Development of a two dimensional stochastic methodology and a computer model to assess response to the nonlinear seismic wave propagation

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Development of a two dimensional stochastic methodology and a computer model to assess response to the nonlinear seismic wave propagation

Submitted by,
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A Thesis Submitted in Partial Fulfillment of the Requirement for Master of Science in Mechanical Engineering

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ABSTRACT

Seismic wave propagation in spatially variable soil continuum can be described by partial differential equations (PDE) with stochastic coefficients. Typical method of analysis in this area is a spectral analysis approach, where time series is presented by a Fourier expansion or a Fourier integral transform. This approach has a limited capability being applicable to the linear problems only.

The novelty of presented method is that it can handle any nonlinear elastic - plastic stochastic constitutive model. The output of the project is the 2D seismic random wave propagation model accounting for the spatial variability of soil properties, described by the linear and nonlinear constitutive models. This model allows accessing the seismic hazard of a region of interest with account of its specific geological and topographic features. Time dependent ground velocities, accelerations, stress components and pressure applied to the walls of an engineering structure (power plant) have been predicted to estimate the seismic lifeline hazard of engineering facilities.

Nonlinear seismic wave propagations are simulated based on a dynamic two dimensional theory of mechanics of continuum with account of nonlinear Hencky-Nadai constitutive models. Boundary conditions relate to the acceleration profile given by accelerometer or seismometer, zero stress components at the ground surface, free surface conditions at the top and non-reflected (absorbed) boundary conditions at distal boundaries.

This model describes heterogeneous spatially distributed ground soil properties, based on a set of nonlinear constitutive equations. Mathematical frame is presented by a coupled set of a nonlinear hyperbolic system of equations, with respect to three components of stress tensor and two components of a velocity vector. Analytical expressions for relating eigenvalues and eigen functions are found using MATLAB symbolic toolbox. The finite volume, characteristically
based Total Variation Diminishing (TVD) method used to predict ground motion wave propagations parameters of interest in a time – space domain as a function of a seismic profile, distance, soil properties. Monte-Carlo simulations are used to model the probability of different outcomes in a process of seismic wave propagation.
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NOMENCLATURE

\( \sigma_{ij} \) - stress tensor

\( \sigma_x, \sigma_y \) - normal stress in x and y respectively

\( \dot{\sigma} \) - derivative wrt time

\( \varepsilon_{ij} \) - strain tensor

\( \varepsilon_{kk} \) - sum of normal components of strain

\( \delta_{ij} \) - kronecker delta

\( \lambda, \mu \) - lame parameters

\( \gamma \) - shear strain

\( \gamma_{oct} \) - octahedral strain

\( \tau \) - shear stress; also in \( \frac{\tau}{h_x, h_y} \) it is the time step for FV scheme

\( \tau_{max} \) - maximum shear stress

\( \tau_{oct} \) - octahedral stress

\( G \) - shear modulus

\( G_{max} \) - maximum shear modulus

\( E \) - modulus of elasticity

\( \bar{E} \) - modified modulus of elasticity

\( \rho \) - density

\( U \) - Velocity in x direction

\( V \) - Velocity in y direction

\( u_x \) - partial derivative of u wrt x

\( v_y \) - partial derivative of v wrt y
\( \nu \) - poisons ratio

\( C \) - p-wave velocity

\( C_g \) - shear wave velocity

\( W \) - Riemann invariants

\( \varphi \) - Function of \( \gamma_{oct} \) and \( \tau_{oct} \)
1. PROBLEM INTRODUCTION

On March 11th 2011, a tsunami caused by a gigantic earthquake hit the Tohoku Region pacific coast in Northern Japan. The seismic center was estimated to be about 130 kilometers east of the Oshika Peninsula of Tohoku, and 24 km underneath the seabed. It extended 500 km along the coastline with a width of 200 km. The intensity of the earthquake was 9.0 on the Richter scale, making it the fourth-largest earthquake recorded since 1900. The earthquake created a gigantic tsunami wave that was about 10 m high at maximum. Once it reached land, it ran up to 40 m above the sea level and intruded 6 km inland, causing catastrophic damage to many people and towns along the coastline. About twenty thousand people lost their lives or are still missing, the major cause of their death being by drowning. And while it is commonly known that several nuclear power plants were lost, the reduction in electrical generating capacity due to the loss of fossil power plants was actually larger.[1]

Presented example is the most destructive and disruptive seismic phenomena occurred recently. Every day there are about fifty earthquakes worldwide that are strong enough to be felt locally and every few days an earthquake occurs that is capable of damaging structures. In countries where the earthquake resistant structural design has been enforced, earthquake fatalities have decreased dramatically. Seismic design of engineering structures is based on the following engineering disciplines:

- Engineering Seismology, dealing with the measurement prediction and characterization of ground motions; with account to the site effects
- Geotechnical Engineering, studying nonlinear soil behavior and site response under cyclic loading;
Although the earthquake did not directly cause structural damage to the power plant but it is important to be prepared for such disasters. The work presented in this thesis aims at predicting the waveforms of acceleration, velocity and stresses at a point on the surface and analyzing based on seismogram data located at a certain distance from the point. This data can be used further by structural/civil engineers to predict the damage that can be caused to a structure and ways to prevent it.
2. LITERATURE

2.1 Related Vocabulary

**Earthquake**: Shaking or trembling of the earth that accompanies rock movements extending anywhere from the crust to 680 km below the Earth’s surface. It is the release of stored elastic energy caused by sudden fracture and movement of rocks inside the Earth. Part of the energy released produces seismic waves, like P, S, and surface waves that travel outward in all directions from the point of initial rupture. These waves shake the ground as they pass by. An earthquake is felt if the shaking is strong enough to cause ground accelerations exceeding approximately 1.0 cm/s².

**Epicenter**: The point on the Earth’s surface directly above the focus of an earthquake.

**Fault**: A fracture or zone of fractures in rock along which the two sides have been displaced relative to each other. If the main sense of movement on the fault plane is up (compressional; reverse) or down (extensional; normal), it is called a dip-slip fault. Where the main sense of slip is horizontal the fault is known as a strike-slip fault. Oblique-slip faults have both strike and dip slip.

**Fault plane**: The plane along which the break or shear of a fault occurs. It is a plane of differential movement, that can be vertical as in a strike slip fault or inclined like a subduction zone fault.

**Fault zone**: Since faults do not usually consist of a single, clean fracture, the term fault zone is used when referring to the zone of complex deformation that is associated with the fault plane.

**Focus**: The point on the fault at which the first movement or break occurred, directly beneath the epicenter.
**Locked fault**: A fault that is not slipping because frictional resistance on the fault is greater than the shear stress across the fault (it is stuck). Such faults may store strain for extended periods that is eventually released in an earthquake when frictional resistance is overcome.

**Seismicity**: The geographic and historical distribution (the “where?” and “how often?”) of earthquakes.

**Tectonics**: Large-scale deformation of the outer part of the Earth resulting from forces in the Earth.

### 2.2 Seismic hazards

Any naturally occurring event such as earthquake, tornado, hurricane and floods which is capable of causing deaths, injuries and property damage is termed as natural hazards. Out of these, the hazards associated with earthquakes are termed as SEISMIC HAZARDS. Following are the most important seismic hazards that frequently occur:

**Ground Shaking**

When an earthquake occurs, seismic waves radiate from the source and rapidly travel in all directions through the earth’s crust. When these waves reach the ground surface, they produce shaking that lasts few seconds or few minutes in severe cases. The strength and duration depends on various factors such as characteristics of soil, intensity of earthquake, depth of the hypocenter.

**Structural Hazards**

This type of hazard usually is the one that comes to mind when one thinks of earthquakes. Structural damage is the leading cause of death and loss to economy in many earthquakes. Falling objects such as brick facings and parapets on the outside of a structure or heavy pictures and shelves within a structure have caused casualties in many earthquakes. Interior facilities such as piping, lighting and storage systems can also be damaged during earthquakes.
Liquefaction
This type of hazard occurs when soils lose their strength and appear to flow as fluids. The soil loses the strength to support structures or remain stable. Since it occurs only in saturated soils, liquefaction is most commonly observed near rivers, bays and water bodies.

Landslides
Strong earthquakes often cause landslides. More often, earthquake induced landslides cause damage by destroying buildings or disrupting bridges and other constructed facilities. Many of the landslides induced by earthquakes often occur due to liquefaction but many simply represent the failures of slopes that were marginally stable under static conditions.

