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Donald Holmes

A. Erhan Mergen

ANKARA UNIVERSITY

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MODELING IN STATISTICAL PROCESS CONTROL

Donald S. Holmes
Stochos, Inc.
14 N. College Street
Schenectady, New York 12305
U.S.A.

A.Erhan Mergen
Rochester Institute of Technology
College of Business, Decision Sciences
107 Lomb Memorial Drive
Rochester, New York 14623-5608
U.S.A.

ABSTRACT

Statistical Process Control (SPC), which is based on statistical theory, helps to monitor the performance of a process. SPC techniques were first introduced by Shewhart [1] in the 1930's. They are used to identify, control and eliminate variation in the process. To control and reduce variation, one should understand its sources. Variation in a process can be grouped into two categories:

1. *Common (Random) cause* variation
2. *Special (Assignable) cause* variation.

A process is said to be operating in a state of statistical control when only *common* sources of variability are present in the process. Such a process is stable and predictable.

SPC is a key component of the total quality philosophy: it is process-oriented, preventive and helpful in identifying the types of variation in the process. This permits identifying who is responsible for controlling and reducing the variation to improve the process. This is accomplished through the use of control charts, either variable or attribute. To monitor a process Shewhart assumed the following model in the control charts:

$$X_t = \mu_x + \varepsilon_t \qquad (1)$$

where X_t is the observation at time t for variable X, μ_x is the mean of X and ε_t's are the error terms which are independent normal random variables with mean 0 and variance σ_ε^2. As can be seen, Shewhart assumed a constant mean for a given variable in his model when the process is in-control and monitored the process through the use of process control charts with upper and lower control limits set usually at ±3 standard deviations around the mean. However, due to mostly recent changes in production methods and technology (e.g., automatic process control systems, just-in-time production methods, etc.), the process model that Shewhart assumed to check the stability of the process may not be a valid model for all the processes. If the right process model is not used, the information provided by the SPC technique could be misleading, i.e., an in-control (stable) process may be declared out-of-control or vice versa.

In this paper, we will discuss some alternative process models that can be used in SPC applications to better explain the behavior of some processes, such as processes with tool wear (or processes that decay in time, e.g., plating operations), autocorrelated processes, and processes that may have multiple and correlated input variables. Box's process adjustment models will also be briefly mentioned as an alternative to SPC methods.

a. Processes that decay in time:

There are processes that, because of their nature, the average of the quality characteristics may show systematic increase or decreases in time, i.e., the averages do not stay constant. This could happen, for example, due to tool wear, depletion of the chemical mixes in plating operations, etc. If the tool wear rate or depletion rate can be modeled, then the deviation from the expected behavior of the process (i.e., the fitted model) can be monitored on a proper control chart (residual analysis). If the chart shows no sign of being out-of-control, we can then declare the process to be stable, i.e., stating that the process mean is changing according to our expectations, which requires no action. As you may notice, in this analysis we are testing a different model, a model that allows the process mean to change in time with a structure.

b. Existence of Autocorrelation:

Autocorrelation, r_k, is a measure of linear association between the data points of the same variable that are k time periods apart. The existence of autocorrelation, which is mainly due to the increasing use of high technology in processes, is an emerging problem for many SPC users. This is especially true in those processes where an automated process control mechanism is in place, and those in which parts are produced very close to each other using high-tech machines. If there is a positive autocorrelation, the regular statistical process control charts (i.e., Shewhart charts) would underestimate the width of the control limits. This, in turn, will cause more unnecessary "out-of-control" signals. The presence of negative autocorrelation, on the other hand, causes the overestimation of the width of the control limits. This would cause the control chart to be less sensitive to process changes. Thus the presence of autocorrelation should be checked as part of an SPC analysis.

If the autocorrelation is part of the process, i.e., the causes cannot be removed, then conventional SPC approaches should be revised to take this into account. In other words, the model in equation (1) is no longer the valid model for the process. Holmes & Gordon [2], Montgomery & Mastrangelo [3], Alwan and Roberts [4], Alwan [5], Maragah [6] and others proposed some ways to deal with the autocorrelation. Holmes and Gordon [2] and others, for example, suggested that the form of autocorrelation, i.e., first order, second order, etc., should be determined and modeled. Then the deviation (i.e., the difference between the actual and the expected value of the statistics being monitored) from this model, i.e., the residuals, should be monitored on the proper control chart(s). Under this approach the question and the model used to test the question is different from the Shewhart model, i.e., the statistics being monitored are no longer constant but change in time with the fitted model and the question being tested is whether the deviations from the model are significant over time.

c. Multiple (and potentially correlated) numbers of quality variables:

In some processes there may be more than one quality characteristic (variable) being monitored. Thus monitoring these characteristics separately requires building and maintaining many control charts (i.e., one chart per characteristic). This is not practical for SPC users. If there are separate control charts for the characteristics and if an out-of-control signal from any of the chart triggers the conclusion that the process is out of control, then the alpha (type I error) will be different from what we planned for each individual chart. The problem will get worse if the characteristics in question are also correlated. If this is the case, using separate control charts for each quality characteristic could lead to erroneous results, i.e., getting out-of-control signals when actually the process has not changed, or getting no signal when the process indeed has changed. The solution to all these problems is to use multivariate analysis techniques. In the multivariate situation, the first thing to do is to check for correlation among the variables using the correlation matrix. If correlations exist, then a multivariate process control technique should be used for more accurate results. T^2 Control charts, a multivariate technique, and principal component analysis are common tools for this purpose. The T^2 statistic, which was first introduced by Hotelling [7], reduces the data on multiple variables into one statistic by taking into account the correlation structure between them, as shown below:

$$T^2 = \left(X - \overline{X}\right)' S^{-1} \left(X - \overline{X}\right) \qquad (2)$$

(p x 1) vector of the averages of the p variables, S is the sample variance-covariance matrix and S^{-1} is the inverse of S. The T^2 control chart monitors this statistic.

d. Process adjustment (regulation) approach:

As Box [8] argued that the objective of process monitoring is to bring the process to a basic state of statistical control without continual adjustment. This implies that the process parameter being monitored stays stable with random variation around it. Though this model works for many processes, it may not work for all the processes. Some processes, due to their nature or despite the best efforts to keep them stable, do not show a stable performance but they wander off their targets. These processes require continual adjustment to keep the process as close as possible to the target. Box makes an analogy, equating process monitoring with statistical significance testing and process adjustment with statistical estimation. Thus, if the process shows these unavoidable deviations from its target, you may be better off estimating the level of deviation and making an adjustment in advance to keep the process close to its target.

In summary, it is crucial to use the right process model to get the benefit of SPC techniques. Failure to do this will cause harm to the process and to the credibility of the SPC techniques.

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