A Mechanized Theory of Communication Analysis in CML

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A Mechanized Theory of Communication Analysis in CML

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A Mechanized Theory of Communication Analysis in CML

by

Thomas Logan

THESIS

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Abstract

A Mechanized Theory of Communication Analysis in CML

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For this master’s thesis, I have developed a formal semantics of a language with concurrent processes (or threads), an initial formal analysis, along with related theorems and formal proofs. The language under analysis is a very simplified version of Concurrent ML. The formal analysis recasts an analysis with informal proofs developed by Reppy and Xiao. It categorizes communication described by programs into simple topologies. One description of topologies is static; that is, it describes all static topologies of a program in a finite number of steps. Another description is dynamic; that is, it describes topologies in terms of running a program for an arbitrary number of steps. The main formal theorem states that the static analysis is sound with respect to the dynamic analysis. Two versions of the static analysis have been developed so far; one with lower precision, and one with higher precision. The higher precision analysis is closer to the work by Reppy and Xiao, but contains many more details making it more challenging to prove formally than the lower precision analysis. The proofs for the soundness theorems of the lower precision analysis have been mechanically verified using Isabelle/HOL, while the higher precision
analysis is currently under development. Indeed, one of the motivations for implementing the analysis in a mechanical setting is to enable gradual extension of analysis and language without introducing uncaught bugs in the definitions or proofs. The definitions used in this formal theory differ significantly from that of Reppy and Xiao, in order to aid formal reasoning. Thus, recasting Reppy and Xiao’s work was far more nuanced than a straightforward syntactic transliteration. Although the definitions are structurally quite different, their philosophical equivalence is hopefully apparent. In this formal theory, the dynamic semantics of Concurrent ML consists of a CEK machine. The static semantics consists of a control flow analysis (0CFA), defined in terms of constraints.
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Chapter 1

Introduction

For this master’s thesis, I have developed a formal semantics of a language with concurrent processes (or threads), an initial formal analysis, along with related theorems and formal proofs. The language under analysis is a very simplified version of Concurrent ML [18]. The formal analysis recasts an analysis with informal proofs developed by Reppy and Xiao [17]. It categorizes communication described by programs into simple topologies. One description of topologies is static; that is, it describes all static topologies of a program in a finite number of steps. Another description is dynamic; that is, it describes topologies in terms of running a program for an arbitrary number of steps. The main formal theorem states that the static analysis is sound with respect to the dynamic analysis. Two versions of the static analysis have been developed so far; one with lower precision, and one with higher precision. The higher precision analysis is closer to the work by Reppy and Xiao, but contains many more details making it more challenging to prove formally than the lower precision analysis. The proofs for the soundness theorems of the lower precision analysis have been mechanically verified using Isabelle/HOL [16], while the higher precision analysis is currently under development. Indeed, one of the motivations for implementing the analysis in a mechanical setting is to enable gradual extension of analysis and language without introducing uncaught bugs in the definitions or proofs. The definitions used in this formal theory differ significantly from that of Reppy
and Xiao, in order to aid formal reasoning. Thus, recasting Reppy and Xiao’s work was far more nuanced than a straightforward syntactic transliteration. Although the definitions are structurally quite different, their philosophical equivalence is hopefully apparent. In this formal theory, the dynamic semantics of Concurrent ML consists of a CEK machine [5]. The static semantics consists of a control flow analysis (0CFA) [19], defined in terms of constraints [14].

1.1 Concurrent ML

In programming languages, concurrency is a program structuring technique that allows evaluation steps to hop back and forth between disjoint syntactic structures within a program. It is useful when conceptually distinct tasks need to overlap in time, but are easier to understand if they are written as distinct structures within the program. Concurrent languages may also allow the evaluation order between steps of terms to be nondeterministic. If it’s not necessary for tasks to be ordered in a precise way, then it may be better to allow a static or dynamic scheduler to pick the most efficient execution order. A common use case for concurrency is for programs that interact with humans, in which a program has to process various requests while remaining responsive to subsequent user inputs, and it must continually provide the user feedback with the latest information it has processed.

Concurrent ML is a particularly elegant programming language for concurrency. It features threads, which are pieces of code allowed to have a wide range of evaluation orders relative to code encapsulated in other threads. Its synchronization mechanism can mandate the execution order between parts of separate threads. It is often the case that synchronization is necessary when data is shared. Thus, in Concurrent ML, synchroniza-
tion is inherent in communication. For asynchronous communication, additional threads may be spawned.

Threads communicate by having shared access to a common channel. A channel can be used to either send data or receive data. When a thread sends on a channel, another thread must receive on the same channel before the sending thread can continue. Likewise, when a thread receives on a channel, another thread must send on the same channel before the receiving thread can continue.

```ocaml
type thread_id
val spawn : (unit -> unit) -> thread_id

type 'a chan
val channel : unit -> 'a chan
val recv : 'a chan -> 'a
val send : 'a chan * 'a -> unit
```

A given channel can have any arbitrary number of threads sending or receiving data on it over the course of the program’s execution. A simple example, derived from Reppy’s book *Concurrent Programing in ML* [18], illustrates these essential features.

The implementation of `Serv` defines a server that holds a number in its state. When a client gives the server a number `v`, the server gives back the number in its state, and updates its state with the number `v`. The next client request will get the number `v`, and so on. Essentially, a request and reply is equivalent to reading and writing a mutable cell in isolation. The function `make` makes a new server, by creating a new channel `reqCh`, and a loop `loop` which listens for requests. The loop expects the request to be composed of a number `v` and a channel `replCh`. It sends its current state’s number on `replCh` and updates the loop’s state with the request’s number `v`, by calling the loop with a that number. The
server is created with a new thread with the initial state 0 by calling spawn (fn () => loop 0). The request channel is returned as the handle to the server. The function call makes a request to a server server with a number v and returns a number from the server. Internally, it extracts the request channel reqCh from the server handle and creates a new channel replCh. It makes a request to the server, sending the number v and the reply channel replCh by calling send (reqCh, (v, replCh)). Then it receives the reply with the new number by calling recv replCh.

```
signature SERV =
sig
type serv
  val make : unit -> serv
  val call : serv * int -> int
end

structure Serv : SERV =
struct
datatype serv = S of (int * int chan) chan

  fun make () =
  let
    val reqCh = channel ()
    fun loop state =
    let
      val (v, replCh) = recv reqCh
      val () = send (replCh, state)
    in
      loop v
    end
  in
    S reqCh
  end

  fun call (server, v) =
  let
    val S reqCh = server
    val replCh = channel ()
    val () = send (reqCh, (v, replCh))
  in
```

4
Concurrent ML actually allows for events other than sending and receiving to occur during synchronization. In fact, the synchronization mechanism is decoupled from events, much in the same way that application is decoupled from functions. Sending and receiving events are represented by `sendEvt` and `recvEvt` and synchronization is represented by `sync`.

```ml
type 'a event
val sync : 'a event -> 'a

val recvEvt : 'a chan -> 'a event
val sendEvt : 'a channel * 'a -> unit event

fun send (ch, v) = sync (sendEvt (ch, v))
fun recv ch = sync (recvEvt ch)
```

An advantageous consequence of decoupling synchronization from events, is that events can be combined with other events via event combinators, and synchronized on exactly once. One such event combinator is `choose`, which constructs a new event consisting of two constituent events, such that when synchronized on, exactly one of the two events may take effect. There are many other useful combinators, such as the `wrap` and `guard` combinators designed by Reppy [17]. Additionally, Donnelly and Fluet extended Concurrent ML with the `thenEvt` combinator described in their work on transactional events [4]. Transactional events enable more robust structuring of programs by allowing non-isolated code to be turned into isolated code via the `thenEvt` combinator, instead of duplicating code with the addition of stronger isolation. When the event constructed by
the \texttt{thenEvt} combinator is synchronized on, either all of its constituent events and functions evaluate in isolation, or none evaluates.

\begin{verbatim}
val choose : 'a event * 'a event -> 'a event
val thenEvt : 'a event * ('a -> 'b event) -> 'b event
\end{verbatim}

\section{Isabelle/HOL}

An interactive theorem proving assistant, or proof assistant, is a machine (typically software) that helps its user specify propositions and prove theorems. Like typical programming systems, it checks that propositions are lexically correct, syntactically correct, and even compositionally correct according to its type system. To determine if a proposition is valid, a proof assistant often requires the user to supply a proof. Once the proof assistant has verified that the user's proof is correct, then the proposition may be considered a theorem. In addition to checking the correctness of proofs and propositions, the proof assistant also assists the user in constructing proofs.

Isabelle/HOL is a popular proof assistant that assists in specifying and proving properties formulated in a higher order logic (HOL). Predicates and functions may be higher order. That is, they may take other predicates or functions as arguments, and functions may return predicates or functions. In HOL, a predicate is actually just a function that returns a value of type boolean, and a proposition is simply a term of type boolean. Terms are deemed compositionally correct according to a system of simple types, similar to that of Standard ML. Isabelle/HOL excels at assisting with proving propositions containing numerous details.

A proof is a sequence of manipulations from axioms to a claim. Typically, unass-
sisted informal proofs, are written in a declarative form. One first states the claim, and then starts off the proof by stating an axiom or theorem. Subsequently derived theorems follow, with or without the details of the manipulation explained. For instance, say you want to prove $P_1 \lor P_2 \rightarrow Q$. You could begin by stating the axiom $P_1 \lor P_2 \vdash P_1 \lor P_2$, commonly written as \texttt{assume $P_1 \lor P_2$}. You can also state $P_1 \vdash P_1$ and $P_2 \vdash P_2$. Since these are intended to form complementary cases of $P_1 \lor P_2$, they are written as \texttt{case $P_1$:} ...	exttt{case $P_2$}. Perhaps you know \texttt{theorem A:} $P_1 \vdash Q$. Using modus ponens, you derive $P_1 \vdash Q$. Let’s say you also know \texttt{theorem B:} $P_2 \vdash Q$. Using modus ponens with theorem B, you derive $P_2 \vdash Q$. These two theorems may be combined into $P_1 \lor P_2 \vdash Q$, which reduces to $\vdash P_1 \lor P_2 \rightarrow Q$.

$$
\vdash P_1 \lor P_2 \rightarrow Q
\begin{proof}
\text{assume } P_1 \lor P_2:\n\text{case } P_1:\n\\text{have } \vdash P_1 \rightarrow Q \text{ by } A
\\text{have } \vdash Q \text{ by modus ponens}
\text{case } P_2:\n\\text{have } \vdash P_2 \rightarrow Q \text{ by } B
\\text{have } \vdash Q \text{ by modus ponens}
\\text{have } P_1 \vdash Q, P_2 \vdash Q
\\text{have } \vdash Q \text{ by disjunction elimination}
\\text{have } P_1 \lor P_2 \vdash Q
\\text{have } \vdash P_1 \lor P_2 \rightarrow Q \text{ by implication introduction}
\text{qed}
\end{proof}
$$

For proving simple propositions, it may be easy to conjure up the theorems and axioms needed to combine and manipulate into the goal. However, for proving complex or unfamiliar propositions, it may be less clear. In addition to the declarative forward proof style, Isabelle/HOL also supports an imperative backwards proof style, in which you start with the goal and break it into simpler and simpler subgoals until all subgoals
are manipulated into axioms. The backwards style of reasoning allows you to focus on the manipulation rule, rather than remembering and gathering up all the propositions that are necessary to combine to reach the goal. The interface of the interactive theorem prover displays the subgoals resulting from applying each manipulation rule.

\[ \vdash P_1 \lor P_2 \rightarrow Q \]
\[ \text{apply (rule impI)} \]
\[ P_1 \lor P_2 \vdash Q \]
\[ \text{apply (erule disjE)} \]
\[ P_1 \vdash Q \]
\[ * P_2 \vdash Q \]
\[ \text{apply (insert A)} \]
\[ P_1, P_1 \rightarrow Q \vdash Q \]
\[ * P_2 \vdash Q \]
\[ \text{apply (erule mp)} \]
\[ P_1 \vdash P_1 \]
\[ * P_2 \vdash Q \]
\[ \text{apply assumption} \]
\[ P_2 \vdash Q \]
\[ \text{apply (insert B)} \]
\[ P_2, P_2 \rightarrow Q \vdash Q \]
\[ \text{apply (erule mp)} \]
\[ P_2 \vdash P_2 \]
\[ \text{apply assumption} \]
\[ \text{done} \]

Note that the syntax shown here differs slightly from that of Isabelle/HOL. In order to make these examples more accessible to those unfamiliar with Isabelle/HOL, I have chosen to use a syntax that is more typical of mathematical logic and language theory literature.

In addition to creating theorems by proving propositions, Isabelle/HOL also allows creating theorems by defining predicates and functions. This feature is critical for constructing inductive propositions that may hold over infinite domains. Infinite domains may be defined as inductive data types, similar to Standard ML. For instance, you can de-
fine the infinite set of natural numbers (with zero) as an inductive data type, and the binary relation that a natural number is less than or equal to another as an inductive predicate.

```plaintext
datatype nat = Z | S nat
predicate lte: nat -> nat -> bool where
  eq: n . Γ ⊢ lte n n
  * lt: n₁ n₂ . Γ ⊢ lte n₁ n₂
    Γ ⊢ lte n₁ (S n₂)
```

Additionally, you can define the infinite set of lists inductively, and the higher order predicate that checks sortedness using a supplied binary relation.

```plaintext
datatype 'a list = Nil | Cons 'a ('a list)
predicate sorted: ('a -> 'a -> bool) -> 'a list -> bool where
  nil: r . Γ ⊢ sorted r Nil
  * uni: r x . Γ ⊢ sorted r (Cons x Nil)
  * cons: r x y ys . r x y,
    Γ ⊢ sorted r (Cons y ys)
    Γ ⊢ sorted r (Cons x (Cons y ys))
```

Each case of the predicate definition is considered a theorem, and you have the option to give it a name. Additionally, these are the only cases that can hold for the predicate. Therefore, the predicate applied to some free variables is equivalent to the disjunction of all the cases, with the variables equal to their respective patterns in each case. This kind of theorem, and other similar theorems for inversion and induction are created with each predicate definition.

```plaintext
theorem sorted.simps: r xs . Γ ⊢ sorted r xs ≡
  (xs = Nil)
  ∨ ∃ x . (xs = (Cons x Nil))
  ∨ ∃ x y ys . (xs = (Cons x (Cons y ys)) ∧ r x y ∧ sorted r (Cons y ys))
```
By composing these definitions, you can state and prove a list of natural numbers

is sorted in non-decreasing order.

\[ ⊢ \text{sorted lte (Cons (Z) (Cons (S Z) (Cons (S (S (S Z))) Nil)))} \]
\[ \text{apply (rule cons)} \]
\[ ⊢ \text{lte Z (S Z)} \]
\[ * ⊢ \text{sorted lte (Cons (S Z) (Cons (S Z) (Cons (S (S (S Z))) Nil)))} \]
\[ \text{apply (rule lt)} \]
\[ ⊢ \text{lte Z Z} \]
\[ * ⊢ \text{sorted lte (Cons (S Z) (Cons (S Z) (Cons (S (S (S Z))) Nil)))} \]
\[ \text{apply (rule eq)} \]
\[ ⊢ \text{sorted lte (Cons (S Z) (Cons (S Z) (Cons (S (S (S Z))) Nil)))} \]
\[ \text{apply (rule cons)} \]
\[ ⊢ \text{lte (S Z) (S (S (S Z)))} \]
\[ * ⊢ \text{sorted lte (Cons (S (S (S Z))) Nil)} \]
\[ \text{apply (rule eq)} \]
\[ ⊢ \text{sorted lte (Cons (S (S (S Z))) Nil)} \]
\[ \text{apply (rule cons)} \]
\[ ⊢ \text{lte (S Z) (S (S Z))} \]
\[ * ⊢ \text{sorted lte (Cons (S (S Z))) Nil)} \]
\[ \text{apply (rule lt)} \]
\[ ⊢ \text{lte (S Z) (S (S Z))} \]
\[ * ⊢ \text{sorted lte (Cons (S (S Z))) Nil)} \]
\[ \text{apply (rule eq)} \]
\[ ⊢ \text{sorted lte (Cons (S (S Z))) Nil)} \]
\[ \text{apply (rule uni)} \]
\[ \text{done} \]

The learning curve for using proof assistants in general and Isabelle/HOL in particular is very steep. Nevertheless, once one has gained some fluency, there are great benefits for certain kinds of projects. In a theory of Concurrent ML, with various propositions about semantics and communication, there are many tedious details that must be specified. The automatic checking of propositions and proofs is excellent at finding errors buried in numerous tedious details. Furthermore, since greater complexity follows from greater number of language features, or greater precision of propositions, it is very
useful to start with simple features and propositions to create a minimal viable theory, and then incrementally increase complexity and modify proofs, accordingly. The proof assistant eases the process incremental extension by pinpointing where the proofs and propositions break as features and complexity are added.

The Isabelle/HOL formalization of this work consists of 12 theory files containing definitions, theorems, and proofs, for the syntax, dynamic semantics, static semantics, soundness of semantics, dynamic communication, static communication, soundness of communication, helper definitions, and lemmas. There are roughly 1421 lines of definitions, and 3052 lines of completed proofs.

1.3 Static Analysis of Concurrent ML

A static analysis that describes communication topologies of channels has practical benefits in at least two ways. It can highlight which channels are candidates for optimized implementations of communication; or in a language extension allowing the specification of specialized channels, it can conservatively verify their correct usage. Without a static analysis to check the usage of the special channels, one could inadvertently use a channel intended for just one sender when really the program has multiple senders, thereby violating the intended semantics.

The utility of the static analysis depends on it being precise, sound, and computable. The analysis is precise if it describes information that isn’t invariantly true for all programs. The analysis is sound if the information it describes about a program is the same or less precise than the information described by the dynamic semantics of the program. The analysis is computable if there is an algorithm that, from an input program,
determines values sufficient for the analysis to hold.

Analyses can be described in a variety of ways. An algorithm that take programs as input and produces information about the behavior as output is ideal for automation. A set of inference rules may be more suitable for clarity of meaning and correctness with respect to the dynamic semantics. However, inference rules can be translated into an algorithm. One rather mechanical method essentially involves specifying a reasoner associated with the rules. First, the reasoner generates a comprehensive set of data structures representing constraints from the rules’ premises; then the reasoner solves the constraints.

For a subset of Concurrent ML without event combinators, Reppy and Xiao developed an efficient algorithm that determines for each channel, all possible threads that send and receive on it. The algorithm depends on each operation in the program being labeled with a program step. A sequence of program steps ordered in a valid execution sequence forms a control path. Distinction between threads in a program can be inferred from whether or not their control paths diverge.

Reppy and Xiao’s algorithm proceeds in multiple steps that produce intermediate data structures used for efficient lookup in the subsequent steps. It starts with a control flow analysis that results in multiple mappings. One mapping is from names to abstract values. Another mapping is from channel-bound names to abstract values that are sent on the respective channels. Another is from function-bound names to abstract values that are the result of the respective function applications. It constructs a control flow graph with possible paths for conditional tests and thread spawning determined directly from the syntax used in the program. Relying on information from the mappings to abstract values, it constructs the possible paths of execution via function application and channel communi-
cation. It uses the graph for live variable analysis of channels, which limits the scope for the remaining analysis and increases precisions. Using the spawn and application edges of the control flow graph, the algorithm then performs a data flow analysis to determine a mapping from program steps to all possible control paths leading into the respective program steps. Using the CFA’s mappings to abstract values, the algorithm determines the program steps for sending and receiving synchronizations per channel name. Then it uses the mapping to control paths to determine all control paths that send or receive on each channel, from which it classifies channels as one-to-many, many-to-one, many-to-many, or one-shot.

The information at each program step is derived from control structures in the program, which dictate how information flows between program steps. Some uses of control structures are literally represented in the syntax, such as the sequencing of namings and assignments in the previous examples. Other uses of control structures may be indirectly represented through names. Function application is a control structure that allows a calling piece of code to flow into a function’s body. Functions can be named, which allows multiple pieces of code to all flow into the same section of code. The name adds an additional step to uncover control structures, and determine data flow. Additionally, in languages with higher order functions and recursion, such as those in the Lisp and ML families, it may be impossible to exactly determine all the values that terms resolve to. However, a control flow analysis can reveal a good approximation of the control structures and values that have been obfuscated by higher order functions. Uncovering the control structures depends on resolving terms to values, and resolving terms to values depends on uncovering the control structures. The mutual dependency means that control flow
analysis is a form of static evaluation. In this work, control flow analysis is used for tracking certain kinds of values, like channels and events, in addition to constructing precise data flow analysis.
Synchronization of sending threads and receiving threads requires determining which threads should wait, and which threads should be dispatched. The greater the information needed to determine this scheduling, the higher the performance penalty. A uniprocessor implementation of synchronization can have very little penalty. Since only one thread can make progress at a time, only one thread requests synchronization at a time, meaning the scheduler won’t waste steps checking for threads competing for the same synchronization opportunity, before dispatching. A multiprocessor implementation, on the other hand, must consider that competing threads may exist, so it must perform additional checks. Additionally, there may be overhead in sharing data between processors due to memory hierarchy designs [10].

