A Method for Generating Random Vibration Using Acceleration Kurtosis and Velocity Kurtosis

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ABSTRACT

Random vibration tests for packaging are conducted to confirm safety during shipping by truck. However, there is a difference between the traditional random vibration tests and the real vibrations on the truck bed. One reason for this difference is the shock caused by road roughness. Hence, many studies have been conducted to improve random vibration testing. In these studies, the root mean square, power spectral density, kurtosis, and probability density of acceleration are considered. In this study, we show that the kurtosis and probability density of velocity are also important factors for such tests and propose a new method for generating vibrations with arbitrary kurtosis of acceleration and velocity. By bringing the kurtosis and probability density of velocity closer to those of real vibration, it is possible to conduct more accurate vibration tests.

KEY WORDS

Transportation, Vibration Test, Velocity, Kurtosis, Probability Density

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INTRODUCTION

Random vibration tests for packaging are conducted to confirm its safety during shipping by truck. However, there are differences between traditional random vibration tests and the real vibrations experienced on the truck bed. One reason for these differences is the shock caused by road roughness, including speed bumps, cracks, and pothole [1]. Hence, many studies have been conducted to improve random vibration testing.

Some researchers have proposed methods focusing on the probability density and kurtosis of acceleration. The probability density of acceleration during traditional random vibration tests has a Gaussian distribution. However, the probability density of acceleration during real transportations is non-Gaussian (high kurtosis value). Hence, methods for controlling the probability density of acceleration during vibration testing have been proposed [2, 3].

Other researchers have proposed methods to divide the truck bed vibration. Singh et al. proposed a method to divide the vibration based on the root mean square (RMS) of acceleration [4]. Griffiths et al. proposed decomposing the vibration by wavelet transformation and then reconstructing it [5]. Zhou et al. proposed to divide the vibration based on a tenth-peak method and a moving crest factor [6].

The factors that are frequently considered in these proposed methods are the RMS, power spectral density (PSD), kurtosis, and probability density of acceleration.

In impact testing for packaging, both maximum acceleration and velocity change are important factors [7]. In earthquake engineering, both maximum acceleration and maximum velocity are correlated with building damages [8]. Hence, velocity is assumed to be an important factor in vibration testing for packaging. However, to our knowledge, no method has yet considered the factors related to velocity in such tests. In this study, we propose a new method for generating vibrations with arbitrary kurtosis of acceleration and velocity and show that the kurtosis and probability density of velocity are both important factors for considering shocks during random vibration testing.

THEORY

Method for generating vibrations

The traditional method for generating random vibrations is expressed as equation (1):

$$a(t) = \sum_{k=1}^{L} A_k \cos(2\pi f_k t + \omega_k),$$  \hspace{1cm} (1)

where $L$, $A_k$, $\Delta f$, and $\phi_k$ are, respectively, the number of frequency components, the amplitude, the frequency resolution, and the k$^{th}$ phase angle. $A_k$ is expressed as equation (2):

$$A_k = \sqrt{2\pi \Delta f P(k \Delta f)},$$  \hspace{1cm} (2)

where $P(k \Delta f)$ is a PSD. In traditional random vibration tests, $\phi_k$ denotes random numbers ranging from 0 to $2\pi$.

Hosoyama et al. proposed a method focusing upon phase angles to approximate the kurtosis and probability density of acceleration [2]. In this method, $\phi_k$ is expressed as equation (3):

$$\phi_k = \phi_{k-1} + \Delta f \tau_{gr}(k \Delta f),$$  \hspace{1cm} (3)

where $\tau_{gr}(k \Delta f)$ is the group-delay time. In this study, $\tau_{gr}(k \Delta f)$ is a random number of which the average value is $m$ and the standard deviation is $\sigma$. $m$ is related to the phase at which the maximum acceleration occurs, and $\sigma$ is related to the envelope curve of vibration. As $\sigma$ increases, this curve becomes sharper, and the kurtosis of acceleration becomes higher.

