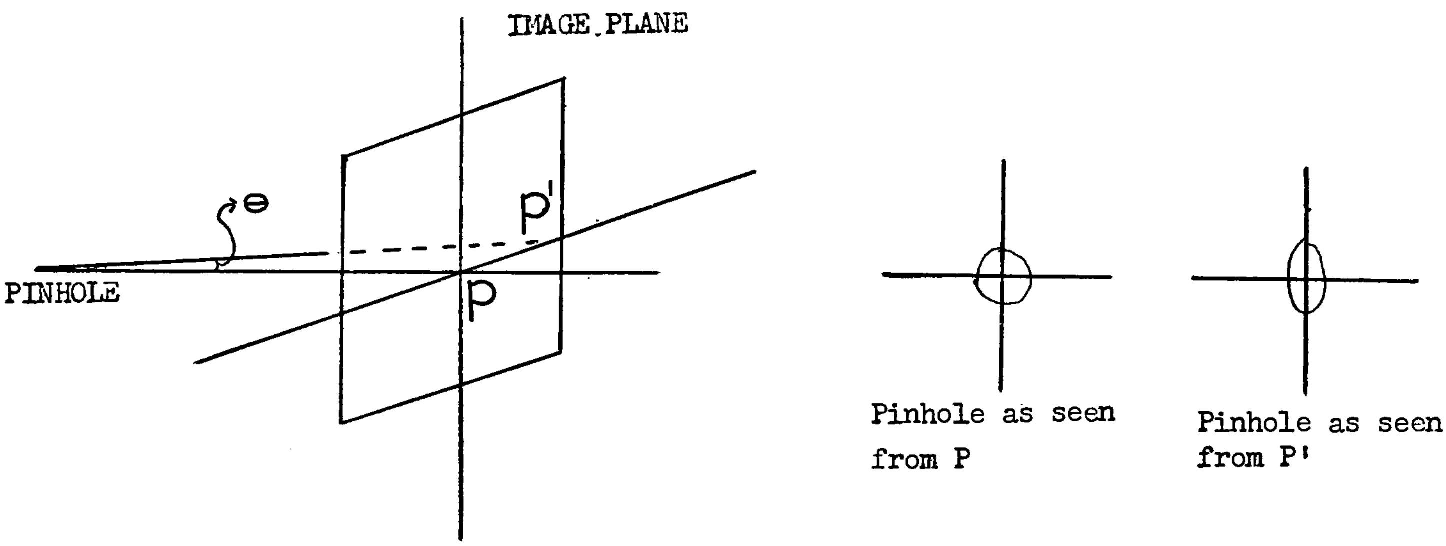


Figure Two. Image of pinhole off-axis

The equation $d^2=ky f$ is for on-axis imaging. Selwyn, Young, and Sayanagi each discussed what happens to off-axis imaging when the pinhole is optimized on-axis.[15,16,17] Selwyn thru physical reasoning showed that as the image moved off-axis, as is shown in Figure Two, the vertical (tangential) diameter of the pinhole remains the same; while, the effective diameter in the horizontal (radial) plane decreases as the cosine of the field angle. The pinhole thus appears as an ellipse. These differences result in astigmatism and curvature of field. Noting that the distance from the pinhole to the film plane is inversely proportional to $\cos\theta$, where θ is the field angle and that f is proportional to d^2 Sayanagi's equations for the position of best tangential focus, $f_t=f\cos^3\theta$, and best radial focus, $f_r=f\cos\theta$ follow after some work. Sayanagi reported monochromatic MTF curves for 30° and 45° off-axis.[18] Young, and Selwyn mentioned that there should be a pinhole size such that curvature of field and astigmatism are minimal over a relatively wide field.[19,20]

The equation $d^2=ky f$ is a function of wavelength and therefore is strictly correct for only monochromatic sources. Most sources are not monochromatic; therefore, Selwyn, Young, and Sayanagi each investigated the consequences of imaging with a broad spectrum. Selwyn and Young, assuming the working visible spectrum to be 400 to 600 nm, noted that if the design of the pinhole was worked out for 500 nm then the optimum pinhole size varies only by about 10%.[21,22] If d is held constant, the optimum length f varies with wavelength leading to longitudinal chromatic aberration. Selwyn noted that this is "excessive;" while, Young thought its "effect is small." Sayanagi gave a detailed representation showing that if the pinhole diameter and focal length are picked according to his one dimensional figure of merit, yielding $d^2=3.8yf$, then the "loss in one-dimensional figure of merit for both extreem wavelengths, 400 to 700 nm, is only 5% as compared to the meddle wavenumber."[23]

The equation $d^2=kyf$ is also based on the object being an infinite distance in front of the pinhole. Selwyn, and Young showed that $1/s + 1/s' = 1/f$ holds for the pinhole camera.[24,25] Although Sayanagi did not include a discussion of finite conjugates in his paper, he did put in information so that it could be worked out.[26]

The MTF curves in the literature are for monochromatic light. Thus it is of interest to develop the polychromatic MTF curves for a specific source and receiver. Standard 5500° K daylight is used as a source and for receivers a typical panchromatic black and white, and a color reversal film are used. With this information a map of the on and off-axis polychromatic MTF, as s' is changed can be made. This map gives information on optimization of the responce over a given field of view as well as furnish information on the depth of focus. The second objective is to use Sayanagi's information on effect of finite object distances to examine depth of field.

PROCEDURE

In figure Three COD is an expanding wavefront from a point source at P entering the pinhole with on-axis point at O. AOB is a reference sphere showing the shape of a wavefront when the on-axis point is at O that converges to a point P' in the image plane. Departures from this reference sphere are known

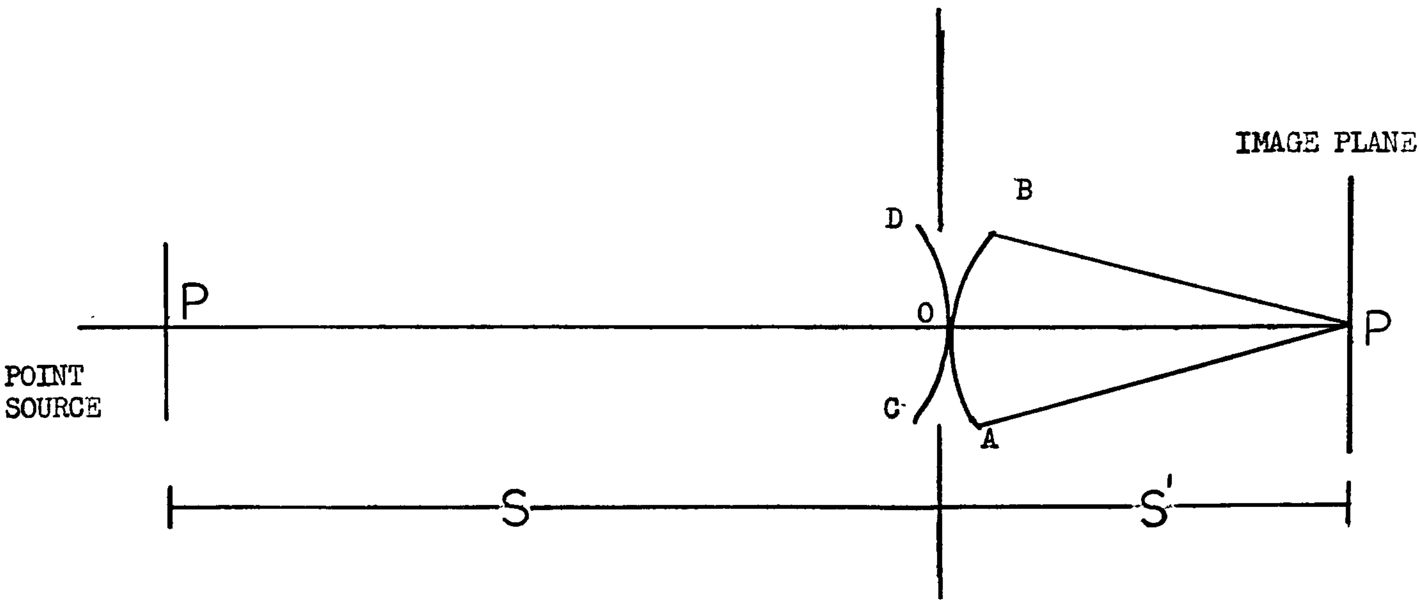


Figure Three. Diagram showing wavefront aberration at pinhole.

as wavefront aberrations and the function which describes it is called the pupil function. When the pupil function is known the MTF can be calculated by using the autocorrelation of the pupil function in terms of suitable coordinates.[27,28]

Since the pinhole camera has an f/# of about 200 on axis it can be thought of as a defocused aberrtion free optical system. Hopkins developed the pupil function for this case and the MTF. Sayanagi modified Hopkin's equation to calculate the MTF of the pinhole. A monochromatic computer program based on this was written.

