The Influence of Tooth Surface Modifications on the Sensitivity of Involute Cylindrical Gears to Manufacturing Errors

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The Influence of Tooth Surface Modifications on the Sensitivity of Involute Cylindrical Gears to Manufacturing Errors

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A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in the Department of Mechanical Engineering of the Kate Gleason College of Engineering

April 24, 2018
Declaration of Authorship

I, Scott Eisele, declare that this thesis titled, “The Influence of Tooth Surface Modifications on the Sensitivity of Involute Cylindrical Gears to Manufacturing Errors” and the work presented in it are my own. I confirm that:

- This work was done wholly or mainly while in candidature for a research degree at this University.

- Where I have consulted the published work of others, this is always clearly attributed.

- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work.

- I have acknowledged all main sources of help.

- Where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself.

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“Projects we have completed demonstrate what we know, future projects decide what we will learn.”

Dr. Mohsin Tiwana
Abstract

The Influence of Tooth Surface Modifications on the Sensitivity of Involute Cylindrical Gears to Manufacturing Errors

by Scott Eisele

Gear drives, like all mechanical devices, are subject to manufacturing errors and defects. Higher levels of precision in manufacturing to increase the quality of the gears are costly, but sometimes required to attain sufficient performance from a gear drive. However, certain intentional micro-geometry modifications to the gear tooth surfaces can be used to reduce the sensitivity of the gear drive to manufacturing errors. These changes in tooth surface design can be simple and cheap to implement, and result in a more robust gear drive which is more indifferent to manufacturing errors. In this thesis, the influence on intentional micro-geometry modifications of the gear tooth surfaces on the mechanical performance of gear drives for different levels of manufacturing errors or associated quality numbers will be investigated. The finite element method will be used to determine the extent to which these gear tooth surface modifications could be used to reduce peak contact and bending stresses in the gear tooth, as well as their influence on the gear drive’s function of transmission errors. Results have shown that the application of micro-geometry modifications to the gear tooth surfaces can drastically decrease the sensitivity of peak contact and bending stresses within a gear drive to manufacturing errors, reducing the manufacturing costs and enabling higher levels of transmitted power.
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List of Abbreviations

AGMA  American Gear Manufacturing Association
DOE   Design of Experiments
ISO   International Standardization Organization
LTE   Loaded Function of Transmission Errors
PTP   Peak-to-peak values, typically used to describe a function of transmission error
List of Symbols

\(C_m\)  Crowning Magnitude  \(\mu m\)
\(L\)  Datum length of tip relief  \(mm\)
\(r_b\)  Gear base radius  \(mm\)
\(r_L\)  Rolling length of tip relief  \(mm\)
\(r_p\)  Tip relief starting radius  \(mm\)
\(\delta\)  Amount of tip relief at tip of gear tooth  \(\mu m\)
\(\epsilon\)  Rolling angle of tip relief  \(rad\)
Chapter 1

Introduction

1.1 Introduction

Gear drives are used to transmit power with change of speed, torque, or direction of motion. Toothed gear drives have been used by humans since the 4th century BCE. There even exists remains of complex gear mechanisms dating as far back as 87 BCE. The Antikythera mechanism is one of the oldest remaining complex mechanisms utilizing gearing, which is theorized to have been used to predict astronomical phenomenon [1]. This mechanism included planetary gearing, coaxial gearing, and what is estimated to be 37 gear wheels which could predict the phase of the moon, the positions of the planets, constellations, and the timing of festivals and events.

FIGURE 1.1: Photograph of the Antikythera Mechanis. (Photo by Brett Seymour)
Gear technology in the ancient world was used for far more than only predicting astronomical events. The earliest ancient Chinese "South Pointing Chariot" dates from sometime between 200 and 265 CE. This mechanism utilized differential gearing to enable a jade statue of a man to face the same direction, regardless of how the chariot was translated or rotated.

![Figure 1.2: Photograph of a recreation of the South Pointing Chariot. (Photo by Andy Dingley)](image)

Although some complex gearing mechanisms exist from ancient times, early gears were mostly made with simple cylindrical cogs or triangular teeth, which do not guarantee smooth, continuous rotation between gears. Gears with such simple teeth tend to be effective at relatively low operating speeds, but are not suitable for the multitude of high speed applications which gears are commonly used for today. Some tooth profiles were developed later on, using metal teeth which could be machined more precisely. Of these early designs, the cycloidal gear drive is the only one still in use today. Cycloidal gear drives utilize teeth defined by epicycloid and hypocycloid curves. Gears with cycloidal teeth have the distinct advantage of maintaining constant angular momentum between the two gears throughout a cycle of meshing, which is critical in high speed applications where teeth are meshing rapidly. Figure 1.3 illustrates the
Mathematician Leonard Euler recognized that gears with teeth traced out of an involute profile could also mesh with a constant ratio of angular velocity between the two gears throughout the entire cycle of meshing [2]. In addition, involute gear drives are relatively easy to manufacture, and the smooth cycle of meshing does not depend on the center distance between the two gears, enabling better performance even if supporting shafts are not perfectly aligned. Since its invention, involute gearing has gained widespread use, since the smooth meshing of involute gear teeth helps them to operate well at high speeds of rotation. Figure 1.4 demonstrates the meshing of two involute gear teeth.

Involute gear drives experience reduced performance under high loads due to deformations experienced by the gear teeth and their influence on the localization of the contact pattern and function of transmission errors. One method of improving gear performance under high loading conditions is to design surface modifications for the gear tooth surfaces. It has been demonstrated that profile modifications on the micron scale, such as tip relief, longitudinal crowning, and profile crowning can improve gear performance under these conditions [3, 4, 5, 6, 7, 8, 9]. In high torque applications, gear teeth and supporting shafts experience relatively large amounts of deflection, resulting in higher contact and bending stresses on tooth surfaces.
Tip relief modifications improve gear performance by allowing for increased clearance in highly loaded regions of the tooth [2, 3, 4, 7, 8, 10]. Longitudinal crowning localizes the point of contact along the teeth, which can reduce the influence of errors of alignment on the gear drive [2, 8]. Profile crowning has a direct influence on the function of transmission errors over the meshing cycle of the gear teeth. Profile crowning creates a parabolic function of transmission errors which can absorb the more unfavorable transmission error functions [9, 11].

![Illustration of an involute gear drive, with the line of action and base circles shown (by Claudio Rocchini).](image)

For high load operation conditions, AGMA and ISO recommend gears of tighter manufacturing tolerances, to minimize the effects of manufacturing errors on gear performance. However, gears of tighter manufacturing tolerances are more expensive and challenging to manufacture than gears of looser tolerances. In fact, gears of higher quality numbers (according to AGMA 2000-A88) feature exponentially higher manufacturing costs [12]. Figure 1.5 shows the relationship between the costs of manufacturing a gear and its quality standard.
The combined effects of gear tooth surface modifications and manufacturing errors are yet to be studied intensively. The only existing work which considered both tooth surface modifications and manufacturing errors utilized an express model for the determination of deformations, and included few trials due to the computational intensity of each model [6]. In this work, it was demonstrated that tooth surface modifications were able to compensate somewhat for manufacturing errors and improve gear performance. It was also determined that surface modifications could negatively impact gear performance if the modifications were non-optimal. However, when applied correctly, this could allow for the utilization of gears of lower quality standards in high load applications, reducing manufacturing costs.

In this thesis, the combined effect of gear manufacturing errors and intentional gear tooth surface modifications on the mechanical performance of spur and helical gear drives in high torque applications is investigated. Different amounts of manufacturing errors have been considered according to the ISO standard for gear tolerances. The influence of pitch deviation, profile deviation, and helix deviation errors on the contact and bending stresses of cylindrical gears is studied.
Chapter 1. Introduction

Gears will be studied. The definitions of these deviations are presented in Chapter 2. Tip relief, profile crowning, and longitudinal crowning are proposed as micro-geometry modifications of gear tooth surfaces to reduce the sensitivity of the mechanical behavior of the gear drive to those mentioned errors. An extended description of these intended gear microgeometry modifications is presented in the Chapter 3.

1.2 Societal context

Gear drives play a large role in mechanical devices everywhere today. Every automobile on the road has a transmission and differential which relies on gears to operate effectively. Transmission errors and high stresses result in high levels of gear noise and unreliability. Nobody likes to have a transmission failure on their vehicle (which demands a costly repair), or the feeling of noise and vibration coming from the gear box. The goal of this work will be to fully characterize the relationship between surface modifications, manufacturing errors, and gear performance, which will enable the use of less expensive gears in the same applications, or superior gears for the same cost. The results of this work will contribute to make gear transmissions to have lower levels of noise and vibration, and absorb the undesirable effects of manufacturing errors, making gear transmissions more reliable and less noisy. Noise pollution from vehicles is an important concern for high traffic areas. Noise from gear drives are primarily caused by the function of transmission errors of the gear drive. The results of this work should provide insight into the relationship between manufacturing error, tooth modifications, and the function of transmission errors. This study could help reduce noise generated by low-cost gear drives, which are consequently very common in many applications today. Noise pollution has vast implications on the environment and quality of life, especially in urban areas. This study will help develop techniques to make gear drives quieter, cheaper, and more reliable.