One way to lower synchronization and communication costs is to use specialized implementations for channels that never have more than one thread ever sending or receiving on them. These specialized implementations would avoid unnecessary checks for competing threads. Concurrent ML does not feature multiple kinds of channels distinguished by their communication topologies, i.e. the number of threads that may end up sending or receiving on the channels. However, channels can be classified into various topologies simply by counting the number of threads per channel during the execution of a program. A many-to-many channel has any number of sending threads and receiving
threads; a one-to-many channel has at most one sending thread and any number of receiving threads; a many-to-one channel has any number of sending threads and at most one receiving thread; a one-to-one channel has one or none of each; a one-shot channel has exactly one sending attempt; a one-sync channel has at most one synchronization.

The following reimplementation of Serv is annotated to indicate the communication topologies derived from its usage. Since there are four threads that make calls to the server, the server’s particular reqCh has four senders. Servers are created with only one thread listening for requests, so the reqCh of this server has just one receiver. So the server’s reqCh is classified as many-to-one. Each application of call creates a distinct new channel replCh for receiving data. The function call receives on the channel once and the server sends on the channel once, so each instance of replCh is one-shot. It could be even more precisely classified as one-sync, since the client function receives on the channel at most once.

```
structure Serv : SERV =
struct

datatype serv = S of (int * int chan) chan

fun make () =
  let
    val reqCh = ManyToOne.channel ()
    fun loop state =
      let
        val (v, replCh) = ManyToOne.recv reqCh
        val () = OneShot.send (replCh, state)
      in
        loop v
      end
    in
      loop 0
    end
    S reqCh
  end
```
fun call (server, v) = 
  let
    val reqCh = server
    val replCh = OneShot.channel ()
    val () = ManyToOne.send (reqCh, (v, replCh))
  in
    OneShot.recv replCh
  end

val server = Serv.make ()

val () = 
  spawn (fn () => Serv.call (server, 35));
  (spawn fn () =>
    Serv.call (server, 12);
    Serv.call (server, 13)
  );
  spawn (fn () => Serv.call (server, 81));
  spawn (fn () => Serv.call (server, 44))

Some hypothetical implementations of specialized and generic Concurrent ML illustrate opportunities for cheaper synchronization. These implementaitons use feasible low-level thread-centric features such as wait and poll. The thread-centric approach allows us to focus on optimizations common to many implementations by decoupling the implementation of communication features from thread scheduling and management. However, a lower level view or scheduler-centric view of synchronization might offer more opportunities for optimization.

In a language with low-level support for concurrency, Concurrent ML could be implemented as a library, which is the case for SML/NJ [3] and MLton [21]. The implementations shown here can be viewed either as a library or as part of a runtime or interpreter.
The benefits of specialization would be much more significant in multiprocessor implementations than in uniprocessor implementations. A uniprocessor implementation could avoid overhead caused by contention to acquire locks, by coupling the implementation of channels with scheduling and only scheduling the sending and receiving operations when no other pending operations have yet to start or have already finished. Reppy’s implementation of Concurrent ML uses SML/NJ’s first class continuations to implement scheduling and communication as one with very low overhead. In contrast, a multiprocessor implementation would allow threads to run on different processors for increased parallelism, therefore it would not be able to mandate when threads attempt synchronization relative to others without losing the parallel advantage. The cost of trying to achieve parallelism is increased overhead due to contention over acquiring synchronization rights.

### 2.1 Many-to-many Synchronization

For many-to-many synchronization, a channel can be in one of three states. Either some threads are trying to send on it, some threads are trying to receive on it, or no threads are trying to send or receive on it. Additionally a channel is composed of a mutex lock, so that sending and receiving operations can yield to each other when updating the channel state. When multiple threads are trying to send on a channel, the channel is associated with a queue consisting of messages to be sent, along with conditions waited
on by sending threads. When multiple threads are trying to receive on a channel, the channel is associated with a queue consisting of initially empty cells that are accessible by receiving threads and conditions waited on by the receiving threads. The channel content holds one of the three potential states and their associated queues and conditions.

The sending operation acquires the channel’s lock to ensure that it updates the channel based on its current state. If the channel is in the receiving state, i.e. there are threads trying to receive from the channel, then the sending operation dequeues an item from the state’s associated queue. The item consists of a condition waited on by a receiving thread and an empty cell that can be accessed by the receiving thread. The sending operation deposits the message in the cell and signals on the receiving state’s condition. Then, if there are no further receiving threads waiting, it updates the channel’s state to inactive; otherwise, it leaves the state in the receiving state. Next, it releases the lock, signals on the receiving state’s condition and returns the unit value. If there are no threads receiving on the channel, the sending operation updates the channel state to the sending state, and enqueues a new condition sendCond and the message. It releases the lock and waits on its condition sendCond. Once a receiving thread signals on its condition, the sending operation returns with the unit value.

The receiving operation acquires the channel’s lock to ensure that it updates the channel based on its current state. If there are threads sending on the channel, the receiving operation dequeues an item from the sending state’s associated queue. The item consists of a condition waited on by a sending thread along with a message. The receiving operation signals on the sending state’s condition. If there are no further sending threads waiting, it updates the channel’s state to inactive; otherwise, it leaves the state in the
sending state. Next, it releases the lock and returns the message from the sending state. If there are no sending threads on the channel, the receiving operation updates the channel state to the receiving state, and enqueues a new condition recvCond and an empty cell. It releases the lock and waits on the its condition recvCond. Once a sending thread signals on its condition, the receiving operation returns with the value deposited in its cell.

structure ManyToManyChan : CHANNEL =
structure

datatype 'a state =
  Send of (condition * 'a) queue
| Recv of (condition * 'a option ref) queue
| Inac

datatype 'a chan =
  Chn of 'a state ref * mutex_lock

fun channel () = Chn (ref Inac, mutexLock ())

fun send (Chn (ctntRef, lock)) m =
  acquire lock;
  (case !ctntRef of
   Recv q =>
     let
       val (recvCond, msgCell) = dequeue q
     in
       msgCell := SOME m;
       if (isEmpty q) then ctntRef := Inac else ();
       release lock;
       signal recvCond
     end
   | Send q =>
     let
       val sendCond = condition ()
     in
       enqueue (q, (sendCond, m));
       release lock;
       wait sendCond
     end
   | Inac =>
     let
val sendCond = condition ()
  in
  ctntRef := Send (queue [(sendCond, m)]); release lock; wait sendCond
end

fun recv (Chn (ctntRef, lock)) = acquire lock;
(case !ctntRef of
  Send q =>
  let
    val (sendCond, m) = dequeue q
  in
    if (isEmpty q) then ctntRef := Inac else ();
    release lock;
    signal sendCond;
    m
  end
  | Recv q =>
    let
      val recvCond = condition ()
      val msgCell = ref NONE
    in
      enqueue (q, (recvCond, msgCell)); release lock;
      wait recvCond;
      valOf (!msgCell)
    end
  | Inac =>
    let
      val recvCond = condition ()
      val msgCell = ref NONE
    in
      ctntRef := Recv (queue [(recvCond, msgCell)]); release lock;
      wait recvCond;
      valOf (!msgCell)
    end
  end
end
2.2 One-to-many Synchronization

Implementation of one-to-many channels, compared to that of many-to-many channels, requires fewer steps to synchronize and can execute more steps outside of critical regions, which reduces contention for locks. A channel is composed of a lock and one of three possible states, as is the case for many-to-many channels. However, the state of a thread trying to send only needs to be associated with one condition and one message, rather than a queue.

The sending operation starts by creating a condition \(\text{sendCond}\), then checks if the channel’s state is inactive and tries to use the compare-and-swap operator to transactionally update the state of the channel to a sending state. If successful, it simply waits on its condition \(\text{sendCond}\). After the receiving thread signals on \(\text{sendCond}\), the sending operation returns the unit value. If the transactional update fails and the channel is in the receiving state, then the sending operation acquires the lock, dequeues an item from the state’s associated queue where the item consists of a receiving condition \(\text{recvCond}\), and a cell for depositing the message to the receiving thread. It deposits the message in the cell. Then, if there are no further items on the queue, the sending operation updates the state to inactive; otherwise, it leaves the state in the receiving state. Next, it releases the lock it, then signals on the receiving condition and returns the unit value.

The lock is acquired after the state is determined to be the receiving state, since the expectation is that the current thread is the only one that tries to update the channel from that state. If the communication classification analysis were incorrect and there were actually multiple threads that could call the sending operation, then there might be data races. Likewise, due to the expectation of a single thread sending on the channel, the
synchronizing operation will never witness the state in the sending state, which would mean another thread is in the process of sending a message.

The receiving operation acquires the lock and checks the state of the channel, just like the receiving operation for many-to-many channels. If the channel is in a state where there is no sending thread waiting, then it updates the state to receiving, behaving the same as the receiving operation of many-to-many channels. If there is already a sending thread waiting, then it updates the state to inactive and releases the lock. Then it signals on the sending state’s condition and returns the message held in the sending state.

```ml
structure OneToManyChan : CHANNEL =
struct
  datatype 'a state =
    Send of condition * 'a
  | Recv of (condition * 'a option ref) queue
  | Inac

  datatype 'a chan =
    Chn of 'a state ref * mutex_lock

  fun channel () = Chn (ref Inac, mutexLock ())

  fun send (Chn (ctntRef, lock)) m =
    let
      val sendCond = condition ()
    in
      case (cas (ctntRef, Inac, Send (sendCond, m))) of
        Inac =>
          (* ctntRef is already set to sending state by cas *)
          wait sendCond
        | Recv q =>
          let
            (* the current thread is the only one that updates from this state *)
          val () = acquire lock
```
val (recvCond, msgCell) = dequeue q

in
  msgCell := SOME m;
  if (isEmpty q) then ctntRef := Inac else ();
  release lock;
  signal (recvCond)
end
| Send _ => raise NeverHappens
end

fun recv (Chn (ctntRef, lock)) =
  acquire lock;
  (case !ctntRef of
    Inac =>
      let
        val recvCond = condition ()
        val msgCell = ref NONE
      in
        ctntRef := Recv (queue [(recvCond, msgCell)]);
        release lock;
        wait recvCond
        valOf (!msgCell)
      end
    | Recv q =>
      let
        val recvCond = condition ()
        val msgCell = ref NONE
      in
        enqueue (q, (recvCond, msgCell));
        release lock; wait recvCond;
        valOf (!msgCell)
      end
    | Send (sendCond, m) =>
      (ctntRef := Inac;
       release lock;
       signal sendCond;
       m)
  )
end
2.3 Many-to-one Synchronization

The implementation of many-to-one channels is very similar to that of one-to-
many channels.

```plaintext
structure ManyToOneChan : CHANNEL =

  struct
    datatype 'a state =
      Send of (condition * 'a) queue
    | Recv of condition * 'a option ref
    | Inac

    datatype 'a chan =
      Chn of 'a state ref * mutex_lock

    fun channel () = Chn (ref Inac, mutexLock ())

    fun send (Chn (cntntRef, lock)) m =
      acquire lock;
      (case !cntntRef of
        Recv (recvCond, msgCell) =>
          (msgCell := SOME m;
           cntntRef := Inac;
           release lock;
           signal recvCond)
      | Send q =>
        let
          val sendCond = condition ()
        in
          enqueue (q, (sendCond, m));
          release lock;
          wait sendCond
        end
      | Inac =>
        let
          val sendCond = condition ()
        in
          cntntRef := Send (queue [(sendCond, m)]);
          release lock;
          wait sendCond
        end
```

25
fun recv (Chn (ctntRef, lock)) = 
  let
    val recvCond = condition ()
    val msgCell = ref NONE
  in
    case cas (ctntRef, Inac, Recv (recvCond, msgCell)) of
      Inac =>
      ( (* ctntRef is already set to receiving state by cas *)
        wait recvCond;
        valOf (!msgCell)
      )
    | Send q =>
      let
        (* the current thread is the only one that updates the state from this state *)
        val () = acquire lock
        val (sendCond, m) = dequeue q
      in
        if (isEmpty q) then ctntRef := Inac else ();
        release lock;
        signal sendCond;
        m
      end
    | Recv _ => raise NeverHappens
  end
end

2.4 One-to-one Synchronization

A one-to-one channel can also be in one of three possible states, but there is no associated lock. Additionally, none of the states is associated with a queue. Instead, the potential states are that of a thread trying to send, with a condition and a message, that of a thread trying to receive with a condition and an empty cell, or the inactive state.
The sending operation creates a condition sendCond and checks if the channel’s state is inactive and tries to use the compare-and-swap operator to transactionally update the state of the channel to a sending state. If successful, it simply waits on its condition sendCond, then returns the unit value. If the transactional update fails and the state is a receiving state, then it deposits the message in the receiving state’s associated cell, updates the channel state to inactive, then signals on the receiving state’s condition and returns the unit value. If the communication analysis for the channel is truly one-to-one, then no other thread will be trying to update the state, so no locks are necessary. Additionally, if the channel is truly one-on-one, the sending operation will never witness a preexisting sending state since it is running on the one and only sending thread.

The receiving operation creates a condition recvCond and an empty cell, then checks if the channel’s state is inactive and tries to use the compare-and-swap operator to transactionally update the state of the channel to the receiving state. If successful, it simply waits on its condition recvCond, then returns the sender’s message in the cell. If the transactional update fails and the state is a sending state, then it updates the channel state to inactive, then signals on the sending state’s condition and returns the message held in the sending state. If the communication analysis for the channel is truly one-to-one, then no other thread will be trying to update the state, so no locks are necessary. Additionally, if the channel is truly one-to-one, the receiving operation will never witness a preexisting receiving state since it is running on the one and only receiving thread.

```
structure OneToOneChan : CHANNEL =
struct
    datatype 'a state =
        Send of condition * 'a
```
datatypé 'a chan = Chn of 'a state ref

fun channel () = Chn (ref Inac)

fun send (Chn ctntRef) m = let
  val sendCond = condition ()
  in
  case cas (ctntRef, Inac, Send (sendCond, m)) of
  Inac => (* ctntRef is already set to sending state by cas *)
    wait sendCond
  | Recv (recvCond, msgCell) =>
    ( (* the current thread is the only one that
        accesses ctntRef for this state
       *)
      msgCell := SOME m;
      ctntRef := Inac;
      signal recvCond )
  | Send _ => raise NeverHappens
  end

fun recv (Chn ctntRef) = let
  val recvCond = condition ()
  val msgCell = ref NONE
  in
  case cas (ctntRef, Inac, Recv (recvCond, msgCell)) of
  Inac =>
    ( (* ctntRef is already set to receiving state by cas *)
      wait recvCond;
      valOf (!msgCell) )
  | Send (sendCond, m) =>
    ( (*
      the current thread is the only one that
      accesses ctntRef for this state
     *)
      ctntRef := Inac;
      signal sendCond )
  | Send _ => raise NeverHappens
  end
2.5 One-shot Synchronization

A one-shot channel consists of the same possible states as a one-to-one channel, but is additionally associated with a mutex lock, to account for the fact that multiple threads may try to receive on the channel, even though only at most one message is ever sent.

The sending operation is like that of one-to-one channels, except that if the state is a receiving state, it simply deposits the message and signals on the receiving state’s condition, without updating the channel’s state to inactive, which would be unnecessary, since no further attempts to send are expected.

The receiving operation creates a condition recvCond and an empty cell, then checks if the channel’s state is inactive and tries to use the compare-and-swap operator to transactionally update the state of the channel to the receiving state. If successful, it simply waits on its condition recvCond, then returns the message deposited in its cell. If the transactional update fails and the state is a sending state, then it acquires the lock, signals on the state’s associated condition and returns the message held in the sending state. It never releases the lock, blocking any additional attempts to receive, which is fine
if there is truly at most one message ever sent on the channel. If the state is a receiving state, then the receiving operation attempts to acquire the lock, but it will never actually acquire it since the thread associated with the receiving state will never release it.

```
structure OneShotChan : CHANNEL =

struct

datatype 'a state =
  Send of condition * 'a
| Recv of condition * 'a option ref
| Inac

datatype 'a chan = Chn of 'a state ref * mutex_lock

fun channel () = Chn (ref Inac, lock ()

fun send (Chn (ctntRef, lock)) m =
  let
    val sendCond = condition ()
  in case
    cas (ctntRef, Inac, Send (sendCond, m)) of
      Inac =>
        (* ctntRef is already set to sending state by cas *)
        wait sendCond
      | Recv (recvCond, msgCell) =>
        (msgCell := SOME m;
         signal recvCond)
      | Send _ => raise NeverHappens
  end

fun recv (Chn (ctntRef, lock)) =
  let
    val recvCond = condition ()
    val msgCell = ref NONE
  in case
    cas (ctntRef, Inac, Recv (recvCond, msgCell)) of
      Inac =>
        (* ctntRef is already set to receiving state by cas *)
        wait recvCond;
        valOf (!msgCell)
```
2.6 One-sync Synchronization

An even more restrictive version of a channel with at most one send could be used if it’s determined that the number of receiving threads is at most one, such as `replCh` in the server example. The one-sync channel is composed of a possibly empty message cell, a condition for a sending thread to wait on, and a condition for a receiving thread to wait on.

The sending operation deposits the message in the cell, signals on the channel’s condition `recvCond`, waits on the condition `sendCond`, and then returns the unit value. The receiving operation waits on `recvCond`, then signals on `sendCond`, then returns the deposited message.

```plaintext
structure OneSyncChan : CHANNEL =
struct
  datatype 'a chan =
    Chn of condition * condition * 'a option ref
end
```
fun channel () = 
  Chn (condition (), condition (), ref NONE)

fun send (Chn (sendCond, recvCond, msgCell)) m =
  (msgCell := SOME m;
   signal recvCond;
   wait sendCond)

fun recv (Chn (sendCond, recvCond, msgCell)) =
  (wait recvCond;
   signal sendCond;
   valOf (!msgCell))
end

2.7 Discussion

The example implementations of generic synchronization and specialized synchronization suggest that cost savings of specialized implementations are significant. For instance, if you know that a channel has at most one sending thread and one receiving thread, then you will lower synchronization costs by using an implementation that is specialized for one-to-one communication. To be certain that the new program with the specialized implementation behaves the same as the original program with the generic implementation, you need to be certain that the specialized program behaves the same, assuming one-to-one or less communication, and that the static classification as one-to-one is sound with respect to the semantics of the program.

Spending your energy to determine the topologies for each unique program and then verifying them for each program would be exhausting. Instead, you would proba-
bly rather have a generic procedure that can compute communication topologies for any program in a language, along with a proof that the procedure is sound with respect to the semantics of the programing language.

This work discusses proofs that a static analysis describing communication topologies is sound with respect to the dynamic semantics. Additionally, it would be important to have proofs that the above specialized implementations are equivalent to the many-to-many implementation under the assumption of particular communication topologies, but such is beyond the scope of this work.
Chapter 3

Mechanized Theory

The definitions and theorems of this work were constructed in the formal language of Isabelle/HOL to enable mechanical verification of the proofs. However, in this presentation, the syntax of the stated definitions and propositions differ from the actual Isabelle/HOL syntax, in order to be more intuitive for those unfamiliar with Isabelle/HOL.

A static analysis for detecting communication topologies in Concurrent ML would be necessary for useful optimizations or checks. The optimization to replace general synchronization with specialized synchronization based on the results of static analysis is one that could have significant performance benefit, as shown empirically by Reppy and Xiao. However, if the static analysis of communication topologies is wrong, then the optimization could produce an incorrect program. What you need is proof that the static analysis is sound with respect to the actual communication topologies that occur when running a program. However, the proof could also be erroneous, and with so many details, it could be extremely difficult for a human to manually check for errors. Isabelle/HOL automatically checks for errors in proofs and specifications and is much more reliable than a human. By using an optimization without a soundness proof, you are trusting that it is sound. If soundness has been proved, then you are trusting that the proof has no errors. If the proof has been verified by Isabelle/HOL, you are trusting that the Isabelle/HOL kernel is sound. The less you need to trust the better.
I extend Reppy and Xiao’s work by developing a mechanically checked specification and proofs for communication topologies in Concurrent ML. The proof checker requires that manipulations are constructed from a small kernel of primitive operations. It’s possible that an intuitive step of reasoning in an informal proof requires multiple tedious steps of precise manipulation in a mechanically checked proof. To avoid the chance of running into these tricky scenarios, I specified the static analysis as inference rules, rather than as an algorithm. In Isabelle/HOL, algorithms are specified as computable functions that must be proved to terminate and be total, either automatically or with additional details supplied by the user. Reppy and Xiao’s algorithm, as with many static analysis algorithms, relies on accumulating values in a growing set with each recursive call until reaching a fixpoint. In Isabelle/HOL it is easy to prove termination for functions that branch on an inductive data type, however it is far from straightforward to prove termination for those that branch on whether or not a fixpoint has been reached. Thus, reformulating the specification as inference rules appears to make formal reasoning more attainable.