Tsunami and Seiche
Rapid vertical seafloor movements caused by fault rupture during earthquakes can produce long-period sea waves called tsunamis. Tsunamis travel great distances at high speeds but are difficult to detect since they usually have less heights (1m) and large wavelength at the point of generation. As the wave approaches the shore the depth of the sea reduces and so does the speed of the wave, increasing its height to several times the original height. Sometimes, the shape of the sea floor in the coastal areas can amplify the wave, producing a nearly vertical wall of water that rushes far inland and causing devastating damage.

Earthquake induced waves in enclosed bodies of water are called seiches. Typically they are caused by long period seismic waves that match the natural period of oscillation of the water in a lake or a reservoir. Another type of seiche can be formed when faulting causes permanent vertical displacement within a lake or reservoir. [2]
2.3 Physics of Seismology

Seismic Waves

When an earthquake occurs, different types of seismic waves are produced: body waves and surface waves. Body waves, which travel through the interior of the earth, are of two types: P-waves and S-waves. Surface waves are of two types: Rayleigh wave and Love wave.

Body Waves

It has been generally accepted that the major part of the ground shaking during an earthquake is due to the upward propagation of body waves from an underlying rock formation. Although surface waves are also involved, their effects are generally considered of secondary importance.[3]

P-Waves

Also known as primary waves/compressional waves/longitudinal waves, involve successive compression and rarefaction of the materials through which they pass. They are analogous to sound waves and the motion of individual particle is parallel to the direction of travel. Like sound waves p-waves can travel through solids and fluids. These types of waves are the first to be recorded on a seismogram. The velocity is in the range of 4.5-6km/s.
Figure 1: Displacements due to P-wave (top) and S-wave (bottom). P-wave results in a volume change and shearing in the material through which they pass, whereas due to S-waves there is pure shear.

**S-waves**

Also known as shear waves/secondary waves/transverse waves, cause shearing deformation as they travel through the material. The motion of individual particle is perpendicular to the direction of s-wave travel. The direction of particle movement can be used to classify the s-waves further into two types: SV-wave (vertical) and SH-wave (horizontal). Since the S-waves cause shearing of the material through which they travel they cannot pass through fluids since fluids cannot resist shear forces. S-waves are the most damaging to structures. S-waves are the second to be recorded on a seismogram with a velocity of around 3km/s.

**Surface waves**

Travelling only through the crust, surface waves are of a lower frequency than body waves, and are easily distinguished on a seismogram as a result. Though they arrive after body waves, it is surface waves that are almost entirely responsible for the damage and destruction associated with earthquakes. This damage and the strength of the surface waves are reduced in deeper earthquakes.
**Love waves**

It's the fastest surface wave and moves the ground from side-to-side. Confined to the surface of the crust, Love waves produce entirely horizontal motion. The velocity is in the range of 2-4.4 km/s in the earth’s crust. These waves are typically faster than Rayleigh waves. The motion of the waves is in both directions i.e. perpendicular to direction of propagation and parallel to the earth’s surface.

![Love wave propagation](image)

**Rayleigh waves**

A Rayleigh wave rolls along the ground just like a wave rolls across a lake or an ocean. Because it rolls, it moves the ground up and down and side-to-side in the same direction that the wave is moving. Most of the shaking felt from an earthquake is due to the Rayleigh wave, which can be much larger than the other waves. The velocity is in the range of 2-4.2 km/s. The motion is in both the directions i.e. in the direction of wave propagation and perpendicular to the direction of wave propagation.[2]
2.4 Soil behavior

Although the majority of publications consider soil as linear elastic medium, the stress-strain curve obtained in a laboratory on a soil sample is a typical constitutive elastic–plastic nonlinear curve. In the \((\tau, \gamma)\) plane the behavior is characterized by a hysteresis loop, the surface and inclination of which depend on the strain amplitude.
Strain amplitudes induced by major earthquakes are capable of creating significant nonlinearity’s and possibly irrecoverable deformations. The curve is explained by four rules called the masing models. For initial loading the soil follows the loading curve along the backbone curve. At the point of stress reversal or unloading (in the above picture corresponding to stress $\tau_1$) the path followed is similar in shape to the backbone curve but that of unloading and displaced by a certain shear strain which is called the residual shear strain. It follows the shape until it intersects the backbone curve after which it follows it until a point of loading. This is followed for the number of loading and unloading cycles.[2] This behavior is called the hysteretic behavior of soil. In our case we have used the hyperbolic backbone curve equation proposed by Arefi et al, 2012,[4] which is expressed as

$$\tau = \frac{G_{\text{max}}\gamma}{1+\beta\left[\frac{G_{\text{max}}}{\tau_{\text{max}}}\right]}$$  where we have used $\alpha = \beta = 1$ as suggested by Hardin and Drnevich  \(1\)
3. OBJECTIVES

1) To develop a linear 2D seismic wave propagation computer model in a time-space domain characterized by constant and variable ground properties using MATLAB.

2) To verify the accuracy of a model by comparing results with harmonic load data.

3) To develop a nonlinear 2D seismic wave propagation computer model in a time-space domain incorporating nonlinear effective stress based soil model.

4) To develop a stochastic counterpart of created deterministic model that accounts for uncertainty in soil properties and oncoming seismic wave angle.
4. LINEAR MODEL DEVELOPMENT

4.1 Governing equations of complete 3D system

It is assumed that the solid is a homogeneous isotropic material. The motion for the elastic medium can be expressed as (with Einstein summation convention), [5]

\[ \rho \frac{\partial^2 u_i}{\partial t^2} = \frac{\partial \sigma_{ij}}{\partial x_j} + \rho f_i, \quad x \in \Omega, \; t > 0, \; i = 1, 2, 3 \] (2)

Where \( \rho \) is the density of the material (taken to be constant), \( f_i \) are components of acceleration due to an applied body force, and the components of stress are given by:

\[ \sigma_{ij} = \lambda (\epsilon_{kk}) \delta_{ij} + 2\mu \epsilon_{ij}, \quad \epsilon_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \quad \text{(general theory of elasticity)} \] (3)

Here, \( \epsilon_{ij} \) and \( \delta_{ij} \) are the components of the (linear) strain tensor and the identity tensor, respectively, \( \epsilon_{kk} = \sum_k \epsilon_{kk} = \nabla \cdot \mathbf{u} \) is the divergence of the displacement, and \( \lambda \) and \( \mu \) are Lame parameters. The latter are related to Young’s Modulus \( E \) and Poisson’s ratio \( \nu \) by \( \mu = \frac{E}{2(1+\nu)} \) and \( \lambda = \nu E / ((1+\nu)(1-2\nu)) \). Initial conditions for the second-order system in (2) are

\[ u(x, 0) = u_0(x), \frac{\partial u_i}{\partial t}(x, 0) = v_0(x), \quad x \in \Omega, \] (4)

In our work we formulate the governing equations as a first order hyperbolic system in a conservative form.

\[ \left\{ \begin{array}{l}
\frac{\partial u_i}{\partial t} = v_i, \\
\frac{\partial v_i}{\partial t} = \frac{1}{\rho} \left( \frac{\partial \sigma_{ij}}{\partial x_j} \right) + f_i, \\
\frac{\partial \sigma_{ij}}{\partial t} = \lambda (\dot{\epsilon}_{kk}) \delta_{ij} + 2\mu \dot{\epsilon}_{ij}
\end{array} \right\} \quad x \in \Omega, \; t > 0, \; i = 1, 2 \ldots n_d, \] (5)

Where \( v(x, t) \) with components \( v_i(x, t) \), is the velocity and the components \( \dot{\epsilon}_{ij} \) of the rate of strain tensor are given by
\[ \dot{e}_{ij} = \frac{1}{2} \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) \] (6)

Initial conditions for displacement and velocity are given by \( u_0(x) \) and \( v_0(x) \) as before, and initial conditions for the components of stress may be derived from (2) applied at \( t=0 \). Note that contrary to what is typically done, we retain the displacements in our formulation of the first order system. Retaining the displacements in the formulation allows the stress-strain relationship (2) to be explicitly imposed at the boundary. In addition it will be useful to have the displacement field when solving fluid-structure interaction problems (to define the fluid-solid interface for grid generation, for example).

The governing equations, whether written as second-order or first-order system, are hyperbolic and represent the motion of elastic waves in the solid. For the second order system, the characteristic wave speeds for a homogeneous material in a periodic or infinite space are \(+C_p\) and \(+C_s\), where the pressure and shear wave speeds are given by \( c_p = \sqrt{\frac{\lambda+2\mu}{\rho}} \) and \( c_s = \sqrt{\frac{\mu}{\rho}} \).