Hosoyama et al. only focused on the value of $\sigma$. In this study, we also focus on the value of $\Delta f$. $\Delta f$ is related to the period $T_d$ at which the maximum
acceleration occurs. The relationship between $\Delta f$ and $T_d$ is expressed as equation (4):

$$T_d = \frac{1}{\Delta f}$$  \hspace{1cm} (4)

Here, $\sigma$ and $T_d$ were changed in the ranges of 0.05 to 2.5 and 1 to 16 s, respectively.

**Method to estimate velocity**

Typically, velocity $v$ can be estimated from acceleration $a$ as equation (5):

$$v_{t+\Delta t} = v_t + (a_t + a_{t+\Delta t}) \frac{\Delta t}{2},$$  \hspace{1cm} (5)

where $\Delta t$ is a sampling period. However, we cannot accurately estimate velocity from the acceleration measured with an accelerometer using equation (5) because of low-frequency noise. In our previous study, we showed a way to more accurately estimate velocity using a low-cut filter [9].

The Fourier transform of acceleration, $A(f)$, is expressed by equation (6):

$$A(f) = \int_{-\infty}^{\infty} a(t)e^{-2\pi ift} dt$$  \hspace{1cm} (6)

That of velocity is expressed by equation (7):

$$V(f) = \frac{A(f)}{2\pi f}$$  \hspace{1cm} (7)

To eliminate low-frequency noise, $V(f)$ is corrected by low-cut filter $L(f)$. The corrected Fourier transform of velocity $V'(f)$ is expressed by equation (8):

$$V'(f) = L(f)V(f)$$  \hspace{1cm} (8)

In this study, a third-order Butterworth filter was used as $L(f)$. The third-order Butterworth filter is expressed by equation (9):

$$L(f) = \frac{1}{1 - 2(\frac{f}{f_c})^2 + (\frac{f}{f_c})^3},$$  \hspace{1cm} (9)

where $f_c$ is the cut-off frequency. In this study, $f_c$ is 0.5 Hz.

Velocity, $v(t)$, can be estimated by an inverse Fourier transform:

$$v(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} V'(f)e^{2\pi ift} df$$  \hspace{1cm} (10)

**Evaluation method**

To evaluate shock intensity during vibration, single-degree-of-freedom (SDOF) structures were used. We assumed that SDOF structures were placed directly on the truck bed (Figure 1). The force balance of an SDOF structure is expressed as equation (11):

$$m\left(\frac{d^2y}{dt^2} - a\right) + c\frac{dy}{dt} + ky = 0,$$  \hspace{1cm} (11)

where $m$, $c$, $k$, and $y$ are, respectively, the mass, viscosity coefficient, spring constant, and relative displacement between the mass and the truck bed. The impulse response function for the displacement per unit impulse force excitation, $h(t)$, is expressed as equation (12):

$$h(t) = \frac{\exp(-\xi \omega_n t) \sin(\omega_d t)}{\omega_d},$$  \hspace{1cm} (12)

where $\xi$, $\omega_n$, and $\omega_d$ are, respectively, the damping factor, undamped natural angular frequency, and damped natural angular frequency of the response [10]. $\omega_n$ is expressed as equation (13):

$$\omega_n = \sqrt{\frac{k}{m}} = 2\pi f_n,$$  \hspace{1cm} (13)

where $f_n$ is the natural frequency. In this study, $f_n$ changed in the range of 1 to 200 Hz. $\xi$ is expressed as equation (14):

$$\xi = \frac{c}{2m\omega_n},$$  \hspace{1cm} (14)
In this study, \( \xi \) was 0.05. \( \varpi_d \) is expressed as equation (15):

\[
\varpi_d = \sqrt{1 - \varpi_n^2}
\]  

(15)

\( y \) is expressed as equation (16):

\[
y(t) = \int_0^t a(\tau)h(t - \tau)d\tau.
\]  

(16)

where \( \tau \) is the parameter. Equation (16) can be calculated by Fourier transformation as

\[
y(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} A(f)H(f)e^{2\pi ift}df,
\]  

(17)

where \( H(f) \) is the Fourier transform of \( h(t) \). The acceleration applied inside the SDOF structure \( a_i \) is proportional to the relative displacement \( y \):

\[
a_i(t) = 4\pi^2f_i^2y(t)
\]  

(18)

**RESULTS AND DISCUSSION**

We generated simulated vibrations using the values shown in Figure 3(c) as the target PSD. The vibration generated using random numbers from 0 to \( 2\pi \) is used to simulate random vibration. Vibrations were also generated from equations (1) to (4). \( T_d \) is set to 1, 2, 4, 8, and 16 s, and \( \sigma \) ranges from 0.05 to 2.5.