Concurrent ML distinguishes itself from other languages for concurrency with its generalized concept of events, event synchronization, and event combinators. Reppy and Xiao’s work produces a static analysis for a subset, which only contained synchronization on sending and receiving events. Originally, I wanted to extend Reppy and Xiao’s analysis to encompass a more generalized notion of events, along with the event combinator for choice. However, I quickly realized that the combination of creating a new specification, in an unfamiliar proof assistant, along with a more complex semantics in the language, would be too much to deal with all at once. Instead, I adhered to Reppy and Xiao’s decision
to construct the specification and proofs for a small subset of Concurrent ML, containing synchronization on just sending and receiving events. Since I constructed these specifications and proofs in Isabelle/HOL, it will be possible to easily extend the semantics and theorems later, by using the proof assistant to pinpoint where the proofs and specifications break upon changes.

The language for the mechanized specification contains features found in untyped lambda calculus with some extensions, such as functions, function application, pairs, first and second selection, left and right cases, case distinction, and unit. Additionally, for concurrency it contains thread spawning, channel creation, sending events, receiving events, and synchronization. These features may be used together by binding the use of one feature to a name, and then sequencing to another term. Additionally, spawning, function application, and case distinction require use of other features.

The dynamic evaluation is represented by a small step relation, and a variant of a CEK machine, in which one pool evaluates to the next pool. A pool consists of many states associated with the paths taken to reach each state. A state consists of a term to be evaluated, an environment for looking up values from names, and a stack of continuations. A valid small step relation always steps from one pool to the pool extended with at least one new state. Terms containing names are not reduced. Instead, an environment can be used to look up the values of names. Names are locally scoped so that they can be reused with a precise meaning, e.g. a function parameter may be bound to a different argument each time it’s called. As such, for any term containing names, its corresponding value is simply the term paired with its own environment.

The initial pool of a running program only contains an empty path associated with
a state containing the program, an empty environment and an empty continuation stack. Full evaluation is represented by the star predicate composed with the small step relation, applied to the initial pool. It states that an initial pool takes zero or more transitive steps to reach another pool.

The static evaluation is represented by a control flow analysis relation. Control flow analysis is a "may" analysis, meaning that the analysis is sound if it holds for all possible evaluations of the dynamic evaluation. It may also hold for evaluations that are impossible to occur in the dynamic evaluation. Instead of associating each term with its own environment, the control flow analysis simply has one static environment for the entire program. If a name takes on different values in different scopes during dynamic evaluation, then the static environment simply considers both to be options.

The soundness theorem of the static evaluation with respect to dynamic evaluation states that if full dynamic evaluation of a program reaches a pool containing some environment, and the static evaluation holds for the program and some static environment, then the static environment is an abstraction of the dynamic environment. That is, if a name maps to some value in the dynamic environment, then, in the static environment, the name maps to a set containing the same value or an abstraction of the value. The static evaluation relates static environments to initial programs, but soundness relates static environments to dynamic environments. Furthermore, the dynamic environment of a state in a pool depends on, the terms, environments, and continuation stacks of states in previous pools. Thus, if the static evaluation holds for a static environment and a program, then there must be a relation that holds for the same static environment and any pool that the program may dynamically evaluate to. Generalizing static evaluation to work for pools,
environments, and stacks, gives a version that can relate the static environment to each component and step of dynamic evaluation. The generalized static evaluation on the initial pool is defined to follow from the static evaluation on the program. The preservation of the static evaluation of a pool across multiple steps of dynamic evaluation is proved by induction on the star relation. The static evaluation on a pool is taken apart into a static evaluation on a dynamic environment, thus relating the dynamic environment with the static environment, which is essentially sound by definition.

The dynamic communication classification consists of five relations to classify channels in a pool, as one-to-many, many-to-one, one-to-one, one-shot, or one-sync. All five classification relations are defined in terms of how any two paths in the pool relate to each other. The one-to-many classification holds if any two paths that send on the channel are ordered. If the sending paths are all ordered or there are no sending paths, then there is simply at most one thread that sends on the channel. Likewise, The many-to-one classification holds if any two paths that receive on the channel are ordered. The one-to-one classification holds if all the receiving paths on the channel are ordered and all the sending paths on the channel are ordered. The one-shot classification holds if there is at most one sending path on a channel, regardless of which paths receive on the channel. The one-sync classification holds if there is at most one sending and one receiving thread on a channel.

The static communication classification relations, as with dynamic communication, are defined in terms of paths that send and receive on a channel of interest. However, these sites of sending and receiving on the channel, are approximations taken from the static environment, whose content is influenced by the rules of static evaluation. The static
flows acceptance relation states that a set of flows, i.e. a graph, accepts a program or term, where each flow is an abstract representation of a small step in the dynamic evaluation. The relation is defined such that whether or not a given finite set of flows holds is decidable. The static paths are formed from the chaining of flows, according to the rules of the static path existence relation. The set of static paths to a sending or receiving synchronization site is bounded by the static flows acceptance relation. Due to cycles in the graph, the set of paths may be infinite. The static one-to-many classification holds if any two static sending paths are uncompetitive, meaning that either they are ordered, like the dynamic classification, or there's no way for the paths to occur in the same run of the program. The many-to-one and one-to-one classification also rely on the notion of noncompetitive paths, rather than simply unordered paths. Likewise, the static one-shot classification holds if any two static sending paths are singular, meaning that either they're equal, like the dynamic classification, or they cannot occur in the same run of the program.

The soundness theorem of the static communication classification relations to their dynamic counterparts states that if a channel is statically classified in some way, then it is dynamically classified the same way. For instance, if the static one-to-many classification holds for a static channel, then the dynamic one-to-many classification holds for all dynamic counterparts of that channel. The static communication classification is a "must" analysis, meaning that if the classification holds statically, then it must hold dynamically. Soundness depends on the soundness and preservation of its constituent parts, including the soundness of static evaluation, the soundness of the uncompetitive relation with respect to the unordered relation, the soundness of the singular relation with respect to equality of paths, and the soundness and preservation of the static path existence re-
lation. As with static evaluation, many proofs of soundness for the static communication classification, rely on bridging the gap between the terms used in dynamic versions and the terms used in the static versions. Generalizations of the static communication classification relations with respect to additional dynamic terms - pools, environments, stacks, and values - are derived to complete these proofs.

This work does not contain formal proofs that the specialized implementations are behaviorally equivalent to a generic implementation, but the example implementations in section 2 should provide good evidence for that.

3.1 Syntax

The syntax used in this formal theory contains a very small subset of Concurrent ML’s features. The features include recursive functions with application, left and right cases with case distinction, pairs with first and second selection, sending and receiving events with synchronization, channel creation, thread spawning, and the unit literal. The syntax is defined in a way to make it possible to relate the dynamic semantics of programs to the syntax of programs. The syntax is defined in administrative normal form (ANF) [6], in which every term is bound to a name. Furthermore, terms only accept names in place of eagerly evaluated inputs.

Restricting the grammar to ANF allows the operational semantics to maintain graph information by associating values with succinct names. Maintaining the values’ ties to the syntax simplifies proofs of soundness, since they must relate dynamic evaluation information to static information based on the syntax.

Additionally, ANF melds nicely with the semantics of control paths, which suc-
cinctly identify the evaluation taken to reach some intermediate result. Instead of relying on additional metalanguage structures to associate atom operations with identifiers, the analysis can simply use the required names of ANF syntax to identify locations in the program.

The ANF syntax is impractical for a programmer to write, yet it is still practical for a language under automated analysis since there is a straightforward procedure to transform more user-friendly syntax into ANF.

```ml
datatype name = Nm string

datatype term =
  Bind name complex term
| Rslt name

and complex =
  Unt
| MkChn
| Atom atom
| Spwn term
| Sync name
| Fst name
| Snd name
| Case name name term name term
| App name name

and atom =
  SendEvt name name
| RecvEvt name
| Pair name name
| Lft name
| Rht name
| Fun name name term
```
3.2 Dynamic Semantics

The dynamic semantics describes how programs evaluate to values. A history of execution is represented by a control path. A control path is a list of steps, where a step is a term’s binding name or resulting name, paired with a mode of control indicating flows by sequencing, spawning, calling, or returning. Channels have no literal representation, but each time a channel is created, it is uniquely identified by the history of the execution up until the step of creation. Atomic terms are not simplified. Instead, atoms are evaluated to closures consisting of the atom syntax, along with an environment that maps its constituent names to their values.

In order to relate the static analyses to the operational semantics, I borrowed Reppy and Xiao’s strategy of stepping between sets of execution paths and their associated terms. The semantics are defined as a CEK machine, rather than a substitution based operational semantics. By avoiding simplification of terms in the operational semantics, it is possible to relate the static evaluations of the static semantics to the evaluations produced by the dynamic semantics, which in turn is relied on to prove soundness of the static semantics.

```ocaml
datatype dynamic_step =
  DSeq name
| DSpwn name
| DCll name
| DRtn name

type dynamic_path = dynamic_step list

datatype chan =
  Chan dynamic_path name

datatype dynamic_value =
  VUnt
```
The evaluation of some complex terms results in sequencing, meaning there is no coordination with other threads, and there is no need to save terms on the continuation stack for later evaluation. These terms are the unit literal, atoms, pairs, and first and second selections. The evaluation depends only on the syntax and an environment for looking up the values of names within the syntax. Additionally, all these terms evaluate to values in a single step.

**predicate** seqEval:
complex -> environment -> dynamic_value -> bool

**where**

* unit: env .
  ⊢ seqEval Unt env VUnt

* atom: a env .
  ⊢ seqEval (Atom a) env (VAtm a env)

* first: env n p n_1 n_2 env_p v .
  env n_p = Some (VAtm (Pair n_1 n_2) env_p),
  env_p n_1 = Some v
  ⊢ seqEval (Fst n_p) env v

* second: env n_p n_1 n_2 env_p v .
  env n_p = Some (VAtm (Pair n_1 n_2) env_p),
  env_p n_2 = Some v
  ⊢ seqEval (Snd n_p) env v

The evaluation of a complex term for application or case distinction results in flows by calling. A calling flow is characterized by the need to save a subterm in the continuation stack for later evaluation. The evaluation depends on the syntax and an environment for looking up the values of names within the syntax. A term is evaluated to
a subterm along with a new environment that will later be used in the evaluation of the subterm. For case distinction, either the left or the right term is called, and the environment is updated with the corresponding name mapped to the value extracted from the left or right pattern. For application, the body of the applied function is called, and the environment is updated with the function’s parameter mapped to the application’s argument. The environment is also updated with the function’s recursive name mapped back to the function.

```
predicate callEval: complex -> env -> term -> env -> bool where

  distincLeft: env n_a n_c env_s v n_l n_r t_l t_r .
  env n_a = Some (VAtm (Lft n_c) env_s),
  env_s n_c = Some v
  ⊢ callEval (Case n_a n_l t_l n_r t_r) env t_l (env(n_l -> v))

  distincRight: env n_a n_c env_s v n_l t_l n_r t_r .
  env n_a = Some (VAtm (Rht n_c) env_s),
  env_s n_c = Some v
  ⊢ callEval (Case n_a n_l t_l n_r t_r) env t_r (env(n_r -> v))

  application: env n_f n_f' n_p t_b env_f n_a v .
  env n_f = Some (VAtm (Fun n_f' n_p t_b) env_f),
  env n_a = Some v
  ⊢ callEval
  (App n_f n_a) env t_b
  (env_f{
    n_f' -> (VAtm (Fun n_f' n_p t_b) env_f),
    n_p -> v
  })
```

The continuation stack maintains a record of terms that should be evaluated once a corresponding called branch of the evaluation has returned. Each continuation in the stack consists of a term, the environment for resolving the term’s names, and an unresolved name, to be resolved when the corresponding branch returns. The initial state of execution consists of a program, an empty environment, and an empty stack of continu-
ations. With each sequential step, the program is reduced to a subterm, and the environment is updated with the term’s name bound to the value of the complex term. Each time a complex term’s inner term is called, the sequenced term is saved as a part of a continuation and pushed onto a stack of continuations. A continuation is popped off the stack when a state’s term is reduced to a result term. A pool of states keeps track of all the states that have been reached through the evaluation of an initial program. Each state is indexed by the dynamic path taken to reach it. A pool’s leaf path indicates a state that has yet to be evaluated. Additionally, the communication between threads is also recorded as a set of correspondences consisting of the path to the sending state, the path to the receiving state, and the channel used for communication.

```plaintext
datatype contin = Ctn name tm env
type stack = contin list
datatype state =
    Stt program env stack
type pool =
    dynamic_path -> state option
predicate leaf: pool -> dynamic_path -> bool where
    intro: pool path stt .
    pool path = Some stt,
    (¬ path’ stt’).
    pool path’ = Some stt’,
    strictPrefix path path’

⇒ leaf pool path
type corresp = dynamic_path * chan * dynamic_path
type communication = corresp set
```

The evaluation of a program may involve evaluation of multiple threads concur-
rently and also communication between threads. Since pools contain multiple states and paths, they can accommodate multiple threads as well. A single evaluation step depends on one pool and evaluates to a new pool based on one or more states in that pool. The initial pool for a program contains just one state indexed by an empty path. The state contains the program, an empty environment, and an empty stack. The pool grows strictly larger with each evaluation step, maintaining a full history. Each step adds new states and paths extended from previous ones, and each step in the path indicates the mode of flow taken to reach the state. Only states indexed by leaf paths are used to evaluate to the next pool.

For the evaluation a leaf path proceeding from a result term, a continuation is popped of the stack. The pool’s new state is formed from the term and environment in the continuation, and the environment is updated with the continuation’s name bound to the result’s value. A sequencing evaluation step of a program picks a leaf state and relies on sequential evaluation of its top complex term. It updates the state’s environment with the value of the complex term and reduces the program to the sequenced term. A calling evaluation step relies on the calling evaluation of a state’s top complex term. The binding name, sequenced term, and environment are pushed onto the stack, and the new state gets its program and environment from the calling evaluation of the complex term. In the case of channel creation, the evaluation updates the state’s environment with the value of the new channel consisting of the path leading to its creation; it leaves the stack unchanged and reduces the program to the sequenced term. In the case of spawning, the evaluation updates the pool with two new paths extending the leaf path. For one, the leaf path is extended with a sequential step whose state has the sequenced term and the environment
updated with binding name bound to the unit value along with the original continuation stack. For the other, the leaf path is extended with a spawning step. Its state has the spawned term, the original environment, and an empty continuation stack.

In the case where two leaf paths in the pool correspond to synchronization on the same channel, and one synchronizes on a sending event and the other synchronizes on a receiving event, the evaluation updates the pool with two new paths and corresponding states. It updates the pool with a new state containing the sending event’s sequenced term, its environment updated with the unit value, and its stack unchanged. It updates the pool with a new state containing the receive event’s sequenced embedded term, the environment updated with the sent value, and its stack unchanged. Additionally, the communication is updated with the sending and receiving paths, and the channel used for communication.

**predicate** dynamicEval:

\[\text{pool} \to \text{communication} \to \text{pool} \to \text{communication} \to \text{bool}\]

where

\[\text{return: pool \ path \ n \ env \ n_k \ t_k \ env_k \ stack' \ v \ comm .}\]

\[\text{leaf pool path,}\]

\[\text{pool path} = \text{Some \ (Stt \ (Rslt n) \ env \ ((Ctn n_k \ t_k \ env_k) \ # \ stack')),}\]

\[\text{env n} = \text{Some v}\]

\[\vdash \text{dynamicEval}\]

\[\text{pool \ comm}\]

\[(\text{pool}(\text{path @ } \text{[DRtn n]} \to (\text{Stt t_k env_j(n_k \to v) stack'}))\]

\[\text{comm}\]

\[\star \text{seq: pool \ path \ n \ c \ t' \ env \ stack \ v .}\]

\[\text{leaf pool path,}\]

\[\text{pool path} = \text{Some \ (Stt \ (Bind n \ c \ t') \ env \ stack)},\]

\[\text{seqEval c env v}\]

\[\vdash \text{dynamicEval}\]

\[\text{pool \ comm}\]
(pool(
    path @ [DSeq n] -> (Stt t' (env(n -> v)) stack)
))

* call: pool path n c t' env stack t_c env_c comm .
  leaf pool path,
  pool path = Some (Stt (Bind n c t') env stack),
  callEval c env t_c env_c
  ⊢ dynamicEval
    pool comm
    (pool(
        path @ [DCll n] -> (Stt t_c env_c ((Ctn n t' env) # stack))
    ))
    comm

* makeChan: pool path n t' env stack .
  leaf pool path,
  pool path = Some (Stt (Bind n MkChn t') env stack)
  ⊢ dynamicEval
    pool comm
    (pool(
        path @ [DSeq n] ->
        (Stt t' (env(n -> (VChn (Chan path n))))) stack)
    ))
    comm

* spawn: pool path n t_c t' env stack comm .
  leaf pool path,
  pool path = Some (Stt (Bind n (Spwn t_c) t') env stack)
  ⊢ dynamicEval
    pool comm
    (pool(
        path @ [DSeq n] -> (Stt t' (env(n -> VUnt)) stack),
        path @ [DSpwn n] -> (Stt t_c env [])
    ))
    comm

* sync: pool path_s n_s n_sc t_s env_s stack_s n_sc n_m
  env_sc path_r n_r n_re t_r env_r stack_r n_re env_re chan comm .
  leaf pool path_s,
  pool path_s = Some
    (Stt (Bind n_s (Sync n_sc) t_s) env_s stack_s),
  env_s n_sc = Some
    (VAtm (SendEvt n_sc n_m) env_sc),
leaf pool $path_r$,
pool $path_r$ = Some
  (Stt (Bind $n_r$ (Sync $n_{rc}$) $t_r$) env$_r$ stack$_r$),
env$_r$ $n_{rc}$ = Some
  (VAtm (RecvEvt $n_{rc}$) env$_{rc}$),
env$_{rc}$ $n_{sc}$ = Some (VChn chan),
env$_{rc}$ $n_{re}$ = Some (VChn chan),
env$_{sc}$ $n_{rm}$ = Some $v_m$ ⊢ dynamicEval
pool comm
  (pool(
    path$_s$ @ [DSeq $n_s$] -> (Stt $t_s$ (env$_s$($n_s$ -> VUnt)) stack$_s$),
    path$_r$ @ [DSeq $n_r$] -> (Stt $t_r$ (env$_r$($n_r$ -> $v_m$)) stack$_r$)
  ))
(comm $\cup$ {{path$_s$, chan, path$_r$}})

3.3 Dynamic Communication

Whether or not two threads compete to synchronize on a channel can be determined by looking at the paths of the pool. If two paths are ordered, that is, one is the prefix of the other or vice versa, then the shorter path synchronizes before the longer path. Two ordered paths either indicates that the two paths occur in the same thread, or that the shorter path precedes the spawning of the thread associated with the longer path. Two paths may indicate two threads that compete to synchronize only if they are unordered.

The dynamic one-to-many classification means that there is no competition on the sending end of a channel; any two paths that synchronize to send on a channel are ordered. The dynamic many-to-one classification means that there is no competition on the receiving end of a channel; any two paths that synchronize to receive on a channel are ordered. The dynamic one-to-one classification means that there is no competition on either the receiving or the sending ends of a channel; any two paths that synchronize on a channel are necessarily ordered for either end of the channel. The dynamic one-
shot classification means there is only one dynamic path that synchronizes and sends on a given channel. The dynamic one-sync classification means there is only one dynamic path that sends on a given channel and at most one thread that receives on it.