Nevertheless formulation (5) seems to be more appropriate, as it allows the use of well-developed mathematical tools for studying various wave propagating boundary value problems based on characteristics theory. Elastic wave propagation measurements in a laboratory experimental model and a field test site have shown similar propagation characteristics despite widely different soil compositions and environmental conditions.[6]

For linear elastic model we apply the same procedure that was introduced for scalar waves to vector wave scattering by a localized elastic inhomogeneity (e.g. Knopoff and Hudson 1964; Miles 1960; Sato 1984 a, b, 1990; Wu 1989; Wu and Aki 1985).

Applying stochastic analysis the spatial variations in Lame’ coefficients and mass density are written as[7]
\[ \lambda(x) = \lambda_0 + \delta \lambda(x), \mu(x) = \mu_0 + \delta \mu(x) \text{ and } \rho(x) = \rho_0 + \delta \rho(x). \quad (7) \]

Where variations of Lame’ parameters and density are modeling as normal or lognormal distributions with a zero mean value. Equations (5) become PDE equations with random coefficients. Numerical modelling is a useful tool to understand the role of different parameters governing site effects.[8]
4.2 Augmented 2-dimensional system in Cartesian coordinates

In the two dimensional case, complete set of equations (5) can be reduced to five equations (8) – (12)

\[
\begin{align*}
\dot{\sigma}_x &= \bar{E}(u_x + \vartheta v_y) \quad (8) \\
\dot{\sigma}_y &= \bar{E}(v_y + \vartheta u_x) \quad (9) \\
\dot{\tau} &= G(u_y + v_x) \quad (10) \\
\dot{u} &= \frac{1}{\rho}(\sigma_{x,x} + \tau_y) \quad (11) \\
\dot{v} &= \frac{1}{\rho}(\tau_x + \sigma_{y,y}) \quad (12)
\end{align*}
\]

The following notations are used: u, v are velocity components, \(\sigma_x, \sigma_y, \tau\) – two dimensional normal and shear stress components, dot above means time derivative, \(E, G, \rho\) – tensile elastic constant, shear elastic constant and density of a soil accordingly, \(\bar{E} = \frac{E}{1-\vartheta^2}\) is the modified elastic constant. Partial derivative of an index free variable by an x or y coordinates is indicated by a relating subscript, whereas for an indexed variable it is defined by the same comma separated subscript.

We adopt the following augmented two-dimensional system in matrix form

\[
\dot{Q} + AQ_x + BQ_y = 0 \quad (13)
\]

Where \(Q\) – is the vector of conservative variables, and A and B 5x5 matrices

\[
Q = \begin{pmatrix}
\sigma_x \\
\sigma_y \\
\tau \\
u \\
v
\end{pmatrix}, \quad A = \begin{bmatrix}
0 & 0 & 0 & E & 0 \\
0 & 0 & 0 & \nu E & 0 \\
0 & 0 & 0 & G & 0 \\
1/\rho & 0 & 0 & 0 & 0 \\
0 & 0 & 1/\rho & 0 & 0
\end{bmatrix}, \quad B = \begin{bmatrix}
0 & 0 & 0 & 0 & \nu E \\
0 & 0 & 0 & E & 0 \\
0 & 0 & 0 & G & 0 \\
0 & 0 & 1/\rho & 0 & 0 \\
0 & 1/\rho & 0 & 0 & 0
\end{bmatrix}
\]
4.3 Eigen structure of the system

We analyze the Eigen structure of matrices A and B, using the MATLAB linear algebra toolbox.

The eigenvalues and eigenvectors of A are found to be

\[ \Lambda^x = \text{diag}(0 \ c \ c_g - c - c_g); \quad c = \frac{\sqrt{E}}{\rho}; \ c_g = \frac{\sqrt{G}}{\rho}; \]

\[ W^x = \left[ (\sigma_y - \nu \sigma_x) \ \frac{u - \frac{\sigma_x}{\rho c}}{2} \ \frac{v - \frac{\tau}{\rho c g}}{2} \ \frac{u + \frac{\sigma_x}{\rho c}}{2} \ \frac{v + \frac{\tau}{\rho c g}}{2} \right] \tag{15} \]

The Eigen structure of matrix B is characterized by the following vectors

\[ \Lambda^y = \text{diag}(0 \ c \ c_g - c - c_g); \]

\[ W^y = \left[ \sigma_x - \nu \sigma_y \ \frac{v - \frac{\sigma_x}{\rho c}}{2} \ \frac{u - \frac{\tau}{\rho c g}}{2} \ \frac{v + \frac{\sigma_y}{\rho c}}{2} \ \frac{u + \frac{\tau}{\rho c g}}{2} \right] \tag{17} \]

4.4 Finite-volume numerical scheme

Consider now a control volume in x-y-t space of dimensions \( h_x = x_{i+1/2} - x_{i-1/2} \); \( h_y = y_{j+1/2} - y_{j-1/2} \); \( \tau = t^{n+1} - t^n \), where fractional index relates to the cell edges, and the whole index – to the cell center.

\[ \text{Figure 5: Schematic of cell used in the Finite Volume Method mesh} \]
A finite volume method for solving (13) reads

\[ \sigma_{x_{i,j}}^{n+1} = \sigma_{x_{i,j}}^n + E \left( \frac{\tau}{h_x} (u_{i+\frac{1}{2},j} - u_{i-\frac{1}{2},j}) + \frac{\partial \tau}{\partial y} (v_{i,j+\frac{1}{2}} - v_{i,j-\frac{1}{2}}) \right) \] (18)

\[ \sigma_{y_{i,j}}^{n+1} = \sigma_{y_{i,j}}^n + E \left( \frac{\tau \partial \tau}{h_x} (u_{i+\frac{1}{2},j} - u_{i-\frac{1}{2},j}) + \frac{\tau}{h_y} (v_{i,j+\frac{1}{2}} - v_{i,j-\frac{1}{2}}) \right) \] (19)

\[ \tau_{i,j}^{n+1} = \tau_{i,j}^n + G \left( \frac{\tau}{h_x} (v_{i+\frac{1}{2},j} - v_{i-\frac{1}{2},j}) + \frac{\tau}{h_y} (u_{i,j+\frac{1}{2}} - u_{i,j-\frac{1}{2}}) \right) \] (20)

\[ u_{i,j}^{n+1} = u_{i,j}^n + \frac{1}{\rho} \frac{\tau}{h_x} (\sigma_{x_{i+\frac{1}{2},j}} - \sigma_{x_{i-\frac{1}{2},j}}) + \frac{\tau}{h_y} (\tau_{i,j+\frac{1}{2}} - \tau_{i,j-\frac{1}{2}}) \] (21)

\[ v_{i,j}^{n+1} = v_{i,j}^n + \frac{1}{\rho} \frac{\tau}{h_x} (\tau_{i+\frac{1}{2},j} - \tau_{i-\frac{1}{2},j}) + \frac{\tau}{h_y} (\sigma_{y_{i,j+\frac{1}{2}}} - \sigma_{y_{i,j-\frac{1}{2}}}) \] (22)

The description of scheme (18)-(22) is complete once expressions for the numerical fluxes relating to the cell edges are provided. Godunov [9] proposed to use the self-similar solution of the Riemann problem to compute numerical fluxes in the direction normal to the cell faces, the algorithm which forms the foundation of a contemporary TVD (total variation diminishing) family of methods, applicable to model propagation of waves of a different physical nature [10].

The numerical procedure is time marching, monotone, implicit, of second order accuracy by space and time coordinates [11].