1.3 The research question

Gear tooth surface modifications have already been shown to aid in reducing stresses and peak-to-peak transmission errors in spur and helical gears [3, 4, 13, 5, 6, 7, 8, 9]. The goal of this
thesis is to understand and characterize the influence of profile modifications on gears when manufacturing errors are considered. The research questions that will be answered with this thesis are:

- Can intentional gear tooth surface modifications reduce the sensitivity of spur and helical gear drives to manufacturing errors?
- Do optimal surface modification parameters exist for a spur or helical gear drive of a given ISO grade, directed to minimize contact and bending stresses in the gear tooth?
- Do optimal surface modification parameters exist for a spur and helical gear drive of a given ISO grade, directed to reduce the peak-to-peak level of transmission errors under load in the gear drive?

1.4 Literature review

Involute gearing is incredibly common in the gearing industry. Gears tooth surfaces with involute profiles, while undeformed and without errors of alignment or manufacture, maintain perfect contact throughout the cycle of meshing in a gear drive and yield no transmission errors. These gears are also very easy to manufacture, and can be generated with high precision. However, involute gears are in line contact at every instant, and are therefore very sensitive to manufacturing errors and errors of alignment [2].

It is already well known and documented that micro-geometry modifications can improve the performance of involute gear systems [3, 4, 5, 6, 7, 8, 9, 13]. Previous works have been directed to optimize these modifications with respect to various aspects of the gear drive. Many of these works consider stress experienced by the gear drive as the primary metric for determination of gear performance. It has been demonstrated through analytical approximations that the optimal value for tip relief intensity, $\delta$, is directly proportional to the torque applied to a gear system [8]. That same study also claimed that the optimal length of tip relief, $L$, is equal to half of the tooth addendum. An experiment by Oswald and Townsend [7] demonstrated that a tip relief of less than 25 microns can reduce fillet bending stresses by up to 25 percent. In their
experiment, both $\delta$ and L were modified in a small spur gear. It was found that a relatively large $\delta$ (23 microns) and beginning the relief at a late roll angle was ideal for minimizing fillet strain. Additionally, they found that beginning tip relief too early could increase bending stresses at low operating torques. More recent studies have sought means of mitigating this poor performance at low operating torques. One such study utilized 2D finite element analysis to evaluate the extent of tip relief’s influence on contact stresses, where it was found that the increased contact stress in that context was relatively small [10]. Tip relief can also negatively impact tooth fillet bending stresses [4]. However, these fillet bending stress increases can be expected to be minimal [4].

In addition to stresses experienced within the gear, transmission errors provide insight into the dynamic performance of a given gear drive. 2D finite element models have been used to show that tip relief can be optimized to reduce peak-to-peak transmission errors in spur gears [3, 9]. In one of these studies, linear relief was utilized with a parabolic fillet, which was able to provide an even smoother function of transmission error than parabolic tip relief alone [3]. Due to the demonstrated relationship between transmission errors and noise levels [14], the design of an optimal tip relief geometry may have a direct impact on the level of sound and vibration of the gear drive. In another study, finite element analysis was used to evaluate the effectiveness of profile and longitudinal crowning on mitigating the influence of errors of alignment on the stress experienced by the pinion of a helical gear drive [11]. It was found that the crowned gear pinion experienced approximately the same peak stress for both 2 arcmin and 3 arcmin of alignment error after modifications.

Aside from tip relief, the influence of longitudinal crowning on gear performance has also been studied. It has been demonstrated that in one gear drive, optimal levels of crowning and tip relief are related [8]. One study utilizing computational methods showed that tip relief and longitudinal crowning are most effective when applied together, because gear teeth can experience higher contact stresses if crowning is considered without tip relief [6].

There have also been many studies which have evaluated the effects of manufacturing errors on gear performance. It has been shown that for a given gear drive with helix deviation
errors meeting ISO grade 5 experienced 72 percent greater fillet stress (in comparison to a hypothe-
cically perfect involute gear drive), and the same gear drive with helix deviation errors meeting ISO grade 7 experienced 140 percent greater fillet stress [6].

Despite the great wealth of work done on both tooth profile modifications and manufactur-
ing errors individually, there exists little work on their combined effects. Using purely analytical
methods, one study analyzed the effects of varying levels of profile crowning and longitudinal
crowning on contact stresses, root stresses, and stress distribution factor for gears with manu-
facturing errors [15]. It was found that the optimal profile modifications for the case of a gear
with no manufacturing errors tended to also produce the best results after manufacturing errors
were introduced, giving a great starting point for gear designers expecting their gear drives to
be manufactured with low quality [15]. Another study examined this behavior in helical gears
using computational methods, but did not examine the evolution of stresses over a full cycle
of meshing [6]. However, their work did consider both tip relief and longitudinal crowning.
It was demonstrated that the two types of modifications can result in increased stresses when
applied individually or haphazardly, and that the two modifications should be utilized together
to ensure performance benefits [6].

The few existing studies aiming to consolidate the effects of manufacturing errors and tooth
surface modifications have utilized express models to estimate stresses [16]. However, these ex-
press models cannot predict the influence of transmission errors accurately. A full finite element
model is required to generate a function of transmission errors over a full cycle of meshing [9].
Often, simplified 2D models are utilized to perform these calculations, which may adequate for
spur gears if longitudinal crowning and helix deviation errors are not considered. Typically, in
these studies, peak-to-peak values of transmission error is presented to summarize gear perfor-
mance. However, it has been demonstrated that peak-to-peak (PTP) transmission error values
are an incomplete metric in describing the influence of the function of transmission errors on
gear drive performance. By taking the fourier transform of the function of transmission errors,
the amplitude spectrum of gear noise can be predicted [5]. It has been shown that TE functions
with the same PTP values can still experience different noise spectra, indicating a different pitch
and volume of noise during operation [5].
Performing such calculations for helical gears implicitly requires a 3D finite element model with several pairs of contact positions. In this work, 3D finite element models are utilized for all calculations involving both spur and helical gears, to develop a full understanding of not only the behavior of the gear drives with respect to helix angles, but helix deviations as well, which has never been studied numerically before.
Chapter 2

Geometry Deviations Due to Manufacturing Errors

2.1 Introduction

ISO and AGMA classify geometry deviations due to manufacturing errors into three main categories. These categories are pitch deviations, profile deviations, and helix deviations [17]. Pitch deviations are errors in the spacing between teeth, profile deviations are errors in the pressure angle of the gear teeth, and helix deviations are variations in the straightness of the gear teeth. Each of these categories contains multiple parameters which determine the quality grade of the gear. The factor between each consecutive ISO grade is $\sqrt{2}$, meaning that moving up two ISO grades equates to tolerance levels twice as large.

In this section, a brief description of the three classes of manufacturing errors examined in this thesis is provided. The mathematical definition and means of measuring each error is discussed.

2.2 Pitch deviations

Pitch between two gear teeth is measured as the transverse circular distance between the leading edge of two teeth. The symbol representing a measured pitch is given by $p_{IM}$. The ISO grade of a gear is determined based on two metrics of pitch deviation. These metrics are the maximum single pitch deviation, and the total cumulative pitch deviation. The individual
single pitch deviation \((f_{pi})\) is the algebraic difference between the actual pitch of a given gear tooth and the corresponding theoretical pitch in the transverse plane, defined on a circle concentric with the gear axis at the measurement diameter \((d_M)\) \cite{17}. In determination of a gear’s ISO grade, the maximum absolute value of all of the individual single pitch deviations is considered to be the single pitch deviation \((f_p)\) of the gear.

The individual cumulative pitch deviation \((F_{pi})\) is the algebraic difference over a sector of several pitches between the actual length and the theoretical length of the relevant arc \cite{17}. Theoretically, it should be equal to the algebraic sum of the single pitch deviations of the same considered pitches. The total cumulative pitch deviation \((F_p)\) is considered to be the largest algebraic difference between the individual cumulative pitch deviation values for a specified flank obtained for all the teeth of a gear. Figure 2.1 illustrates how these quantities are measured with respect to a spur gear.

\[\text{Figure 2.1: Diagram of ISO metrics to measure pitch deviation, from ISO 1328-1 [17].}\]

In this thesis, pitch deviations are modeled through a sinusoidal function that takes into account the total cumulative pitch deviation and the single pitch deviation. A sinusoidal function is used for the location of each single tooth on the reference circle and this means that each tooth has to be generated individually for the modelling of the pitch deviation.
2.3 Profile deviations

Profile deviation is the amount by which a measured tooth profile deviates from the design profile \[18\]. As shown in Figure 2.2, the total profile deviation $F_\alpha$ is the superposition of the profile form deviation $f_{f,\alpha}$ and the profile slope deviation $f_{H,\alpha}$. The profile evaluation range, $L_\alpha$, according to \[17\], is considered equal to 92% of the active length of the tooth profile.