**Predicate** `isSendPath: pool -> chan -> dynamic_path -> bool` where

- **Intro**: `pool path n n e t' env stack n_sc n_m env_e chan .`
- `pool path = Some (Stt (Bind n (Sync n_e t') env stack), env n_e = Some (VAtm (SendEvt n_sc n_m env_e), env_e n_sc = Some (VChn chan)) ⊢ isSendPath pool chan path`

**Predicate** `isRecvPath: pool -> chan -> dynamic_path -> bool` where

- **Intro**: `pool path n n e t' env stack n_sc n_m env_e chan .`
- `pool path = Some (Stt (Bind n (Sync n_e t') env stack), env n_e = Some (VAtm (RecvEvt n_sc n_m env_e), env_e n_sc = Some (VChn chan)) ⊢ isRecvPath pool chan path`

**Predicate** `forEveryTwo: ('a -> bool) -> ('a -> 'a -> bool) -> bool` where

- **Intro**: `p r . ∀ path1 path2 . p path1 ∧ p path2 → r path1 path2 ⊢ forEveryTwo p r`

**Predicate** `ordered: 'a list -> 'a list -> bool` where

- **First**: `path1 path2 . prefix path1 path2 ⊢ ordered path1 path2`
- **Second**: `path2 path1 . prefix path2 path1 ⊢ ordered path1 path2`

**Predicate** `oneToMany: tm -> chan -> bool` where

- **Intro**: `t_0 chan pool comm . star dynamicEval [[] -> (Stt t_0 [->] [])] {} pool comm, forEveryTwo (isSendPath pool chan) ordered ⊢ oneToMany t_0 chan`

**Predicate** `manyToOne: tm -> chan -> bool` where

- **Intro**: `t_0 chan pool comm . star dynamicEval [[] -> (Stt t_0 [->] [])] {} pool comm, forEveryTwo (isRecvPath pool chan) ordered ⊢ manyToOne t_0 chan`

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predicate oneToOne: tm -> chan -> bool where
intro: t₀ chan pool comm .
    star dynamicEval [[] -> (Stt t₀ [-] [])] {} pool comm ,
    forEveryTwo (isSendPath pool chan) ordered ,
    forEveryTwo (isRecvPath pool chan) ordered
⊢ oneToOne t₀ chan

predicate oneShot: tm -> chan -> bool where
intro: t₀ chan pool comm .
    star dynamicEval [[] -> (Stt t₀ [-] [])] {} pool comm ,
    forEveryTwo (isSendPath pool chan) (op =)
⊢ oneShot t₀ chan

predicate oneSync: tm -> chan -> bool where
intro: t₀ chan pool comm .
    star dynamicEval [[] -> (Stt t₀ [-] [])] {} pool comm ,
    forEveryTwo (isSendPath pool chan) (op =),
    forEveryTwo (isRecvPath pool chan) ordered
⊢ oneSync t₀ chan

3.4 Static Semantics

The static semantics describes an estimation of the intermediate static values and terms that might result from running a program. Although the estimations are imprecise with respect to the dynamic semantics, they are certainly accurate, which is confirmed by the mechanically checked proofs of soundness. The static semantics enable deduction of static information about channels and events, which is crucial for statically deducing information about synchronization on channels and communication classification. The static values consist of the static unit value, static channels, and static atom values. The static unit value is no less precise than the dynamic unit value, but static channels and static atom values are imprecise versions of their dynamic counterparts. A static channel is identified only by the name it binds to at creation time, rather than the full path that leads up to its creation. A static atom value is simply an atomic term without an environ-
ment for looking up its constituent names. A static environment contains the intermediate evaluation results by associating names to multiple potential static values. Thus, in addition to some static values being imprecise, the results of evaluation may have even lower precision by containing multiple potential static values. In order to find the return value of a term, it is useful to fetch the name embedded within the term’s eventual result term, which is formally defined by `resultName`.

```haskell
datatype static_value =
    SUn | SChn name
| SAtm atom

type static_value_map =
    name -> static_value set

fun resultName: term -> name where
    n .
    ⊢ resultName (Rslt n) = n
* n c t’.
    ⊢ resultName (Bind n c t’) = (resultName t)
```

The static evaluation is a control flow analysis (CFA) that describes a relation between a program term and two maps to static values. The first map is the static environment, which contains names mapped to the evaluations of terms they bind to. The second map is the static communication, containing names of channels mapped to values that might be sent over the channels identified by those names.

The definition of static evaluation is syntax-directed, meaning the proof of a static evaluation is defined to be structurally inductive following the self-similar structure of the syntax. It should be possible to decide if a static evaluation holds by unraveling the program term into smaller and smaller terms, until reaching a term without any smaller terms. Additionally, for any given program, there should be instances of static environ-
ments, such that the static evaluation holds, in which case, there is likely an algorithm to compute the static environments from a program, by following the basic structure of the definitional proof of static evaluation. This certainly appears likely, but it has not been formally proven in this work.

The static evaluation relation is defined in a single definition. The definition is fairly uniform and mimics the structure of the syntax. The static evaluation for each syntactic form is very similar. For instance, if a term has a sequenced term, the static evaluation of the term is defined by the static evaluation of its sequenced term, whether the original term is a spawning term, a function term, or a case distinction term. In contrast, in the definition of dynamic evaluation, the evaluation of certain syntactic forms is more similar to some forms than others. Case distinction terms are evaluated similarly to application terms. They both require saving some terms on the continuation stack, while evaluating other terms. Function terms are dynamically evaluated similarly to other atomic terms. The static evaluation has only a single global environment for looking up values of names in the whole program, whereas the dynamic evaluation associates local environments with different terms in the program, allowing the same names to resolve to different values depending on the context. Therefore, the static evaluation is less precise.

\[
\text{predicate} \ \text{staticEval}: \\
\text{static_value_map} \rightarrow \text{static_value_map} \rightarrow \text{term} \rightarrow \text{bool} \\
\text{where} \\
\text{result: staticEnv staticComm n} . \\
\vdash \text{staticEval staticEnv staticComm (Rslt n)} \\
\ast \ \text{unit: staticEnv n staticComm t’} . \\
\text{SUnt} \in \text{staticEnv n}, \\
\text{staticEval staticEnv staticComm t’} \\
\vdash \text{staticEval staticEnv staticComm (Bind n Unt t’)}
\]
* makeChan: n staticEnv staticComm t’. 
  (SChn n) ∈ staticEnv n,
  staticEval staticEnv staticComm t’
  ⊢ staticEval staticEnv staticComm (Bind n MkChn t’)

* sendEvt: n_c n_m staticEnv n staticComm t’. 
  (SAtm (SendEvt n_c n_m)) ∈ staticEnv n,
  staticEval staticEnv staticComm t’
  ⊢ staticEval staticEnv staticComm (Bind n (Atom (SendEvt n_c n_m)) t’)

* recvEvt: n_c staticEnv n staticComm t’. 
  (SAtm (RecvEvt n_c)) ∈ staticEnv n,
  staticEval staticEnv staticComm t’
  ⊢ staticEval staticEnv staticComm (Bind n (Atom (RecvEvt n_c)) t’)

* pair: n_1 n_2 staticEnv n staticComm t’. 
  (SAtm (Pair n_1 n_2)) ∈ staticEnv n,
  staticEval staticEnv staticComm t’
  ⊢ staticEval staticEnv staticComm (Bind n (Atom (Pair n_1 n_2)) t’)

* left: n_a staticEnv n staticComm t’. 
  (SAtm (Lft n_a)) ∈ staticEnv n,
  staticEval staticEnv staticComm t’
  ⊢ staticEval staticEnv staticComm (Bind n (Atom (Lft n_a)) t’)

* right: n_a staticEnv n staticComm t’. 
  (SAtm (Rht n_a)) ∈ staticEnv n,
  staticEval staticEnv staticComm t
  ⊢ staticEval staticEnv staticComm (Bind n (Atom (Rht n_a)) t’)

* function: n_f n_1 t_b staticEnv staticComm n t’. 
  (SAtm (Fun n_f n_1 t_b)) ∈ staticEnv n_f,
  staticEval staticEnv staticComm t_b,
  (SAtm (Fun n_f n_1 t_b)) ∈ staticEnv n,
  staticEval staticEnv staticComm t’
  ⊢ staticEval staticEnv staticComm (Bind n (Atom (Fun n_f n_1 t_b)) t’)

* spawn: n_f n_1 t_b staticEnv staticComm n t’. 
  SUnt ∈ staticEnv n,
  staticEval staticEnv staticComm t_c,
  staticEval staticEnv staticComm t’
  ⊢ staticEval staticEnv staticComm (Bind n (Spwn t_c) t’)

* sync: staticEnv n_c n staticComm t’. 
  ∀ n_k n_m n_c .

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(SAtm (SendEvt n_{ac} n_{m})) \in \text{staticEnv} n_{c}
\rightarrow \text{SChn} n_{c} \in \text{staticEnv} n_{ac}
\rightarrow \text{SUn} \in \text{staticEnv} n \land \text{staticEnv} n_{m} \subseteq \text{staticComm} n_{c},
\forall n_{rc} n_{c} .
(SAtm (RecvEvt n_{rc})) \in \text{staticEnv} n_{c}
\rightarrow \text{SChn} n_{c} \in \text{staticEnv} n_{rc}
\rightarrow \text{staticComm} n_{c} \subseteq \text{staticEnv} n,
\text{staticEval} \text{staticEnv} \text{staticComm} t'
\vdash \text{staticEval} \text{staticEnv} \text{staticComm} (\text{Bind} n (\text{Sync} n_{c}) t')

* first: \text{staticEnv} n_{f} n \text{staticComm} t' .
\forall n_{1} n_{2} .
(SAtm (Pair n_{1} n_{2})) \in \text{staticEnv} n_{f}
\rightarrow \text{staticEnv} n_{1} \subseteq \text{staticEnv} n,
\text{staticEval} \text{staticEnv} \text{staticComm} t'
\vdash \text{staticEval} \text{staticEnv} \text{staticComm} (\text{Bind} n (\text{Fst} n_{1}) t')

* second: \text{staticEnv} n_{f} n \text{staticComm} t' .
\forall n_{1} n_{2} .
(SAtm (Pair n_{1} n_{2})) \in \text{staticEnv} n_{f}
\rightarrow \text{staticEnv} n_{2} \subseteq \text{staticEnv} n,
\text{staticEval} \text{staticEnv} \text{staticComm} t'
\vdash \text{staticEval} \text{staticEnv} \text{staticComm} (\text{Bind} n (\text{Snd} n_{1}) t')

* distinction: \text{staticEnv} n_{s} n_{t} n_{l} n \text{staticComm} n_{r}, t', t' .
\forall n_{c} .
(SAtm (Lft n_{c})) \in \text{staticEnv} n_{s}
\rightarrow \text{staticEnv} n_{c} \subseteq \text{staticEnv} n_{l},
\text{staticEnv} (\text{resultName} t_{l}) \subseteq \text{staticEnv} n,
\text{staticEval} \text{staticEnv} \text{staticComm} t_{l},
\forall n_{c} .
(SAtm (Rht n_{c})) \in \text{staticEnv} n_{s}
\rightarrow \text{staticEnv} n_{c} \subseteq \text{staticEnv} n_{r},
\text{staticEnv} (\text{resultName} t_{r}) \subseteq \text{staticEnv} n,
\text{staticEval} \text{staticEnv} \text{staticComm} t_{r},
\text{staticEval} \text{staticEnv} \text{staticComm} t'
\vdash \text{staticEval} \text{staticEnv} \text{staticComm} (\text{Bind} n (\text{Case} n_{s} n_{l} t_{l} n_{r} t_{r}) t')

* application: \text{staticEnv} n_{f} n_{a} n \text{staticComm} t' .
\forall n_{f}' n_{t} t_{b} .
(SAtm (Fun n_{f}' n_{a} t_{b})) \in \text{staticEnv} n_{f}
\rightarrow \text{staticEnv} n_{a} \subseteq \text{staticEnv} n_{t},
\text{staticEnv} (\text{resultName} t_{b}) \subseteq \text{staticEnv} n,
\text{staticEval} \text{staticEnv} \text{staticComm} t'
\vdash \text{staticEval} \text{staticEnv} \text{staticComm} (\text{Bind} n (\text{App} n_{f} n_{a}) t')

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It is straightforward to follow the rules of static evaluation in order to build up functions mapping names to static values for the static environment and the static communication. Recasting the example server implementation into the ANF syntax demonstrates this informal procedure. The unit value, and left and right case constructors are used to represent natural numbers.

```ml
bind u1 = unit
bind r1 = right u1
bind l1 = left r1
bind l2 = left l1

bind mkSrv = fun _ x2 =>
  (bind k1 = mkChn
   bind srv = fun srv' x3 =>
     (bind e1 = recvEvt k1
      bind p1 = sync e1
      bind v1 = fst p1
      bind k2 = snd p1
      bind e2 = sendEvt k2 x3
      bind z5 = sync e2
      bind z6 = app srv' v1
      rslt z6)
   bind z7 = spawn
     (bind z8 = app srv r1
      rslt z8)
   rslt k1)

bind rqst = fun _ x4 =>
  (bind k3 = fst x4
   bind v2 = snd x4
   bind k4 = mkChn
   bind p2 = pair v2 k4
   bind e3 = sendEvt k3 p2
   bind z9 = sync e3
   bind e4 = recvEvt k4)
```
bind v3 = sync e4
    rslt v3
)

bind srvr = mksr u1
bind z10 = spawn
  ( bind p3 = pair srvr l1
      bind z11 = app rqst p3
      rslt z11
    )
bind p4 = pair srvr l2
bind z12 = app rqst p4
rslt z12

Let's see how an informal procedure can produce the static environments by following the structure of the definitional proof structure of static evaluation. We start at the top of the program and pick the rule from the definition of static evaluation that might hold true for the current syntactic form. Then we choose the smallest environment that satifies that rule's conditions. In the server implementation, the program starts with bind u1 = unt in ..., which only unifies with the rule concluding with staticEval staticEnv staticComm (Bind n Unt ...), with n = (Nm "u1"). The conditions for that rule require SUnt ∈ staticEnv (Nm "u1"), staticEval staticEnv staticComm .... We choose the smallest static environment, for which SUnt ∈ staticEnv (Nm "u1") holds, and that happens to be \( \lambda n \cdot \text{if } n = (Nm "u1") \text{ then } \{SUnt\} \text{ else } \{\} \). Since there's no condition that directly states what's required of the static communication, we can simply choose an empty environment to start with. The second condition is static evaluation on a smaller term, which indicates that we should repeat this procedure again for the remainder of the program, incrementally adding more static values for each binding name in the program. We continually repeat this procedure from the top of the program until there's nothing
more we can add to the static environments. The rule for synchronization is the only rule in which there are conditions on the static communication environment. So we will only add to the static communication environment when we encounter synchronization terms.

The following static environments result from following this informal procedure on the example ANF server implementation. To make the presentation clear, the syntactic sugar 
\((r1 \rightarrow \{\text{rht } u1\}, \ldots)\) is used to mean \(\lambda n. \text{if } n = (\text{Nm } "r1") \text{ then } \{\text{SAtm } (\text{Rht } (\text{Nm } "u1"))\} \text{ else } \ldots \text{ else } \{}\). The representation of static values closely resembles the concrete syntax for complex terms.

```plaintext
val serverStaticEnv: name -> static_value set = 
{
  u1 -> \{unt\},
  r1 -> \{rht u1\},
  l1 -> \{lft r1\},
  l2 -> \{lft l1\},
  mksr -> \{fun _ x2 => \ldots\},
  x2 -> \{unt\},
  k1 -> \{chn k1\},
  srv -> \{fun srv' x3 => \ldots\},
  srv' -> \{fun srv' x3 => \ldots\},
  x3 -> \{rht u1, lft r1, lft l1\},
  e1 -> \{recvEvt k1\},
  p1 -> \{pair v2 k4\},
  v1 -> \{lft r1, lft l1\},
  k2 -> \{chn k4\},
  e2 -> \{sendEvt k2 x3\},
  z5 -> \{unt\},
  z7 -> \{unt\},
  u5 -> \{unt\},
  rqst -> \{fun _ x4 => \ldots\},
  x4 -> \{pair srrv l1, pair srrv l2\},
  k3 -> \{chn k1\},
  v2 -> \{lft r1, lft l1\},
  k4 -> \{chn k4\},
  p2 -> \{pair v2 k4\},
  e3 -> \{sendEvt k3 p2\},
  z9 -> \{unt\},
  e4 -> \{recvEvt k4\},
  v3 -> \{rht u1, lft r1, lft r2\},
```
The static reachability describes terms that might be reachable from larger terms, during dynamic evaluation. A sound approximation for dynamically reachable terms are terms that are transitively embedded within larger terms. A term is statically reachable from itself, and an initial term can statically reach any term that its embedded terms can statically reach.

**predicate** staticReachable: term -> term -> bool where
- refl: t .
- `⊢ staticReachable t t`
- spawn: t, t z n t' .
  - `staticReachable t c t z`
- `⊢ staticReachable (Bind n (Spwn t c) t') t z`
- distinctLeft: t z n n z n l f t, t' .
  - `staticReachable t c t z`
- `⊢ staticReachable (Bind n (Case n (s n l t l n r t r) t') t') t z`
- distinctRight: t z n n z n l f t, t' .
  - `staticReachable t f t z`
- `⊢ staticReachable (Bind n (Case n (s n l t l n r t r) t') t') t z`
- function: t h t z n n f n l f t, t' .
  - `staticReachable t h t z`
- `⊢ staticReachable (Bind n (Atom (Fun n f n l t h)) t') t z`
- seq: t' t z n c .
  - `staticReachable t' t z`
- `⊢ staticReachable (Bind n c t') t z`
3.5 Static Communication

To describe communication statically, it is helpful to identify each term with a short description. The term identifier of a binding term is the binding name, and indication of its use in a binding. The term identifier of a result term is the embedded name, and indication of its use in a result.

```
datatype tm_id =
  IdBind name
| IdRslt name

fun termId: term -> tm_id where
  n c t'.
  ⊢ termId (Bind n c t') = IdBind n
  * n .
  ⊢ termId (Rslt n) = IdRslt n

type tm_id_map = tm_id -> name set
```

The static communication describes a sound approximation of the static paths that communicate on static channels. The static sending identifier classification means that a term identifier might represent a synchronization to send on a given static channel. The static receiving identifier classification means that a term identifier might represent a synchronization to receive on a given static channel.

```
predicate staticSendId:
  static_value_map -> term -> name -> tm_id -> bool
  where
  intro: t₀ n n_e t' n_sc n_m staticEnv n_c .
  staticReachable t₀ (Bind n (Sync n_e) t'),
  (SAtm (SendEvt n_sc n_m)) ⊆ staticEnv n_c,
  (SChn n_e) ∈ staticEnv n_sc
  ⊢ staticSendId staticEnv t₀ n_c (IdBind n)

predicate staticRecvId:
  static_value_map -> term -> name -> tm_id -> bool
  where
  intro: t₀ n n_e t' n_rc staticEnv n_c .
```
In the server implementation, the static channel identified by the name \( k_1 \) is waited on by the server. It has one receiving identifier in the server function at identifier \( \texttt{bind} \ p_1 \) and a sending identifier in the request function at identifier \( \texttt{bind} \ z_9 \). The channel identified by the name \( k_4 \) is sent with a client’s request for the server to reply on. It has a receiving identifier in the request function at \( \texttt{bind} \ v_3 \) and a sending identifier in the server function at \( \texttt{bind} \ z_5 \).

Reppy and Xiao’s work relies on detecting the liveness of channels in order to gain higher precision in the static classification of communication. Since formal proofs are inherently complicated with numerous details, it was easier to first formally prove soundness for a version without the added complication of considering liveness of channels.

The definitions are purposely structured to allow adding live channel analysis to the definition fairly easily with just a few alterations. Section 4 expands on these alterations and outlines a strategy that is likely to result in formal proofs of soundness.

For the lower precision version without the liveness of channels, there are four modes - sequencing, calling, spawning, and returning - indicating how the term identifier flows to the identifier of the next term. A flow is a triplet of a term identifier, a mode of flow, and a term identifier for the next term. A static step is just a term identifier along with the mode it uses to flow to the next term. A static path is a list of static steps.

```plaintext
datatype mode =
```

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The evaluation of a term results in the flow to new terms, via sequencing, calling, returning, or spawning. The static flows acceptance describes a set of all the flows that could be traversed during a program’s evaluation. It depends on the static environment. For a result term, there are no demands on the flow graph. For all bind terms, except those binding to case distinction and function application, the sequential flow from the top term to the sequenced term accept the term, and the accepting flows are also the accepting flows for the sequenced term. For binding to a function, the accepting flows are also accepting flows for the body of the function. For binding to thread spawning, the spawning flow from the top term to the spawned term might be traversed, and the accepting flows are also accepting flows for the spawned term.

In the case of case distinction, the calling flow from the case distinction term to its left case’s term accept the case distinction term, and the calling flow from a term to the right case’s term accept the case distinction term. The returning flow from the result of the left case’s term to the sequenced term accept the result term, and the returning flow from the result of the right case’s term to the sequenced term also accept the result term. Additionally, the accepting flows for a term are also accepting flows for its left case’s term,
right case’s term, and the sequenced term.

In the case of application, if the applied name is actually bound to a function, then a calling flow from the application term to the function’s body accept the application term, and the returning flow from the result of the function to the sequenced term accept the application term. Additionally, the accepting flows for the application term are also accepting flows for the sequenced term.