Below we present the algorithm of calculating a few of flux terms based on a centered cell values

\[ u_{i+\frac{1}{2},j} = W^x_{2_{i,j}} + W^x_{4_{i+1,j}} \] (23)

\[ \sigma_{x_{i+\frac{1}{2},j}} = \rho c (W^x_{2_{i,j}} + W^x_{4_{i+1,j}}) \] (24)

\[ v_{i+\frac{1}{2},j} = W^x_{3_{i,j}} + W^x_{5_{i+1,j}} \] (25)

\[ \tau_{i+\frac{1}{2},j} = \rho c_g (W^x_{3_{i,j}} + W^x_{5_{i+1,j}}) \] (26)
4.5 Solution propagation

![Figure 6: Propagation of fluxes to the cell boundaries schematic representation](image)

The above figure represents how the flux propagates to the cell boundaries and the advance of the solution. In the previous section we presented the transformation of the problem to Riemann [12] invariants with fluxes at the boundaries related to fractional indices and properties at cell centers related by whole indices. At any time instant the properties at the cell centers are dependent on the properties of the cell at the previous instant of time and the fluxes along the boundaries of the cells. They are related by the expressions in equation (18)-(22). The Riemann variable presented in equation (15) and (17) has the characteristic speed $\pm C$ and $\pm C_g$ where $g$ represents the shear wave speed. These variables propagate with positive speeds in the positive direction and similarly in the negative direction. The characteristics carry the information to the cell boundaries where the fluxes with fractional indices are calculated. The domain of the problem is accounted for in the finite-volume approach. This ensures the satisfaction of the physical law over the finite region rather than at a point. The discretization has the quality to conserve the properties over the finite space. [13]

For example,

$$W_{x4}(i,j) = W_{x4}(i,j) + \frac{1}{2} \left( u_{i,j+\frac{1}{2}} + \frac{1}{\rho c} \sigma_{x,i,j+\frac{1}{2}} \right),$$

$$W_{x2}(i,j + 1) = W_{x2}(i,j + 1) + \frac{1}{2} \left( u_{i,j+\frac{1}{2}} - \frac{1}{\rho c} \sigma_{x,i,j+\frac{1}{2}} \right).$$
Similarly the process repeats for every cell and every property that is normal stress, shear stress and velocities in X and Y directions.

![Schematic of soil mesh with powerplant](image)

**Figure 7: Mesh of the soil and the power-plant structure**

### 4.6 Boundary conditions

**1) Downward**

At the depth, the kinematic boundary conditions identify relating flux components as the measured components of velocities

\[
\begin{align*}
    u_{i=\frac{1}{2},j} &= u_0(t,y) \quad (27) \\
    v_{i=\frac{1}{2},j} &= v_0(t,y) \quad (28)
\end{align*}
\]

Relating stress components are calculated based on a similarity solution for x – propagating invariants

\[
W_{x4}(i = 1, j) = W_{x4} \left( i = \frac{1}{2}, j \right) = \left( u_{i=\frac{1}{2},j} + \frac{1}{\rho c} \sigma_{x,i=\frac{1}{2},j} \right)^{\frac{1}{2}} \quad (29)
\]
\[
\sigma_{x,i=\frac{1}{2}j} = \rho c (2W_{x4,i=\frac{1}{2}j} - u_{i=\frac{1}{2}j}) 
\]

(30)

\[
W_{x5}(i = 1,j) = W_{x5} \left( i = \frac{1}{2}j \right) = \left( v_{i=\frac{1}{2}j} + \frac{1}{\rho c} \tau_{i=\frac{1}{2}j} \right) \frac{1}{2} 
\]

(31)

\[
\tau_{i=\frac{1}{2}j} = \rho c (2W_{x5,i=1,j} - v_{i=\frac{1}{2}j}) 
\]

(32)

### (2) Free Surface

At the free surface the normal and tangential stress components are assumed to be equal to zero.

This is in accordance with the study by Maria Paola Santisi d’Avila and Jean-Francois Semblat.[14]

\[
\sigma_{x,Nx+\frac{1}{2}j} = 0 
\]

(33)

\[
\tau_{x,Nx+\frac{1}{2}j} = 0 
\]

(34)

Relating velocity components are calculated based on x – propagating invariants

\[
W_{x2,i=Nx,j} = \frac{1}{2} \left( u_{Nx+\frac{1}{2}j} - \frac{1}{\rho c} \sigma_{x,Nx+\frac{1}{2}j} \right) 
\]

(35)

\[
u_{Nx+\frac{1}{2}j} = 2W_{x2,i=Nx,j} 
\]

(36)

\[
W_{x3,i=Nx,j} = \frac{1}{2} \left( v - \frac{r}{\rho c g} \right)_{Nx+\frac{1}{2}j} 
\]

(37)

\[
v_{Nx+\frac{1}{2}j} = 2W_{x3,Nx,j} 
\]

(38)

### (3) Left Boundary Condition

Non-reflected (absorbed) boundary conditions are implemented, which means that the reflected (right invariants) are equal to zero. These conditions are supplemented by the left invariants arriving to the left boundary
\[ W_{y2,j=\frac{1}{2}} = \left( v - \frac{\sigma_y}{\rho c} \right)_{i,j=\frac{1}{2}} = 0; \quad v_{i,j=\frac{1}{2}} = W_{y4,i,j=\frac{1}{2}} \] (39)

\[ W_{y4,j=\frac{1}{2}} = \left( v + \frac{\sigma_y}{\rho c} \right)_{i,j=\frac{1}{2}}; \quad \sigma_{y,i,j=\frac{1}{2}} = \rho c \cdot v_{i,j=\frac{1}{2}} \] (40)

\[ W_{y3,l,j=\frac{1}{2}} = \left( u - \frac{\tau}{\rho c} \right)_{i,j=\frac{1}{2}} = 0 \] (41)

\[ W_{y5,l,j=\frac{1}{2}} = \left( u + \frac{\tau}{\rho c} \right)_{i,j=\frac{1}{2}} \] (42)

\[ u_{i,\frac{1}{2}} = W_{y5,l,j=1} \]

\[ \tau = \rho c g u_{i,j=\frac{1}{2}} \] (43)

(4) Right Boundary conditions

Non-reflected (absorbed) boundary conditions are implemented, which means that the reflected (left invariants) are equal to zero. These conditions are supplemented by the right invariants arriving to the right boundary

\[ W_{y4,i,j=Ny+\frac{1}{2}} = \left( v + \frac{\sigma_y}{\rho c} \right)_{i,j=Ny+\frac{1}{2}} \] (44)

\[ W_{y2,i,j=Ny+\frac{1}{2}} = \left( v - \frac{\sigma_y}{\rho c} \right)_{i,j=Ny+\frac{1}{2}} \] (45)

\[ v_{i,Ny+\frac{1}{2}} = W_{y2,i,j=Ny} \]

\[ \sigma_y = -\rho c v_{i,Ny+\frac{1}{2}} \] (46)

\[ W_{y5,l,j=Ny+\frac{1}{2}} = \left( u + \frac{\tau}{\rho c} \right)_{i,j=Ny+\frac{1}{2}} = 0 \] (47)

\[ W_{y3,l,j=Ny+\frac{1}{2}} = \left( u - \frac{\tau}{\rho c} \right)_{i,j=Ny+\frac{1}{2}} \] (48)

\[ u_{i,Ny+\frac{1}{2}} = W_{y3,i,j=Ny} \]

\[ \sigma_y = -\rho c g u_{i,Ny+\frac{1}{2}} \] (49)
4.7 1D testing case

Assume that deformation of the medium is one-dimensional, and only one axial component of the stress tensor and velocity vector is nonzero. The physical and numerical models are described by equations (8), (11) and (18), (21) accordingly. We run the scheme (18), (21) with the following input data:

The total length $L=200\text{m}$; density $\rho=2000\text{kg/m}^3$; time of simulation $T=5\text{s}$; speed of pressure waves propagation $c=1636\text{m/s}$; frequency of a propagating wave $\omega = \frac{\pi L}{c}$; Courant-Friedrichs-Lewy number $\text{cfl}=0.5$. It simply states that the method must be used in such a way that information has a chance to propagate at the correct physical speeds, as determined by the eigenvalues of the flux.[10] The linear elastic Young modulus was calculated based on a speed of sound and density $E = \rho c^2$.