![Figure 2.2: Diagram of ISO metrics to measure profile deviation for gear with an unmodified involute profile, from ISO 1328-1 \[17\].](image)

In this thesis, it is assumed that the profile form deviation $f_{f,\alpha}$, mainly caused by the roughness or small undulations along the profile direction, is zero. The profile slope deviation $f_{H,\alpha}$ is then the only deviation affecting the considered value of the profile deviation. Related with profile deviation, the influence of positive or negative profile slope deviations will be considered. The profile deviation direction is positive when the profile line shows an increase of the material towards the tooth tip, relative to the design profile, corresponding to a decreased pressure angle of the tooth.

The total profile deviation for a gear of accuracy grade ISO 5 is obtained by considering the following equation:

$$ F_\alpha = 3.2 \sqrt{m} + 0.22 \sqrt{d} + 0.7 $$

(2.1)

where $m$ is the module and $d$ is the pitch radius of the gear. Once the profile deviation and the profile evaluation distance are known, the variation of the pressure angle to simulate the profile
deviation $\Delta \alpha$ is determined as:

$$\Delta \alpha = \arctan \left( \frac{F_\alpha}{L_\alpha \tan(\alpha)} \right)$$

(2.2)

where $\alpha$ is the pressure angle.

### 2.4 Helix deviations

Helix deviation is the amount by which a measured helix deviates from the design helix [18]. As shown in Figure 2.3, the total helix deviation $F_\beta$ is the superposition of the helix form deviation $f_{f\beta}$ and the helix slope deviation $f_{H\beta}$. In Figure 2.3, $L_\beta$ is the helix evaluation range, that is given by the face width shortened at both sides by the smaller value of the 5% of the face width or a length equal to one module.

In this thesis, it will be assumed that the helix form deviation $f_{f\beta}$, mainly caused by the roughness of the surface, is zero. Therefore, the helix slope deviation $f_{H\beta}$ is the only deviation affecting the helix deviation. In order to consider the most unfavorable scenario for simulations, the value of the total helix deviation $F_\beta$ according to ISO will be considered for the determination of the effective variation of the helix angle $\Delta \beta$, and therefore,

$$\Delta \beta = \arctan \left( \frac{F_\beta}{L_\beta} \right)$$

(2.3)
2.4. Helix deviations

The total helix deviation for an accuracy grade 5 is obtained by considering the following equation:

\[ F_\beta = 0.1\sqrt{d} + 0.63\sqrt{b} + 4.2 \quad (2.4) \]

where \( d \) is the pitch radius of the gear and \( b \) is its face width.
Chapter 3

Intentional Geometry Modifications

3.1 Introduction

It is already well known and documented that micro-geometry modifications can improve the performance of involute gear systems [13, 6, 7, 8, 9]. With an increasing emphasis on the strength-to-weight ratio of gear drives, gears are pushed to their limits of performance, and subjected to increased levels of stress in usage. Geometry modifications have been demonstrated to be especially important in gear drives experiencing high stresses and strains. When gear drives are subjected to increased loads, the deflection experienced by the teeth can be large enough to significantly alter the involute profile of the teeth. This unwanted deformation can cause undesirable effects such as a discontinuous function of transmission errors, and damage to gear teeth in the form of pitting of the teeth flanks. However, by compensating for the anticipated deflection through micro-geometry alterations, a superior gear drive can be designed to tackle these high load applications.

In this chapter, the various geometry modifications which have been examined in this thesis are described. These descriptions are intended primarily as a reference for non-experts.

3.2 Tip relief

AGMA [19] defines tip relief as “a modification of a tooth profile whereby a small amount of material is removed near the tip of the gear tooth.” It is used to avoid premature contacts between contacting teeth due to the elastic deformation of gear tooth surfaces and pitch or
profile errors. Tip relief can be defined for cylindrical gears with involute profiles by defining
the amount of tip relief at the addendum radius, $\delta$, shape of tip relief (linear or parabolic) and
starting point on the involute profile. For definition of the starting point, the following three
definition methods are used by considering: (i) the datum length for tip relief, $L$, (ii) the rolling
length, $r_L$, or (iii) the rolling angle, $\epsilon$. As shown in Figure 3.1, the relation between the rolling
length and corresponding rolling angle is:

$$r_L = r_b \epsilon$$  \hfill (3.1)

where $r_b$ is the base radius of the gear. Given one of the following tip relief definition factors,
the radius $r_p$ of the starting point of tip relief can be obtained as follows:

- Datum length, $L$:

$$r_p = r_a - L$$  \hfill (3.2)

- Rolling length, $r_L$:

$$r_p = \sqrt{r_b^2 + r_L^2}$$  \hfill (3.3)

- Rolling angle, $\epsilon$:

$$r_p = r_b \sqrt{1 + \epsilon^2}$$  \hfill (3.4)

Figure 3.2 shows a tip relief with parabolic shape applied to a spur gear tooth, including the
amount of tip relief at the addendum radius, $\delta$, and the datum length for tip relief, $L$.

3.3 Longitudinal crowning

Longitudinal crowning is often referred to simply as “crowning.” AGMA [19] defines crowning
as “teeth which have surfaces modified in the length-wise direction to produce localized
contact or to prevent contact at their ends.” It has been shown that at high torques, crowning
can benefit gear performance by localizing contact stresses in the center of the gear tooth, away
from the edges [20]. In that study, it was also shown that there exist optimal crowning parameters for a given loading condition and gear geometry. Longitudinal crowning has been also
3.3. Longitudinal crowning

FIGURE 3.1: Diagram of geometric parameters involved in tip relief definition.

FIGURE 3.2: Representation of tip relief with parabolic shape in a spur gear drive.
analytically demonstrated to contribute to the reduction of stress distribution factors [8]. A visual representation of longitudinal crowning of parabolic shape applied to the tooth surfaces of a spur gear is shown in Figure 3.3.

![Geometry comparison of a longitudinally crowned spur gear flank with respect to the standard geometry.](image)

**Figure 3.3:** Geometry comparison of a longitudinally crowned spur gear flank with respect to the standard geometry.

### 3.4 Profile crowning

Profile crowning, also known as barreling, involves changing the chordal thickness of the tooth along its axis. This modification eliminates end bearing by offering a contact bearing in the center of the gear. It has the additional benefit of introducing a pre-defined continuous parabolic function of transmission error. This continuous function of transmission errors can help absorb the more unfavorable, discontinuous functions of transmission errors associated with manufacturing errors and high operating loads. Gears with profile crowning may not need the simultaneous application of a tip relief.
3.5 Front and back relief

Front and back relief is a modification similar to longitudinal crowning, with the added parameter of length of application across the face width. Gears with front and back relief will have material removed from the teeth along the sides of the gear teeth, to help localize the contact path towards the center of the tooth. This modification can be preferable to longitudinal crowning since it can push the contact path into a specific location along the gear tooth, rather than simply pushing the contact path to the center of the gear drive. Figure 3.5 demonstrates this modification for a front and back relief of $\delta = 12 \, \mu m$ and $L = 4 \, mm$. 

**Figure 3.4:** Geometry comparison of a barreled (profile crowning) spur gear flank with respect to the standard geometry.
Figure 3.5: Geometry comparison of a gear flank with front and back relief with respect to the standard geometry.
Chapter 4

Methodology and Computational Tools

4.1 Introduction

For gear researchers, performing empirical tests can be costly and time consuming. Finding the optimal level of micro-geometry modifications to a given gear drive can be especially so, given the cost of manufacturing and measuring the multitude of individual gears to be tested. Some analytical models exist to approximate optimal levels of micro-geometry modification, but many of these models do not provide sufficient accuracy to both determine the optimal levels of modification, and evaluate the performance of the gear drive, as measured by the function of loaded transmission error and the von Mises stress experienced by the gear tooth at all points of contact. To accurately determine the optimal levels of gear tooth surface modification, as well as evaluate the function of loaded transmission error of the gear, finite element modelling is necessary.

4.2 Design of experiments

Design of experiments refers to the utilization of a systematic methodology to determine the relationship between inputs and outputs of a process. By varying the input parameters systematically, complex relationships between the factors and the output can be determined with a minimal number of trials. In this study, a full factorial experiment design is utilized. A full factorial experiment is one which varies each factor between two pre-determined levels one at a time, until all possible combinations have been tried. A full factorial experiment was selected
because this format allows the investigator to study the effect of each factor individually on the response variable, as well as determine the influence interactions between factors will have on the response variable.

In this work, the response variables considered are the peak contact stress experienced in the tooth flanks of both the pinion and the wheel, the peak bending stress experienced by both the pinion and the wheel, and the excitation of certain meshing frequencies within the LTE function. DOEs are designed considering the level of manufacturing error of each type as the factors. Each class of manufacturing error is varied between what was expected for a gear of ISO Grades 2 and 6. For each experiment manufacturing errors are only simulated in the pinion. No manufacturing errors are simulated in the wheel.

