Experimental Tests and Numerical Simulations for Failure Investigation on Corrugated Boxes Used on Household Appliance Packaging

Diego Rodrigues*  
Whirlpool Latin America

José Pereira  
Federal University of Santa Catarina-Brazil

ABSTRACT

Packages made of corrugated paper are fundamental to the protection, transportation and handling of the appliance product market. During the storage and sales stages of a product, the package must resist compressive loads in different directions beyond moderate impacts. In this context, the objective of this work is to develop and implement a post-processor that allows the simultaneous analysis of two of the most common failure modes of packages made of corrugated paper: failure due to tensile or compressive stress limit, and failure due to local buckling, when the buckling of the faces of the corrugated paper between two peaks of the fluting waves occurs. It was realized that the current procedure proposed by the literature expend a series of operations, what became the failure analysis for corrugated boxes too lengthy for the immediacy of the industry. Thus, it was chosen to apply a procedure to simplify the failure limit surface. Tensile tests were performed for the characterization of corrugated material used in numerical simulations. Experimental tests were performed on corrugated boxes used on household appliance products. The Tsai-Wu criteria for the material failure evaluation was applied and a modified Nyman-Gustafsson criteria for the local buckling analysis on the numerical simulations was proposed. A good correlation between experimental tests and numerical results was obtained. This work brings high expectations to the agility requested by the industry in the launching of new products.

KEY WORDS

Household appliances, corrugated paper, packaging, local buckling, Tsai-Wu.

*Diego Rodrigues
Corresponding Author
diego_f_rodrigues@whirlpool.com
INTRODUCTION

The development of corrugated paper as structural material for packages began during World War II due to a shortage of materials, and consequently the need to rationalize its use. In this context, the corrugated paper gained importance because of its easy manufacturing, excellent mass/strength relationship, light weight, low cost, easy assembling and disassembling, besides being easily transported before and after use. In the years after the end of the war, its use was extended to all kinds of industries [1]. At present, its use in packaging goes from non-durable consumer goods, such as transport of fruits and vegetables to products with high added value such as electronic equipment and household appliances. In the so-called white line that contemplates the market share where it is inserted, the washing machines, the refrigerators, the air conditioners and the electric ovens and stoves, the use of corrugated paper for packaging is only being compared to the use of the expanded polystyrene (EPS) for expanded packaging, together with the thermoformed plastic film. However, the latter alternative is technically more complex and economically more expensive, normally used under special conditions, such as excessive humidity in transportation or products with higher added value.

Packaging plays a key role in product protection for its transport and handling, during the steps of storage and the marketing of their life cycle, which includes resisting compressive loads in different directions, besides moderate impacts. There is also an aesthetic importance associated with the packaging of a product on the white line, because the packaging is the customer’s first contact with the product just purchased. Thus, not just the graphic arts have to please the consumer, but also the physical integrity of the packaging must be preserved, because if there are traces of kneading or damage on it, the consumer could interpret that the product has also been damaged and consequently request its exchange. This would entail costs, mainly logistic ones, besides harming the company’s image, whose bias may be even more severe.

As well as any other structure that should resist mechanical loads, the corrugated boxes should also be designed to resist the transportation and handling loads. Thus, numerical simulation models are good alternatives for the prediction of mechanical performance of corrugated boxes. Commercial softwares are frequently used for failure mode identification in corrugated paper packaging such as: a) material failure, in which the papers fail by tension or compression according to the Tsai-Wu criterion for composite materials and b) global buckling, in which failure occurs by instability of one plane face of the packaging due to compressive loads. However, there are still other failure modes that are not identified on it. One of these failure modes corresponds to local buckling, which is the buckling of one of the outer faces of the packaging between two adjacent peaks.