**predicate** staticFlowsAccept:
{static_value_map -> graph -> term -> bool

**where**

result: staticEnv graph n .
\[ \vdash \text{staticFlowsAccept staticEnv graph (Rslt n)} \]

* unit: n t’ graph staticEnv .
  (IdBind n, MSeq, termId t’) \(\in\) graph,
  staticFlowsAccept staticEnv graph t’
\[ \vdash \text{staticFlowsAccept staticEnv graph (Bind n Unt t’)} \]

* makeChan: n t’ graph staticEnv .
  (IdBind n, MSeq, termId t’) \(\in\) graph,
  staticFlowsAccept staticEnv graph t’
\[ \vdash \text{staticFlowsAccept staticEnv graph (Bind n MkChn t’)} \]

* sendEvt: n t’ graph staticEnv n c n m .
  (IdBind n, MSeq, termId t’) \(\in\) graph,
  staticFlowsAccept staticEnv graph t’
\[ \vdash \text{staticFlowsAccept staticEnv graph (Bind n (Atom (SendEvt n c n m)) t’)} \]

* recvEvt: n t’ graph staticEnv n c .
  (IdBind n, MSeq, termId t’) \(\in\) graph,
  staticFlowsAccept staticEnv graph t’
\[ \vdash \text{staticFlowsAccept staticEnv graph (Bind n (Atom (RecvEvt n c)) t’)} \]

* pair: n t’ graph staticEnv n_1 n_2 .
  (IdBind n , MSeq, termId t’) \(\in\) graph,
  staticFlowsAccept staticEnv graph t’
\[ \vdash \text{staticFlowsAccept staticEnv graph (Bind n (Atom (Pair n_1 n_2)) t’)} \]
* left: $n \ t' \ \text{graph staticEnv } n_s$.
  
  \[(IdBind n, MSeq, termId t') \in \text{graph},
  \text{staticFlowsAccept staticEnv graph } t'\]
  \[\vdash \text{staticFlowsAccept staticEnv graph } (\text{Bind } n \ (\text{Atom } (\text{Lft } n_s)) \ t')\]

* right: $n \ t' \ \text{graph staticEnv } n_s$.
  
  \[(IdBind n, MSeq, termId t') \in \text{graph},
  \text{staticFlowsAccept staticEnv graph } t'\]
  \[\vdash \text{staticFlowsAccept staticEnv graph } (\text{Bind } n \ (\text{Atom } (\text{Rht } n_s)) \ t')\]

* function: $n \ t' \ \text{graph staticEnv } t_b \ n_f \ n_t$.
  
  \[(IdBind n, MSeq, termId t') \in \text{graph},
  \text{staticFlowsAccept staticEnv graph } t',
  \text{staticFlowsAccept staticEnv graph } t_b\]
  \[\vdash \text{staticFlowsAccept staticEnv graph } (\text{Bind } n \ (\text{Fun } n_f \ n_t \ t_b) \ t')\]

* spawn: $n \ t' \ t_c \ \text{graph staticEnv }$. 
  
  \{\(\begin{array}{l}
  (IdBind n, MSeq, termId t')
  
  
  (IdBind n, MSPwn, termId t_c)
  \end{array}\)\}
  \[\subseteq \text{graph},
  \text{staticFlowsAccept staticEnv graph } t_c,
  \text{staticFlowsAccept staticEnv graph } t'\]
  \[\vdash \text{staticFlowsAccept staticEnv graph } (\text{Bind } n \ (\text{Spwn } t_c) \ t')\]

* sync: $n \ t' \ \text{graph staticEnv } n_{sc}$.
  
  \[(IdBind n, MSeq, termId t') \in \text{graph},
  \text{staticFlowsAccept staticEnv graph } t'\]
  \[\vdash \text{staticFlowsAccept staticEnv graph } (\text{Bind } n \ (\text{Sync } n_{sc}) \ t')\]

* first: $n \ t' \ \text{graph staticEnv } n_f$.
  
  \[(IdBind n, MSeq, termId t') \in \text{graph},
  \text{staticFlowsAccept staticEnv graph } t',\]
  \[\vdash \text{staticFlowsAccept staticEnv graph } (\text{Bind } n \ (\text{Fst } n_f) \ t')\]

* second: $n \ t' \ \text{graph staticEnv } n_t$.
  
  \[(IdBind n, MSeq, termId t') \in \text{graph},
  \text{staticFlowsAccept staticEnv graph } t',\]
  \[\vdash \text{staticFlowsAccept staticEnv graph } (\text{Bind } n \ (\text{Snd } n_t) \ t')\]

* distinction: $n \ t_l \ t_r \ t' \ \text{graph staticEnv } n_s$.
  
  \{\(\begin{array}{l}
  (IdBind n, MCll, termId t_l)
  
  (IdBind n, MCll, termId t_r)
  \end{array}\)\}
  \[\subseteq \text{graph},
  \text{staticFlowsAccept staticEnv graph } t_l,
  \text{staticFlowsAccept staticEnv graph } t_r\]
  \[\vdash \text{staticFlowsAccept staticEnv graph } (\text{Bind } n \ (\text{Distinguish } n_s) \ t')\]

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\[
\begin{align*}
(\text{IdRslt (resultName } t_l), & \text{ MRtn, termId } t'), \\
(\text{IdRslt (resultName } t_r), & \text{ MRtn, termId } t')
\end{align*}
\] \subseteq \text{graph},
\text{staticFlowsAccept staticEnv graph } t_l,
\text{staticFlowsAccept staticEnv graph } t_r,
\text{staticFlowsAccept staticEnv graph } t' \\
\vdash \text{staticFlowsAccept staticEnv graph } (\text{Bind } n \ (\text{Case } n \ n_l \ n_r \ t_r \ t_r) \ t')
\]

\[
* \text{ application: } n \ t' \ \text{graph staticEnv } n_f \ n_a .
\forall n_f', n_b .
\ (\text{SAtm (Fun } n_f' \ n_t \ n_a)) \in \text{staticEnv } n_f
\rightarrow
\{ \\
(\text{IdBind } n, \text{ MCll, termId } t_b), \\
(\text{IdRslt (resultName } t_b), \text{ MRtn, termId } t')
\} \subseteq \text{graph},
\text{staticFlowsAccept staticEnv graph } t' \\
\vdash \text{staticFlowsAccept staticEnv graph } (\text{Bind } n \ (\text{App } n_f \ n_a) \ t')
\]

The smallest graph of flows that accepts a program is finite. Additionally, the static acceptance relation is syntax-directed, which offers guidance towards computing the graph from a program. Thus, to statically determine communication classifications, it should be possible to compute the two shortest paths that send or receive on the same channel. The server implementation represented as a control flow graph illustrates how static flows acceptance can interpret a graph from a program.
The static path traceability means that a static path with a given starting term identifier, and ending condition, can be traced by traversing the flows in a graph. The
empty path statically exists if the starting term identifier meets the ending condition. Otherwise, a path statically exists if the last static step corresponds to a flow that meets the ending condition, and the longest strict prefix of the path statically exists.

**Predicate staticTraceable:**

\[
\begin{align*}
\text{flow set} & \rightarrow \text{tm_id} \rightarrow (\text{tm_id} \rightarrow \text{bool}) \rightarrow \text{static_path} \rightarrow \text{bool} \\
\text{where} & \\
\text{empty: start graph isEnd} . \\
\text{isEnd start} & \\
\vdash \text{staticTraceable graph start isEnd} [] \\
\text{snoc: graph star middle path isEnd end mode} . \\
\text{staticTraceable graph start} (\lambda \ell . \ell = \text{middle}) \text{ path,} \\
\text{isEnd end,} \\
(\text{middle, mode, end}) \in \text{graph} & \\
\vdash \text{staticTraceable graph start isEnd} (\text{path} \ @ \ [(\text{middle, mode})])
\end{align*}
\]

In the graph of the server implementation, there are two paths each corresponding to its own thread that lead to sending on static channel \texttt{chn k1} and a potentially infinite number of paths that lead to receiving on channel \texttt{chn k1}, but all on the same thread. There are an infinite number of paths that lead to sending on static channel \texttt{chn k4}, and two paths that lead to receiving on static channel \texttt{chn k4}. This is analysis is better than nothing, but it’s still somewhat imprecise. The static \texttt{chn k4} corresponds to multiple distinct dynamic channels, each with just one sender and one receiver. The higher precision analysis discussed in section 4 addresses this issue.

The static inclusion means that two static paths might be traced in the same run of a program. Ordered paths might be inclusive, and also a path that diverges from another at a spawn point might be inclusive. This concept is useful for achieving greater precision, since if two paths cannot occur in the same run of a program, only one needs to be counted towards the communication classification. The predicates are intended to be applied to paths starting from the beginning of a program.
**predicate** staticInclusive: static_path -> static_path -> bool **where**

  first: path1 path2 .
  prefix path1 path2
  ⊢ staticInclusive path1 path2
  * second path2 path1 .
  prefix path2 path1
  ⊢ staticInclusive path1 path2
  * spawnFirst: path n path1 path2 .
  ⊢ staticInclusive
    (path @ [(IdBind n, MSpwn)] @ path1)
    (path @ [(IdBind n, MSeq)] @ path2)
  * spawnSecond: path n path1 path2 .
  ⊢ staticInclusive
    (path @ [(IdBind n, MSeq)] @ path1)
    (path @ [(IdBind n, MSpwn)] @ path2)

The uncompetitiveness means that two paths can’t compete during a run of a program. Either they are ordered or they cannot occur in the same run of a program. The singularity means that two paths are the same or only of them can occur in a given run of a program.

**predicate** uncompetitive: static_path -> static_path -> bool **where**

  ordered: path1 path2 .
  ordered path1 path2
  ⊢ uncompetitive path1 path2
  * notInclus: path1 path2 .
  ¬ (staticInclusive path1 path2)
  ⊢ uncompetitive path1 path2

**predicate** singular: static_path -> static_path -> bool **where**

  refl: path .
  ⊢ singular path path
  * notInclus: path1 path2 .
  ¬ (staticInclusive path1 path2)
  ⊢ singular path1 path2

The static one-to-many classification means that there is at most one thread that attempts to send on a given static channel at any time during a run of a given program, but there may be many threads that attempt to receive on the channel.
**predicate** staticOneToMany: term -> name -> bool where

intro: staticEnv staticComm t graph n c.
staticEval staticEnv staticComm t,
staticFlowsAccept staticEnv graph t,
forEveryTwo (staticTraceable graph (termId t)
  (staticSendId staticEnv t n c)) uncompetitive
⊢ staticOneToMany t n c.

The static many-to-one predicate means that there may be many threads that attempt to send on a static channel, but there is at most one thread that attempts to receive on the channel for any time during a run of a given program.

**predicate** staticManyToOne: term -> name -> bool where

intro: staticEnv staticComm t graph n c.
staticEval staticEnv staticComm t,
staticFlowsAccept staticEnv graph t,
forEveryTwo (staticTraceable graph (termId t)
  (staticRecvId staticEnv t n c)) uncompetitive
⊢ staticManyToOne t n c.

The static one-to-one classification means that there is at most one thread that attempts to send and at most one thread that attempts to receive on a given static channel for any time during a run of a given program.

**predicate** staticOneToOne: term -> name -> bool where

intro: staticEnv staticComm t graph n c.
staticEval staticEnv staticComm t,
staticFlowsAccept staticEnv graph t,
forEveryTwo (staticTraceable graph (termId t)
  (staticSendId staticEnv t n c)) uncompetitive,
forEveryTwo (staticTraceable graph (termId t)
  (staticRecvId staticEnv t n c)) uncompetitive
⊢ staticOneToOne t n c.

The static one-shot classification means that there is at most one attempt to synchronize to send on a static channel in any run of a given program.

**predicate** staticOneShot: term -> name -> bool where

intro: staticEnv staticComm t graph n c.
staticEval staticEnv staticComm t,
staticFlowsAccept staticEnv graph t,
forEveryTwo (staticTraceable graph (termId t)
  (staticSendId staticEnv t n c)) singular
⊢ staticOneShot t n c.

The static one-sync classification means that there is at most one attempt to send
on a static channel and at most one thread that attempts to receive on a static channel in
any run of a given program.

**predicate** staticOneSync: term -> name -> bool where
  intro: staticEnv staticComm t graph n. 
  staticEval staticEnv staticComm t,
  staticFlowsAccept staticEnv graph t,
  forEveryTwo (staticTraceable graph (termId t)
    (staticSendId staticEnv t n c)) singular,
  forEveryTwo (staticTraceable graph (termId t)
    (staticRecvId staticEnv t n c)) uncompetitive
  ⊢ staticOneSync t n c.

### 3.6 Formal Reasoning

The semantics and analyses must contain many details. To ensure the correctness
of proofs, it is necessary to check that there are no subtle errors in either the definitions
or the proofs. Proofs require many subtle manipulations of symbols. The difference be-
tween a false statement and a true statement can often be difficult to spot, since the two
may be very similar lexically. However, a mechanical proof checker, such as the one in
Isabelle/HOL, has no difficulty discerning between valid and invalid derivations. Mechani-
cal checking of proofs can notify users of errors in the proofs or definitions far better and
faster than manual checking. This work has greatly benefited from Isabelle’s proof checker
in order to correctly define the language semantics, control flow analysis, communication
analysis, and other helpful definitions. For instance, some bugs in the defininitions were
found while trying to prove soundness. The proof checker would not accept the proof unless I provided facts that should be false, indicating that the definitions did not state my intentions. After correcting the errors in the definitions, the proof was completed such that the proof checker was satisfied.

The reasoning involved in proving the soundness of each communication classification is based around breaking the goal into simpler subgoals, and generalizing assumptions to create useful induction hypotheses. It is often useful to create helper definitions that can be deduced from premises of the theorem being proved and enable general reasoning across arbitrary programs. A frequent pattern is to define predicates in terms of semantic structures, like the environment, stack, and pool, and deduce the instantiation of these predicates on the initial program state.

Some aspects of the generalized predicate definitions exist simply to prove that they imply instantiations of the term based predicates on the initial program. The generalized definitions are necessary in order to allow direct access to properties that would otherwise be deeply nested in an inductive structure and inaccessible by a predictable number of logical steps for an arbitrary program.

One of the most difficult aspects of formal reasoning is in developing adequate definitions. It is often possible to define a single semantics in multiple ways. For instance, the sortedness of a list could be defined in terms of the sortedness of its tail or in terms of the sortedness of its longest strict prefix. To prove theorems relating sortedness to other relations, it may be important that the other relations are inductively defined on the same subpart of the list. Some relations may only be definable on the tail, while others can be defined only on the strict prefix. In such cases, it is necessary to define sortedness in
two ways, and prove their equivalence, in order to prove theorems relating to less flexible relations.

**predicate** sortedLeft: nat list -> bool **where**

- empty: \(\vdash\) sortedLeft []
- uni: x . \(\vdash\) sortedLeft [x]
- cons: x y zs .
  
  n \(\leq\) y, 
  
  sortedLeft (y # zs)
  
  \(\vdash\) sortedLeft (x # y # zs)

**predicate** sortedRight: nat list -> bool **where**

- empty: \(\vdash\) sortedRight []
- uni: z . \(\vdash\) sortedRight [z]
- snoc: xs y z .
  
  sortedRight (xs @ [y]),
  
  y \(\leq\) z
  
  \(\vdash\) sortedRight (xs @ [y] @ [z])

**lemma** sortedEquiv: xs .

\(\vdash\) sortedLeft xs \(\equiv\) sortedRight xs

### 3.7 Soundness

The theorem for soundness of static one-to-many classification states that if a static channel is statically classified as one-to-many for a given program, then any corresponding dynamic channel is classified as one-to-many for the same program. The soundness of classification of many-to-one, one-to-one, one-shot, and one-sync all follow the same pattern.

**theorem** staticOneToManySound: t \(\vdots\) n c path c .

\(\vdash\) staticOneToMany t \(\vdots\) n c (Chan path c n c)

**theorem** staticManyToOneSound: t \(\vdots\) n c path c .

\(\vdash\) staticManyToOne t \(\vdots\) n c (Chan path c n c)
staticManyToOne \ t_0 \ n_c \\
\vdash \ \text{manyToOne} \ t_0 \ (\text{Chan} \ \text{path}_c \ n_c)

\textbf{theorem} \ \text{staticOneToOneSound}: \ t_0 \ n_c \ \text{path}_c \\
\text{staticOneToOne} \ \text{staticEnv} \ t_0 \ n_c \\
\vdash \ \text{oneToOne} \ t_0 \ (\text{Chan} \ \text{path}_c \ n_c)

\textbf{theorem} \ \text{staticOneShotSound}: \ t_0 \ n_c \ \text{path}_c \\
\text{staticOneShot} \ t_0 \ n_c \\
\vdash \ \text{oneShot} \ t_0 \ (\text{Chan} \ \text{path}_c \ n_c)

\textbf{theorem} \ \text{staticOneSyncSound}: \ t_0 \ n_c \ \text{path}_c \\
\text{staticOneSync} \ t_0 \ n_c \\
\vdash \ \text{oneSync} \ t_0 \ (\text{Chan} \ \text{path}_c \ n_c)

The formal proofs of soundness of each static classification follow a similar structure. Let's examine in some detail the formal proof of soundness of static one-to-many classification, by unwinding the theorem into the lemmas that it follows from. The following diagram illustrates the key dependencies of the theorems and lemmas in used in the derivations.
The soundness of static one-to-many classification is proved by a few simpler lemmas and the definitions of static and dynamic one-to-many classification. There is an isomorphic correspondence between the paths of dynamic evaluation and the paths of static evaluation, by definition. The static paths are derived from the static flow graph. Although it is possible to directly derive the dynamic paths from the static flow graph, deriving an isomorphic path structure keeps the paths’ relation to the flow graph clear. Additionally, for the higher precision analysis, static paths are not isomorphic to dynamic paths.

The three main lemmas state the soundness of static path existence, the soundness of static inclusiveness, and the soundness of static sending identifier classification. These
lemmas depend on a correspondence between static paths and dynamic paths. The lemma for soundness of static inclusiveness states that any two dynamic paths traced by running a program correspond to statically inclusive static paths. It follows from a straightforward case analysis of static inclusivity. The lemma for soundness of static path existence states that for any dynamic path traced by running a program, there is a corresponding static path that is statically traceable. The lemma for soundness of static sending identifier classification states that if running a program reaches a synchronization on a sending event, then that identifier for that synchronization is statically classified as sending.

**Predicate**

```
predicate pathsCorrespond: dynamic_path -> static_path -> bool where
  empty:
  ⊢ pathsCorrespond [] []
  * seq: path staticPath n .
    pathsCorrespond path staticPath
    ⊢ pathsCorrespond
      (path @ [DSeq n])
      (staticPath @ [(IdBind n, MSeq)])
  * spawn: path staticPath n .
    pathsCorrespond path staticPath
    ⊢ pathsCorrespond
      (path @ [DSpwn n])
      (staticPath @ [(IdBind n, MSpwn)])
  * call: path staticPath n .
    pathsCorrespond path staticPath
    ⊢ pathsCorrespond
      (path @ [DCll n])
      (staticPath @ [(IdBind n, MCll)])
  * return: path staticPath n .
    pathsCorrespond path staticPath
    ⊢ pathsCorrespond
      (path @ [DRtn n])
      (staticPath @ [(IdRslt n, MRtn)])
```

**Lemma**

```
lemma staticTraceableSound: t_0 pool comm path t
env stack staticEnv staticComm graph isEnd .
star dynamicEval [[] -> (Stt t_0 [->] [])] pool comm,
pool path = Some (Stt t env stack),
staticEval staticEnv staticComm t_0,
staticFlowsAccept staticEnv graph t_0,
```

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isEnd (termId t)
⊢ exists staticPath .
pathsCorrespond path staticPath
∧ staticTraceable graph (termId t₀) isEnd staticPath

**lemma** staticInclusiveSound: t₀ pool comm path₁ stt₁ path₂ stt₂
staticPath₁ staticPath₂ .
star dynamicEval [[] -> (Stt t₀ [->] [] [])] {} pool comm
pool path₁ = Some stt₁,
pool path₂ = Some stt₂,
pathsCorrespond path₁ staticPath₁,
pathsCorrespond path₂ staticPath₂
⊢ staticInclusive staticPath₁ staticPath₂

**lemma** staticSendIdSound: t₀ pool comm path n nₑ t’ env
stack nₓ c nₓ _env’ path c₁ staticEnv staticComm .
star dynamicEval [[] -> (Stt t₀ [->] [] [])] {} pool comm,
pool path = Some (Stt (Bind n (Sync nₑ) t’) env stack),
env nₑ = Some (VAtm (SendEvt nₓ c nₓ e) env’),
env’ nₓ = Some (VChn (Chan path c₁ nₑ)),
staticEval staticEnv staticComm t₀
⊢ staticSendId staticEnv t₀ n c₁ (IdBind n)

The soundness of static path existence is proved by generalizing static flows acceptance and static evaluation over pools, such that information about a step in the program can be deduced by a fixed number of logical steps regardless of the location of the program step or the size of the program. Without such generalization, it would be possible to prove soundness for a fixed program, but not any arbitrary program.

The generalization of static flows acceptance is comprised of static flows acceptance of values, static flows acceptance of environments, static flows acceptance of stacks, and static flows acceptance of pools. In most cases, it simply states that a subterm of some semantic element is also statically accepting. The exception is in the case of static flows acceptance of a non-empty stack, where there is an additional condition that the graph contains a flow from a result identifier to the term identifier of the continuation. This
information is consistent with static flows acceptance of terms, but provides direct information about a flow in the graph, which would otherwise only be deducible by a varying number of logical steps depending on the program.