An exact solution for the one-dimensional case was adopted in a form

$$U(x, t) = \sin\left(\frac{\pi x}{L}\right)\sin(\omega t);$$

$$\sigma(x, t) = -\rho c \cos\left(\frac{\pi x}{L}\right)\cos(\omega t);$$

Boundary conditions, compliant with the exact solution are:

$$U(0, t) = 0; \quad \sigma(L, t) = \rho c \cos(\omega t).$$

We ran simulations with three different meshes of 20, 50 and 100 cells. Figure 9 presents time dependent distributions of velocity (left) and normal stress (right) at the first, middle and the last cells. The plots of exact and numerical solutions are non-distinguishable for all three numerical meshes
The snapshots of velocity and normal stress waveforms are presented in Figure 9 for three different meshes at five instances of time: T/5, 2T/5, 3T/5, 4T/5, T for numerical (dash lines) and exact solutions (solid lines). It appears that space numerical distributions are more sensitive to the cell count, matching exact solution (lines non-distinguishable) for the 100 cells at CFL=0.5. It should be noted that CFL =1 results in much more accurate results, but we intentionally use CFL=0.5, which provides an extra stability margin for the nonlinear cases.
Figure 9: 1D Comparison of waveforms of Velocity (U) and normal Stress sigma vs Distance to surface for mesh size considering 20(top), 50(middle), 100(bottom) points
4.8 2D pure shear dynamic deformation test

Assume that deformation of the medium is two-dimensional, and only one component of a stress tensor – shear stress, and both components of a velocity vector are nonzero. The physical and numerical models are described by equations (10) - (12) and (20) - (22) accordingly. We run the scheme (20), (21), (22) with the following input data:

The total length of the area L=400m; depth H=400m; density p=2000kg/m$^3$; time of simulation T=100s; speed of pressure waves propagation c=1636m/s; Poisson coefficient $\vartheta = 0.4$; Shear wave propagation velocity was calculated based on a Poisson correction $c_g = \frac{c}{\sqrt{E(1+\vartheta)}}$; Frequency of a propagating wave $\omega = \frac{\pi L}{c}$; Courant-Friedrichs-Lewy number $cfl=0.5$. The linear elastic and shear moduli have been calculated based on relating velocities of sound and density $E = \rho c^2$, $G = \rho c_g^2$.

An exact solution for the one-dimensional case was adopted in a form

$$U(x,y,t) = \cos\left(\frac{\pi x}{H}\right)\sin\left(\frac{\pi y}{L}\right)\cos(\omega t); \quad (50)$$

$$V(x,y,t) = \sin\left(\frac{\pi x}{L}\right)\cos\left(\frac{\pi y}{L}\right)\cos(\omega t); \quad (51)$$

$$\tau(x,y,t) = \frac{2G}{\omega L} \cos\left(\frac{\pi x}{L}\right)\cos\left(\frac{\pi y}{L}\right)\sin(\omega t); \quad (52)$$

Boundary conditions, compliant with the exact solution are:

$$V(0,y,t) = 0; \quad \tau(L,y,t) = -\frac{2G}{\omega L} \cos\left(\frac{\pi y}{L}\right)\sin(\omega t) \quad (53)$$

$$U(x,0,t) = 0; \quad \tau(x,L,t) = -\frac{2G}{\omega L} \cos\left(\frac{\pi x}{L}\right)\sin(\omega t) \quad (54)$$

The snapshots of a vertical velocity $U$ and a shear stress $\tau$ waveform are presented in Figure.10 for three different meshes 10x10, 30x30 and 60x60 at four instances of time T/4, T/2, 3T/4 and 3T/2.
T. Numerical solution relating to cfl=0.5 is presented by dash lines, and exact solutions by solid lines. Evidently, that the finest mesh provides the better match. Similar to the one–dimensional case, application of cfl =1 results in much more accurate results, but we intentionally use cfl=0.5, which provides an extra stability margin for the nonlinear cases. Additional plots, including visualization of all components are presented in Figure. 11.
Figure 10: 2D pure shear test. Comparison of waveforms of Velocity (U) and normal Stress sigma vs Distance to surface for mesh size considering 10(top), 30(middle), 60(bottom) points.
Figure 11: 2D pure shear test. Comparison of waveforms of vertical velocity, horizontal velocity, shear stress vs Distance to surface for mesh size considering 10(top), 30(middle), 60(bottom) points
4.9 2D harmonic load constant properties

The physical and numerical models are described by equations (8) - (12) and (18) - (22) accordingly. We ran the scheme (18)-(22) with the following input data:

The total length of the area $L=400\text{m}$; depth $H=200\text{m}$; density $\rho = 2000\text{kg/m}^3$; time of simulation $T=10\text{s}$; speed of pressure waves propagation $c = 1636\text{m/s}$; Poisson coefficient $\nu = 0.4$; Shear wave propagation velocity was calculated based on a Poisson correction $c_g = \frac{c}{\sqrt{E(1+\nu)}}$; Courant-Friedrichs-Lewy number $\text{CFL}=0.5$. The linear elastic and shear moduli have been calculated based on relating velocities of sound and density $E = \rho c^2$, $G = \rho c_g^2$. A stability criterion prescribes the “safe” time step as $\Delta t = \frac{\text{CFL}}{c(\frac{1}{h_x} + \frac{1}{h_y})}$. Three types of meshes have been employed: 10x10, 30x30 and 60x60 cells. To prove monotonicity of a numerical scheme the artificial acceleration load in a y-direction has been approximated as a harmonic function $\text{ACCEL}_Y = 0.2\pi \ast \frac{4}{T} \cos\left(\frac{4\pi t}{T}\right) \sin\left(\frac{\pi y}{L}\right)$. Since boundary conditions in our model should be specified in a form of either velocity or the stress tensor components, acceleration function was integrated to obtain velocity profile $\text{VEL}_Y = 0.2 \sin\left(\frac{4\pi t}{T}\right) \sin\left(\frac{\pi y}{L}\right)$. In addition we applied the $x$ – component of velocity assuming that the angle between the wave front and the horizontal direction is $6^\circ$, so that $\text{VEL}_X = \text{VEL}_Y \ast \tan\left(6^\circ\right)$. Multiple results of parametric analysis on different meshes are presented in Figures 12 – 17. In this case we do not have an exact solution, but we can check the monotonicity, coherence, boundary condition satisfaction, convergence. Figure 12 and 13 present the time distribution of horizontal and vertical components of velocities accordingly. Each figure contains nine subplots, relating to the mesh 10x10 (upper line), 30x30 (middle line) and 60x60 (bottom line). Each subplot corresponds to the selected characteristic
point, indicated by legend, inside calculation domain. All distributions are monotone and coherent. Figures 12-16 present time evolution of three stress components, calculated at the same characteristic points, using the same three computational meshes. It is easy to see that the x component of normal stress and a shear stress component tend to zero approaching the ground surface. The small deviation from zero is explained by the fact that the stress components in our algorithm relate to the cell centers, whereas boundary conditions are applied to the cell side.

The last Figure 17 visualizes the snapshots of the velocity x-component, normal stress in x-direction and a shear stress component. All components are monotone, and satisfy to the applied boundary conditions at the free surface, where the normal stress and shear stress should be absent, and applied conditions at the depth where both components of velocities are specified. Convergence on a sequence of meshes is observed based on a fact that the boundary conditions are satisfied perfectly well on a fine mesh.
Figure 12: Comparison of waveforms of horizontal velocity for mesh size considering 10(top), 30(middle), 60(bottom) points
Figure 13: Comparison of waveforms of vertical velocity for mesh size considering 10(top), 30(middle), 60(bottom) points
Figure 14: Comparison of waveforms of normal stress in X direction for mesh size considering 10(top), 30(middle), 60(bottom) points
Figure 15: Comparison of waveforms of normal stress in Y direction for mesh size considering 10(top), 30(middle), 60(bottom) points
Figure 16: Comparison of waveforms of shear stress $\tau$ for mesh size considering 10(top), 30(middle), 60(bottom) points
Figure 17: Comparison of waveforms of velocity/normal stress in X direction and shear stress TAU vs Distance for mesh size considering 10(top), 30(middle), 60(bottom) points.
4.10 2D real load constant properties

The physical and numerical models are described by equations (8) - (12) and (18) - (22) accordingly. We ran the scheme (18)-(22) with the input data described in the previous section. For the present case the real acceleration profile applied at the depth of 200 m is used. Black indicates all 4000 input data points, whereas red color profile corresponds to the uniformly distributed 400 input point (Figure 18, left). Figure 18, right describes distribution of velocity $VEL = \int_0^t ACCEL(\tau) d\tau$. The coarse mesh based distribution of input acceleration and velocity skip a large number of local peaks inherent for the fine mesh, which can result in a notable inaccuracy.

Multiple results of parametric analysis on different meshes are presented in Figures 20–25. Figures 20 – 24 present time distributions of horizontal and vertical components of velocities, and three components of a stress tensor accordingly. Each figure contains nine subplots, relating to the mesh 10x10 (upper line), 30x30 (middle line) and 60x60 (bottom line). Each subplot
corresponds to the selected characteristic point, indicated by legend, inside calculation domain. All distributions are non-monotone since the input data is non-monotone. Figures 20-24 present time evolution of three stress components, calculated at the same characteristic points, using the same three computational meshes. It is again easy to see that the x component of normal stress and a shear stress component tend to zero approaching the ground surface. The small deviation from zero is explained by the fact that the stress components in our algorithm relate to the cell centers, whereas boundary conditions are applied to the cell side.