From the DOE, the influence of each class of manufacturing error on peak contact stress can be found individually. Two types of plots will be used to illustrate results, Pareto charts and plots of fitted means. A Pareto chart of the effects is a histogram which plots the influence of each factor individually and combined against each other for a single designed experiment. Figure 4.1 illustrates how to read a Pareto chart of the effects of a DOE.

![Figure 4.1: Sample Pareto chart of the effects of a DOE, to illustrate how a Pareto chart of effects should be interpreted.](image-url)
A plot of fitted means compares the average peak contact stress at each level for each factor for a given DOE. Figure 4.2 illustrates how to interpret these plots.

**Figure 4.2:** Sample plot of fitted means, to illustrate how fitted means plots should be interpreted.

### 4.3 IGD - Integrated Gear Design

Integrated Gear Design (IGD), is a computer program for gear design, analysis, optimization and troubleshooting of gear drives [21]. IGD integrates a virtual gear generator with algorithms for tooth contact analysis, backlash analysis, free-form design and automatic generation of finite element models for stress analysis.

IGD takes both advanced gear design and gear manufacturing into consideration. In IGD, virtual gears can be generated by virtual cutting tools mimicking a real manufacturing processes. The models generated by IGD can be exported to CAD computer programs in multiple formats. Additionally, IGD can be used to generate finite element models for stress and deformation analysis.

To assist in the generation of the hundreds of finite element models considered in this study,
IGD has been utilized to automatically generate models of gears with various levels of manufacturing errors and tooth modifications. These models were then exported to Abaqus for finite element analysis.

### 4.3.1 IGD modeling of manufacturing errors

Figure 4.3 demonstrates the IGD manufacturing error input window.

![Screenshot from the manufacturing errors function in IGD.](image)

In IGD, pitch deviations are modeled through a sinusoidal function that takes into account the total cumulative pitch deviation and the single pitch deviation. A sinusoidal function is used for the location of each single tooth on the reference circle and this means that each tooth has to be generated individually for the modelling of the pitch deviation. Once the entire gear has been generated from this sinusoidal function, a reduced number of teeth are considered in the finite element model of the gear drive. The subset of teeth considered for the finite element model is selected such that the two teeth with the largest single pitch deviation is always included in the model.

IGD models profile deviations by variation of the pressure angle of the gear. It is assumed that the profile form deviation $f_{fr}$, mainly caused by the roughness or small undulations along the profile direction, is zero, and therefore the profile slope deviation $f_{Ha}$ is the only deviation affecting the considered value of the profile deviation. Related with profile deviation, the influence of positive or negative profile slope deviations can both be considered. The profile slope
4.3. IGD - Integrated Gear Design

development direction is positive when the profile line shows an increase of the material towards
the tooth tip, relative to the design profile, corresponding to a negative pressure angle deviation.

Helix deviations are modeled in IGD by variation of the helix angle of the gear. Similarly to
profile deviations, it is assumed that the helix form deviation $f_{f\beta}$ is zero, so that the helix slope
deviation $f_{H\beta}$ is the only deviation affecting the helix deviation. In order to consider the most
unfavorable scenario for simulations, the value of the total helix deviation $F_{\beta}$ according to ISO
will be considered for the determination of the effective variation of the helix angle $\Delta\beta$ for all
teeth in the generated gear. Helix deviation may be applied as either a left-handed deviation
or a right-handed one. For a left-handed helical gear, a left-handed helix deviation results in an
increased helix angle.

4.3.2 IGD modeling of geometry modifications

Freeform tooth geometries can be generated in IGD. A tooth modification function enables
the application of various tooth modifications simultaneously. Figure 4.4 shows a screenshot of
the freeform geometry function from IGD.

![Figure 4.4: Screenshot from the tooth geometry modification function in IGD.](image)

In IGD, each modification is mathematically defined differently, depending on the type of
modification specified. IGD considers geometry modifications by the same formulas as pre-
sent in Chapter 3.
4.4 Finite element models for stress analysis

For the case of spur gears, a typical finite element model has been considered. These models consist of 3D 8-node linear incompatible mode elements, denoted with the code "C3D8I" in Abaqus. Elements of this type are three dimensional first order elements with incompatible modes to improve their bending behavior [22]. Specifically, the issues of shear locking and volumetric locking, which often result in inaccurate results for linear elements in bending, are addressed with this element type. Quadratic elements will typically still yield more accurate results, but C3D8I elements typically require much less computation time. These elements Figure 4.5 shows an example of a mesh for a spur gear used in this thesis for finite element analysis.

Figure 4.5: Example of a 3D finite element mesh of a gear drive.

For the helical gear DOEs, twice as many trials are required due to the addition of "hand of helix deviation" as a factor. To help reduce computational times for the helical gear geometries, a multipoint-constraint based finite element model was utilized. These finite element models gradually increase mesh density, to enable a dense mesh along the points of contact between teeth, but a less dense mesh on the side of the tooth which does not experience any contact. By gradually increasing mesh density, a more accurate result can be obtained without the computational intensity of a dense mesh throughout the entire tooth.

Figure 4.6 illustrates how multipoint-constraint based models are defined in this work.
To vary mesh density so rapidly, transitional nodes have been defined to aid the transition to a finer mesh. These nodes are referred to as dependent nodes, while nodes which align with the previous, coarser mesh are referred to as independent nodes. Mesh transitions require additional constraints of the degrees of freedom the dependent nodes as linear functions of the degrees of freedom of the independent nodes, in order to assure the continuity in the field of displacements. The dependent nodes are then defined as a linear interpolation function of the independent nodes. The elements are 3D 8-node linear isoparametric elements, denoted with the code "C3D8" in Abaqus. Previous works have shown that this strategy of mesh refinement can produce accurate results [23].
Chapter 5

Spur Gears and Their Sensitivity to Manufacturing Errors

5.1 Introduction

When asked to picture a gear in one’s head, spur gears are typically what comes to mind. Spur gears can be considered a special case of helical gears where the helix angle is equal to zero. Due to their symmetric shape, they can be simulated with two-dimensional models [9]. However, some manufacturing defects and modifications, such as helix deviations, can only be simulated with a three-dimensional model.

In this chapter, the sensitivity of the peak contact stress of two spur gear geometries to manufacturing errors will be established, and then compared to the sensitivity of the same geometry with a variety of standard tooth modifications applied. The two geometries considered are similar, but Geometry 1 will have a pressure angles of 20° and Geometry 2 will have a pressure angle of 25°. The modifications considered will include tip relief, longitudinal crowning, and profile crowning. With the help of these geometry modifications, it will be determined if the sensitivity of peak stresses experienced in the gear drive due to any individual factor can be reduced. Additionally, the amount of reduction expected will be compared between two geometries. Lastly, the ability of profile crowning to absorb the discontinuous functions of transmission errors will be examined and quantified for the two spur gear geometries.
5.2 Definition of gear geometries

Two spur gear geometries have been considered as Geometries 1 and 2, with pressure angles of 20 degrees and 25 degrees, respectively. Table 5.1 shows the macro-geometry design parameters of the two simulated spur gear drives.

<table>
<thead>
<tr>
<th></th>
<th>GEOMETRY 1</th>
<th>GEOMETRY 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PINION</td>
<td>WHEEL</td>
</tr>
<tr>
<td>Number of teeth</td>
<td>21</td>
<td>37</td>
</tr>
<tr>
<td>Module [mm]</td>
<td>4.0</td>
<td>4.0</td>
</tr>
<tr>
<td>Normal pressure angle [deg]</td>
<td>20</td>
<td>25.0</td>
</tr>
<tr>
<td>Addendum coefficient [-]</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Dedendum coefficient [-]</td>
<td>1.25</td>
<td>1.25</td>
</tr>
<tr>
<td>Root radius coefficient [-]</td>
<td>0.38</td>
<td>0.38</td>
</tr>
<tr>
<td>Profile shift coefficient [-]</td>
<td>0.2060</td>
<td>-0.2060</td>
</tr>
<tr>
<td>Generating shift coefficient [-]</td>
<td>0.1802</td>
<td>-0.2317</td>
</tr>
<tr>
<td>Face width [mm]</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>Center distance [mm]</td>
<td>116.0</td>
<td>116.0</td>
</tr>
</tbody>
</table>

Table 5.1: Macro-geometry parameters of Geometries 1 and 2.

The derived data of the simulated gears is shown in Table 5.2.