**predicate** staticFlowsAcceptValue:
\[ \text{static\_value\_map} \to \text{graph} \to \text{dynamic\_value} \to \text{bool} \]

**where**

\[
\begin{align*}
\text{unit: staticEnv graph} & \quad \vdash \text{staticFlowsAcceptValue} \; \text{staticEnv graph} \; \text{VUnt} \\
\text{* chan: staticEnv graph n, c} & \quad \vdash \text{staticFlowsAcceptValue} \; \text{staticEnv graph} \; (\text{VChn n, c}) \\
\text{* sendEvt: staticEnv graph env n, n_m, staticEnv graph env} & \quad \vdash \text{staticFlowsAcceptVal} \; \text{staticEnv graph} \; (\text{VAtm (SendEvt n, n_m, env)}) \\
\text{* recvEvt: staticEnv graph env n, staticEnv graph env} & \quad \vdash \text{staticFlowsAcceptVal} \; \text{staticEnv graph} \; (\text{VAtm (RecvEvt n, env)}) \\
\text{* left: staticEnv graph env n, p} & \quad \vdash \text{staticFlowsAcceptVal} \; \text{staticEnv graph} \; (\text{VAtm (Lft n, p, env)}) \\
\text{* right: staticEnv graph env n, p} & \quad \vdash \text{staticFlowsAcceptVal} \; \text{staticEnv graph} \; (\text{VAtm (Rht n, p, env)}) \\
\text{* function: staticEnv graph t, b, env n_f, n_p} & \quad \vdash \text{staticFlowsAcceptVal} \; \text{staticEnv graph} \; (\text{VAtm (Fun n_f, n_p, t, b, env)}) \\
\text{* pair: staticEnv graph env} & \quad \vdash \text{staticFlowsAcceptVal} \; \text{staticEnv graph} \; (\text{VAtm (Pair n_1, n_2, env)})
\end{align*}
\]
The flows described by the various versions of static flows acceptance depend on static environments in order to look up the flow in the case where the term is a function. The static environment is constrained by the static evaluation of the program that is dynamically evaluated. Generalized versions of static evaluation that related the static en-
environment to other elements of dynamic evaluation enable further deduction about flows. As with the generalized versions of static flows acceptance, the generalized versions of static evaluation are designed to preserve static environments across dynamic evaluations of pools. They also provide direct access to binding information from names to static values in a fixed number of logical steps. Static evaluation of terms correlates terms to static values, but the generalized static evaluations correlate dynamic evaluation structures of values, environments, and stacks, to static values. The abstraction function relates dynamic values to static values and helps with the larger goal of relating dynamic semantic elements to static values and static environments.

```haskell
fun abstract: dynamic_value -> static_value where
  ⊢ abstract VUnt = SUnt
  * path n .
    ⊢ abstract (VChn (Chan path x)) = SChn x
  * atom env .
    ⊢ abstract (VAtm atom env) = SAtm atom

predicate staticEvalValue:
  static_value_map -> static_value_map -> dynamic_value -> bool
  where

    unit: staticEnv staticComm .
    ⊢ staticEvalValue staticEnv staticComm VUnit

    * chan: staticEnv staticComm chan .
    ⊢ staticEvalValue staticEnv staticComm (VChn chan)

    * sendEvt: staticEnv staticComm env n c n_m .
      staticEvalEnv staticEnv staticComm env
      ⊢ staticEvalValue staticEnv staticComm
      (VAtm (SendEvt n_c n_m) env)

    * recvEvt: staticEnv staticComm env n_c .
      staticEvalEnv staticEnv staticComm env
      ⊢ staticEvalValue staticEnv staticComm
      (VAtm (RecvEvt n_c) env)

    * left: staticEnv staticComm env n .
```

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staticEvalEnv \textit{staticEnv} \textit{staticComm} \textit{env}~~\vdash~~\textit{staticEvalValue} \textit{staticEnv} \textit{staticComm} (\text{VAtm} \textit{Lft} \textit{n} \textit{env})

* right: \textit{staticEnv} \textit{staticComm} \textit{env} \textit{n} .
  \textit{staticEvalEnv} \textit{staticEnv} \textit{staticComm} \textit{env} \textit{n} .
  \textit{staticEvalEnv} \textit{staticEnv} \textit{staticComm} \textit{env} \textit{n} .
  \textit{staticEvalValue} \textit{staticEnv} \textit{staticComm} (\text{VAtm} \textit{Rht} \textit{n} \textit{env})

* function: \textit{n}_f \textit{n}_p \textit{t}_b \textit{staticEnv} \textit{staticComm} \textit{env} .
  \{\text{SAtm} (\text{Fun} \textit{n}_f \textit{n}_p \textit{t}_b)\} \subseteq \textit{staticEnv} \textit{f},
  \textit{staticEval} \textit{staticEnv} \textit{staticComm} \textit{t}_b,
  \textit{staticEvalEnv} \textit{staticEnv} \textit{staticComm} \textit{env} \textit{n} .
  \textit{staticEvalValue} \textit{staticEnv} \textit{staticComm} (\text{VAtm} (\text{Fun} \textit{n}_f \textit{n}_p \textit{t}_b) \textit{env})

* pair: \textit{staticEnv} \textit{staticComm} \textit{env} \textit{n}_1 \textit{n}_2 .
  \textit{staticEvalEnv} \textit{staticEnv} \textit{staticComm} \textit{env} \textit{n}_1 \textit{n}_2 .
  \textit{staticEvalValue} \textit{staticEnv} \textit{staticComm} (\text{VAtm} (\text{Pair} \textit{n}_1 \textit{n}_2) \textit{env})

\textbf{predicate\ staticEvalEnv:}
\begin{align*}
\textit{static_value_map} & \rightarrow \textit{static_value_map} \rightarrow \textit{env} \rightarrow \textit{bool} \\
\textbf{where} \quad & \textit{intro: staticEnv staticComm env} . \\
& \forall \textit{n v} . \\
& \quad \text{env n = Some v} \\
& \rightarrow \{\text{abstract v}\} \subseteq \textit{staticEnv n} \\
& \quad \wedge \textit{staticEvalValue staticEnv staticComm v} \\
& \vdash \textit{staticEvalEnv staticEnv staticComm env}
\end{align*}

\textbf{predicate\ staticEvalStack:}
\begin{align*}
\textit{static_value_map} & \rightarrow \textit{static_value_map} \\
& \rightarrow \textit{static_value set} \rightarrow \textit{stack} \rightarrow \textit{bool} \\
\textbf{where} \quad & \textit{empty: staticVals n staticEnv staticComm t env stack’} . \\
& \vdash \textit{staticEvalStack staticEnv staticComm staticVals []} \\
\textbf{* cons: staticVals staticEnv staticComm stack’} . \\
& \textit{staticVals} \subseteq \textit{staticEnv n}, \\
& \textit{staticEval} \textit{staticEnv staticComm t}, \\
& \textit{staticEvalEnv staticEnv staticComm env}, \\
& \textit{staticEvalStack staticEnv staticComm (staticEnv (resultName t)) stack’} \\
& \vdash \textit{staticEvalStack staticEnv staticComm staticVals ((Ctn n t env) # stack ’)}
\end{align*}
**predicate** staticEvalPool:
static_value_map -> static_value_map -> pool -> bool

**where**
intro: pool staticEnv staticComm .
∀ path t env stack .
  pool path = Some (Stt t env stack)
  →
  staticEval staticEnv staticComm t
  ∧ staticEvalEnv staticEnv staticComm env
  ∧ staticEvalStack staticEnv staticComm (staticEnv (resultName t)) stack
⊢ staticEvalPool staticEnv staticComm pool

A variant of star that inducts on the left of the transitive connection is helpful for relating dynamic path existence to static path existence, since it mirrors the direction that way paths grow, which influenced the choice of induction on the longest strict prefix of paths in the definition of static path existence.

**predicate** starLeft: ('a -> 'a -> bool) -> 'a -> 'a -> bool

**where**
refl: r z z .
⊢ starLeft r z z

* trans: r x y z .
  starLeft r x y,
  r y z
  ⊢ starLeft r x z

**lemma** starImpliesStarLeft: r x z .
  star r x z
  ⊢ starLeft r x z

**lemma** starLeftTrans: r x y z .
  starLeft r x y,
  starLeft r y z
  ⊢ starLeft r x z

The lemma for soundness of static path existence follows from the generalized definitions of static flows acceptance, the definition of static path existence, and the preservation of static flows acceptance across multiples steps of evaluation.
Lemma \textit{staticFlowsAcceptPoolPreserved}: $t_0$ pool comm staticEnv staticComm

graph .

star dynamicEval [[] \rightarrow (Stt t_0 \rightarrow [])] \{} pool comm,
staticEval staticEnv staticComm $t_0$,
staticFlowsAcceptPool staticEnv graph [[] \rightarrow (Stt t_0 \rightarrow [])] \%
\%

\% The preservation of static flows acceptance over pools is proved by the equivalence between star and its leftward variant, and induction on the leftward variant. The preservation of static evaluation of pools over multiple steps is also relied upon.

Lemma \textit{staticEvalPoolPreserved}: pool comm pool' comm' staticEnv staticComm .

star dynamicEval pool comm pool' comm'
staticEvalPool staticEnv staticComm pool
\%

\% The soundness of static inclusiveness is derived from various lemmas that preserve relations from pairs of dynamic paths to pairs of corresponding static paths. Some of these lemmas are the preservation of the strict prefix relation from static to dynamic paths, and the preservation of static inclusiveness over extension of static paths.

Lemma \textit{strictPrefixPreservedCorresp}: staticPath1 staticPath2 dynamicPath1
dynamicPath2 .
strictPrefix staticPath1 staticPath2,
pathsCorrespond dynamicPath1 staticPath1,
pathsCorrespond dynamicPath2 staticPath2
\%

\% These various preservation lemmas are derived from the basic properties of lists and straightforward properties of path correspondence, such as commutativity, as well as foundational principles like induction of corresponding paths.
The lemma for soundness of static sending identifier classification $\text{staticSendIdSound}$ is proved using the lemma for soundness of static evaluation for synchronization of a send event, and the lemma for soundness of static evaluation. Since only sending identifiers are relevant, the soundness of static reachability is used to ensure that the static step is indeed a sending identifier.

**Lemma** \( \text{sendChanStaticEvalSound} : t_0 \text{pool} \text{comm} \text{staticEnv} \text{staticComm} \text{path} \)

\[
\vdash SChn n_e \in \text{staticEnv} n_{sc}
\]

**Lemma** \( \text{staticEvalSound} : t_0 \text{pool} \text{comm} \text{staticEnv} \text{staticComm} \text{path} \)

\[
\vdash \text{abstract v} \in \text{staticEnv} n
\]

**Lemma** \( \text{staticReachableSound} : t_0 \text{pool} \text{comm} \text{staticEnv} \text{staticComm} \text{path} \)

\[
\vdash \text{staticReachable} t_0 t
\]

Both the soundness of static evaluation on the synchronization of a send event, and the soundness of static evaluation follow from the preservation of static evaluation over multiple steps of dynamic evaluation.

The lemma for soundness of static reachability relies on a reformulation of static reachability defined by proofs that induct on a larger term containing the reachable term. This definition is useful for forward derivations of reachability relations, however it doesn’t
offer much guidance for deciding reachability. In contrast, the definition of the original
tstatic reachability relation is syntax-directed in order to portray a clear connection to a
computable algorithm that can determine the reachable term from an initial program. To
show that a term is reachable from the initial program, it is necessary to show that each
intermediate term is reachable from the initial term. Thus, the induction needs to enable
unraveling the goals from the end of the program to the beginning, maintaining the initial
program state in context for each subgoal. Because the static reachable relation is a "may"
analysis, the generalized relations hold unless there is a clear reason that it could never
hold. For instance, in the relation over atoms, for all atoms other than functions, it holds
without any additional demands.

**predicate** staticReachableForward: term -> term -> bool where

refl: t₀.
 ⊢ staticReachableForward t₀ t₀

* spawn: t₀ n tₖ t’.
  staticReachableForward t₀ (Bind n (Spwn tₖ) t’)
  ⊢ staticReachableForward t₀ tₖ

* distincLeft: t₀ n nₛ nₜ nₙₗ tₙₗ tₙᵣ t’.
  staticReachableForward t₀ (Bind n (Case nₛ nₜ nₙₗ nₙᵣ t’) t’)
  ⊢ staticReachableForward t₀ tₙₗ

* distincRight: t₀ n nₛ nₜ nₙₗ tₙₗ nₙᵣ t’.
  staticReachableForward t₀ (Bind n (Case nₛ nₜ nₙₗ nₙᵣ t’) t’)
  ⊢ staticReachableForward t₀ tₙᵣ

* function: t₀ n nₕ nₚ tₜ t’.
  staticReachableForward t₀ (Bind n (Atom (Fun nₕ nₚ tₜ)) t’)
  ⊢ staticReachableForward t₀ tₜ

* seq: t₀ n nₜ nₚ tₚ t’.
  staticReachableForward t₀ (Bind n c t’)
  ⊢ staticReachableForward t₀ t’

**predicate** staticReachableAtom: term -> atom -> bool where
sendEvt: t_0 n_c n_m .
⊢ staticReachableAtom t_0 (SendEvt n_c n_m)

* recvEvt: t_0 n_c .
⊢ staticReachableAtom t_0 (RecvEvt n_c)

* pair: t_0 n_1 n_2 .
⊢ staticReachableAtom t_0 (Pair n_1 n_2)

* left: t_0 n_l .
⊢ staticReachableAtom t_0 (Lft n_l)

* right: t_0 n_r.
⊢ staticReachableAtom t_0 (Rht n_r)

* function: t_0 t_b n_f n_p t_b .
  staticReachableForward t_0 t_b
  ⊢ staticReachableAtom t_0 (Fun n_f n_p t_b)

predicate staticReachableVal: term -> dynamic_value -> bool where
  unit: t_0 .
  ⊢ staticReachableValue t_0 VUn

* chan: t_0 c .
  ⊢ staticReachableValue t_0 (VChn c)

* atom: t_0 t env .
  staticReachableAtom t_0 t,
  staticReachableEnv t_0 env
  ⊢ staticReachableValue t_0 (VAtm t env)

predicate staticReachableEnv: term -> env -> bool where
  intro: t_0 env
  ∀ n v .
  env n = Some v
  → staticReachableValue t_0 v
  ⊢ staticReachableEnv t_0 env

predicate staticReachableStack: term -> stack -> bool where
  empty: t_0 .
  ⊢ staticReachableStack t_0 []

* cons: t_0 t_k env_k stack'.
  staticReachableForward t_0 t_k,
  staticReachableEnv t_0 env_k,
  staticReachableStack t_0 stack'
  ⊢ staticReachableStack t_0 ((Ctn n_k t_k env_k) # stack')
**predicate** staticReachablePool: term -> pool -> bool where

intro: t0 pool .

∀ path t env stack .
  pool path = Some (Stt t env stack)
→ staticReachableForward t0 t
∧ staticReachableEnv t0 env
∧ staticReachableStack t0 stack
⊢ staticReachablePool t0 pool

The soundness of static reachability follows from the definitions its generalized form of static reachability of pools and its soundness.

**lemma** staticReachablePoolSound: t0 pool .

star dynamicEval [ ] -> (Stt t0 [ ] [ ]), {}, pool comm
⊢ staticReachablePool t0 pool

The soundness of static reachability of pools follows from the lemma that the forward static reachability implies the syntax-directed static reachability, and the equivalence between star and the leftward star. It relies on induction of the leftward star and constructs the static reachability proposition using the forward definition.

**lemma** staticReachableForwardImpliesStaticReachable: t0 t.

staticReachableForward t0 t
⊢ staticReachable t0 t

**lemma** staticReachableTrans: t1 t2 t3 .

staticReachable t1 t2,
staticReachable t2 t3
⊢ staticReachable t1 t3

The lemma that the forward variant of static reachability implies the syntax-directed static reachability follows from induction on the forward static reachability and the transitivity of static reachability, which follows from induction on static reachability.
Chapter 4

Higher Precision Static Analysis

In many programs, like in the server example, channels are created within functions. The functions may be applied multiple times, creating multiple distinct channels with each application. It may be that each channel is used just once and then discarded. However, the static analysis described so far would identify all the distinct channels by the same name, since each distinct channel is created by the same piece of syntax. Thus, it would classify those channels as being used more than once, when actually they might be used at most once each.

It is possible to be more precise by trimming the program under analysis down to just the part where the static channel is live. The static channel cannot be live between the last use of a corresponding dynamic channel and the creation of a new dynamic channel with the same name. Thus, each truncated program would have just one dynamic channel corresponding to the static channel under analysis.

The higher precision analysis uses a trimmed down graph to better differentiate between distinct channels. A trimmed graph is specialized for a particular dynamic channel. From the creation step, it must contain transitive flows to all the program steps where the channel is live. It should also be as small as possible, for higher precision.

In the whole graph used in the previous analysis, a spawning flow connects a
child thread to the rest of the program. For a trimmed graph, it may be that the channel of interest is not created until after some spawn step, so there is no need to include the spawning flow in the trimmed graph. However, later on in the program it may become apparent that the channel of interest is sent via another channel to that spawned thread. Since there is no spawning flow already connecting that thread to the trimmed graph, a flow with a sending mode is used between the sending identifier and the receiving identifier of synchronization in order to link the part of the thread with the live channel into the graph. In additions to the new mode of sending, previous modes of sequencing, calling, returning, and spawning are also included.

```plaintext
datatype mode =
    MSeq
  | MSpwn
  | MSend name
  | MCll
  | MRtn

type flow = tm_id * mode * tm_id

type static_step = tm_id * mode

type staticPath = static_step list
```

To demonstrate some key concepts of the higher precision analysis, an additional loop function \( \ell p \) is added to the example server implementation. The loop basically just wastes steps, but it is used to demonstrate how liveness analysis treats functions that don’t contain any channel of interest.

```plaintext
bind u1 = unt
bind r1 = rht u1
bind l1 = lft r1
bind l2 = lft l1

bind \ell p = fun \ell p’ x1 =>
(}
```
bind z1 = case x1 of
  lft y1 =>
    (bind z2 = app lp' y1
     rslt z2)
| rht y2 =>
    (bind u2 = unt
     rslt u2)
bind u3 = unt
rslt u3

bind mksr = fun _ x2 =>
  (bind k1 = mkChn
   bind z4 = lp l2
   bind srv = fun srv' x3 =>
     (bind e1 = recvEvt k1
      bind p1 = sync e1
      bind v1 = fst p1
      bind k2 = snd p1
      bind e2 = sendEvt k2 x3
      bind z5 = sync e2
      bind z6 = app srv' v1
      rslt z6)
   bind z7 = spawn
     (bind z8 = srv r1
      rslt z8)
   rslt k1)

bind rqst = fun _ x4 =>
  (bind k3 = fst x4
   bind v2 = snd x4
   bind k4 = mkChn
   bind p2 = pair v2 k4
   bind e3 = sendEvt k3 p2

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The static flows acceptance for higher precision is similar to that of the lower precision analysis. However, it must additionally consider flows with the sending mode. To accommodate the sending flow from sender to receiver, the initial program must be carried each inference rule.