All vibrations last 160 s, which is the same time as that for a real vibration. Random vibration lasts 1 s (1000 Hz) per frame and consists of 160 frames. Each frame generated from equations (1) to (4) is taken with a period of \( T_d \). For example, the vibration with a \( T_d \) of 1 s consists of 160 frames, and that with a \( T_d \) of 16 s consists of 10 frames.

Figure 4 shows the relationship between \( T_d, \sigma \), and the kurtosis of acceleration. As \( \sigma \) decreased, the kurtosis of acceleration increased. As \( \sigma \) increased, the kurtosis of acceleration decreased and converged to 3, which was the same value as the normal distribution. Even with the same value of \( \sigma \), the kurtosis of acceleration tends to increase as \( T_d \) increases.

**EXPERIMENT**

The vertical vibration data on the truck bed was used as a target. A DER-1000 accelerometer (Shinyei Testing Machinery Co., Ltd.) was fixed to the truck bed. Figure 2 shows the truck and the accelerometer. We ran the truck on the road for 160 s. Figure 3(a), (b), and (c) show, respectively, the acceleration, velocity, and acceleration PSD of real vibration.

![Fig. 2: Truck and accelerometer.](image)

![Fig. 3: Real vibration (a) acceleration; (b)velocity; (c)PSD of acceleration.](image)
Figure 4 shows the relationship between $\sigma$, $T_d$, and the kurtosis of acceleration for different values of $T_d$: (a) $T_d = 1$ s; (b) $T_d = 2$ s; (c) $T_d = 4$ s; (d) $T_d = 8$ s; (e) $T_d = 16$ s.

Figure 5 shows the relationship between $\sigma$, $T_d$, and the kurtosis of velocity for different values of $T_d$: (a) $T_d = 1$ s; (b) $T_d = 2$ s; (c) $T_d = 4$ s; (d) $T_d = 8$ s; (e) $T_d = 16$ s.

Figure 5 shows the relationship between $T_d$, $\sigma$, and the kurtosis of velocity. As $\sigma$ decreased, the kurtosis of velocity increased. As $\sigma$ increased, the kurtosis of velocity decreased and converged to 3, which is the same value as in the normal distribution. The kurtosis of acceleration and velocity showed the same trend.

Figure 6 shows the relationship between the kurtosis of acceleration and that of velocity. When the value of $T_d$ was small, the amount of increase in the kurtosis of velocity was small with respect to that of acceleration. As the value of $T_d$ increased, the amount of increase in the kurtosis of velocity was bigger. Even if $T_d$ increased, the kurtosis of velocity did not significantly exceed that of acceleration.

Table: Vibration statistics.

<table>
<thead>
<tr>
<th>$T_d$</th>
<th>Acceleration RMS (m/s$^2$)</th>
<th>Kurtosis</th>
<th>Velocity RMS (m/s)</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 s</td>
<td>1.88</td>
<td>11.61</td>
<td>0.0613</td>
<td>11.90</td>
</tr>
<tr>
<td></td>
<td>random vibration</td>
<td>1.90</td>
<td>2.93</td>
<td>0.0701</td>
</tr>
<tr>
<td>4 s</td>
<td>1.90</td>
<td>11.67</td>
<td>0.0653</td>
<td>4.66</td>
</tr>
<tr>
<td>8 s</td>
<td>1.88</td>
<td>11.52</td>
<td>0.0608</td>
<td>9.08</td>
</tr>
<tr>
<td>16 s</td>
<td>1.90</td>
<td>11.63</td>
<td>0.0624</td>
<td>11.91</td>
</tr>
</tbody>
</table>

To compare in more detail, we extracted the vibrations whose kurtosis of acceleration is close to real vibration. Table shows the statistics of the vibrations. The acceleration kurtosis of extracted non-random vibrations was close to 11.6 ($\pm$0.1). On the other hand, the kurtosis of velocity differed for all generated vibrations. The vibration with a $T_d$ of 16 s had a velocity kurtosis close to that of real vibration. Vibrations with $T_d$ values of 1 and 4 s had small velocity kurtosis compared to real vibration.
Fig. 7: Time series of acceleration  
(a) random vibration; (b) $T_d = 1 \text{ s}$;  
(c) $T_d = 4 \text{ s}$; (d) $T_d = 16 \text{ s}$.