<table>
<thead>
<tr>
<th></th>
<th>GEOMETRY 1</th>
<th>GEOMETRY 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PINION</td>
<td>WHEEL</td>
</tr>
<tr>
<td>Pitch [mm]</td>
<td>12.5664</td>
<td>12.5664</td>
</tr>
<tr>
<td>Base pitch [mm]</td>
<td>11.8085</td>
<td>11.3890</td>
</tr>
<tr>
<td>Transverse contact ratio [-]</td>
<td>1.6089</td>
<td>1.4491</td>
</tr>
<tr>
<td>Circumferential backlash [mm]</td>
<td>0.150</td>
<td>0.150</td>
</tr>
<tr>
<td>Normal backlash [mm]</td>
<td>0.141</td>
<td>0.141</td>
</tr>
<tr>
<td>Reference diameter [mm]</td>
<td>84.0000</td>
<td>148.0000</td>
</tr>
<tr>
<td>Tip diameter [mm]</td>
<td>93.6477</td>
<td>154.3523</td>
</tr>
<tr>
<td>Root diameter [mm]</td>
<td>75.4417</td>
<td>136.1462</td>
</tr>
<tr>
<td>Base diameter [mm]</td>
<td>78.9342</td>
<td>139.0745</td>
</tr>
<tr>
<td>Root form diameter [mm]</td>
<td>79.5104</td>
<td>140.7741</td>
</tr>
<tr>
<td>Addendum [mm]</td>
<td>4.8239</td>
<td>3.1761</td>
</tr>
<tr>
<td>Dedendum [mm]</td>
<td>4.2792</td>
<td>5.9269</td>
</tr>
<tr>
<td>Tooth thickness [mm]</td>
<td>6.8079</td>
<td>5.6085</td>
</tr>
<tr>
<td>Space width [mm]</td>
<td>5.7585</td>
<td>6.9579</td>
</tr>
</tbody>
</table>

Table 5.2: Derived data of Geometries 1 and 2.
5.2. Design of Experiments

For the spur gear geometries, four factors were included in the generation of the DOE: magnitude of helix deviation, magnitude of pitch deviation, magnitude of profile deviation, and profile deviation direction. As discussed in Chapter 3, there are multiple measures utilized in determination of ISO grade based on helix, pitch, and profile deviations. All factors were varied between the maximum levels of error which would be expected for a gear of grade ISO 2 and what would be expected for a gear of grade ISO 6. Table 5.3 lists the amount of manufacturing error considered for these DOEs.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Minimum Value</th>
<th>Maximum Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Pitch Deviation</td>
<td>mm</td>
<td>0.007</td>
<td>0.028</td>
</tr>
<tr>
<td>Total Profile Deviation</td>
<td>mm</td>
<td>0.0034</td>
<td>0.013</td>
</tr>
<tr>
<td>Profile Deviation Direction</td>
<td>-</td>
<td>Negative</td>
<td>Positive</td>
</tr>
<tr>
<td>Total Helix Deviation</td>
<td>mm</td>
<td>0.003</td>
<td>0.012</td>
</tr>
</tbody>
</table>

Table 5.3: DOE parameters for Geometries 1 and 2.

Setting up a full factorial experiment from these values yielded sixteen runs, and the details of each run is demonstrated in table 5.4.

<table>
<thead>
<tr>
<th>Total Pitch Dev. [µm]</th>
<th>Total Profile Dev. [µm]</th>
<th>Total Helix Dev. [µm]</th>
<th>Profile Dev. Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.0</td>
<td>3.4</td>
<td>Positive</td>
<td>3.0</td>
</tr>
<tr>
<td>28.0</td>
<td>3.4</td>
<td>Positive</td>
<td>3.0</td>
</tr>
<tr>
<td>7.0</td>
<td>13.0</td>
<td>Positive</td>
<td>3.0</td>
</tr>
<tr>
<td>28.0</td>
<td>13.0</td>
<td>Positive</td>
<td>3.0</td>
</tr>
<tr>
<td>7.0</td>
<td>3.4</td>
<td>Positive</td>
<td>12.0</td>
</tr>
<tr>
<td>28.0</td>
<td>3.4</td>
<td>Positive</td>
<td>12.0</td>
</tr>
<tr>
<td>7.0</td>
<td>13.0</td>
<td>Positive</td>
<td>12.0</td>
</tr>
<tr>
<td>28.0</td>
<td>13.0</td>
<td>Positive</td>
<td>12.0</td>
</tr>
<tr>
<td>7.0</td>
<td>3.4</td>
<td>Negative</td>
<td>3.0</td>
</tr>
<tr>
<td>28.0</td>
<td>3.4</td>
<td>Negative</td>
<td>3.0</td>
</tr>
<tr>
<td>7.0</td>
<td>13.0</td>
<td>Negative</td>
<td>3.0</td>
</tr>
<tr>
<td>28.0</td>
<td>13.0</td>
<td>Negative</td>
<td>3.0</td>
</tr>
<tr>
<td>7.0</td>
<td>3.4</td>
<td>Negative</td>
<td>12.0</td>
</tr>
<tr>
<td>28.0</td>
<td>3.4</td>
<td>Negative</td>
<td>12.0</td>
</tr>
<tr>
<td>7.0</td>
<td>13.0</td>
<td>Negative</td>
<td>12.0</td>
</tr>
<tr>
<td>28.0</td>
<td>13.0</td>
<td>Negative</td>
<td>12.0</td>
</tr>
</tbody>
</table>

Table 5.4: DOE Conditions
5.2.2 Mesh selection for Geometries 1 and 2

For Geometries 1 and 2, a fully constrained finite element model was utilized. As mentioned in Chapter 4, the mesh selected was generated in IGD. A standard finite element mesh was used for the simulation of these geometries. Spur gears are mostly symmetrical, experiencing only line contact between the two contacting teeth. The mesh parameters for each geometry are listed in Table 5.5.

<table>
<thead>
<tr>
<th>Geometry</th>
<th>Longitudinal Elements</th>
<th>Transverse Elements</th>
<th>Fillet Elements</th>
<th>Elements Under the Active Surface</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometry 1 Pinion</td>
<td>59</td>
<td>34</td>
<td>17</td>
<td>3</td>
</tr>
<tr>
<td>Geometry 1 Wheel</td>
<td>54</td>
<td>29</td>
<td>17</td>
<td>3</td>
</tr>
<tr>
<td>Geometry 2 Pinion</td>
<td>59</td>
<td>34</td>
<td>17</td>
<td>3</td>
</tr>
<tr>
<td>Geometry 2 Wheel</td>
<td>54</td>
<td>29</td>
<td>17</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 5.5: Mesh parameters for Geometries 1 and 2.

5.2.3 Optimal tip relief for Geometries 1 and 2

The first modification considered for the experiment will be tip relief. As discussed in Chapter 3, tip relief can improve gear performance by preventing premature contact between teeth throughout a cycle of meshing. Each geometry is expected to have an optimal tip relief design which depends on several factors, such as the number of teeth, the operating torque, and the face width of the gears [8]. A parabolic application of tip relief will be assumed, since linear tip relief can result in increased stress experienced by the fillet [4]. The optimal value of the other two parameters, \( \delta \) and \( L \), will be found by testing a range of values for each parameter.

To find the optimal tip relief, first, simulations were ran varying tip relief intensity (\( \delta \)) and length (\( L \)) for a gear with no manufacturing errors. In this context, the optimal design was defined as the combination of tip relief at the addendum radius and datum length providing the lowest maximum Von-Mises contact stress in the tooth surfaces. The results for the case of Geometry 1 can be seen in Figures 5.1 and 5.2. From the evolutions of stress shown in these figures, an optimal tip relief of \( \delta = 12 \, \mu\text{m} \) and \( L = 1.0 \, \text{mm} \) was selected for Geometry 1.
5.2. Definition of gear geometries

The optimal tip relief was also found for Geometry 2. This was done in the same way as described for Geometry 1. The plots of the evolution of contact stresses are shown in figures.
5.3 and 5.4. From this data, an optimal tip relief of $\delta = 6\mu m$ and $L = 1.0\text{mm}$ was selected for Geometry 2.

![Graph showing contact stresses and tip relief](image)

**Figure 5.3**: Pinion flank contact stresses at various amounts of tip relief at the addendum radius ($\delta$) for Geometry 2. (See Figure 3.2)

![Graph showing contact stresses and tip relief datum length](image)

**Figure 5.4**: Pinion flank contact stresses at various levels of tip relief datum length ($L$) for Geometry 2. (See Figure 3.2)

### 5.3 Baseline sensitivity to manufacturing errors (no modifications)

Prior to evaluating the influence of tooth modifications to gear performance with various levels of manufacturing error, a baseline sensitivity to manufacturing errors must be established. To do so, the DOE outlined in Table 5.3 was performed without the consideration of any tip relief or longitudinal crowning. This DOE allowed the individual effects of each class of
manufacturing error to be determined, to gain a better understanding of how each class of error is affected by the addition of tooth modifications. For Geometries 1 and 2, Figures 5.5 and 5.6 respectively demonstrate the sensitivity of peak von Mises stress along the pinion tooth flank, in MPa, to each class of manufacturing error.

**Figure 5.5:** Sensitivity of pinion peak contact stress to manufacturing errors for Geometry 1 with no flank modifications.

**Figure 5.6:** Sensitivity of pinion peak contact stress to manufacturing errors for Geometry 2 with no flank modifications.