Barn's has shown that the polychromatic MTF, P(S), can be calculated by using the following equation.[29]

$$P(s) = \frac{\int_0^\infty R_y S_y T_y M_y(s)\, dy}{\int_0^\infty R_y S_y T_y\, dy}$$

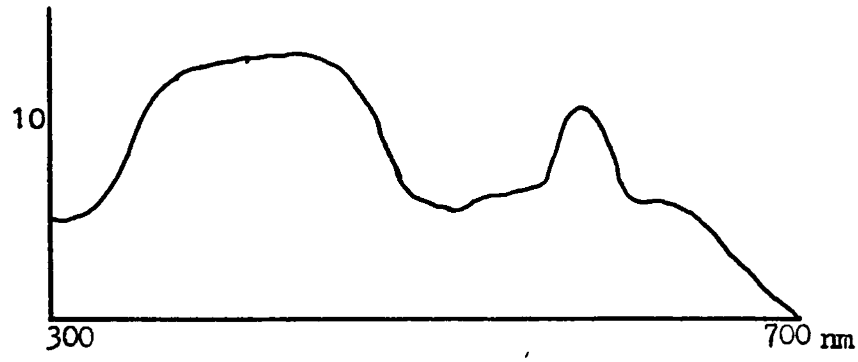
Ry is the spectral responce of the detector as a function of wavelength. Ty is the spectral transmission of the optical system, one in the case of a pinhole. Sy is the energy distribution of the source and My(S) is the monochromatic MTF. This equation was evaluated by approximating the integrals with a finite sum using seventeen intervals. A computer program was written that calculates the My(S) weights them according to the appropriate weighting factor then summ them to form the polychromatic MTF.

Off-axis both radial and tangential images must be considered. For the radical case, as Figure One shows, the distance OP' increases inversely proportionally to the cosine of the field angle Θ. For tangential case both the pinhole diameter and the distance OP' vary as the cosine of the field angle. Using one program for each, the polychromatic MTF program was generalized to calculate the on and off-axis MTF.

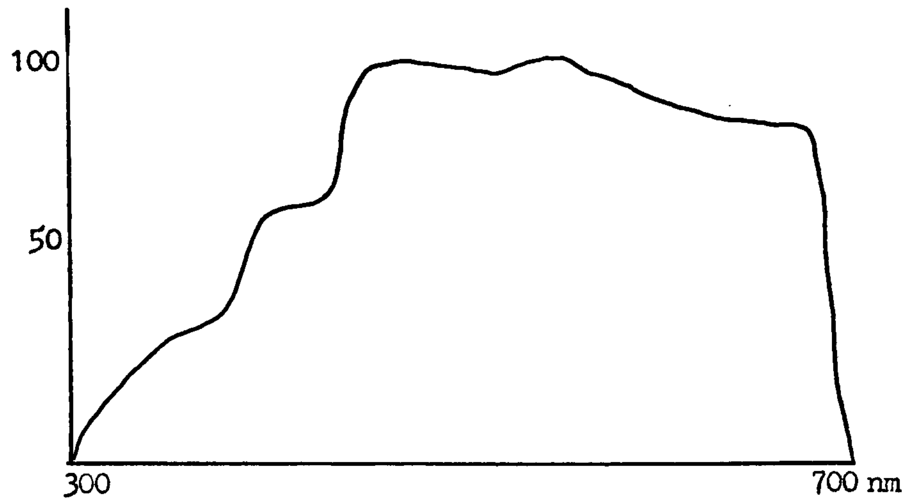
To calculate MTF's for finite object distances, Sayanagi's derivation of $d^2=kyf$ was worked thru using the aberration coefficient for finite conjugates. After doing so, $d^2=kyf(s+f)/s$. A monochromatic program was modified to calculate MTFs for finite conjugates.

DATA AND CONCLUSIONS

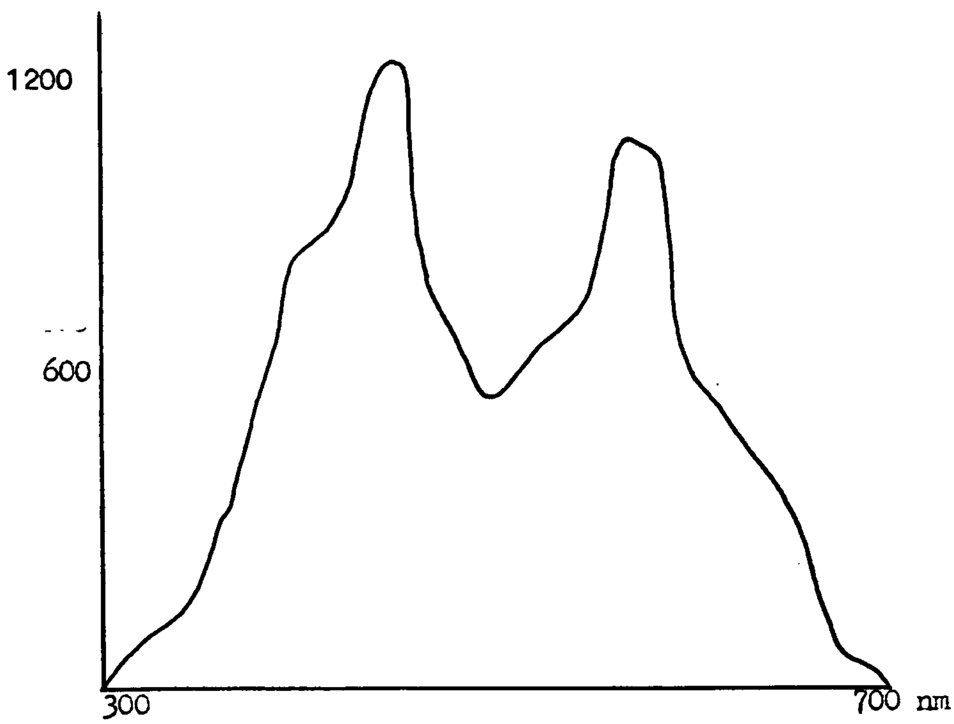
Figure Four shows the weighting function and its conponents for a typical black and white panchromatic film. Figure Five shows on and off- axis polychromatic MTF for a pinhole of diameter equal to 0.2 mm and focal length of 20.9 mm using this weighting function. In Figure Five the MTFs for on-axis and 15^o off-axis are essentially identical. This is as indicated from Sayanagi's equations for position of best tangential and radial focus. When s' equals