Fig. 8: Time series of velocity  
(a) random vibration; (b) $T_d = 1 \text{ s}$;  
(c) $T_d = 4 \text{ s}$; (d) $T_d = 16 \text{ s}$.
Figure 9 shows the PSD of acceleration. All vibrations had almost the same PSD.

Figure 10 shows the probability density of acceleration. As shown in Figure 10(a), the random vibration, for which kurtosis is close to that of a normal distribution, had a small probability-density-acceleration spread. Other vibrations had nearly the same probability density of acceleration.

Figure 11 shows the velocity probability density. Despite having nearly the same probability density of acceleration, the probability density of the velocity was different from each other. As shown in Figure 11(a), there was a small acceleration probability density spread under random vibration. Figure 11(b) shows that the vibration with a $T_d$ of 1 s closely approximated the spread with random vibration. This was consistent with the kurtosis of the velocity being 4.66, which is close to the normal distribution. The probability densities of the vibration velocity with a $T_d$ of 16 s (Figure 11(d)) and the real vibration (Figure 11(e)) had similar spreads.

Figure 12 shows $a_i$ obtained by equations (12)–(18) when $f_n$ is 5 Hz. As shown in Figs. 12(c) and (d), the period of $T_d$ could be confirmed. However, a period of 1 s could not be clearly confirmed in Fig. 12(b).
Fig. 9: PSD of acceleration
(a) random vibration; (b) $T_d = 1$ s; (c) $T_d = 4$ s; (d) $T_d = 16$ s; (e) real vibration.

Fig. 10: Probability density of acceleration (a) random vibration; (b) $T_d = 1$ s; (c) $T_d = 4$ s; (d) $T_d = 16$ s; (e) real vibration.

Fig. 11: Probability density of velocity (a) random vibration; (b) $T_d = 1$ s; (c) $T_d = 4$ s; (d) $T_d = 16$ s; (e) real vibration.
Fig. 12: Time series of $a_i$ when $f_n$ is 5 Hz (a) random vibration; (b) $T_d = 1$ s; (c) $T_d = 4$ s; (d) $T_d = 16$ s; (e) real vibration.

Fig. 13: The relationship between $f_n$ and $\text{max } |a_i|$ (a) random vibration; (b) $T_d = 1$ s; (c) $T_d = 4$ s; (d) $T_d = 16$ s; (e) real vibration.
Figure 13 shows the relationship between the maximum absolute values of $|a_i|$ ($\max |a_i|$) and $f_n$. It is supposed that, the larger $\max |a_i|$ is, the more damage is incurred to packages.

Figure 14 shows $\max |a_i|$ divided by $\max |a_i|$ of real vibration. Random vibration had a low $\max |a_i|$ compared to real vibration in all $f_n$ ranges. Hence, it is supposed that only weak shock occurred during random vibration, as compared to real vibration. As shown in Figs. 14(b)–(d), as $T_d$ increased, $\max |a_i|$ divided by $\max |a_i|$ of real vibration approached a value of 1.0 in many $f_n$ ranges. The vibration with a $T_d$ of 16 s, for which kurtosis was the closest to that of real vibration, also had $\max |a_i|$ closest to that of real vibration.

**CONCLUSIONS**

We generated vibrations with arbitrary kurtosis of acceleration and velocity by changing the factors $\sigma$ and $T_d$.

Using this method, we generated vibrations with the same acceleration kurtosis, probability density of acceleration, and PSD, but different kurtosis and probability density of velocity. This showed that the kurtosis and probability density of velocity are important factors for considering shocks during random vibration testing. By bringing the kurtosis and probability density distribution of velocity closer to that of real vibration, it is possible to conduct vibration tests more like real vibrations.

The next task is to verify the effect through actual vibration tests.
REFERENCES