The last Figure 25 visualizes the snapshots of all velocity and stress components. All components satisfy to the applied boundary conditions at the free surface, where the normal stress and shear stress should be equal to zero, and applied conditions at the depth where both components of velocities are specified. Convergence on a sequence of meshes is observed based on a fact that the boundary conditions are satisfied perfectly well on a fine mesh.
Figure 19: Real Data-Comparison of waveforms of horizontal velocity for mesh size considering 10(top), 30(middle), 60(bottom) points
Figure 20: Real Data-Comparison of waveforms of vertical velocity for mesh size considering 10(top), 30(middle), 60(bottom) points
Figure 21: Real Data-Comparison of waveforms of normal stress in X direction for mesh size considering 10(top), 30(middle), 60(bottom) points
Figure 22: Real Data-Comparison of waveforms of normal stress in Y direction for mesh size considering 10(top), 30(middle), 60(bottom) points
Figure 23: Real Data-Comparison of waveforms of shear stress TAU for mesh size considering 10(top), 30(middle), 60(bottom) points
Figure 24: Real Data-Comparison of waveforms of vertical velocity/normal stress in X direction/shear stress TAU for mesh size considering 10(top), 30(middle), 60(bottom) points
4.11 2D real load multi layered linear soil structure

Among the most important effects of seismic waves is the strong dependence of damage on its location, though the distance from different sites to the oncoming wave is the same. This phenomenon is due to the variability of site local properties, which alter the characteristics of seismic motion and cause concentrations of damage during earthquakes. There are two ways of describing the sedimentary infilling properties: the basin is either composed of distinct homogeneous geological layers or through properties progressively changing as a function of depth.[8] We have used a similar model as the latter one by discretizing the soil into a system of horizontal layers parallel to x-y plane except that the properties for different layers have been assumed unlike a function of depth. Local site effects have an important effect on the surface ground motion. The soil profile characteristics from the used site are presented in Table 1. These are average properties taken from the Geophysical Journal International.[14]

<table>
<thead>
<tr>
<th>Layer #</th>
<th>Thickness (m)</th>
<th>Mass density (kg/m³)</th>
<th>Elastic modulus (MPa)</th>
<th>Poisson ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td>2000</td>
<td>4.49*10³</td>
<td>0.4</td>
</tr>
<tr>
<td>2</td>
<td>30</td>
<td>2200</td>
<td>5.19*10³</td>
<td>0.4</td>
</tr>
<tr>
<td>3</td>
<td>50</td>
<td>2500</td>
<td>5.49*10³</td>
<td>0.4</td>
</tr>
<tr>
<td>4</td>
<td>100</td>
<td>2900</td>
<td>6.4*10³</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Table 1: Soil properties for different layers

The rest of properties and a seismic load are identical to the ones presented in the previous section.

The physical and numerical models are described by equations (8) - (12) and (18) - (22) accordingly. The only difference in the algorithm applied to the multi layered structure is that solution of the Riemann problem (23) – (26), relating to the cell flux, is generalized, becoming dependent on different mechanical properties of cells adjacent to the evaluating cell side.
Multiple results of parametric analysis on different meshes are presented in Figures 26–31. Figures 26 – 27 present time distributions of horizontal and vertical components of velocities, and three components of a stress tensor accordingly. Each figure contains nine subplots, relating to the mesh 10x10 (upper line), 30x30 (middle line) and 60x60 (bottom line). Each subplot corresponds to the selected characteristic point, indicated by legend, inside calculation domain. All distributions are non-monotone since the input data is non-monotone. Figure 27, 28 and 29 present time evolution of three stress components, calculated at the same characteristic points, using the same three computational meshes. We can see that the x component of normal stress and a shear stress component tend to zero approaching the ground surface. The small deviation from zero is explained by the fact that the stress components in our algorithm relate to the cell centers, whereas boundary conditions are applied to the cell side.

The last Figure 30 visualizes the snapshots of all velocity and stress components. All components satisfy to the applied boundary conditions at the free surface, where the normal stress and shear stress should be equal to zero, and applied conditions at the depth where both components of velocities are specified. Convergence on a sequence of meshes is observed based on a fact that the boundary conditions are satisfied perfectly well on a fine mesh.
Figure 25: Real Data-Comparison of waveforms of horizontal velocity for mesh size considering 10(top), 30(middle), 60(bottom) points
Figure 26: Real Data multiple layers—Comparison of waveforms of vertical velocity for mesh size considering 10(top), 30(middle), 60(bottom) points.
Figure 27: Real Data multiple layers-Comparison of waveforms of normal stress SIGMA in X direction for mesh size considering 10(top), 30(middle), 60(bottom) points.
Figure 28: Real Data multiple layers-Comparison of waveforms of normal stress SIGMA in Y direction for mesh size considering 10(top), 30(middle), 60(bottom) points
Figure 29: Real Data multiple layers-Comparison of waveforms of shear stress TAU for mesh size considering 10(top), 30(middle), 60(bottom) points
Figure 30: Real Data multiple layers—Comparison of waveforms of vertical velocity/normal stress SIGMA in X direction/shear stress TAU vs Distance to the surface for mesh size considering 10(top), 30(middle), 60(bottom) points.
4.12 2D nonlinear properties seismic ground motion wave propagation

It is obvious that soil does not react elastically. Contrary to the conventional modeling, which assumes linear elastic behavior of the soil, the present approach uses an elastic-plastic model reproducing a variety of experimentally observed hysteretic soil behavior. Structures which are located in areas where large nonlinear behavior of soil is observed face not only horizontal and vertical components of inertial forces by earthquake shaking but can also experience large differential motions and rotations of their foundations. This needs a systematic approach and research focusing on the development of advanced numerical simulation models. [15] Seismologists have had a general realization that nonlinear effects of soil are more common than what was previously assume. It is of prime importance to create or use the appropriate mathematical model to predict these effects. Nonlinear soil models track the seismic load in the stress-strain space by making use of several stress-strain relationships.[16]

Two different basic theories create the foundation of theory of plasticity: incremental theory (Hencky-Nadai)[17], specifying relationship between increments of deviatoric components of stress and strain, and deformation theory (Hencky – Nadai - Ilyushin), defining a single effective stress – effective strain curve, whose shape is governed by the simple uniaxial tension, or a simple shear deformation test. The latter approach is adopted in the present work, and briefly described below.

According to Hencky-Nadai, constitutive equations for stress components for 2D case can be presented in the following form:

\[
\varepsilon_x = \varphi (\sigma_x - \frac{1}{2} \sigma_y)
\]

(55)
\[ \varepsilon_y = \varphi (\sigma_y - \frac{1}{2} \sigma_x) \tag{56} \]

\[ \gamma = 3 \varphi \tau \tag{57} \]

Where parameter \( \varphi \) is calculated based on a dependence of a united stress strain curve in terms of octahedral stress \( \tau_{oct} \) and octahedral strain \( \gamma_{oct} \)

\[ \varphi = \frac{\gamma_{oct}}{\tau_{oct}} ; \tag{58} \]

\[ \gamma_{oct}^2 = \frac{2}{3} \left[ \varepsilon_x^2 + \varepsilon_y^2 - \varepsilon_x \varepsilon_y + 3 \gamma^2 \right] \tag{59} \]

\[ \tau_{oct}^2 = \frac{2}{3} \left[ \sigma_x^2 + \sigma_y^2 - \sigma_x \sigma_y + 3 \tau^2 \right] \tag{60} \]

Following Arefi et al, 2012, a hyperbolic equation for the backbone curve can be defined as[4]

\[ \tau = \frac{G_{max} \gamma}{1 + \beta \left| \frac{G_{max} \gamma}{\tau_{max}} \right|^2} \tag{61} \]

Where, \( \alpha \) and \( \beta \) are dimensionless factor. Following Hardin and Drnevich we adopt \( \alpha = \beta = 1 \).

In this case \( G_{max} \) is interpreted as the tangent shear modulus at \( \gamma \rightarrow 0 \) (the largest modulus), and \( \tau_{max} \) is the largest tangential stress at \( \gamma \rightarrow \infty \).