Spectral Sensitivity of Receiver



Energy Distribution of Source



Weighting Factor

Figure Four. Weighting Function and Components

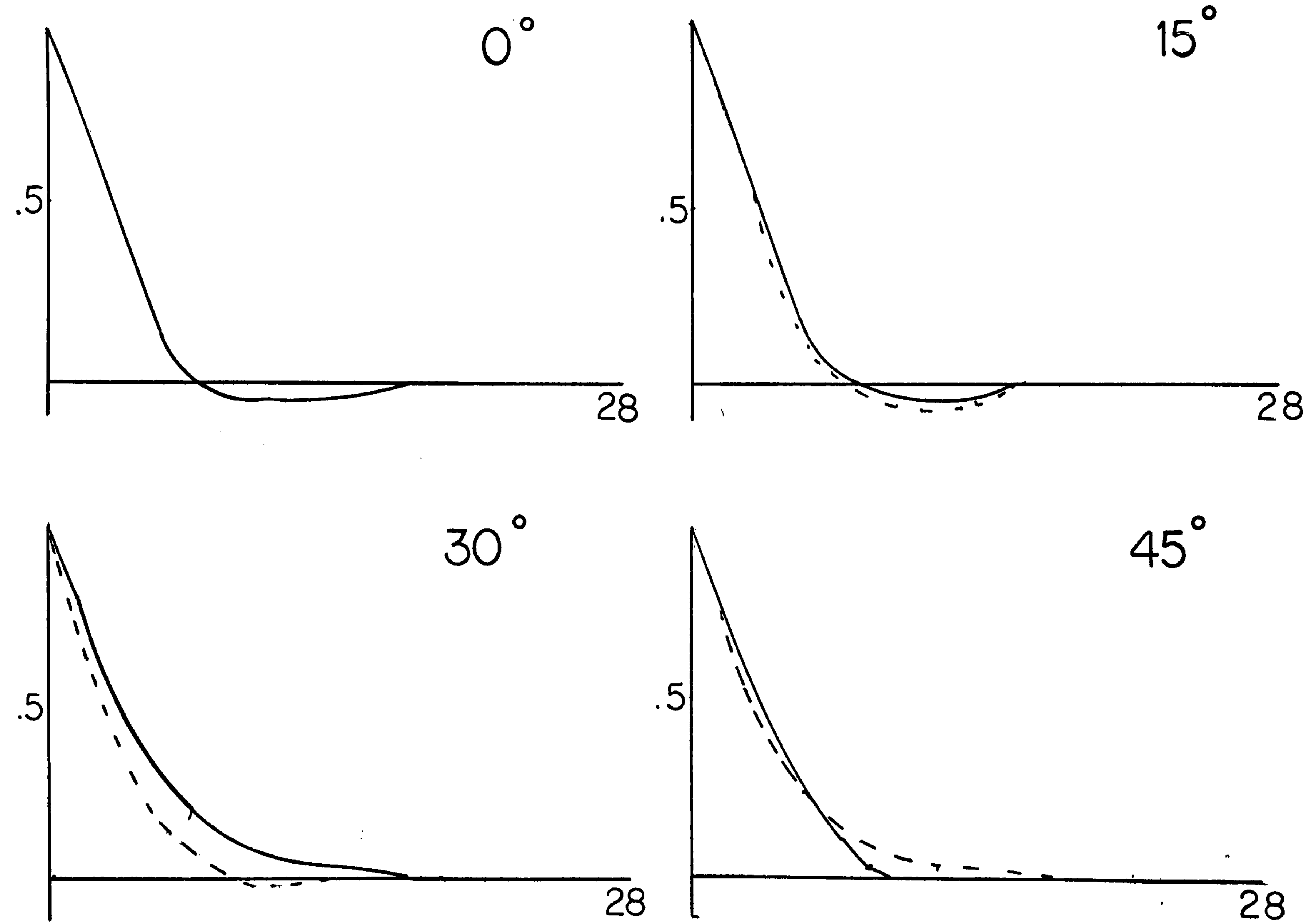


Figure Five A. On and Off-axis MTFs when s'= 12.6 mm. Radial MTF dotted.

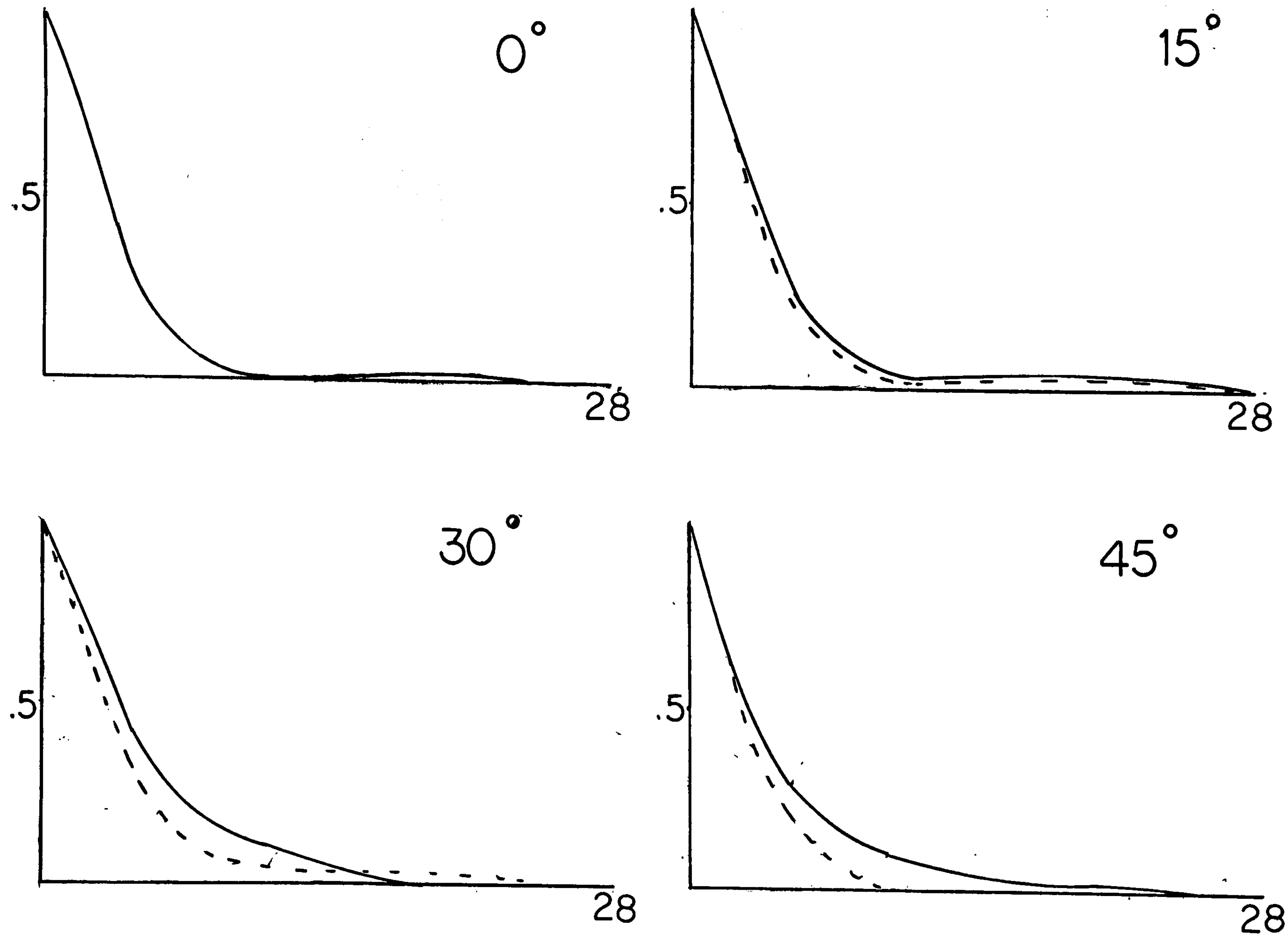


Figure Five B. On and Off-axis MTFs when s' = 15.7 mm Radial MTF dotted.