In this context of structural integrity of corrugated boxes applied to packaging of household appliance products, the goal of this work is to develop and implement a procedure to analyse the failure due to material strength and the failure due to local buckling. Corrugated paper was characterized by standard tensile tests and experimental tests on corrugated box packaging were performed. For the material strength failure evaluation, the Tsai-Wu criteria was applied and for the local buckling failure evaluation, a modified Nyman-Gustafsson criteria is proposed. A good correlation between numerical results and experimental tests were obtained. The results of this research bring positive expectations for the request of the industry in the fast design of corrugated boxes of new products.
BACKGROUND AND EARLIER WORK

The history of corrugated paper for packaging and the nomenclature related to this subject is well placed by Benjamin Frank in his literature survey [2]. In works shown by Andreas Allansson, Björn Svärd and Tomas Nordstrand, additional features related to corrugated board such as the manufacturing process, components and the configurations are presented [3], [4] and [5]. A single-wall board of a corrugated paper has two face sheets, called liners or facings, which are bonded to a wave shaped, called fluting, medium or core, as shown in Figure 1. The fluting waves are oriented in a machine-direction (x, MD), the transversal direction of the waves is oriented in the cross-direction (y, CD) and the orientation through the thickness of the board is denoted z. This definition of principal directions of a corrugated paper is equivalent to the directions of a composite material defined on the orthotropic directions (1, 2, 3) [6] and [7].

Most of the research on the structural performance of corrugated paper are concentrated on the transport dynamics and the compressive loading for packaging damage investigation. In the first subject, one can essentially find laboratory transport tests for identification of vibration levels and the impact intensities on the transport and handling during the steps of storage. The eigenmodes of a single box and a stacked packaging unit of 40 boxes were carried out [8]. The effect of truck speed during transportation on vibration and shock levels was also investigated [9]. It was observed when the speed is above 45 km/h, no significant changes in the peak of acceleration of vibration were found. The handling during the storage is frequently investigated in corrugated paper for packaging. The influence of low-intensity repeated impacts on the cushioning performance of honeycomb with different dropping height was tested [10].

On the subject concerning the compressive loading packaging damage of corrugated board buckling are frequently investigated. Corrugated board buckling can occur in two distinct ways. One is called global buckling, when the associated deformation occurs throughout the entire thickness of the board. The other form is called local buckling, where the associated deformation occurs on the liners between the waves of the core. Both modes of buckling can be seen in Figure 2 [3].

Fig. 1: Single-wall of a corrugated board.

Fig. 2: Failure on corrugated paper: (a) global buckling and (b) local buckling.
The global buckling of corrugated board boxes is simpler to analyse. Commercial softwares can be used on the numerical simulation and the virtual model can represent the geometric shape of the corrugated paper or can be a simple plate with homogenized properties. Global and local buckling on corrugated board and corrugated boxes have been investigated recently by various researchers [11]-[20]. In some references, the critical load into global buckling analysis is correlated to the homogenized bending stiffness of the board and the local buckling is predicted by a unit cell approach designed with the finite element method [11]. A mesh-free Galerkin method based on the first-order shear deformation theory is used for the elastic buckling analysis of stiffened and un-stiffened corrugated plates [12]. An analytical plate model to predict the elastic post-buckling behaviour of corrugated board boxes with measured kinematic boundary conditions is proposed [13]. A minimization energy procedure for the local buckling on corrugated board facings is proposed and combined with the Tsai-Wu criterion for the material failure [14] and [15]. Local buckling and material failure were also combined to investigate the collapse mechanism of biaxially loaded corrugated board cylinders [16].

Because of the manufacturing process of corrugated board which contain various processing-induced geometrical, it can be related to composite materials and its elastic properties are defined on the orthotropic directions (1, 2, 3) as shown in Figure 1. Thus, the strain-stress relationship for corrugated board can be defined as:

For the determination of the Young’s moduli in the directions 1 and 2 ($E_1$, $E_2$) of papers applied on corrugated paper, a standard test can be administered as ASTM D642 [22] and ABNT NBR NM-ISO 1924-2 [23]. Remaining parameters can be approximated [3] and [5]: Besides standard tests for corrugated board characterization, vibration testing can be used. In this case, the non-destructive Chladni-pattern method can be utilized accurately for quality control of the boards during manufacturing [24].