**Predicate staticFlowsAcceptTm:**

\[
\text{static\_value\_map} \rightarrow \text{graph} \rightarrow \text{term} \rightarrow \text{term} \rightarrow \text{bool}
\]

**Where**

\[
\begin{align*}
\text{result: staticEnv graph} \ n \ . \\
& \vdash \text{staticFlowsAcceptTm staticEnv graph} \ t \ 0 \ (\text{Rslt} \ n) \\
\end{align*}
\]

* **unit:** \( n \ t' \ \text{graph staticEnv} \ . \\
(\text{IdBind} \ n \ , \ \text{MSeq, termId} \ t') \in \text{graph}, \\
\text{staticFlowsAcceptTm staticEnv graph} \ t_0 \ t' \\
& \vdash \text{staticFlowsAcceptTm staticEnv graph} \ t_0 \ (\text{Bind} \ n \ \text{Unt} \ t')
\]

* **makeChan:** \( n \ t' \ \text{graph staticEnv} \ . \\
(\text{IdBind} \ n \ , \ \text{MSeq, termId} \ t') \in \text{graph}, \\
\text{staticFlowsAcceptTm staticEnv graph} \ t_0 \ t' \\
& \vdash \text{staticFlowsAcceptTm staticEnv graph} \ t_0 \ (\text{Bind} \ n \ \text{MkChn} \ t')
\]

* **sendEvt:** \( n \ t' \ \text{graph staticEnv} \ n_c \ n_m \ . \\
(\text{IdBind} \ n \ , \ \text{MSeq, termId} \ t') \in \text{graph}, \\
\text{staticFlowsAcceptTm staticEnv graph} \ t_0 \ t'
\]
⊢ staticFlowsAccept
  staticEnv graph
  (Bind n (Atom (SendEvt n c n m)) t')

* recvEvt: n t' graph staticEnv n_c
  (IdBind n , MSeq, termId t') ∈ graph,
  staticFlowsAcceptTm staticEnv graph t_0 t'
  ⊢ staticFlowsAcceptTm staticEnv graph t_0 (Bind n (Atom (RecvEvt n_c)) t')

* pair: n t' graph staticEnv n_1 n_2
  (IdBind n , MSeq, termId t') ∈ graph,
  staticFlowsAcceptTm staticEnv graph t_0 t'
  ⊢ staticFlowsAcceptTm staticEnv graph t_0 (Bind n (Pair n_1 n_2)) t'

* left: n t' graph staticEnv n_s
  (IdBind n , MSeq, termId t') ∈ graph,
  staticFlowsAcceptTm staticEnv graph t_0 t'
  ⊢ staticFlowsAcceptTm staticEnv graph t_0 (Bind n (Lft n_s)) t'

* right: n t' graph staticEnv n_s
  (IdBind n , MSeq, termId t') ∈ graph,
  staticFlowsAcceptTm staticEnv graph t_0 t'
  ⊢ staticFlowsAcceptTm staticEnv graph t_0 (Bind n (Rht n_s)) t'

* function: n t' graph staticEnv t_b n_f n_p
  (IdBind n , MSeq, termId t') ∈ graph,
  staticFlowsAcceptTm staticEnv graph t_0 t',
  staticFlowsAcceptTm staticEnv graph t_0 t_b
  ⊢ staticFlowsAcceptTm staticEnv graph t_0 (Bind n (Fun n_f n_p t_b)) t'

* spawn: n t' t_c graph staticEnv.
  {{IdBind n, MSeq, termId t'}},
  (IdBind n, MSpwn, termId t_c)} ⊆ graph,
  staticFlowsAcceptTm staticEnv graph t_0 t_c,
  staticFlowsAcceptTm staticEnv graph t_0 t'
  ⊢ staticFlowsAcceptTm staticEnv graph t_0 (Bind n (Spwn t_c)) t'

* sync: n t' graph staticEnv n_sc
  (IdBind n , MSeq, termId t') ∈ graph,
  ∀ n_ac n_m n_c n_y.
  (SAAtm (SendEvt n_sc n_m)) ∈ staticEnv nSE,
  → (SChn n_c) ∈ staticEnv n_ac
  → staticRecvId staticEnv t_0 n_c (IdBind n_y)
  → (IdBind n, MSend n_ac, IdBind n_y) ∈ graph),
  staticFlowsAcceptTm staticEnv graph t_0 t'
\[⊢ \text{staticFlowsAcceptTm} \text{staticEnv} \text{graph} \ t_0 \ (\text{Bind} \ n \ (\text{Sync} \ n_{ac}) \ t')\]

* first: \(n \ t' \ \text{graph} \ \text{staticEnv} \ n_p\).
  \(\{\text{IdBind} \ n \ , \ \text{MSeq}, \ \text{termId} \ t'\} \in \text{graph},\)
  \(\text{staticFlowsAcceptTm} \text{staticEnv} \text{graph} \ t_0 \ t',\)
  \[⊢ \text{staticFlowsAcceptTm} \text{staticEnv} \text{graph} \ t_0 \ (\text{Bind} \ n \ (\text{Fst} \ n_p) \ t')\]

* second: \(n \ t' \ \text{graph} \ \text{staticEnv} \ n_p\).
  \(\{\text{IdBind} \ n \ , \ \text{MSeq}, \ \text{termId} \ t'\} \in \text{graph},\)
  \(\text{staticFlowsAcceptTm} \text{staticEnv} \text{graph} \ t_0 \ t',\)
  \[⊢ \text{staticFlowsAcceptTm} \text{staticEnv} \text{graph} \ t_0 \ (\text{Bind} \ n \ (\text{Snd} \ n_p) \ t')\]

* distinction: \(n \ t_l \ t_r \ t' \ \text{graph} \ \text{staticEnv} \ n_s\).
  \(\{\text{IdBind} \ n, \ \text{MCll}, \ \text{termId} \ t_l\},\)
  \(\{\text{IdBind} \ n, \ \text{MCll}, \ \text{termId} \ t_r\},\)
  \(\{\text{IdRslt} (\text{resultName} \ t_l), \ \text{MRtn}, \ \text{termId} \ t'\},\)
  \(\{\text{IdRslt} (\text{resultName} \ t_r), \ \text{MRtn}, \ \text{termId} \ t'\}\subseteq \text{graph},\)
  \(\text{staticFlowsAcceptTm} \text{staticEnv} \text{graph} \ t_0 \ t_l,\)
  \(\text{staticFlowsAcceptTm} \text{staticEnv} \text{graph} \ t_0 \ t_r,\)
  \(\text{staticFlowsAcceptTm} \text{staticEnv} \text{graph} \ t_0 \ t',\)
  \[⊢ \text{staticFlowsAcceptTm} \text{staticEnv} \text{graph} \ t_0 \ (\text{Bind} \ n \ (\text{Case} \ n_s \ n_l \ n_r \ t_l \ t_r) \ t')\]

* application: \(\text{staticEnv} \ n \ t' \ \text{graph} \ n_f, n_a\).
  \(\forall \ n_f' \ n_p \ t_b \ . \)
  \(\{\text{SAAtm} (\text{Fun} \ n_f' \ n_p \ t_b)\} \subseteq \text{staticEnv} \ n_f\)
  \[→\]
  \(\{\text{IdBind} \ n, \ \text{MCll}, \ \text{termId} \ t_b\},\)
  \(\{\text{IdRslt} (\text{resultName} \ t_b), \ \text{MRtn}, \ \text{termId} \ t'\}\subseteq \text{graph},\)
  \(\text{staticFlowsAcceptTm} \text{staticEnv} \text{graph} \ t_0 \ t'\)
  \[⊢ \text{staticFlowsAcceptTm} \text{staticEnv} \text{graph} \ t_0 \ (\text{Bind} \ n \ (\text{App} \ n_f \ n_a) \ t')\]

**predicate** \text{staticFlowsAccept}:
\(
\text{static} \_\text{value} \_\text{map} \rightarrow \text{graph} \rightarrow \text{term} \rightarrow \text{bool}
\)

**where**
\[\begin{align*}
\text{intro: staticEnv graph n} \ . \ & \text{staticFlowsAcceptTm staticEnv graph} \ t_0 \ t_0 \\
& ⊢ \text{staticFlowsAccept staticEnv graph} \ t_0
\end{align*}\]

The server implementation represented as a graph illustrates how static accep-
tance by flows can interpret a program as a flow graph.

For the liveness of channel analysis, it is necessary to track any name built on a channel. A name is built on a channel if the name binds to a static value containing a channel of interest or a static value that contains names that are built on the channel. For
the name to be considered built on a channel in the case where the tracked name possibly
binds to a function, the channel simply needs to be live in the body of the function. This
condition is represented formally as the requirement that there is a name, such that it is a
free variable in the function, and it’s built on the channel.

```plaintext
fun freeVarsAtom: atom -> name set where
n₁ n₂ .
| freeVarsAtom (SendEvt n₁ n₂) = {n₁, n₂}
* n₁ .
  freeVarsAtom (RecvEvt n₁) = {n₁}
* n₂ .
  freeVarsAtom (Pair n₁ n₂) = {n₁, n₂}
* n .
  freeVarsAtom (Lft n) = {n}
* n .
  freeVarsAtom (Rht n) = {n}
* n₁ n₂ .
  freeVarsAtom (Fun n₁ n₂) = freeVarsTerm t₂ \ {n₁, n₂}

and freeVarsComplex: complex -> name set where
| freeVarsComplex Unt = {} *
| freeVarsComplex MkChn = {}
* atom .
  freeVarsComplex (Atom atom) = freeVarsAtom atom
* t .
  freeVarsComplex (Spwn t) = freeVarsTerm t
* n .
  freeVarsComplex (Sync n) = {n}
* n .
  freeVarsComplex (Fst n) = {n}
* n .
  freeVarsComplex (Snd n) = {n},

* n₁ n₂ t₁ n₃ t₃ \ t₄ .
  freeVarsComplex (Case n₁ n₂ t₁ n₃ t₄) =
    {n₁} \ freeVarsTerm t₁ \ freeVarsTerm t₃ \ {n₃, n₄}
* n₅ .
  freeVarsComplex (App n₅) = {n₅, n₆},

and freeVarsTerm: term -> name set where
n c t .
| freeVarsTerm (Bind n c t) = freeVarsComplex c \ freeVarsTerm t \ {n}
```

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\( \vdash \text{freeVarsTerm} (\text{Rslt} n) = \{n\} \)

**predicate** staticBuiltOnChan: static_value_map \( \to \) name \( \to \) name \( \to \) bool

where

\[
\text{chan}: n_c \ \text{staticEnv} \ n .
\]
\[
\text{SChn} \ n_c \in \text{staticEnv} \ n
\]
\[
\vdash \text{staticBuiltOnChan} \ \text{staticEnv} \ n_c \ n
\]

* sendEvt: \( n_{sc} \) \( n_m \) \text{staticEnv} \ n \ n_c .
\[
(\text{SAtm} (\text{SendEvt} n_{sc} n_m)) \in \text{staticEnv} \ n,
\]
\[
(\text{staticBuiltOnChan} \ \text{staticEnv} \ n_c \ n_{sc}
\]
\[
\lor \ \text{staticBuiltOnChan} \ \text{staticEnv} \ n_c \ n_m
\]
\[
\vdash \text{staticBuiltOnChan} \ \text{staticEnv} \ n_c \ n
\]

* recvEvt: \( n_{rc} \) \text{staticEnv} \ n \ n_c .
\[
(\text{SAtm} (\text{RecvEvt} n_{rc})) \in \text{staticEnv} \ n,
\]
\[
\text{staticBuiltOnChan} \ \text{staticEnv} \ n_c \ n_{rc}
\]
\[
\vdash \text{staticBuiltOnChan} \ \text{staticEnv} \ n_c \ n
\]

* pair: \( n_1 \) \( n_2 \) \text{staticEnv} \ n \ n_c .
\[
(\text{SAtm} (\text{Pair} n_1 n_2)) \in \text{staticEnv} \ n,
\]
\[
(\text{staticBuiltOnChan} \ \text{staticEnv} \ n_c \ n_1
\]
\[
\lor \ \text{staticBuiltOnChan} \ \text{staticEnv} \ n_c \ n_2
\]
\[
\vdash \text{staticBuiltOnChan} \ \text{staticEnv} \ n_c \ n
\]

* left: \( n_a \) \text{staticEnv} \ n \ n_c .
\[
(\text{SAtm} (\text{Lft} n_a)) \in \text{staticEnv} \ n,
\]
\[
\text{staticBuiltOnChan} \ \text{staticEnv} \ n_c \ n_a
\]
\[
\vdash \text{staticBuiltOnChan} \ \text{staticEnv} \ n_c \ n
\]

* right: \( n_a \) \text{staticEnv} \ n \ n_c .
\[
(\text{SAtm} (\text{Rht} n_a)) \in \text{staticEnv} \ n,
\]
\[
\text{staticBuiltOnChan} \ \text{staticEnv} \ n_c \ n_a
\]
\[
\vdash \text{staticBuiltOnChan} \ \text{staticEnv} \ n_c \ n
\]

* function: \( n_f \) \( n_p \) \( t_b \) \( n_{fv} \).
\[
(\text{SAtm} (\text{Fun} n_f n_p t_b)) \in \text{staticEnv} \ n,
\]
\[
n_{fv} \in \text{freeVarsAtom} (\text{Fun} n_f n_p t_b),
\]
\[
\text{staticBuiltOnChan} \ \text{staticEnv} \ n_{fv} \ n
\]
\[
\vdash \text{staticBuiltOnChan} \ \text{staticEnv} \ n_c \ n
\]
The static channel liveness describes entry functions and exit functions. The entry function maps a term identifier to a set of names built on the given channel, if those names are live at the entry of that term identifier. The exit function maps a term identifier to a set of names built on the given channel, if those names are live at the exit of that term identifier.

The following diagram illustrates the entry and exit sets of each term id for channel chn k1 in the server example. Each entry set appears right above its related term identifier, and each exit set appears right below its related term identifier.
The following diagram illustrates the entry and exit sets of each term id for channel chn k4 in the server example. Each entry set appears right above its related term identifier, and each exit set appears right below its related term identifier.
**Predicate** staticChanLive:

\[
\text{static\_value\_map} \to \text{tm\_id\_map} \to \text{tm\_id\_map} \to \text{name} \to \text{term} \to \text{bool}
\]

**Where**

\[
\text{result: staticEnv \text{entr n}_c \text{n}_y \text{exit} .}
\]

(\[
\text{(staticBuiltOnChan staticEnv n}_c \text{n}_y)}
\]
→ \{n_y\} ⊆ \text{entr (IdRslt \ n_y)}
)
⊢ \text{staticChanLive staticEnv entr exit \ n_c (Rslt \ n_y)}

* \text{unit: exit } \ n \ \text{entr } \ t' \ \text{staticEnv } \ n_c .
  \begin{align*}
  & (\text{exit (IdBind } n) \setminus \{n\} \subseteq \text{entr (IdBind } n), \\
  & \text{entr (termId } t') \subseteq \text{exit (IdBind } n), \\
  & \text{staticChanLive staticEnv entr exit } \ n_c \ t' \\
  \end{align*}
⊢ \text{staticChanLive staticEnv entr exit } \ n_c \ (\text{Bind } n \ \text{Unt } t')

* \text{makeChan: exit } \ n \ \text{entr } \ t' \ \text{staticEnv } \ n_c .
  \begin{align*}
  & (\text{exit (IdBind } n) \setminus \{n\} \subseteq \text{entr (IdBind } n), \\
  & \text{entr (termId } t') \subseteq \text{exit (IdBind } n), \\
  & \text{staticChanLive staticEnv entr exit } \ n_c \ t' \\
  \end{align*}
⊢ \text{staticChanLive staticEnv entr exit } \ n_c \ (\text{Bind } n \ \text{MkChn } t')

* \text{sendEvt: exit } \ n \ \text{entr } \ t' \ \text{staticEnv } \ n_c \ n_{sc} \ n_m \ t' \ \text{\ n_c} .
  \begin{align*}
  & (\text{exit (IdBind } n) \setminus \{n\} \subseteq \text{entr (IdBind } n), \\
  & \text{staticBuiltOnChan staticEnv } \ n_c \ n_{sc} \\
  & \rightarrow \{n_{sc}\} \subseteq \text{entr (IdBind } n) \\
  \end{align*}
, \begin{align*}
  & \text{staticBuiltOnChan staticEnv } \ n_c \ n_m \\
  & \rightarrow \{n_m\} \subseteq \text{entr (IdBind } n) \\
  \end{align*}
, \begin{align*}
  & \text{entr (termId } t') \subseteq \text{exit (IdBind } n), \\
  & \text{staticChanLive staticEnv entr exit } \ n_c \ t' \\
  \end{align*}
⊢ \text{staticChanLive staticEnv entr exit } \ n_c \ (\text{Bind } n \ (\text{Atom (SendEvt } n_{sc} \ n_m)) \ t')

* \text{recvEvt: exit } \ n \ \text{entr } \ t' \ \text{staticEnv } \ n_c \ n_r \ n_{rc} .
  \begin{align*}
  & (\text{exit (IdBind } n) \setminus \{n\} \subseteq \text{entr (IdBind } n), \\
  & \text{staticBuiltOnChan staticEnv } \ n_c \ n_r \\
  & \rightarrow \{n_r\} \subseteq \text{entr (IdBind } n) \\
  \end{align*}
, \begin{align*}
  & \text{entr (termId } t') \subseteq \text{exit (IdBind } n), \\
  & \text{staticChanLive staticEnv entr exit } \ n_c \ t' \\
  \end{align*}
⊢ \text{staticChanLive staticEnv entr exit } \ n_c \ (\text{Bind } n \ (\text{Atom (RecvEvt } n_{rc})) \ t')

* \text{pair: exit } \ n \ \text{entr } \ t' \ \text{staticEnv } t_c \ n_1 \ n_2 \ t' .
  \begin{align*}
  & (\text{exit (IdBind } n) \setminus \{n\} \subseteq \text{entr (IdBind } n), \\
  \end{align*}

\[
\begin{align*}
\text{staticBuiltOnChan } & \text{staticEnv } n_c \text{ } n_1 \rightarrow \{n_1\} \subseteq \text{entr (IdBind } n) \\
\text{} & ), \\
\text{} & (\text{staticBuiltOnChan } \text{staticEnv } n_c \text{ } n_2 \\
\text{} & \rightarrow \{n_2\} \subseteq \text{entr (IdBind } n) \\
\text{} & ), \\
\text{entr (termId } t') & \subseteq \text{exit (IdBind } n), \\
\text{staticChanLive } & \text{staticEnv } \text{entr exit } n_c \text{ } t' \\
\vdash \text{staticChanLive } & \text{staticEnv } \text{entr exit } n_c \text{ } (\text{Bind } n \text{ } (\text{Atom (Pair } n_1 \text{ } n_2)) \text{ } t')) \\
\end{align*}
\]

* left: \text{exit } n \text{ } \text{entr staticEnv } n_c \text{ } n_a \text{ } t'.
\text{} \text{(exit (IdBind } n) \setminus \{n\} \subseteq \text{entr (IdBind } n), \\
\text{} \text{staticBuiltOnChan } \text{staticEnv } n_c \text{ } n_a \\
\text{} \rightarrow \{n_a\} \subseteq \text{entr (IdBind } n) \\
\text{} ), \\
\text{entr (termId } t') & \subseteq \text{exit (IdBind } n), \\
\text{staticChanLive } & \text{staticEnv } \text{entr exit } n_c \text{ } t' \\
\vdash \text{staticChanLive } & \text{staticEnv } \text{entr exit } n_c \text{ } (\text{Bind } n \text{ } (\text{Atom (Lft } n_a)) \text{ } t'))
\]

* right: \text{exit } n \text{ } \text{entr staticEnv } n_c \text{ } n_a \text{ } t'.
\text{} \text{(exit (IdBind } n) \setminus \{n\} \subseteq \text{entr (IdBind } n), \\
\text{} \text{staticBuiltOnChan } \text{staticEnv } n_c \text{ } n_a \\
\text{} \rightarrow \{n_a\} \subseteq \text{entr (IdBind } n) \\
\text{} ), \\
\text{entr (termId } e) & \subseteq \text{exit (IdBind } n), \\
\text{staticChanLive } & \text{staticEnv } \text{entr exit } n_c \text{ } t \\
\vdash \text{staticChanLive } & \text{staticEnv } \text{entr exit } n_c \text{ } (\text{Bind } n \text{ } (\text{Atom (Rht } n_a)) \text{ } e)
\]

* function: \text{exit } n \text{ } \text{entr } t_b \text{ } n_p \text{ } n \text{ } \text{staticEnv } n_c \text{ } t' \text{ } n_f.
\text{} \text{(exit (IdBind } n) \setminus \{n\} \subseteq \text{entr (IdBind } n), \\
\text{} \text{(entr (termId } t_b) \setminus \{n_f, n_p\} \subseteq \text{entr (IdBind } n), \\
\text{staticChanLive } & \text{staticEnv } \text{entr exit } n_c \text{ } t_b, \\
\text{entr (termId } t') & \subseteq \text{exit (IdBind } n), \\
\text{staticChanLive } & \text{staticEnv } \text{entr exit } n_c \text{ } t' \\
\vdash \text{staticChanLive } & \text{staticEnv } \text{entr exit } n_c \text{ } (\text{Bind } n \text{ } (\text{Atom (Fun } n_f \text{ } n_p \text{ } t_b)) \text{ } t'))
\]

* spawn: \text{exit } n \text{ } \text{entr } t' \text{ } t_c \text{ } n_c \text{ } \text{staticEnv}.
\text{} \text{(exit (IdBind } n) \setminus \{n\} \subseteq \text{entr (IdBind } n), \\
\text{} \text{(entr (termId } t') \subseteq \text{exit (IdBind } n), \\
\text{} \text{(entr (termId } t_c) \subseteq \text{exit (IdBind } n), \\
\text{staticChanLive } & \text{staticEnv } \text{entr exit } n_c \text{ } t_c,
\[
\text{staticChanLive \ staticEnv \ entr \ exit \ n_c \ t'}
\]
\[\vdash \text{staticChanLive \ staticEnv \ entr \ exit \ n_c} \]
\[\quad \text{(Bind \ n \ (Spwn \ t_c) \ t')}\]

* sync: exit n entr staticEnv n_c n_c t'.
\[
\text{(exit (IdBind n) \ \{n\}) \subseteq entr (IdBind n),}
\]
\[
\text{staticBuiltOnChan staticEnv \ n_c \ n_c}
\]
\[
\rightarrow \ \{n_c\} \subseteq entr \ (IdBind n)
\]
\[
\text{entr (termId t') \subseteq exit (IdBind n),}
\]
\[
\text{staticChanLive staticEnv \ entr \ exit \ n_c} \]
\[
\quad \text{(Bind \ n \ (Sync \ n_c) \ t')}
\]

* first: exit n entr staticEnv n_c n_a t'.
\[
\text{(exit (IdBind n) \ \{n\}) \subseteq entr (IdBind n),}
\]
\[
\text{staticBuiltOnChan staticEnv \ n_c \ n_a}
\]
\[
\rightarrow \ \{n_a\} \subseteq entr \ (IdBind n)
\]
\[
\text{entr (termId t') \subseteq exit (IdBind n),}
\]
\[
\text{staticChanLive staticEnv \ entr \ exit \ n_c} \]
\[
\quad \text{(Bind \ n \ (Fst \ n_a) \ t')}
\]

* second: exit n entr staticEnv n_c n_a t'.
\[
\text{(exit (IdBind n) \ \{n\}) \subseteq entr (IdBind n),}
\]
\[
\text{staticBuiltOnChan staticEnv \ n_c \ n_a}
\]
\[
\rightarrow \ \{n_a\} \subseteq entr \ (IdBind n)
\]
\[
\text{entr (termId t') \subseteq exit (IdBind n),}
\]
\[
\text{staticChanLive staticEnv \ entr \ exit \ n_c} \]
\[
\quad \text{(Bind \ n \ (Snd \ n_a) \ t')}
\]

* distinction: exit n entr t_l n_l t_r \ staticEnv n_c \ n_s \ t'.
\[
\text{(exit (IdBind n) \ \{n\}) \subseteq entr (IdBind n),}
\]
\[
\text{entr (termId t_l) \ \{n_l\} \subseteq entr (IdBind n),}
\]
\[
\text{entr (termId t_r) \ \{n_r\} \subseteq entr (IdBind n),}
\]
\[
\text{staticBuiltOnChan staticEnv \ n_c \ n_s}
\]
\[
\rightarrow \ \{n_s\} \subseteq entr \ (IdBind n)
\]
\[
\text{staticChanLive staticEnv \ entr \ exit \ n_c \ t_l,}
\]
\[
\text{staticChanLive staticEnv \ entr \ exit \ n_c \ t_r,}
\]
\[
\begin{align*}
\text{entr } (\text{termId } t') & \subseteq \text{exit } (\text{IdBind } n), \\
\text{exit } (\text{IdRslt } (\text{resultName } t)) & \subseteq \text{entr } (\text{tmId } t'), \\
\text{exit } (\text{IdRslt } (\text{resultName } t_r)) & \subseteq \text{entr } (\text{tmId } t'), \\
\text{staticChanLive } \text{staticEnv } \text{entr } \text{exit } n_c \; t' & \vdash \text{staticChanLive } \text{staticEnv } \text{entr } \text{exit } n_c \; (\text{Bind } n \; (\text{Case } n_s \; n_l \; t_l \; n_r \; t_r) \; t') \\
\end{align*}
\]

* application: \(\text{exit } n \; \text{entr } \text{staticEnv } n_c \; n_a \; n_f \; t'\).

\[
\begin{align*}
(\text{exit } (\text{IdBind } n) \setminus \{n\}) & \subseteq \text{entr } (\text{IdBind } n), \\
(\text{staticBuiltOnChan } \text{staticEnv } n_c \; n_a & \rightarrow \{n_a\} \subseteq \text{entr } (\text{IdBind } n)) \\
(\text{staticBuiltOnChan } \text{staticEnv } n_c \; n_f & \rightarrow \{n_f\} \subseteq \text{entr } (\text{IdBind } n) ) \\
\text{entr } (\text{termId } t') & \subseteq \text{exit } (\text{IdBind } n), \\
(\text{SAtm } (\text{Fun } n_f' \; x_p \; t_b) & \in \text{staticEnv } n_f \\
& \rightarrow \text{exit } (\text{IdRslt } (\text{resultName } t_b)) \subseteq \text{entr } (\text{tmId } t'); \\
\text{staticChanLive } \text{staticEnv } \text{entr } \text{exit } n_c \; t' & \vdash \text{staticChanLive } \text{staticEnv } \text{entr } \text{exit } n_c \; (\text{Bind } n \; (\text{App } n_f \; n_a) \; t') \\
\end{align*}
\]

The following diagram illustrates the graph of the server example, containing only live flows for channel \text{chn} \; k1.
The following diagram illustrates the graph of the server example, containing only live flows for channel k4.
The static path liveness for the higher precision analysis states that a channel is live for an entire static path found within a graph.