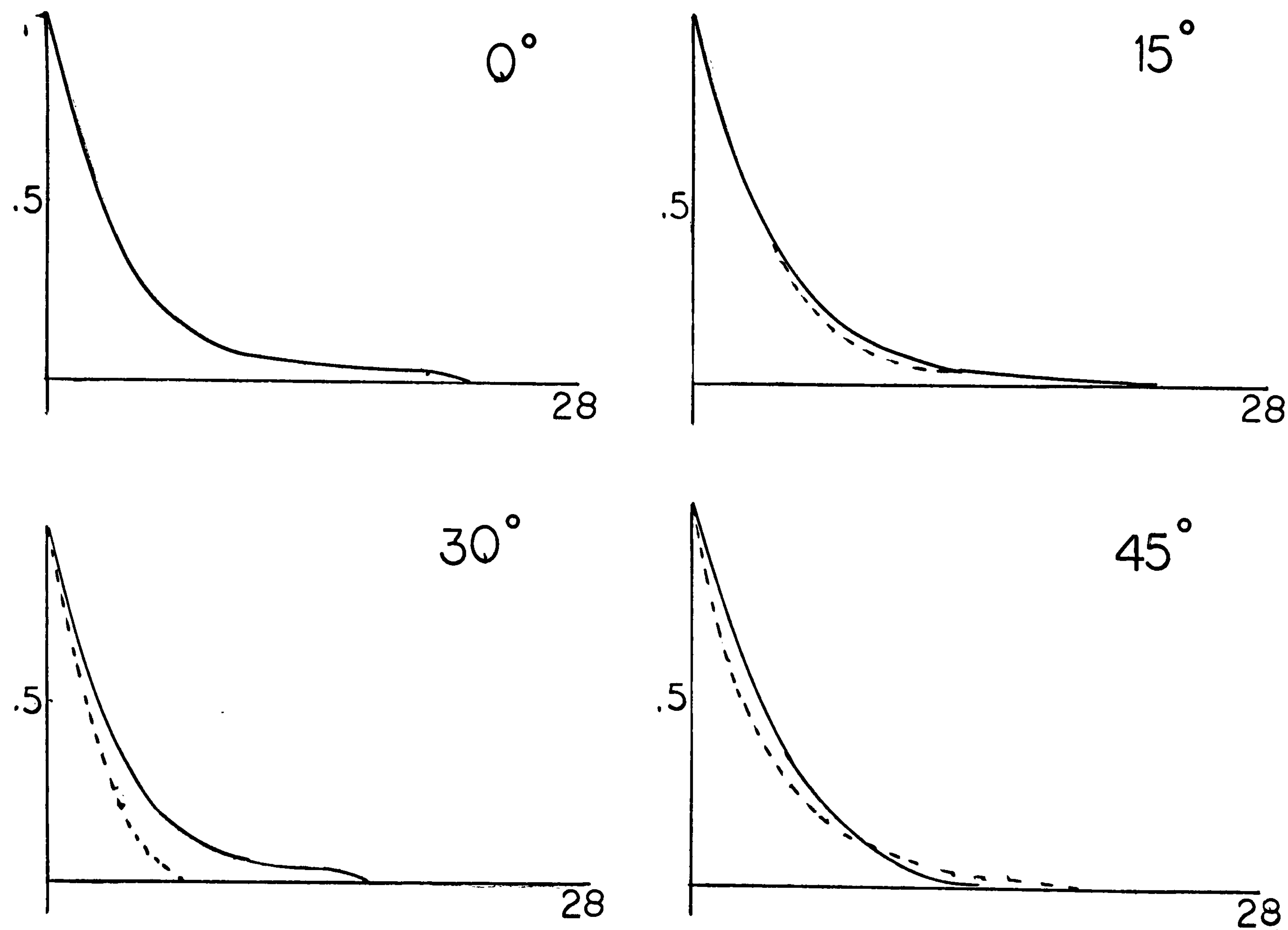


Figure Five. C. On and Off-axis MTFs when s' = 20.9 mm, Radial MTF dotted.