**TSAI-WU CRITERION FOR MATERIAL FAILURE**

The Tsai-Wu failure criterion for plane stress (1,2) developed by Tsai and Wu is the criterion most applied for determining material failure in facings of corrugated paper since it accounts for difference in material behaviour in tension and compression [6]. It is stated that material failure occurs when the left side of the equation below is equal to one:

$$F_1\sigma_{11} + F_2\sigma_{22} + F_{11}\sigma_{11}^2 + F_{22}\sigma_{22}^2 + F_{66}\sigma_{12}^2 + 2F_{12}\sigma_{11}\sigma_{22} = 1$$

(6)
Where the constants $F_i$ and $F_{ij}$ are related to the tensile (T), compressive (C) and shear (S) strengths as shown:

\[
\begin{align*}
F_1 &= \frac{1}{X^T_1} + \frac{1}{X^C_1} \\
F_2 &= \frac{1}{X^T_2} + \frac{1}{X^C_2} \\
F_{11} &= -\frac{1}{X^T_1 X^C_1} \\
F_{22} &= -\frac{1}{X^T_2 X^C_2} \\
F_{66} &= \frac{1}{S^2} \\
\end{align*}
\]  

(7)

According to Nordstrand [5], the coupling factor and the shear strength can be approximated by:

\[
\begin{align*}
F_{12} &= -0.36 \sqrt{F_{11} F_{22}} \\
S^c &= 0.78 \sqrt{X^T_1 X^C_1} \\
\end{align*}
\]  

(8)

An alternative to equation (6) on the material failure analysis is to transform the Cartesian to spherical coordinate stresses and thereby obtain the length of a stress vector for their Tsai-Wu criteria which represents a surface of an ellipsoid in the stress space, see Figure 3:

\[
\begin{align*}
\sigma_{11} &= \sigma^R \sin \phi \cos \theta \\
\sigma_{12} &= \sigma^R \sin \phi \sin \theta \\
\sigma_{22} &= \sigma^R \cos \phi \\
\end{align*}
\]  

(10)

Thus, the Tsai-Wu criteria in a spherical coordinate system has the form:

\[
\begin{align*}
\left(F_{11} \eta_{11}^2 + F_{22} \eta_{22}^2 + 2F_{12} \eta_{12} \eta_{22} + 2F_{12} \eta_{11} \eta_{22}\right) \left(\sigma^R_{TW}\right)^2 \\
+ \left(F_{11} \eta_{11} + F_{22} \eta_{22}\right) \sigma^R_{TW} - 1 &= 0 \\
\sigma^R_{TW} &> 0
\end{align*}
\]  

(11)

Where:

\[
\begin{align*}
\eta_{11} &= \sin \phi \cos \theta \\
\eta_{12} &= \sin \phi \sin \theta \\
\eta_{22} &= \cos \phi
\end{align*}
\]  

(12)

**NYMAN-GUSTAFSSON CRITERIA FOR LOCAL BUCKLING FAILURE**

In order to analyse local buckling behavior, Nyman and Gustafsson developed a method to predict their identification based on a higher order shear stress theory [14] and [15]. In this method, the stress state is calculated by using the equation below:

\[
\sigma^R_{NO} = \frac{\sum_i^3 g_i}{60a^2 \tau \lambda^2 \left(-c_2\eta_{11} + 2a^2u \eta_{12}\right)} \sum_{i=1}^{27} h_i
\]

\[
\sigma^R_{NO} > 0, \quad [\lambda, u] \in x_{cr}
\]  

(13)
Where \( \alpha \) is the wavelength of the corrugate core, \( t \) is the thickness of the facing, \( \lambda \) is the half buckling wavelength and \( \mu \) is the inclination of the nodal lines, see Figure 4. The coefficients \( g_i \) and \( h_i \) in the sums are presented in the work developed by Nyman and Gustafsson [15]. In equation (13), \( x_{cr} \) is the solution of the half buckling wave length \( \lambda \) and the inclination of the nodal lines \( \mu \) found by numerical minimization.

**COMBINED FAILURE CRITERION**

When analyzing the stress state on the facing of a corrugated board, one must take into account both critical stress obtained by Tsai-Wu, with equation (11), and the local buckling critical stress, with equation (13), choosing the most significant failure criteria or the smallest of critical stress. Thus, the minimum stress that leads to failure is defined as:

\[
\sigma^R = \min \{ \sigma_{\text{NG}}^R, \sigma_{\text{NG}}^R \} 
\] (14)

Thus, for a given shear stress \( \sigma_{12} \), the normal stress \( \sigma_{11} \) and \( \sigma_{22} \) associated to the stress state, the limit values for the stress failure are obtained, see Figure 5.