We need to specify constitutive equations in a form (8) – (10) to apply methodology developed for the elastic deformation

In a reverse form constitutive equations Hencky-Nadai read

\[ \sigma_x = \frac{4}{3} \varphi^{-1} (\varepsilon_x + \frac{1}{2} \varepsilon_y) \tag{62} \]
\[ \sigma_y = \frac{4}{3} \varphi^{-1} \left( \varepsilon_y + \frac{1}{2} \varepsilon_x \right) \]  \hspace{1cm} (63)

\[ \tau = \frac{\varphi^{-1}}{3} \gamma \]  \hspace{1cm} (64)

Where \( \varphi^{-1} = \frac{G_{\text{max}}}{1 + \beta \left| \frac{G_{\text{max}}}{\gamma_{\text{oct}}} \right|^\alpha} \)

To present constitutive equations with respect to the components of stress rates, we differentiate equations above by time, arriving at the following augmented system of five differential equations in a matrix form, identical (13)

\[ \dot{Q} + A Q_x + B Q_y = 0 \]

\[ A = \begin{bmatrix}
0 & 0 & 0 & a_{11} & a_{13} \\
0 & 0 & 0 & a_{21} & a_{22} \\
0 & 0 & 0 & a_{31} & a_{32} \\
\frac{1}{\rho} & 0 & 0 & 0 & 0 \\
0 & 0 & 1/\rho & 0 & 0
\end{bmatrix} \quad B = \begin{bmatrix}
0 & 0 & 0 & a_{13} & a_{13} \\
0 & 0 & 0 & a_{23} & a_{22} \\
0 & 0 & 0 & a_{33} & a_{32} \\
0 & 0 & 1/\rho & 0 & 0 \\
0 & 1/\rho & 0 & 0 & 0
\end{bmatrix} \]

Where

\[ a_{11} = \frac{4}{3} \varphi^{-1} + \frac{4}{3} (\varphi^{-1})_y (\varepsilon_x + \frac{1}{2} \varepsilon_y) \frac{2}{g_{\text{oct}}} (\varepsilon_x - \varepsilon_y) \]

\[ a_{12} = \frac{2}{3} \varphi^{-1} + \frac{4}{3} (\varphi^{-1})_y (\varepsilon_x + \frac{1}{2} \varepsilon_y) \frac{2}{g_{\text{oct}}} (\varepsilon_y - \varepsilon_x) \]

\[ a_{13} = \frac{4}{3} (\varphi^{-1})_y (\varepsilon_x + \frac{1}{2} \varepsilon_y) \frac{2}{3 g_{\text{oct}}} \]

\[ a_{21} = \frac{2}{3} \varphi^{-1} + \frac{4}{3} (\varphi^{-1})_y (\varepsilon_y + \frac{1}{2} \varepsilon_x) \frac{2}{g_{\text{oct}}} (\varepsilon_x - \varepsilon_y) \]

\[ a_{22} = \frac{4}{3} \varphi^{-1} + \frac{4}{3} (\varphi^{-1})_y (\varepsilon_y + \frac{1}{2} \varepsilon_x) \frac{2}{g_{\text{oct}}} (\varepsilon_y - \varepsilon_x) \]

\[ a_{23} = \frac{4}{3} (\varphi^{-1})_y (\varepsilon_y + \frac{1}{2} \varepsilon_x) \frac{2}{3 g_{\text{oct}}} \]
\[ a_{31} = \frac{1}{3}(\varphi^{-1})_y \gamma \frac{2}{\gamma_{oct}} (\varepsilon_x - \varepsilon_y) \]

\[ a_{32} = \frac{1}{3}(\varphi^{-1})_y \gamma \frac{2}{\gamma_{oct}} (\varepsilon_y - \varepsilon_x) \]

\[ a_{33} = \frac{\varphi^{-1}}{3} + \frac{1}{3}(\varphi^{-1})_y \gamma \frac{2\gamma}{\gamma_{oct}} \]

We analyze the Eigen structure of matrices A and B, using the MATAALB linear algebra toolbox, and apply it to the time marching algorithm according to (18) – (22). Boundary conditions and implementation remain identical to the ones described in (27) – (49). In general the algorithm at every time step can be interpreted as a generalization of its linear elastic counterpart by introducing local “quasi-elastic” parameters, different at each cell, and at each time instant. The algorithm can be described as the following:

1) Initial approach- all zeroes.
2) Calculate \( \varphi^{-1} = G_{max} \text{(automatically at } \gamma = 0) \)
3) Apply boundary conditions, calculate invariants, fluxes, update parameters at each cell
4) Calculate octahedral components at each cell, \( \varphi^{-1} = \frac{\tau_{oct}}{\gamma_{oct}} = \frac{G_0}{1 + \beta \left| \frac{\varepsilon_{oct}}{\gamma_{oct}} \right|^\alpha} \)
5) Go to 3

The input data, pertaining to the nonlinear case, is the following:

\( L=400\text{m}; \ H=200\text{m}; \) density \( p=2000\text{kg/m}^3; \) \( G_{max} = 1600\text{MPa}; \) \( \tau_{max} = 10^4\text{Pa}; \) time of simulation \( T=10\text{s}; \) Courant-Friedrichs-Lewy number \( \text{cfl}=0.5. \) \( \Delta t = \min \left( \frac{\text{cfl}}{h_x + h_y} \right). \)

Three types of meshes have been employed: 10x10, 30x30 and 6x60 cells. Multiple results of parametric analysis on different meshes are presented in Figures 32 – 35. Figures 32-34 present the time distribution of a horizontal component of velocity, normal and shear stresses accordingly. Each
figure contains nine subplots, relating to the mesh 10x10 (upper line), 30x30 (middle line) and 60x60 (bottom line). Each subplot corresponds to the selected characteristic point, indicated by legend, inside calculation domain. The last two figures visualize the snapshots of the velocity x-component, normal stress in x-direction and a shear stress component for the nonlinear case and a relating linear case at $\tau_{\text{max}} \to \infty$. All components satisfy to the applied boundary conditions at the free surface, where the normal stress and shear stress should be absent, and applied conditions at the depth where both components of velocities are specified. Convergence on a sequence of meshes is observed based on the fact that the boundary conditions are satisfied perfectly well on a fine mesh.
Figure 31: Comparison of waveforms of horizontal velocity for a mesh size considering 10(top) /30(middle) /60(bottom) points in X and Y direction each
Figure 32: Comparison of waveforms of normal stress in X direction for a mesh size considering 10(top) /30(middle) /60(bottom) points in X and Y direction each
Figure 33: Comparison of waveforms of shear stress $\tau$ for a mesh size considering 10(top) /30(middle) /60(bottom) points in X and Y direction each
Figure 34: Comparison of waveforms of vertical velocity, normal stress in X direction, shear stress $\tau$ for a mesh size considering 10(top) / 30(middle) / 60(bottom) points in X and Y direction each
4.13 Stochastic seismic wave propagation in nonlinear soil with uncertain properties

Conventionally seismic hazards are analyzed using the technique called Probabilistic Seismic Hazard Analysis (PSHA). It is based on the total probability theorem. This method gives the output of the magnitude of earthquake, peak ground acceleration and return period of an earthquake due to all possible sources close to the site under consideration. The input for this method is the seismicity data from all the sources which can potentially cause an earthquake at the site. However, all this can also be obtained by using the Monte-Carlo simulation method or stochastic modelling. It has been used in several studies by seismologists around the world. It offers variety of advantages to seismic hazard prediction such as adaptability to different seismicity models, powerful handling of uncertainty, adaptability to risk analysis and is conceptually straightforward. The results obtained are also easy to comprehend for a layman. The only drawback it has is, the computational time to run the simulation.[18] This chapter investigates the effects of random variations of soil properties and oncoming wave angle on propagating seismic waves using Monte-Carlo simulation. Studies by Bazzurro and Cornell, 2004 have also presented a probabilistic model for the effect of layered soil structure with uncertain soil properties.[19] A similar study has been presented by Badaoui et al where the focus is on stochastic seismic response of a layered soil site.[20] Their model uses a layered soil with log-normal distribution of layer heights. In our model soil properties, including tangent shear modulus at zero shear strain - $G$, maximum shear stress constant - $\tau_M$, soil density $\rho$ as well as an incident angle $\alpha$ of oncoming wave are considered as random in the numerical modeling. Any soil property $X$ is presented as

$$X(\xi) = m_X(\xi) + \sigma_X(\xi)U$$
In which

\( \xi \) – Vector spatial coordinate

\( m_X(\xi) \) – mean value depending on a vector spatial coordinate

\( \sigma_X(\xi) \) – Standard deviation depending on a vector spatial coordinates

\( U \) – A zero mean random, unit variance field

The soil properties are assumed to be normally distributed. If they are assumed to be a log-normally distributed the same procedure hold but for the logarithms of the properties. Other distributions like Weibull, binomial, Poisson, Maxwell, etc. can be introduced.