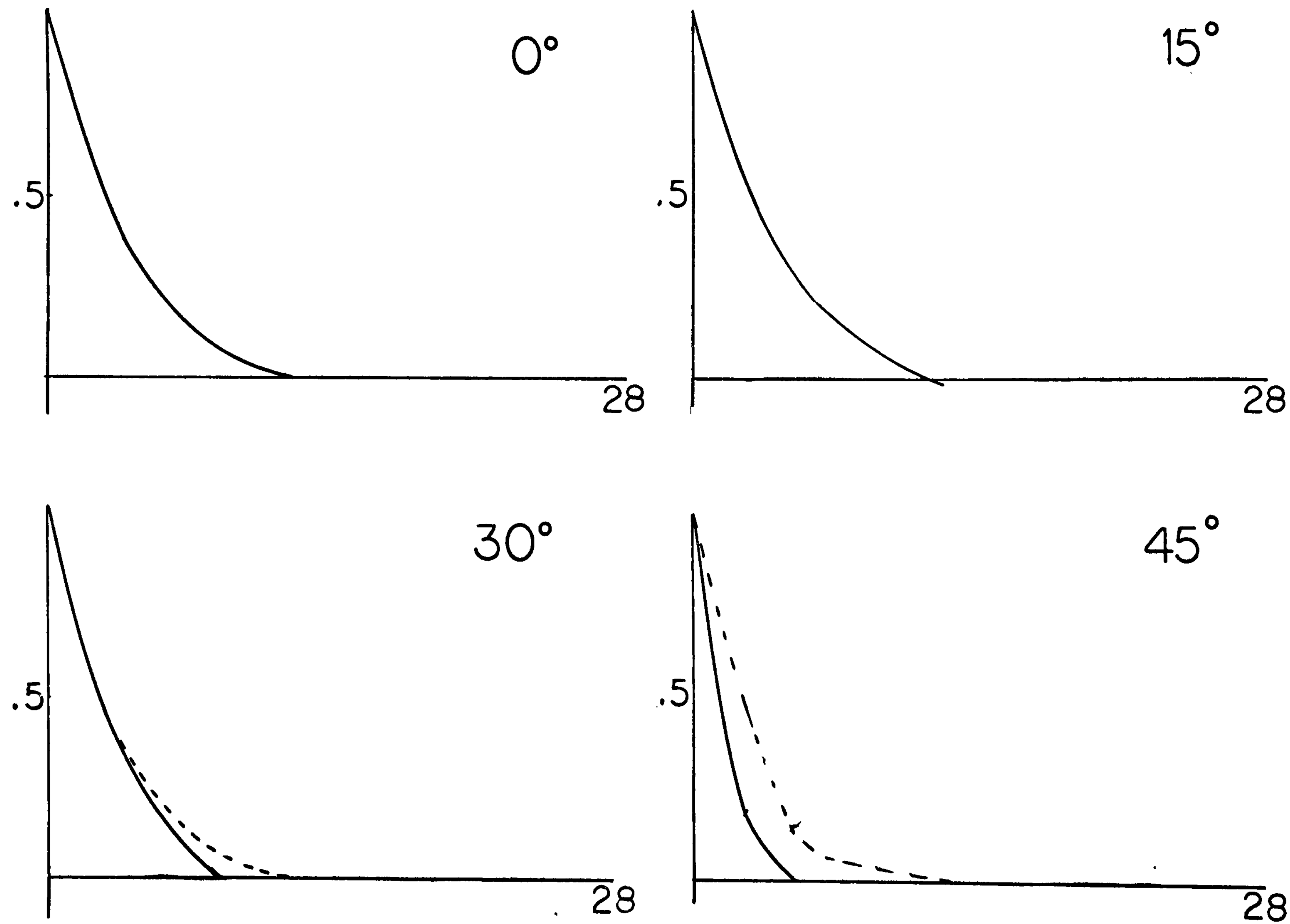


Figure Five D. On and Off-axis MTFs when $S' = 31.4$ mm, Radial MTF is dotted

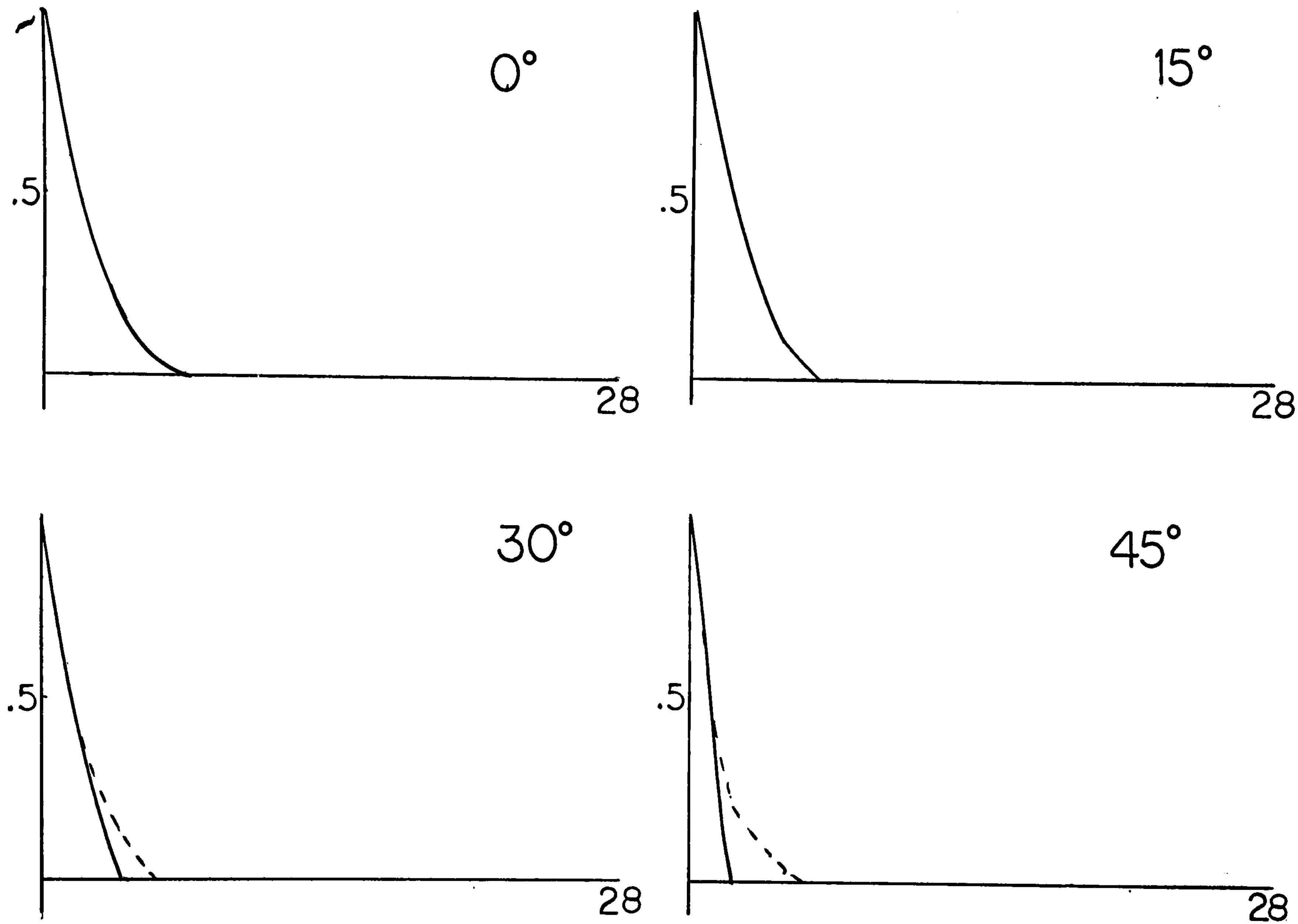


Figure Five E. On and Off-axis when s' = 62.8 mm, Radial MTF dotted.

15.7 mm on-axis, the on-axis MTF is the limiting case; that is, all off-axis MTFs for this film lane position give higher modulation for equal lines per millimeter and a higher cut off frequency. This confirms Selwyn's and Young's prediction that such a film plane position exists. Sayanagi predicted the MTF of the middle wavelength of the source would be a good approximation for the polychromatic MTF. Figure Five shows that the 20.9 mm film plane position has the bes MTF on- axis which is the position for best focus at 500 nm using Sayanagi's one dimensional figure of merit criterion. Figure Six shows that for on-axis the monochromatic MTF is a good approximation. Figure Seven is a plot of the lines per millimeter in-axis versus the position of the film plane. This gives an indication of the on-axis depth of focus. If acceptable focus is taken as 50% of themaximum lines per millemeter, then the depth of focus is over 25 mm. Five shows that it is better to be on the near side of the on-axis optimum since the off- axis MTF shows greater modula ion and gigher cut off frequencies.

Figure Eight shows the weighting function and its components for a typical color reversal film. Figure Nine is a comparison of thw weighting functions for color reversal and back and white after normalizing by considering the largest value to be one. All results for the panchromatic black and white film hold for the color film. Differences reflect differences in the weighting functions. Figure Ten is a comparison of a black and white MTF with a color MTF under identical conditions except for film.

Figure Eleven compares MTFs calculated for a pinhole of 0.2 mm and 20.9 mm focal length with s equal to 5f and 20f.with the MTF based on an infinite