A failure index can be calculated from the stress state obtained with a finite element model and can be defined as

\[
\Phi(\sigma_{11}, \sigma_{12}, \sigma_{22}) = \frac{\|\sigma\|}{\sigma_f^R} 
\] (15)

This failure index takes the unity value or greater than one at failure for the most significant failure mode.

**SIMPLIFIED MODEL FOR LOCAL BUCKLING FAILURE**

The local buckling identification on corrugated board by using equation (13) from the Nyman-Gustafsson method is difficult to apply in real applications in industries, since the minimization process of this equation is performed in each element of the mesh of the virtual model of the corrugated box. Thereby, this present work proposes a simplified solution of the critical stress for the local buckling failure investigation without need of minimization. Consequently, this simplified solution is proposed as:

\[
\hat{\sigma}_{\text{NG}}^R = F(\sigma_{11}, \sigma_{12}, \sigma_{22}) = F(\phi, \theta) 
\] (16)
Experimental Tests and Numerical Simulations for Failure Investigation

Where $\theta$ and $\phi$ are angles of the stress vector $\sigma^b$ in respect to $\sigma_{11}$ and $\sigma_{22}$ axis in the stress space, respectively, see Figure 3. The function F is obtained by a mode Frontier routine which follows the restriction of Equation (10).

NUMERICAL AND EXPERIMENTAL VALIDATION FOR HOUSEHOLD APPLIANCE PACKAGING

In this section, an industrial corrugated box used on household appliance products is analyzed in respect to material failure and local buckling failure. The material failure is investigated by using Equation (11) and local buckling is investigated by using the simplified method described by Equation (17) based on the Nyman-Gustafsson method.

Experimental tests on corrugated box samples were performed in a laboratory in order to validate this simplified method proposed in this present work.

The dimension of the corrugated box samples used in the experimental tests are presented in Figure 6 and the geometric shape of the board with the nominal dimension of its components are presented in Figure 7. The corrugated board tested was the Type B. The cardboard itself is used for cooktop range products in Brazil's market and it is widely available for testing.

\[
\hat{\sigma}_{\text{R}}^b = 216.88 - 72.149 \phi - 254.01 \theta - 8.3886 \phi^2 + 50.277 \phi \theta + 139.92 \theta^2 \\
- 68.554 \phi^3 + 131.96 \phi^2 \theta - 78.827 \phi \theta^2 - 33.378 \theta^3 \\
+ 59.694 \phi^4 - 67.459 \phi^3 \theta - 7.3464 \phi^2 \theta^2 + 22.252 \phi \theta^3 + 3.3936 \theta^4 \\
- 16.482 \phi^5 + 9.1043 \phi^4 \theta + 10.572 \phi^3 \theta^2 - 4.2783 \phi^2 \theta^3 - 1.6901 \phi \theta^4 - 0.15422 \theta^5 \\
+ 1.5074 \phi^6 - 9.023076 \phi^5 \theta - 1.4240 \phi^4 \theta^2 + 0.021438 \phi^3 \theta^3 + 0.333401 \phi^2 \theta^4 \\
- 0.0091237 \phi \theta^5 + 0.0094645 \theta^6
\] (17)

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Fig. 6: Dimension of the corrugated box used on the household appliance packaging.

Fig. 7: Geometric shape of the corrugated board.
The components of the corrugated board are Kraft paper with a density of 140 g/m³ on the facings and a recycled paper with a density of 160 g/m³ on the core. Elastic coefficients were obtained according to ABNT NBR NM-ISO 1924-2 [22] performing standard tensile tests. At least 10 proofs for each component is recommended: 5 in the machine-direction (x, MD) and 5 in the cross-direction (y, CD). The proofs were strictly controlled at a temperature of 20 °C and 50% humidity. Compressive properties were obtained according to ABNT NBR NM-ISO 14260 [24] performing Ring Crush Tests-RTC by the supplier of the raw material. The results from these mechanical tests are presented in Table 1. Figure 8 illustrates the mechanical tests employed for the characterization of the corrugated board.