Figures 5.7 and 5.8 show the fitted means of each factor considered by the DOE, in MPa, for Geometries 1 and 2. A sample plot illustrating how to interpret these plots can be found in Section 4.2, Figure 4.2.
Chapter 5. Spur Gears and Their Sensitivity to Manufacturing Errors

From Figures 5.5, 5.6, 5.7, and 5.8, it can be seen that the factor with the largest influence on peak stress in the pinion flank for Geometries 1 and 2 is the profile deviation direction. Profile deviation direction had an influence of 245.2 MPa for Geometry 1, and 129.34 MPa for Geometry 2. Overall, Geometry 2 was less sensitive to manufacturing errors than Geometry 1. For both geometries, a positive profile deviation direction yielded lower stresses in the pinion flank than a negative profile deviation. The factor with the second largest influence was helix deviation. In both geometries, lower levels of helix deviation resulted in reduced peak contact stress. For Geometry 1, helix deviation had an influence of 167.23 MPa, and for Geometry 2, helix deviation had an influence of 104.76 MPa.
5.4 Sensitivity to manufacturing errors with only tip relief

Next, the DOE outlined in Table 5.3 was performed with the consideration of tip relief. For both Geometries 1 and 2, the optimal level of tip relief as identified in Subsection 5.2.3 was applied to the pinion for all DOE runs. The sensitivity of pinion peak contact stress is shown in Figures 5.9 and 5.10.

![Pareto Chart of the Effects](image)

**Figure 5.9:** Sensitivity of pinion peak contact stress to manufacturing errors for Geometry 1 with tip relief.

![Pareto Chart of the Effects](image)

**Figure 5.10:** Sensitivity of pinion peak contact stress to manufacturing errors for Geometry 2 with tip relief.

Previous to the implementation of tip relief, maximum contact stress is mostly sensitive to
profile deviation direction, total pitch deviation, and total helix deviation. However, after tip relief is applied to the gear, maximum von mises stress in the gear tooth surface becomes largely unaffected by any factor other than total helix deviation.

In both geometries, lower levels of helix deviation resulted in reduced peak contact stress. For Geometry 1 helix deviation had an influence of 69.66 MPa, and for Geometry 2 helix deviation had an influence of 69.56 MPa.

For the case of no tip relief, Geometry 1 demonstrated an increased sensitivity to manufacturing errors than Geometry 2 in general. However, once tip relief was applied, the sensitivity of the two Geometries was substantially closer.

5.5 Sensitivity to manufacturing errors with both tip relief and longitudinal crowning

With the knowledge that tip relief could compensate for manufacturing errors other than helix deviation, the next geometry modification considered was longitudinal crowning. Before applying some amount of crowning to each geometry, the optimal crowning needed to be found. The criteria for optimization was again the minimization of maximum Von Mises stress in the pinion tooth flank. Simulations were ran for gears of varying helix deviation and magnitudes of longitudinal crowning. Helix deviation was varied to reflect that of different ISO grades, and crowning magnitudes were incremented by 0.5 $\mu$m until an optimal was found. The results are demonstrated in Figure 5.13, which compares the two curves of optimal crowning versus level of helix deviation for Geometries 1 and 2.

After determining a relationship for the optimal amount of crowning as a function of helix deviation, the same factorial experiment was considered as in determining the effectiveness of tip relief in reducing sensitivity. The DOE simulations were ran with a longitudinal crowning intensity of 6 $\mu$m applied to the pinion for both geometries. The optimal levels of tip relief as outlined in Subsection 5.2.3 were also applied to the pinion for both geometries. Manufacturing errors were the same as described in Table 5.3. Figures 5.11 and 5.12 demonstrate the sensitivity results.
5.5. Sensitivity to manufacturing errors with both tip relief and long...

For both geometries, no factor contributed to peak contact stress by more than 40 MPa. The factor which had the most influence on peak stress experienced by the pinion flank was helix deviation for both geometries.
5.5.1 Optimal Longitudinal Crowning vs Level of Helix Deviation

As mentioned in Section 5.5, the criteria for optimization was again the minimization of maximum Von Mises stress in the pinion tooth flank. Simulations were ran for gears of varying helix deviation and magnitudes of longitudinal crowning. Helix deviation was varied to reflect that of different ISO grades, and crowning magnitudes were incremented by 0.5 µm until an optimal was found. In agreement with other works [8], optimal levels of longitudinal crowning changed with increasing levels of helix deviation within a given gear drive. Apparently, optimal crowning and helix deviation are linearly related for these spur gear drives. Figure 5.13 compares the two curves of optimal crowning for Geometries 1 and 2.

![Figure 5.13: Comparison of optimal longitudinal crowning between Geometries 1 and 2.](image)

A regression analysis yielded the trend line shown in the image, which is described by equation 5.1 and features an $r^2$ value of 0.9984.

$$\text{Optimal Crowning Magnitude} = 2.7095 + 0.3803 \times F_{\beta T} \quad (5.1)$$

Also interesting to note, is that Geometries 1 and 2 shared very similar optimal levels of crowning for a given loading condition and identical levels of helix deviation.
5.6 Influence of manufacturing errors on the loaded function of transmission errors

The function of loaded transmission errors (LTEs) can be challenging to calculate, since it accounts for the deformation of the teeth throughout the cycle of meshing. However, the function of loaded transmission errors can be calculated by combining the results of finite element analysis with unloaded transmission error results, which can be obtained from the undeformed gear models. Figures 5.14 and 5.15 show the function of loaded transmission errors for Geometry 1 at various ISO grades, separated by the direction of the profile deviation.

![Figure 5.14: LTEs for Geometry 1 at various ISO grades with positive profile deviation.](image-url)
Figures 5.14 and 5.15 demonstrate the influence of the direction of the profile deviation on the function of loaded transmission errors. For gear drives with a positive directional profile deviation, the LTE quickly spikes up, then slowly drops for each contacting tooth. For gear drives with a negative profile deviation, the LTE slowly rises, then quickly drops when the tooth comes out of contact. Another initial observation is that the peak-to-peak (PTP) values of the LTE function is greatest for gear drives with the most manufacturing error, or the highest ISO grade. Typically, "peak-to-peak" values are a good metric of overall gear performance, since greater PTP values indicate a less smooth motion of the gear drive throughout the cycle of meshing. However, PTP values are not the only metric which can be used to evaluate the function of transmission errors. By looking at the harmonics of a gear drive’s LTE, an approximation of gear noise can be made. These harmonics have been found by taking the discrete Fourier transform of the function of transmission errors at multiples of the frequency of meshing. The units of this spectrum is in arcsec. Greater magnitudes of harmonics indicate a noisy gear drive with a high level of vibration. Figure 5.16 demonstrates the amplitude spectrum as obtained via the discrete Fourier transform of the LTE function for DOE Trial 1 of Geometry 1.
5.6. Influence of manufacturing errors on the loaded function of transmission errors

**Figure 5.16:** Amplitude spectrum as obtained by the discrete Fourier transform for Geometry 1, Trial 1.

Introducing a pre-defined parabolic function of transmission errors through the use of profile crowning has been demonstrated to help compensate for a discontinuous and noisy function of transmission errors. Figures 5.17 and 5.18 show the response of the LTE function to various amounts of profile crowning for Geometry 1 with deviations corresponding to ISO grade 4.

**Figure 5.17:** LTEs for Geometry 1 at various levels of profile crowning and positive profile deviation, ISO 4.
Chapter 5. Spur Gears and Their Sensitivity to Manufacturing Errors

Figure 5.18: LTEs for Geometry 1 at various levels of profile crowning and negative profile deviation, ISO 4.

Figures 5.19 and 5.20 show the response with deviations corresponding to ISO grade 8.

Figure 5.19: LTEs for Geometry 1 at various levels of profile crowning and positive profile deviation, ISO 8.
5.6. Influence of manufacturing errors on the loaded function of transmission errors

Interestingly, the profile crowning tends to have a different influence on the LTE function for the cases of positive and negative profile deviation. Gear drives simulated with positive profile deviations tended to have their PTP value reduced more significantly than those with negative profile deviations, but gears with a negative profile deviation tended to have a more symmetrical and smooth LTE function.

After assembling some preliminary results, a DOE was performed to determine the influence of each class of manufacturing error on the magnitude of the first three harmonics of the gear drive outlined in Geometry 1. The DOE is the same one as outlined in Table 5.3. Figures 5.21 and 5.22 show the calculated LTEs, and Figures 5.23 thru 5.25 shows the fitted means of the magnitude of the first three harmonics (in arcsec) for the two levels of error outlined by the DOE for each factor.
These results indicate that profile deviation had the most impact on the excitation of the gear drive’s meshing frequencies from the LTE function. Additionally, profile deviation direction
5.6. Influence of manufacturing errors on the loaded function of transmission errors

had a significant influence on the excitation of the second and third meshing frequencies, with a positive profile deviation resulting in a greater vibrational response from the gear drive.

**Figure 5.23:** Sensitivity of the excitation of the first meshing frequency (350Hz) to manufacturing errors for Geometry 1.

**Figure 5.24:** Sensitivity of the excitation of the second meshing frequency (700Hz) to manufacturing errors for Geometry 1.

**Figure 5.25:** Sensitivity of the excitation of the third meshing frequency (1050Hz) to manufacturing errors for Geometry 1.
Figures 5.26 thru 5.28 shows the fitted means of the magnitude of the first three harmonics (in arcsec) for the case of 14 µm of profile crowning applied to the pinion.