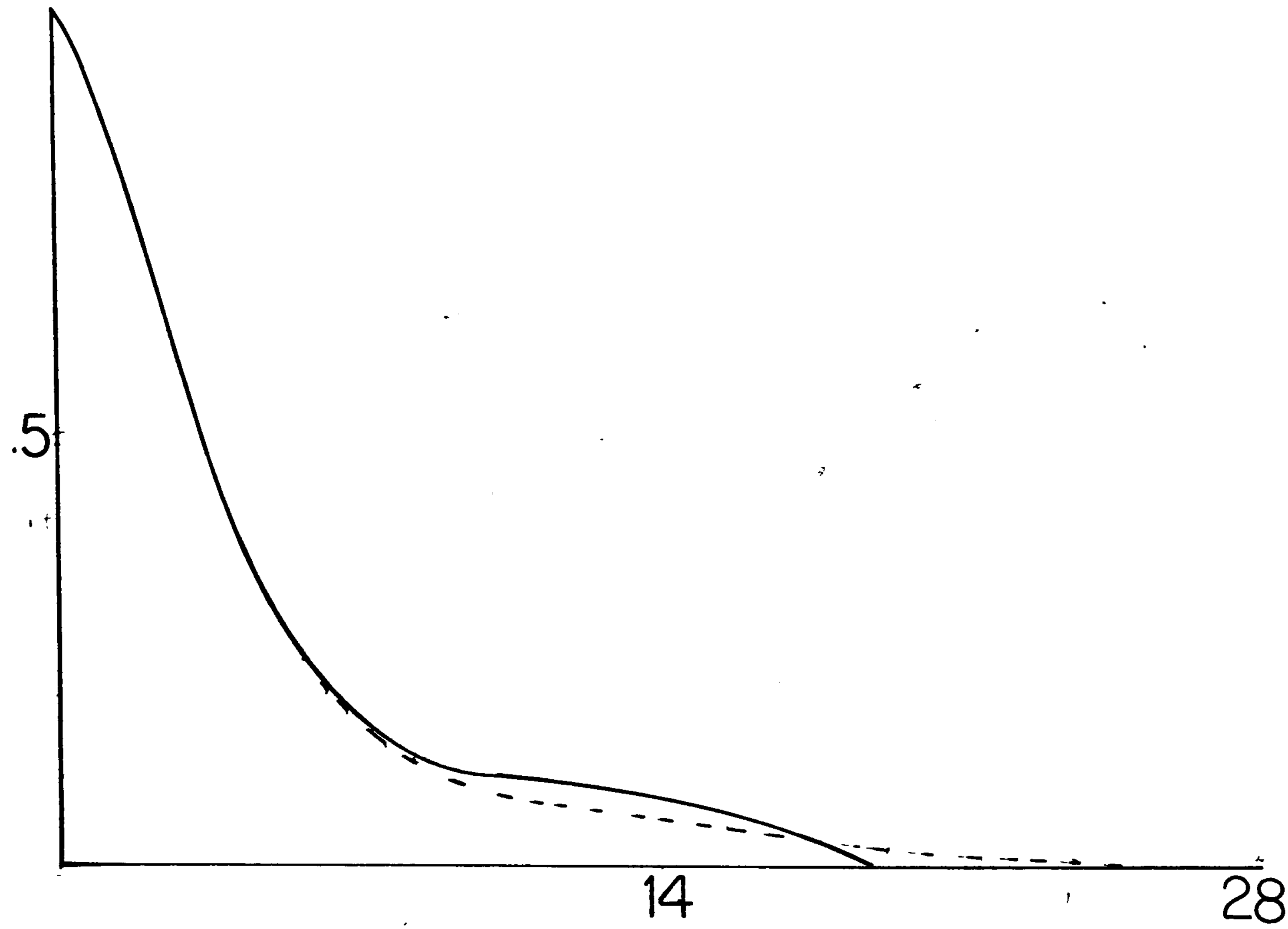


Figure Six. Comparison of Polychromatic MTF with MTF of middle wavelength, 500 nm.

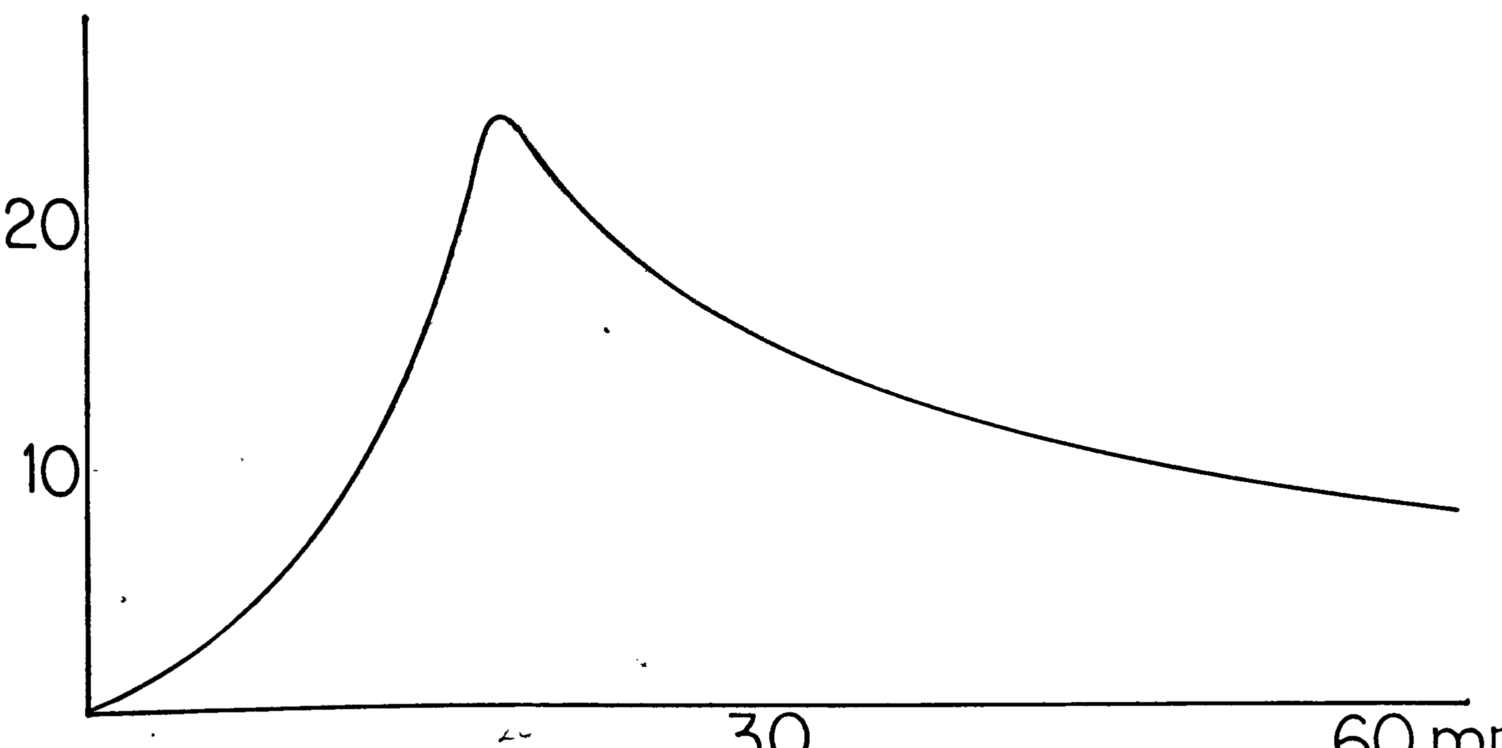
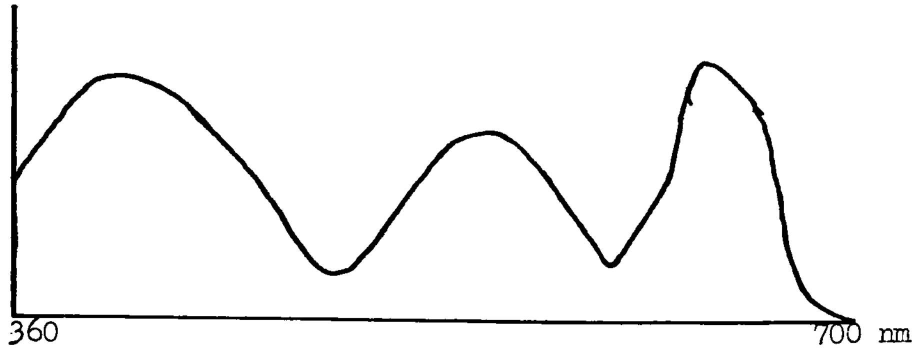
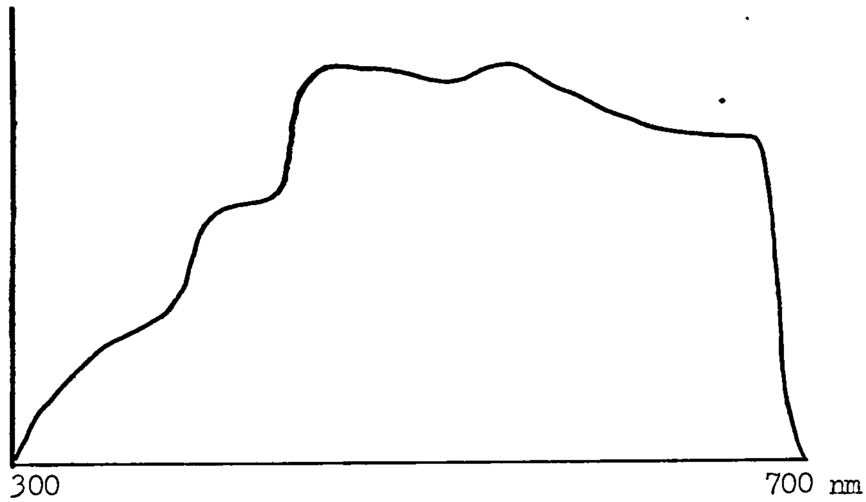


Figure Seven. On-axis depth off field.