Table 1: Mechanical properties for the corrugated board components.

<table>
<thead>
<tr>
<th></th>
<th>Facings (Kraft 140)</th>
<th>Core (recycled 160)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$ (mm)</td>
<td>0.216 ± 0.0045</td>
<td>0.214 ± 0.00121</td>
</tr>
<tr>
<td>$E_1$ (MPa)</td>
<td>2793 ± 122</td>
<td>2054 ± 223</td>
</tr>
<tr>
<td>$E_2$ (MPa)</td>
<td>1554 ± 219</td>
<td>1191 ± 156</td>
</tr>
<tr>
<td>$X_{11}$ (MPa)</td>
<td>39 ± 3.0</td>
<td>19 ± 2.7</td>
</tr>
<tr>
<td>$X_{22}$ (MPa)</td>
<td>27 ± 1.9</td>
<td>30 ± 1.5</td>
</tr>
<tr>
<td>$X_{12}$ (MPa)</td>
<td>21 ± 1.5</td>
<td>10 ± 0.9</td>
</tr>
<tr>
<td>$X_{13}$ (MPa)</td>
<td>17 ± 1.6</td>
<td>20 ± 2.1</td>
</tr>
</tbody>
</table>

According to Allansson e Svärd [21], the remaining elastic coefficients for corrugated boards can be approximated by using Equations (2) to (5) previously presented:

Table 2: Mechanical properties for the corrugated board components.

<table>
<thead>
<tr>
<th></th>
<th>Facings (Kraft 140)</th>
<th>Core (recycled 160)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_{12}$ (MPa)</td>
<td>806</td>
<td>805</td>
</tr>
<tr>
<td>$G_{13}$ (MPa)</td>
<td>51</td>
<td>37</td>
</tr>
<tr>
<td>$G_{23}$ (MPa)</td>
<td>0.40</td>
<td>0.30</td>
</tr>
<tr>
<td>$v_{12}$</td>
<td>0.34</td>
<td>0.34</td>
</tr>
<tr>
<td>$v_{13}$</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>$v_{33}$</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>$S_6$ (MPa)</td>
<td>16</td>
<td>19</td>
</tr>
</tbody>
</table>

Prior to the validation of the industrial corrugated box shown in Figure 6, the simplified method for the local buckling analysis presented in this work is validated for a corrugated board under biaxial stress with the geometric shape shown in Figure 7 and the mechanical properties presented in Tables (1) and (2).
In Figure 9, the Nyman-Gustafsson method and the simplified method in this condition are illustrated. As expected, the simplified method shows a continuum evolution of the critical stress for any stress state. In addition, the results presents a good correlation between both methods.

In Figure 10, the combined failure criterion, the material failure by Tsai-Wu criteria, and the local buckling are demonstrated by the Nyman-Gustafsson minimization and simplified methods. As observed, the simplified method developed in this work can be used on industrial corrugated boxes for local buckling analysis without loss of accuracy.

As follows, the corrugated box samples of a household appliance packaging product presented in Figure 6 are analyzed with respect to the combined failure criterion. Experimental tests on 3 samples were performed as shown in Figure 11. The samples are positioned between a fixed table and a sliding table with a vertical velocity of 10 mm/s until a failure is identified on the lateral faces.

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**Fig. 9:** Nyman-Gustafsson criterion: (a) simplified and (b) minimization.

**Fig. 10:** Combined failure criterion.

**Fig. 11:** Experimental tests of a corrugated box packaging.
The compressive load is measured by load cells placed over a fixed table. In Figure 12, the deformation evolution of a sample for different displacements of the sliding table is shown.

For the virtual model development, a multilayer shell with three layers through the thickness was used with the geometry and mechanical properties presented earlier. The size of the elements was approximately 4 mm. The rotations in the normal axes for the edges were removed and only the rotations in the parallel axis were maintained. In Figure 13, the compressive load versus the displacement of the sliding table from experimental tests and virtual experiment are illustrated.