**Predicate** staticPathLive:
- graph -> tm_id_map -> tm_id_map -> tm_id
- (tm_id -> bool) -> static_path -> bool
**Where**
- empty: graph entr exit start isEnd .
  ⊢ staticPathLive graph entr exit start isEnd []
- edge: graph entr exit start middle path isEnd end .
  staticPathLive graph entr exit start (λ l . l = middle) path,
  isEnd end,
  (middle, MSeq, end) ∈ graph,
  ¬ Set.is_empty (exit middle),
  ¬ Set.is_empty (entr end)
  ⊢ staticPathLive graph entr exit start isEnd (path @ [(middle, edge)])

As with the lower precision analysis, the higher precision analysis relies on recognizing whether or not two paths can actually occur within in a single run of a program.

The static inclusiveness states which paths might occur within the same run of the pro-
gram. In contrast to the analogous definition for the lower precision analysis, the higher precision definition needs to consider paths containing the sending mode. As mentioned earlier, the path from the synchronization on sending to the synchronization on receiving is necessary to ensure that all uses of a channel are reachable from the channel’s creation identifier. The singularity means that only one of the two given paths can occur in a run of program. The uncompetitiveness means that the two given paths do not compete in any run of a program.

**predicate** staticInclusive: static_path -> static_path -> bool where

ordered: path1 path2 .

prefix path1 path2 \lor path2 path1

\[\vdash \text{staticInclusive} \text{ path1 path2} \]

* spawnLeft: path n path1 path2 .

\[\vdash \text{staticInclusive} \]

(path @ [(IdBind n, MSpwn)] @ path1)

(path @ [(IdBind n, MSeq)] @ path2)

* spawnRight: path n path1 path2 .

\[\vdash \text{staticInclusive} \]

(path @ [(IdBind n, MSeq)] @ path1)

(path @ [(IdBind n, MSpwn)] @ path2)

* sendLeft: path n path1 path2 .

\[\vdash \text{staticInclusive} \]

(path @ [(IdBind n, MSend n \text{e})] @ path1)

(path @ [(IdBind n, MSeq)] @ path2)

* sendRight: path n path1 path2 .

\[\vdash \text{staticInclusive} \]

(path @ [(IdBind n, MSeq)] @ path1)

(path @ [(IdBind n, MSend n \text{e})] @ path2)

**predicate** singular: static_path -> static_path -> bool where

refl: path .

\[\vdash \text{singular path path} \]

* notInclus: path1 path2 .

\[\neg \text{(staticInclusive path1 path2)} \]

\[\vdash \text{singular path1 path2} \]

**predicate** uncompetitive: static_path -> static_path -> bool where

ordered: path1 path2 .

ordered path1 path2

\[\vdash \text{uncompetitive path1 path2} \]
The communication classifications are described using the liveness properties, but are otherwise similar to the lower precision classifications.

**Predicate** staticOneToMany: term -> name -> bool where
intro: staticEnv staticComm t graph entr exit n
staticEval staticEnv staticComm t,
staticFlowsAccept staticEnv graph t,
staticChanLive staticEnv entr exit n t,
forEveryTwo (staticPathLive graph entr exit (IdBind n) (staticSendId staticEnv t n)) uncompetitive
⊢ staticOneToMany t n

**Predicate** staticManyToOne: term -> name -> bool where
intro: staticEnv staticComm t graph entr exit n
staticEval staticEnv staticComm t,
staticFlowsAccept staticEnv graph t,
staticChanLive staticEnv entr exit n t,
forEveryTwo (staticPathLive graph entr exit (IdBind n) (staticRecvId staticEnv t n)) uncompetitive
⊢ staticManyToOne t n

**Predicate** staticOneToOne: term -> name -> bool where
intro: staticEnv staticComm t graph entr exit n
staticEval staticEnv staticComm t,
staticFlowsAccept staticEnv graph t,
staticChanLive staticEnv entr exit n t,
forEveryTwo (staticPathLive graph entr exit (IdBind n) (staticSendId staticEnv t n)) uncompetitive,
forEveryTwo (staticPathLive graph entr exit (IdBind n) (staticRecvId staticEnv t n)) uncompetitive
⊢ staticOneToOne t n

**Predicate** staticOneShot: term -> name -> bool where
intro: staticEnv staticComm t graph entr exit n
staticEval staticEnv staticComm t,
staticFlowsAccept staticEnv graph t,
staticChanLive staticEnv entr exit n t,
forEveryTwo (staticPathLive graph entr exit (IdBind n) (staticSendId staticEnv t n)) singular

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\[\vdash \text{staticOneShot } t \ n \ c\]

**predicate** staticOneSync: term -> name -> bool where
intro: staticEnv staticComm t graph entr exit \(n_c\).  
staticEval staticEnv staticComm t,  
staticFlowsAccept staticEnv graph t,  
staticChanLive staticEnv entr exit \(n_c\) t,  
forEveryTwo (staticPathLive graph entr exit (IdBind \(n_c\)) (staticSendId staticEnv t n c)) singular,  
forEveryTwo (staticPathLive graph entr exit (IdBind \(n_c\)) (staticRecvId staticEnv t n c)) uncompetitive  
\[\vdash \text{staticOneSync } t \ n \ c\]

### 4.1 Higher Precision Soundness Proof Strategy

To prove soundness of the static communication classification, it should be possible to use previous techniques of generalizing propositions over pools and other semantic components, along with finding equivalent representations of propositions that vary in their inductive subcomponent. One thing that will make carrying out the formal proof particularly tricky is that dynamic paths in the dynamic semantics need to correspond to static paths from the trimmed graphs, which might also contain sending flows, instead of the spawning flows of the dynamic paths. The correspondence between these dynamic paths and static paths is not bijective, as it is for the lower precision analysis. However, finding a satisfactory correspondence for each dynamic and static path is critical for proving soundness.

Essentially, it will be necessary to show that static properties that hold for some static path are preserved for corresponding dynamic paths. However, in the higher precision analysis these paths correspond modulo the channel of interest. An outline of the derivation of soundness of one-shot classification demonstrates the strategy so far.
Theorem staticOneShotSound: t₀ nₙ pathₙ.
staticOneShot t₀ nₙ ⊢ oneShot t₀ (Chan pathₙ nₙ)

The theorem for soundness of one-shot classification depends on correlating dynamic paths with static paths.

Predicate pathsCorrespond: dynamic.path -> static.path -> bool where

empty: pathsCorrespond [] []

* seq: path staticPath n.
  pathsCorrespond path staticPath ⊢ pathsCorrespond
  (path @ [DSeq n])
  (staticPath @ [[IdBind n, MSeq]])

* spawn: path staticPath n.
  pathsCorrespond path staticPath
⊢ pathsCorrespond
  (path @ [DSpwn n])
  (staticPath @ [(IdBind n, MSpwn)])

* call: path staticPath n .
  pathsCorrespond path staticPath
⊢ pathsCorrespond
  (path @ [DCll n])
  (staticPath @ [(IdBind n, MCll)])

* return: path staticPath n .
  pathsCorrespond path staticPath
⊢ pathsCorrespond
  (path @ [DRtn n])
  (staticPath @ [(IdRslt n, MRtn)])

**predicate** dynamicBuiltOnChanVal val -> chan -> bool where
  chan: chan .
⊢ dynamicBuiltOnChanVal (VChn chan) chan

* closure: env c atom .
  dynamicBuiltOnChanEnv env chan
⊢ dynamicBuiltOnChanVal (VClsr atom env) chan

and dynamicBuiltOnChanEnv: environment -> chan -> bool where
  intro: env n v chan .
  env n = Some v,
  dynamicBuiltOnChanVal v chan
⊢ dynamicBuiltOnChanEnv env chan

**predicate** pathsCorrespondModChan:
  tm -> chan -> dynamic_path -> static_path -> bool
where
  chan: tr z nz path c fx stt staticPath comm .
  pathsCorrespond ((DSeq nz) # path c fx) staticPath
⊢ pathsCorrespondModChan
  t0 (Chan path, nz)
  (path c @ (DSeq nz) # path c fx) staticPath

* send: t0 pool comm n, path c fx stt path, nz, nsc stt s, env, stack sy, stack ry, nrc t ry env ry stack ry chan c chan staticPath c staticPath c sfx .
  star dynamicEval [] -> (Stt t0 [-] []) [] pool comm,
  pool path, = Some (Stt (Bind n, Sync nsc) t sy) env, stack sy,
  pool path, = Some (Stt (Bind n, Sync nrc) t ry) env, stack ry),
\{(path_s, chan_c, path_r)\} \subseteq \text{comm},
\text{env}_{\text{ry}} \text{ chan } n_r = \text{Some } v_{\text{ry}}
\text{dynamicBuiltOnChanVal } v_{\text{ry}} \text{ chan},
\text{pathsCorrespondModChan } t_0 \text{ chan path}_s \text{ staticPath}_{pfx},
\text{pathsCorrespond path}_{sfx} \text{ staticPath}_{sfx}
\vdash \text{pathsCorrespondModChan } t_0 \text{ chan}
\text{ (path}_s \ast (\text{DSeq } n_r) \# \text{path}_{sfx})
\text{ (staticPath}_{pfx} \ast [(\text{IdBind } n_s, \text{MSend } n_{sc}), (\text{IdBind } n_r, \text{MSeq})] \ast 
\text{staticPath}_{sfx})

Additionally the soundness theorem follows from the soundness of static path liveness, the soundness of static inclusiveness, and the soundness of sending identifier classification. The reasoning about the sending identifier classification is identical to that of the lower precision analysis, but the reasoning for the former two is significantly more complicated and not yet completed. The complication arises from the correlation between dynamic paths and static paths. The proofs depend on finding a static path that depends on a given dynamic path. In the lower precision analysis the correlation was straightforward. There was only one possible static path to choose for it to correlate with the given dynamic path. In the higher precision analysis, the relationship between the two kinds of paths is not so simple, and finding a description of the static path that correlates with the dynamic path is more challenging.

The proposition isSendPath pool' (Chan path_c n_c) path' is derived by unfolding the definition of oneShot. It is generalized by pool' path' = Some (Stt t' env' stack'),
dynamicBuiltOnChanState (Stt t' env' stack') (Chan path_c n_c) in the soundness of static path liveness. Since the static chan liveness depends on names being statically built on the channel, the soundness theorem also depends on the soundness of static built-on-channel classification with respect to the dynamic built-on-channel classification.
**predicate** dynamicBuiltOnChanStack: contin list -> chan -> bool where
  env: env_k chan n_k t_k stack’.
  dynamicBuiltOnChanEnv env_k chan
  ⊢ dynamicBuiltOnChanStack (Ctn n_k t_k env_k # stack’) chan
  * stack: env_k chan n_k t_k stack’.
  dynamicBuiltOnChanStackStack stack’ chan
  ⊢ dynamicBuiltOnChanStack (Ctn n_k t_k env_k # stack’) chan

**predicate** dynamicBuiltOnChanState: state -> chan -> bool where
  env: env chan t stack’,
  dynamicBuiltOnChanEnvEnv env chan
  ⊢ dynamicBuiltOnChanState (Stt t env stack) chan
  * stack: stack chan t env’.
  dynamicBuiltOnChanStackStack stack chan
  ⊢ dynamicBuiltOnChanState (Stt t env stack) chan

**lemma** staticPathLiveSound: t_0 pool comm path t env stack
  staticEnv staticComm entr exit n_c graph isEnd path_c.
  star dynamicEval [] -> (Stt t_0 [-> []]) {} pool comm,
  pool path = Some (Stt t env stack),
  dynamicBuiltOnChanState (Stt t env stack) (Chan path_c n_c)
  staticEval staticEnv staticComm t_0,
  staticChanLive staticEnv entr exit n_c t_0,
  staticFlowsAccept staticEnv graph t_0,
  isEnd (tmId t)
  ⊢ exists staticPath.
  pathsCorrespondModChan pool comm (Chan path_c n_c) path staticPath
  ∧ staticTraceable graph entr exit (IdBind n_c) isEnd staticPath

**lemma** staticInclusiveSound: t_0 pool comm staticEnv entr exit n_c graph
  staticComm
  path1 stt1 path_c staticPath1 path2 stt2 staticPath2.
  star dynamicEval [] -> (Stt t_0 [-> []]) {} pool comm,
  staticChanLive staticEnv entr exit n_c t_0,
  staticFlowsAccept staticEnv graph t_0,
  staticEval staticEnv staticComm t_0,
  pool path1 = Some stt1,
  pathsCorrespondModChan pool comm (Chan path_c n_c) path1 staticPath1,
  staticTraceable graph entr exit
  (IdBind n_c) (staticSendId staticEnv t_0 n_c) staticPath1,
  pool path2 = Some stt2,
  pathsCorrespondModChan pool comm (Chan path_c n_c) path2 staticPath2,
  staticTraceable graph entr exit

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Additional lemmas are needed to prove the soundness of static path liveness. These lemmas include preservation of the correspondence of paths, preservation of the static chan liveness, and soundness of a name statically built on a static channel.

**Lemma** pathsCorrespondModChanPreservedSnoc:
\[
\begin{align*}
t_0 & \text{ chan path staticPath site step}, \\
\text{pathsCongruentModChan} \ t_0 \text{ chan path staticPath,} \\
\text{pathsCongruent} \ [\text{site}] \ [\text{step}] & \quad \vdash \quad \text{pathsCongruentModChan} \ t_0 \text{ chan (path @ [site]) (staticPath @ [step])}
\end{align*}
\]

**Lemma** staticChanLivePoolPreserved: staticEnv entr exit x c t_0 pool’ comm’.
\[
\begin{align*}
\text{staticLiveChanPool} \ \text{staticEnv} \ \text{entr exit} \ x \ c \ [] & \to (\text{Stt} \ t_0 \ [-] \ []), \\
\text{staticEval} \ \text{staticEnv} \ \text{staticComm} \ t_0, \\
\text{star} \ \text{dynamicEval} \ [] & \to (\text{Stt} \ t_0 \ [-] \ [])) \} \ \text{pool’ comm’} \\
\vdash \quad \text{staticLiveChanPool} \ \text{staticEnv} \ \text{entr exit} \ x \ c \ \text{pool’}
\end{align*}
\]

**Lemma** staticBuiltOnChanSound: v path c n env n pool path
\[
\begin{align*}
t’ \ \text{env’ stack’} & \text{ staticEnv staticComm pool graph t_0,} \\
\text{dynamicBuiltOnChanVal} \ v (\text{Chan path c n}), \\
\text{env n} & = \text{Some v,} \\
\text{pool path} & = \text{Some (Stt t’ env’ stack’),} \\
\text{staticEvalPool} \ \text{staticEnv staticComm pool,} \\
\text{staticChanLivePool} \ \text{staticEnv entr exit} \ n_c \ \text{pool,} \\
\text{staticFlowsAcceptPool} \ \text{staticEnv graph t_0 pool} \\
\vdash \quad \text{staticBuiltOnChan} \ \text{staticEnv} \ n_c \ n
\end{align*}
\]

The formal proofs for these lemmas are under active development at the time of this writing.
Chapter 5

Conclusion

In this thesis, I developed a formal theory for a subset of Concurrent ML. The main contributions consist of formal specifications of dynamic and static evaluation, dynamic and static communication classification, dynamic and static channel liveness, and various related concepts. The formal specifications enabled the creation of mechanically checked formal proofs of soundness of static evaluation and static communication classification. Through the process of developing these formal proofs, I noticed a number of important reasoning patterns - generalization of propositions to related concepts on alternate structures; skewing the direction of induction to different subparts of structures; changing the direction of inference from forward to backward. This work is a small contribution based on a significant amount of previous related work. Additionally, there are many ways for this work to be extended.

5.1 Related Work

There has been much research on both dynamic and static analysis of concurrent languages. The formal communication classification analysis and soundness proofs in this work are based on the analysis and proofs of Specialization of CML message-passing primitives by Reppy and Xiao [17]. The mechanization of concurrency analyses is prevalent, and mechanization is typically the main goal when developing the analyses. Examples include
type checkers in compilers, model checking tools for concurrency models, such as Lustre [9] and Kind [20], and also verification libraries in proof assistants, such as Affeldt et al’s Coq library [2]. These systems can verify certain properties of concurrency programs or models, but they don’t make any guarantees about the analysis itself. Rather than focus on mechanizing the analysis, this work has focused on mechanizing the theory of analyses for concurrent languages, i.e. the meta-theory of concurrency. There have been a number of works on the meta-theory of Concurrent ML, such as the work of Reppy and Xiao, Nielson et al [15], Kobayashi et al [11], and Gasser et al [7]. There has been relatively little work to mechanize theories of Concurrent ML; however, there has been much work in the mechanization of the theories of $\pi$-calculus [13], such as the work by Gay [8] and Melham [12].

5.2 Future Work

The formal syntax, semantics, and communication analysis of this work form the basis of a framework for studying Concurrent ML events, synchronization mechanisms, and their applications. These language features enable the construction of reactive programs, which have separation of parts that are conceptually distinct, yet still depend on each other.

This work has kicked off the framework with a formal communication analysis that has practical applications in aiding optimizations for parallel computation. In the future, additional analyses could be built on the existing semantics, in order to verify the correctness of language extensions or optimizations. Extending the semantics to handle event combinators for choosing events, sequencencing events, guarding events, among
others, would be an important next step.

Concurrency is a double edged sword. Without specification of ordering, programs may describe their behavior more clearly or allow parallelism for faster execution. On the other hand, unspecified orderings may also lead to nondeterministic behavior, which may be not be wanted. To gain the benefits of concurrency without its hindrance, the language could be extended with syntax to identify blocks of code that are required to be deterministic, along with a corresponding static analysis that checks if such code is actually deterministic. The determinism analysis could rely on the static communication analysis to ensure that all synchronized receiving events receive from at most one channel, and that channel is sent on by at most one thread, and that thread is also deterministic. Other analyses could aid optimizations for incremental computation [1]. One possible optimization could transform a program into one that checks for altered dependencies and only recomputes the data that depends on altered dependencies.
Bibliography