Spectral Sensitivity of Receiver



Energy Distribution of Source

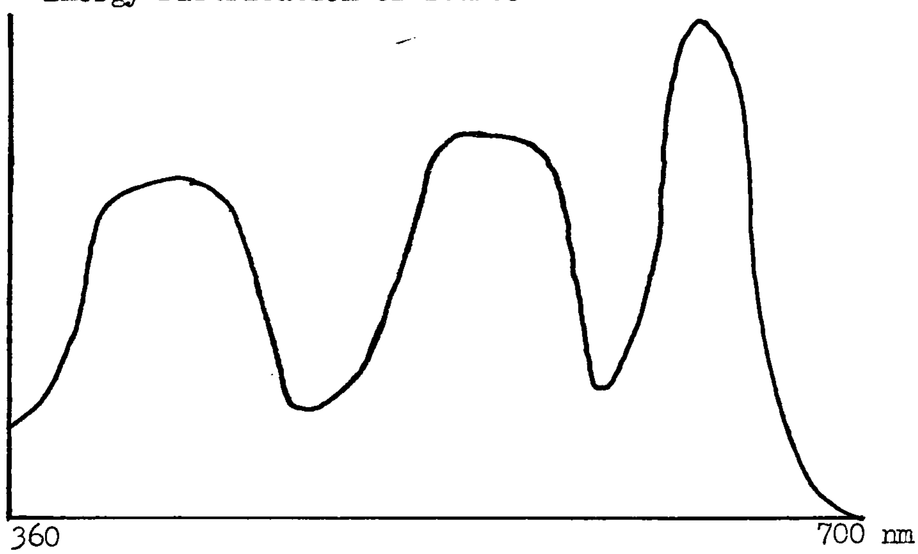


Figure Eight. Weighting Function and Compoents for Color Reversal Film

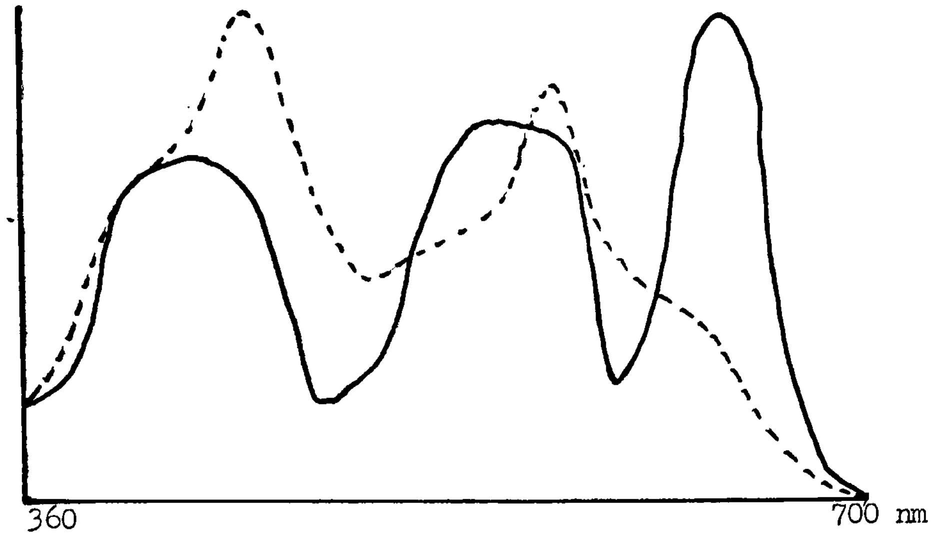


Figure Nine. Comparison of Black and White Weighting Function (dots) with Color Reversal Weighting Function.