In the virtual experiment, a limit of 5% of volume failure was arbitrary defined, based on the failure plot. It is observed a good relationship between experimental and virtual tests in respect to the compressive load. It can be seen that, although similar, the compressive critical load is quite variable, this is mainly due to the dimensional variation and variations of the mechanical properties of the packaging. It can be considered that for this purpose it has been minimized, since the packaging with the closest possible dimensions of the simulated geometry were chosen as samples.

![Fig. 12: Deformation of corrugated box packaging sequence.](image)

![Fig. 13: Load vs. displacement from experimental tests and virtual experiment.](image)
In Figure 14, the displacements in directions (x, y, z) and the failure index (see Equation (15)) obtained with the virtual model are plotted.

From a qualitative analysis, it can also be seen that there is a correlation between the failure mode shown in the simulation shown in Figure 14 and the failure mode shown in the experimental test shown in Figure 12. In Figure 12 it is shown the deformation of the box during the test, while in Figure 14 it is shown the displacement in three different directions and the failure criteria at Virtual Load Limit. These pictures should be compared with Figure 12(f) as shown in Figure 15. As observed in Figure 12(f) there is approximately a symmetry in the failure path. Thus, in Figure 15 a deformation of the corrugated box sample and the failure index are placed side by side. As observed in Figure 12(a) and Figure 15(a), the failure starts on the higher corner on the larger face of the corrugated box and the path of failure propagation is along the center of the face. In the virtual experiment test shown in Figure 15(b), the same phenomenon is observed.

Even though the local buckling is not directly observed in the experimental test of the corrugated box samples, one can conclude based on the corrugated board under biaxial stress validation, and on the virtual and experimental test that the modified method can speed up the design of household appliance packaging in industrial applications.
CONCLUSION

The objective of this study was to know the behavior of packaging made from corrugated paper type B, as well as to study their failure criteria, the methods used for their prediction and implement them into a commercial software in a practical and viable way for the added high value product industry. Commercial softwares for simulation of corrugated paper packaging behavior are used for material failure identification, in which the papers fail by tension or compression, and global buckling, in which failure occurs by instability of one plane face of the packaging due to compressive loads. However, the failure mode corresponding to local buckling, which is the buckling of one of the outer faces of the packaging between two adjacent peaks is not identified on it.

The local buckling behavior was initially studied by Nyman and Gustafsson and a method to predict their identification based on a higher order shear stress theory based on a numerical minimization of variables was developed. The local buckling identification on corrugated board by the Nyman-Gustafsson method is difficult to apply in real applications, since this minimization process is performed in each element of the mesh of the virtual model of the corrugated box, which can take a long time for the industry's need. In this context, a simplified Nyman-Gustafsson method for the local buckling investigation on corrugated boxes used on household appliance packaging has been presented. Standard tensile tests for facings and core characterization were carried out. Besides that compressive tests on corrugated box samples of household appliance packaging were carried out.

This simplified method has to be validated with the Nyman-Gustafsson minimization method for a corrugated board under biaxial stress with the geometric shape of the corrugated board and the mechanical properties of the components of the corrugated box in the study. For the mechanical characterization of the facings and the core tensile, compressive tests on standardized proofs have to be performed and humidity and temperature control is strictly required.

The simplified Nyman-Gustafsson method for the local buckling investigation was combined with the Tsai-Wu criterion for material failure analysis. In addition, the most significant failure mode of a corrugated box can be identified. This procedure of failure investigation can be implemented in commercial software on industrial application of corrugated boxes with reliable accuracy since the Nyman-Gustafsson minimization method requires elevated utilization of processing time.

This approach was applied on a corrugated paper type B of household appliance packaging and cannot be applied directly on combinations of flutes, once theirs parameters affect the criteria developed by Nyman-Gustafsson. In these cases, different sizes and box proportions have to tested.

For other industrial applications of corrugated boxes, especially frozen foods, where humidity and temperature are more stringent, these variables along with geometry variability and deformation rate have to be carefully observed in the mechanical tests for the material characterization for a positive correlation with the experimental tests.
REFERENCES