In general the parameters such as mean, standard deviation, correlation parameters should be inferred from in situ test. In our simulations, based on a review published by Baker [21] we adopted the following parameters

\( m_G = 1600 \text{MPa} \)

\( \sigma_G = 240 \text{MPa} \)

\( m_{\tau_M} = 10000 \text{Pa} \)

\( \sigma_{\tau_M} = 1500 \text{Pa} \)

\( m_\rho = 2000 \text{kg/m}^3 \)

\( \sigma_\rho = 300 \text{kg/m}^3 \)

\( m_\alpha = 30^\circ \)
\[ \sigma_\alpha = 5^\circ \]

The output seismic parameter considered here is the peak ground acceleration. The input parameters rather than being replaced by averaged values are described by appropriate function distributions, using an in situ test estimate for mean and standard deviation quantities. The schematic of a Monte-Carlo simulation, described in the Figure, contains of the following steps: (1) Specify the stochastic nature of input variables, (2) Generate random field for all inputs, (3) Run multiple simulations for the entire set of input data, (4) Analyze statistical information, (5) Verify results

![Figure 35: Schematic of a Monte-Carlo simulation.](image)

The relating MATLAB script is presented in the appendix, Annexure A.
Monte Carlo simulation of a peak of a surface acceleration has been performed for three different sample sizes: 100, 1000 and 10000. Relating histograms with superimposed normal, lognormal, Weibull, Student, extreme value and smoothing kernel density of probability distributions are presented in Figures 37, 38 & 39. We ran the Chi-square goodness-of-fit test for each of mentioned above PDFs, which returned a test decision for the null hypothesis that the data in vector ACCEL comes from a current pdf with a mean and variance estimated from ACCEL, using the chi-square goodness-of-fit test. The alternative hypothesis is that the data does not come from such a distribution. The result h for all tested probability density distributions was 1, which means that the test rejected the null hypothesis at the 5% significance level. The measured goodness-of-fit (or p value) typically summarizes the difference between the population values and the values for the distribution in question. [22]

Figure 36: Histogram and superimposed PDFs for sample size nrand=100
Figure 37: Histogram and superimposed PDFs for sample size n_rand=1000
The table below contains mean, standard deviation, skewness and Kurtosis characteristics for the peak of acceleration obtained based on a 100, 1000 and 10,000 sample size. It appeared that the difference in samples 1,000 and 10,000 does not exceed 3% for mean, standard deviation, variance and 10% for skewness and Kurtosis.

**Mean** → \( \bar{X} = \frac{1}{n} \sum_{i=1}^{n} x \) *Where n is the sample size and x corresponds to the observed value*

**Variance** → \( S^2 = \frac{1}{n-1} \sum_{i=1}^{n} (X - \bar{X})^2 \)

**Standard Deviation** → \( S = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (X - \bar{X})^2} \)

**Skewness** → \( g = \frac{\sum_{i=1}^{n} (X - \bar{X})^3 / N}{S^3} \) *where N is the sample size, S the standard deviation*

**Kurtosis** → \( \frac{\sum_{i=1}^{n} (X - \bar{X})^4 / N}{S^4} \)
<table>
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<th>Sample</th>
<th>Mean</th>
<th>STD Deviation</th>
<th>Variance</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
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<td>0.0027</td>
<td>7.26e-6</td>
<td>-1.068</td>
<td>4.01</td>
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<td>0.0028</td>
<td>7.5798e-6</td>
<td>-0.6368</td>
<td>2.9849</td>
</tr>
</tbody>
</table>

Table 2: Peak acceleration statistical characteristics
5. CONCLUSION

On complete analysis and after reviewing the results the following conclusions can be drawn:

- For testing case for 1D and 2D harmonic loading the plots for exact and numerical solutions match completely. Comparing the closed form solution the boundary conditions at X=0 and X=L, show zero velocity at the input and zero stresses at the free surface which was also obtained from the numerical model.

- For the 2D real load application for a linear soil model and the 2D multilayered model as well the results obtained converged very well by increasing the mesh size for the finite volume scheme. The normal stress at the free surface was assumed to be zero and the output was very close to zero, not exactly zero because the methodology gives an output of the values at the cell centers and not the cell boundaries. The values should converge to zero if a finer mesh is used.

- In the 2D real load application to the nonlinear soil model where we used the Hencky-Nadai-Ilyushin deformation theory for soil model which was assumed to behave in an elasto-plastic manner, also the results converged perfectly at the free surface. The plots for shear stress at the left and right boundary show a small finite value which is a result of the corner cells which have characteristics of the left or right boundary and the top surface. This means that only the normal component of stress and the shear stress at free surface is zero but the shear stress at the left/right boundaries are not zero which cause a combined effect at the corner cells. As the mesh of the soil model is refined the results converge better which agrees with the Godunov numerical scheme used in the methodology.

- The stochastic model where we vary three soil properties and the oncoming wave angle in a Monte-Carlo simulation methodology gives an output of the range of peak ground acceleration that can be observed at the free surface. The properties are assumed to have a
normal distribution which gives an output acceleration that is observed to have a kernel distribution fit based on a chi-squared goodness of fit test. The MC simulation relies on the sampling of the model which means that the solution converges on increasing the size of the sample. This was evident in running three different sample sizes of 100, 1000 and 10000. The difference in the mean, standard deviation and variance was within 3% for the sample size of 1000 and 10000. The skewness and kurtosis values with 10% difference for the latter two sample sizes.
6. FUTURE WORK

Although the results obtained were exactly as were expected but there is always a scope for getting better results and investing more into the research work. The topic of seismic wave assessment has a broad scope for development. The model can be extended to 3D system by considering the effects of stresses not only in X and Y but in Z direction as well. The effect of reflection, refraction of waves can be accounted for in the model at the boundaries. The use of a finer mesh can lead to better convergence of the solutions. Using a finer mesh and extending to 3 dimensions demands immense computing power with high performance computing or parallel processing. The stochastic methodology where we used a sample size of 10,000 for four variables can be extended to 100,000 by varying even more factors which affect the seismic wave propagation. This model provides solutions for the P and S waves. In future the effect of surface waves can also be consolidated into the system to provide a more accurate system of solution.
7. SOCIETAL CONTEXT

Earthquakes which occur on a high magnitude and a large intensity have been known to cause massive damage to the environment and the human life. If we take a look at the history of the earthquakes we can see that lives in figures of hundreds and thousands have been lost. Few of the major accidents include the Chili Earthquake of 1960 and the nuclear disaster at Fukushima in 2011. Due to advancements in technology and better methods of prediction of earthquakes we are now able to identify with greater probability the occurrence and the location of earthquakes. With the help of this novel method highlighted in our study, velocity and stresses waveforms induced can be calculated on structures on earth’s surface. This type of study is very important while designing structures which have a large impact on the environment and the society in case of a natural calamity like an earthquake. This will help to evaluate the safety of the structure and the possible threats or hazards that can be caused due to the failure and how that can be prevented. The acceleration recorded on a seismogram need not necessarily always be at the exact location of construction. Hence it is very important to understand as to how a wave propagates to a particular site from the recorded values. The acceleration, velocity and stress profile helps a structural or civil engineer to make the design decisions in accordance to this data. With the growing population and the rate of development the number of people affected in case of a large scale calamity is quite high compared to hundred years back. Hence it is of prime importance to limit the loss and damage to a minimum possible.
8. ANNEXURE

for irand=1:NRAND

G=GMEAN+randn(irand,1)*SIGMAG;

\[ \tau_M = \tau_M \text{MEAN} + \text{randn}(irand,2) \times \text{SIGMA}_{\tau_M}, \]

\[ \rho = \rho \text{MEAN} + \text{randn}(irand,3) \times \text{SIGMA}_\rho; \]

\[ \alpha = \alpha \text{ MEAN} + \text{randn}(irand,4) \times \text{SIGMA}_\alpha; \]

ACCEL(irand)=SEISMIC(G, \tau_M, \rho, \alpha)

end

MEAN=mean(ACCEL)

STD=std(ACCEL)

VAR=var(ACCEL)

SKEWNESS=skewness(ACCEL)

KURTOSIS=kurtosis(ACCEL)

subplot(2,3,1)

histfit (ACCEL,10)

title('Normal Distribution')

legend('Histogram','Normal PDF')

xlabel('Maximum Acceleration m/s^2')

'Test for Normal'

h = chi2gof(ACCEL)

'test for Alpha'

h = chi2gof(ACCEL,'Weibull')
9. REFERENCES