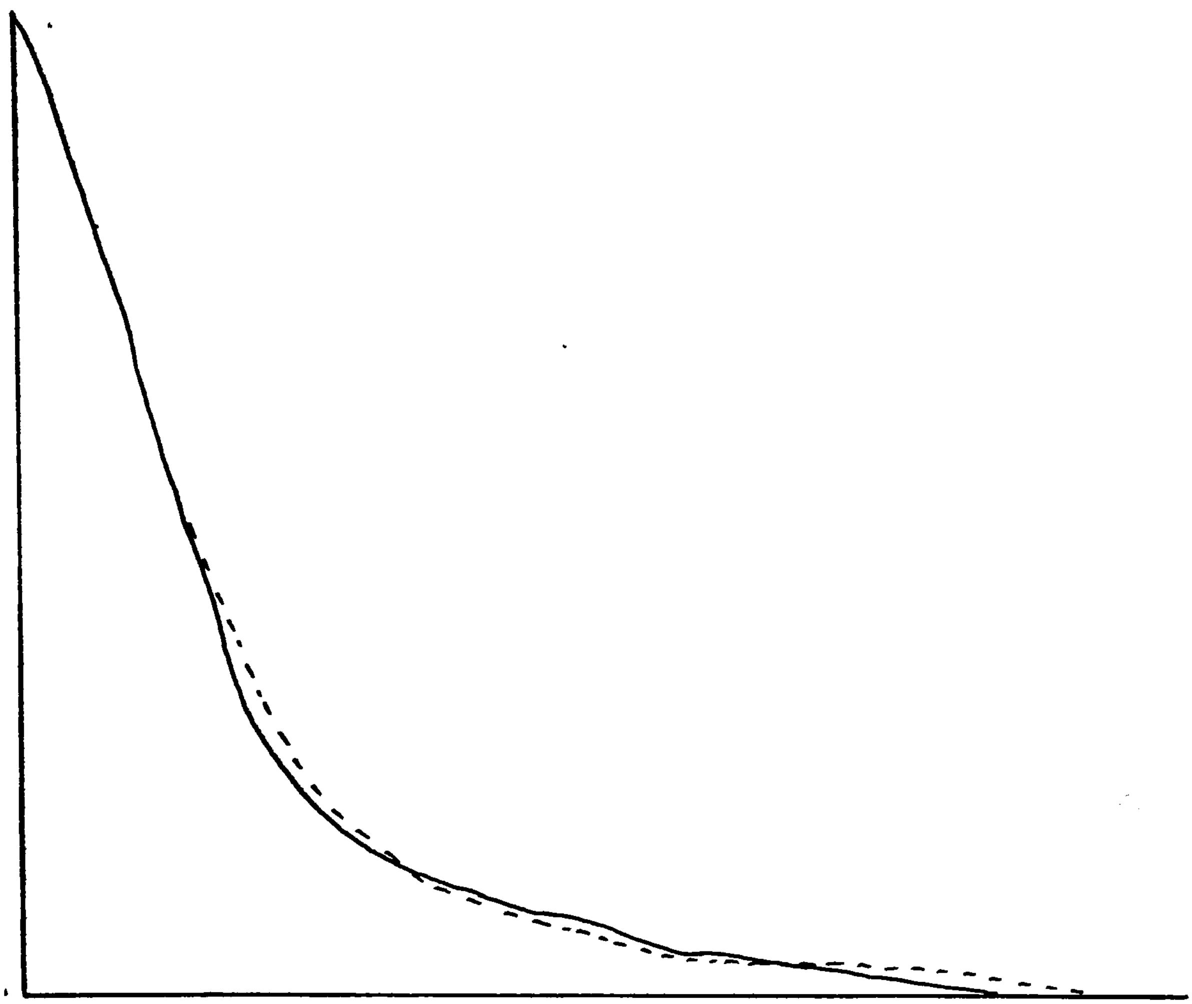
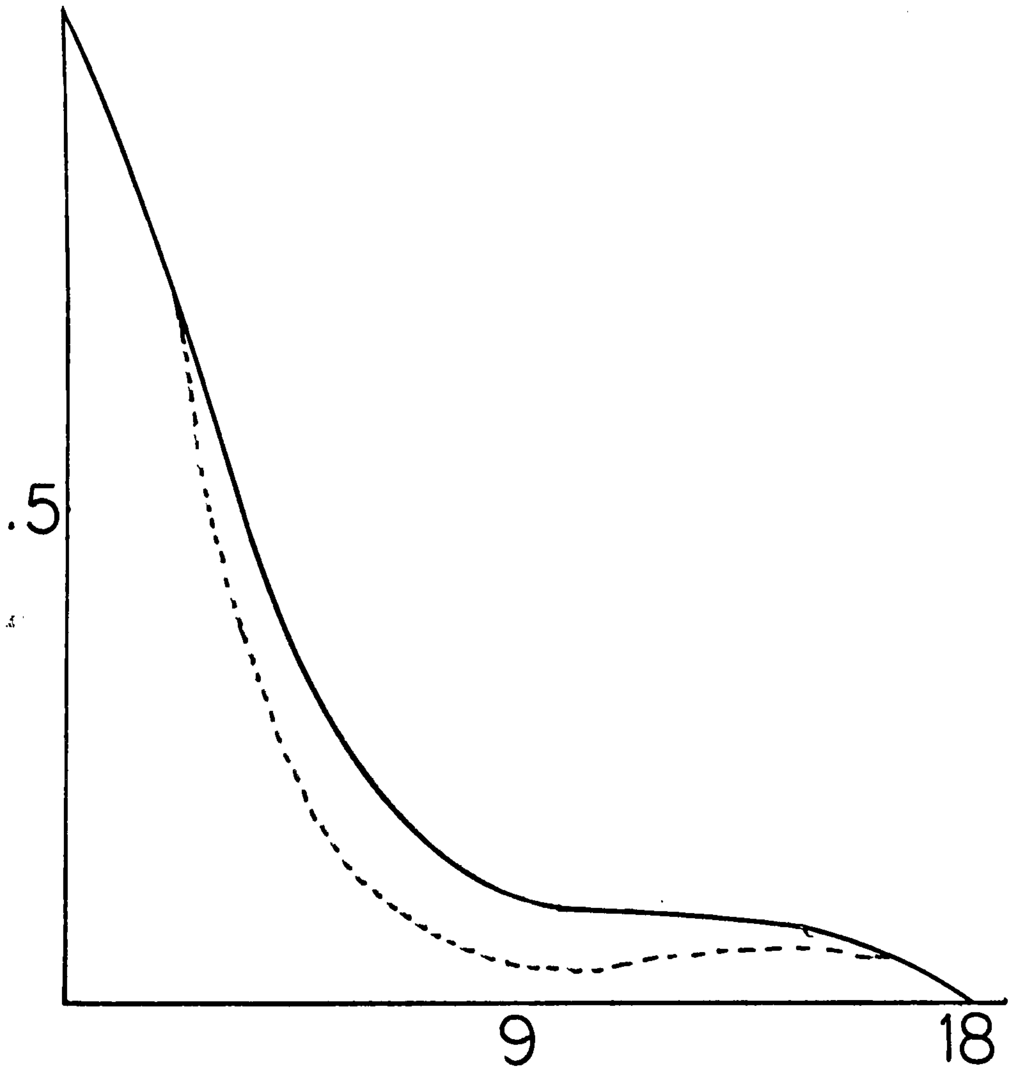
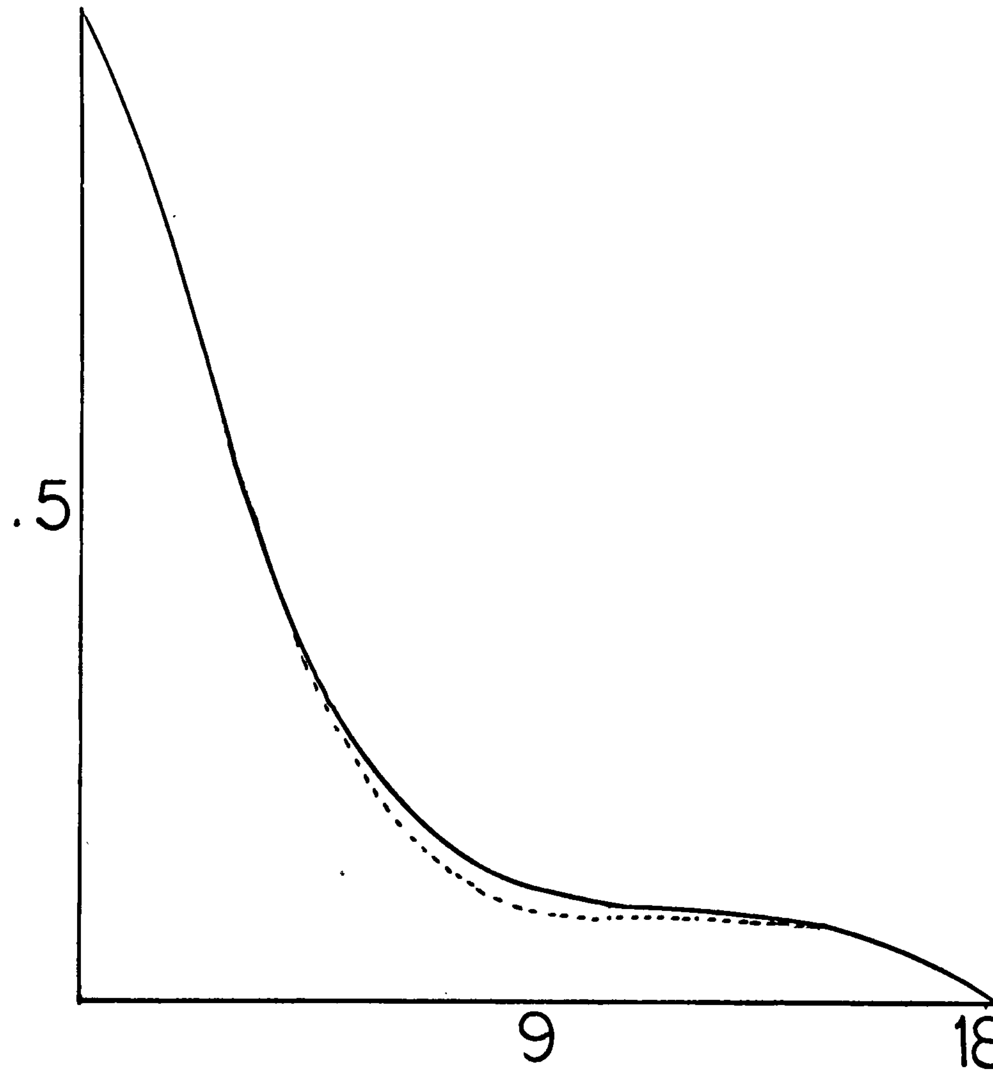


Figure Ten. Comparison of MTF of Color Reversal Film with MTF of Black and White Film. (dots).



MTFs for objects at 5f and infinity



MTFs for objects at 20f and infinity

Figure Eleven. Depth of Field for Pinhole.

object distance. S' equals 20.9 mm. This shows that the depth of field can be considered practicall in finite. Yet, it showd that it is best to optimize the camera for the particular object distance if s is under 10f.

Using the weighting function shown in Figure Four a map using a focal length of 162 mm was made. Outside of less lines per millimeter and larger depth of focus, the same general conclusions can be drawn for this focal length as for 20.9 mm.

CONCLUSIONS

In this paper the on and off-axis polychromatic MTF for apinhole camera was developed. It was demonstrated that there is a film plane position such that the on and off- axis MTF have approximately similar MTF's. It was shown that the depth of field can be considered infinite; yet, when s is under 10f it is best to optimize for this object distance in order to increase modulation.

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