**Figure 5.26**: Sensitivity of the excitation of the first meshing frequency (350Hz) to manufacturing errors with 14 µm profile crowning for Geometry 1.

**Figure 5.27**: Sensitivity of the excitation of the second meshing frequency (700Hz) to manufacturing errors with 14 µm profile crowning for Geometry 1.

**Figure 5.28**: Sensitivity of the excitation of the third meshing frequency (1050Hz) to manufacturing errors with 14 µm profile crowning for Geometry 1.
After profile crowning was applied, the sensitivity of the 350Hz response to profile deviation magnitude was reduced significantly. The only factor which had a significant impact on the 350Hz response was profile deviation direction. The 700Hz and 1050Hz responses were also less sensitive to the level of deviation to all factors than the response without profile crowning. In addition to the sensitivity reduction, the mean response of all frequencies was reduced after profile crowning was applied.

5.7 Summary of results

In this Chapter, two spur gear geometries were considered as outlined in Table 5.1. Geometry 1, with a pressure angle of 20°, was most sensitive to the profile deviation direction for the case of no tooth modifications. Interestingly, Geometry 1 demonstrated little sensitivity to the amount of profile deviation by itself, with an average influence of only 13.68 MPa between ISO grades 2 and 6 for the case of no tooth modifications. To recall, a gear of grade ISO 6 will have tolerances four times as loose as that of ISO 2. Geometry 2, with a pressure angle of 25°, was also most sensitive to the profile deviation direction for the case of no tooth modifications. However, unlike Geometry 1, Geometry 2 demonstrated relatively high sensitivity to the amount of profile deviation, with an average influence of 60 MPa between ISO grades 2 and 6 for the case of no tooth modifications. With no tooth modifications, Geometry 2 demonstrated less sensitivity overall to manufacturing errors than Geometry 1.

Once tip relief was considered, the sensitivity the peak contact stress experienced in a cycle of meshing by Geometries 1 and 2 to manufacturing errors decreased dramatically. This modification was most beneficial for Geometry 1, where sensitivities were high to begin with. The only factor which contributed substantially to peak contact stress in Geometries 1 and 2 with tip relief was helix deviation, with an influence of 69.66 MPa for Geometry 1 and 65.56 MPa for Geometry 2. Longitudinal crowning benefited these two gear drives further, reducing the sensitivity to any individual factor to less than 40 MPa.

Geometries 1 and 2 also demonstrated a linear relationship between the level of helix deviation and the optimal magnitude of longitudinal crowning. A regression analysis was performed on the optimal crowning magnitude data for both Geometries 1 and 2, and it was observed that
two curves varied very little from each other over a wide range of helix deviations, ranging from gears of grade ISO 0 to ISO 10.

The excitation of the meshing frequencies in the LTE function for Geometry 1 was shown to be highly sensitive to manufacturing errors for the case of no modifications. For Geometry 1 with no modifications, profile deviation had the most impact on the excitation of the gear drive’s meshing frequencies from the LTE function. However, the application of 14 \( \mu \text{m} \) of longitudinal crowning was able to reduce the sensitivity of the excitation of the first meshing frequency between ISO Grades 2 and 6 to all factors but profile deviation direction, which increased.
Chapter 6

Helical Gears and Their Sensitivity to Manufacturing Errors

6.1 Introduction

As mentioned previously, spur gears are a special case of helical gears where the helix angle is equal to zero. In many applications, the use of helical gears can result in improved performance of the gear drive [2]. Unlike in spur gearing, contact between teeth in a helical gear drive does not begin along the entire tooth at once. This enables teeth to mesh gradually, and can mitigate the vibrations and cyclic loads which the teeth are subjected to in a spur gear drive. However, helical gears will generate an additional axial load due to the contact lines not remaining parallel to the direction of rotation.

Chapter 5 discussed the influence of manufacturing errors and geometry modifications on spur gears, but did not address these factors for the case of helical gears. The inclusion of a helix angle in a study examining geometry modifications and manufacturing errors is more challenging to simulate due to the large number of elements needed to accurately determine stresses and strains, as well as the 3D mesh required to capture the helix angle.

6.2 Definition of gear geometries

Tables 6.1 and 6.2 describe the macro-geometry design parameters of the four helical gear drives simulated for the designed experiment, as outlined in Table 6.3. Only those parameters
relevant for the virtual generation of the gear drives are shown.

<table>
<thead>
<tr>
<th>PINION</th>
<th>WHEEL</th>
<th>PINION</th>
<th>WHEEL</th>
</tr>
</thead>
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<td>21</td>
<td>37</td>
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<table>
<thead>
<tr>
<th>GEOMETRY 3</th>
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</thead>
<tbody>
<tr>
<td>Number of teeth</td>
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<tr>
<td>Module [mm]</td>
<td>4.0</td>
</tr>
<tr>
<td>Normal pressure angle [deg]</td>
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</tr>
<tr>
<td>Helix angle [deg]</td>
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<tr>
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<td>1.00</td>
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<tr>
<td>Dedendum coefficient [-]</td>
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<tr>
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<td>Generating shift coefficient [-]</td>
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<tr>
<td>Face width [mm]</td>
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</tr>
<tr>
<td>Center distance [mm]</td>
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</tr>
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</table>

TABLE 6.1: Macro-geometry parameters of Geometries 3 and 4.

<table>
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<tr>
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<th>PINION</th>
<th>WHEEL</th>
</tr>
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<tbody>
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<td>37</td>
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</tbody>
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<table>
<thead>
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<th>GEOMETRY 6</th>
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</thead>
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<td>Module [mm]</td>
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<tr>
<td>Normal pressure angle [deg]</td>
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</tr>
<tr>
<td>Helix angle [deg]</td>
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<tr>
<td>Addendum coefficient [-]</td>
<td>1.00</td>
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</tr>
<tr>
<td>Center distance [mm]</td>
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</tr>
</tbody>
</table>

TABLE 6.2: Macro-geometry parameters of Geometries 5 and 6.

6.2.1 Design of Experiments for Geometries 3, 4, 5, and 6

For Geometries 3, 4, 5, and 6, the additional parameter of "Hand of Helix Deviation" was considered. For the case of spur gears, with a helix angle of zero, it is expected that the direction of the helix deviation would be unimportant, since the gear drive is otherwise symmetric.
6.2. Definition of gear geometries

However, in helical gears, this is not the case. Table 6.3 lists the parameters and values used for levels of manufacturing error for the helical gear DOEs.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Minimum Value</th>
<th>Maximum Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Pitch Deviation</td>
<td>mm</td>
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<td>0.028</td>
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<tr>
<td>Total Profile Deviation</td>
<td>mm</td>
<td>0.0034</td>
<td>0.013</td>
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<tr>
<td>Profile Deviation Direction</td>
<td>-</td>
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<td>Positive</td>
</tr>
<tr>
<td>Total Helix Deviation</td>
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<td>0.012</td>
</tr>
<tr>
<td>Hand of Helix Deviation</td>
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<td>Right</td>
</tr>
</tbody>
</table>

**Table 6.3**: DOE parameters for Geometries 3, 4, 5, and 6.

Setting up a full factorial experiment from these values yielded thirty two runs, and the details of each run is demonstrated in Tables 6.4 and 6.5.

<table>
<thead>
<tr>
<th>Total Pitch Dev. [µm]</th>
<th>Total Profile Dev. [µm]</th>
<th>Total Helix Dev. [µm]</th>
<th>Profile Dev. Direction</th>
<th>Helix Dev. Direction</th>
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</thead>
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</table>

**Table 6.4**: DOE conditions for helical geometry Trials 1 through 16.
Chapter 6. Helical Gears and Their Sensitivity to Manufacturing Errors

<table>
<thead>
<tr>
<th>Total Pitch Dev. [μm]</th>
<th>Total Profile Dev. [μm]</th>
<th>Total Helix Dev. [μm]</th>
<th>Profile Dev. Direction</th>
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<tr>
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<td>28.0</td>
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<td>7.0</td>
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<tr>
<td>28.0</td>
<td>13.0</td>
<td>Negative</td>
<td>12.0</td>
<td>Right</td>
</tr>
</tbody>
</table>

Table 6.5: DOE conditions for helical geometry Trials 17 through 32.

6.2.2 Mesh selection for Geometry 3

To find a mesh with sufficient density to provide accurate results for the stresses experienced by both the wheel and the pinion in Geometry 3, three preliminary meshes were generated, and ran in Abaqus to test for convergence. These meshes were generated in IGD based upon a gear drive with no micro-geometry modifications or manufacturing errors considered, for simplicity. Table 6.6 lists the properties of the preliminary multipoint-constraint finite element models attempted for Geometry 3 for the pinion and wheel respectively. Each FEM utilizes one pinion-wheel pairing.

<table>
<thead>
<tr>
<th>Mesh Number</th>
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<th>Fillet Elements</th>
</tr>
</thead>
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<td>16</td>
<td>8</td>
</tr>
<tr>
<td>Mesh 1 Wheel</td>
<td>16</td>
<td>16</td>
<td>8</td>
</tr>
<tr>
<td>Mesh 2 Pinion</td>
<td>48</td>
<td>32</td>
<td>16</td>
</tr>
<tr>
<td>Mesh 2 Wheel</td>
<td>32</td>
<td>32</td>
<td>8</td>
</tr>
<tr>
<td>Mesh 3 Pinion</td>
<td>64</td>
<td>48</td>
<td>16</td>
</tr>
<tr>
<td>Mesh 3 Wheel</td>
<td>48</td>
<td>32</td>
<td>8</td>
</tr>
</tbody>
</table>

Table 6.6: Pinion mesh properties for Geometry 3 mesh refinement.
With each the models outlined in Table 6.6, the evolution of stresses was found. Figures 6.1, 6.2, 6.3, and 6.4 compare the evolution of stresses across each model.

**Figure 6.1:** Pinion flank stresses at various mesh densities for Geometry 3.

**Figure 6.2:** Wheel flank stresses at various mesh densities for Geometry 3.
Chapter 6. Helical Gears and Their Sensitivity to Manufacturing Errors

The evolution of stresses throughout a full cycle of meshing is expected to be a perfectly periodic function in these cases, since the gears simulated in this preliminary mesh refinement had no simulated manufacturing errors. This is best evaluated from the calculated evolutions of fillet stresses, in Figures 6.3 and 6.4. It is expected that Mesh 3, with the densest meshes for both the pinion and wheel, will produce the most accurate result. However, Mesh 2 produces a similarly accurate result with substantially fewer elements, resulting in a shorter computation time. From this data, Mesh 2 was selected to be used in the DOEs for Geometry 3.
6.2.3 Mesh selection for Geometry 4

To find a mesh with sufficient density to provide accurate results for the stresses experienced by both the wheel and the pinion in Geometry 4, three preliminary meshes were generated, and ran in Abaqus to test for convergence. These meshes were generated in IGD based upon a gear drive with no micro-geometry modifications or manufacturing errors considered, for simplicity. Table 6.7 lists the properties of the preliminary multipoint-constraint finite element models attempted for Geometry 4.

<table>
<thead>
<tr>
<th>Mesh Number</th>
<th>Longitudinal Elements</th>
<th>Transverse Elements</th>
<th>Fillet Elements</th>
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<td>Mesh 2 Wheel</td>
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</tr>
<tr>
<td>Mesh 3 Wheel</td>
<td>64</td>
<td>48</td>
<td>16</td>
</tr>
</tbody>
</table>

| Table 6.7: Mesh properties for Geometry 4 mesh refinement. |

Since Geometry 4 has the same helix angle as Geometry 3, it is expected that the geometries will require similar meshes to achieve convergence in the solution.

![Figure 6.5: Pinion flank stresses at various mesh densities for Geometry 4.](image)
Chapter 6. Helical Gears and Their Sensitivity to Manufacturing Errors

The evolution of stresses throughout a full cycle of meshing is expected to be a perfectly periodic function in these cases, since the gears simulated in this preliminary mesh refinement...
had no simulated manufacturing errors. Mesh 3, with the finest mesh density, is also expected to yield the most accurate results. However, for Geometry 4, the results for Mesh 2 were sufficiently close to Mesh 3 at all contacting positions for the purposes of evaluating the sensitivity to manufacturing errors, with the maximum deviation between the two meshes of 16.9 percent, and an average deviation of only 6.2 percent. Since Mesh 2 was less dense, the computation time was shorter than Mesh 3, so for Geometry 4’s DOE, Mesh 2 was selected.

### 6.2.4 Mesh selection for Geometry 5

To find a mesh with sufficient density to provide accurate results for the stresses experienced by both the wheel and the pinion in Geometry 5, four preliminary meshes were generated, and ran in Abaqus to test for convergence. These meshes were generated in IGD based upon a gear drive with no micro-geometry modifications or manufacturing errors considered, for simplicity. Table 6.8 lists the properties of the preliminary multipoint-constraint finite element models attempted for Geometry 5.

<table>
<thead>
<tr>
<th>Mesh Number</th>
<th>Longitudinal Elements</th>
<th>Transverse Elements</th>
<th>Fillet Elements</th>
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</tr>
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<td>Mesh 4 Wheel</td>
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<td>48</td>
<td>16</td>
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</table>

**Table 6.8:** Mesh properties for Geometry 5 mesh refinement.

Since Geometries 5 and 6 have a greater helix angle than Geometries 3 and 4, it is expected that the geometries will require more elements in the longitudinal direction to achieve convergence in the solution.
Chapter 6. Helical Gears and Their Sensitivity to Manufacturing Errors

![Figure 6.9: Pinion flank stresses at various mesh densities for Geometry 5.](image1)

![Figure 6.10: Wheel flank stresses at various mesh densities for Geometry 5.](image2)

![Figure 6.11: Pinion fillet stresses at various mesh densities for Geometry 5.](image3)
6.2. Definition of gear geometries

6.2.5 Mesh selection for Geometry 6

To find a mesh with sufficient density to provide accurate results for the stresses experienced by both the wheel and the pinion in Geometry 6, four preliminary meshes were generated, and ran in Abaqus to test for convergence. These meshes were generated in IGD based upon a gear drive with no micro-geometry modifications or manufacturing errors considered, for simplicity. Table 6.9 lists the properties of the preliminary multipoint-constraint finite element models attempted for Geometry 6.

<table>
<thead>
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<th>Longitudinal Elements</th>
<th>Transverse Elements</th>
<th>Fillet Elements</th>
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<tr>
<td>Mesh 1 Wheel</td>
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<td>Mesh 2 Pinion</td>
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</tbody>
</table>

Table 6.9: Mesh properties for Geometry 6 mesh refinement.
Figure 6.13: Pinion flank stresses at various mesh densities for Geometry 6.

Figure 6.14: Wheel flank stresses at various mesh densities for Geometry 6.

Figure 6.15: Pinion fillet stresses at various mesh densities for Geometry 6.
6.2. Definition of gear geometries

6.2.6 Optimal tip relief for Geometries 3, 4, 5, and 6

As discussed in Chapter 3, tip relief can improve gear performance by preventing contact near the leading edge of the teeth throughout a cycle of meshing, (which typically leads to high contact stresses). In helical gears, contact at the tip of the tooth is maintained throughout the entire cycle of meshing, making tip relief modifications particularly important for helical gear drives. Similarly to spur gear Geometries 1 and 2, a parabolic shape of tip relief will be assumed (linear tip relief can result in increased stress experienced by the fillet [4]). However, for Geometries 3, 4, 5, and 6, tip relief will be considered on both the pinion and the wheel in all cases.

To find the optimal tip relief, first, simulations were ran varying tip relief intensity and length for the case of no manufacturing errors. Similar in Geometries 1 and 2, optimization was defined as the combination of tip relief at the addendum radius and datum length providing the lowest maximum von-Mises contact stress in the tooth surfaces. For Geometries 3 thru 6, $L$ was varied between 1 mm and 1.5 mm, and $\delta$ varied between $10\mu$m, $15\mu$m, and $20\mu$m.

The results for the case of Geometry 3 can be seen in Figures 6.17 and 6.18. Varying levels of tip relief had a near insignificant impact on fillet stresses, much less than the precision of the finite element simulation as established in Chapter 4. For Geometry 3, the fillet stresses have also been shown in Figures 6.19 and 6.20 to illustrate this, but plots of fillet stress are not included for Geometries 4, 5, and 6.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure6_16}
\caption{Wheel fillet stresses at various mesh densities for Geometry 6.}
\end{figure}
Chapter 6. Helical Gears and Their Sensitivity to Manufacturing Errors

The combination of $\delta = 20 \mu m$ and $L = 1.5$ mm yielded the lowest peak contact stress in the pinion flank, and was selected as the optimal tip relief for Geometry 3.
6.2. Definition of gear geometries

For Geometry 4, the same combinations of $L$ and $\delta$ were tested. Figures 6.21 and 6.22 compare the evolution of stresses across different levels of tip relief for Geometry 4.
Chapter 6. Helical Gears and Their Sensitivity to Manufacturing Errors

Figure 6.21: Evolution of pinion flank contact stresses at different levels of tip relief for Geometry 4.

Based on these results, an optimal tip relief was determined to be $\delta = 15 \, \mu m$ and $L = 1.5 \, mm$.

Figures 6.23 and 6.24 compare the evolution of contact stresses across different amounts of tip relief for Geometry 5. Based on these results, an optimal tip relief was determined to be $\delta = 15 \, \mu m$ and $L = 1.5 \, mm$.
6.2. Definition of gear geometries

Figures 6.23 and 6.24 compare the evolution of contact stresses across different levels of tip relief for Geometry 5. Based on these results, an optimal tip relief was determined to be $\delta = 15 \mu m$ and $L = 1.5 \text{ mm}$. 

Figures 6.25 and 6.26 compare the evolution of contact stresses across different levels of tip relief for Geometry 6. Based on these results, an optimal tip relief was determined to be $\delta = 15 \mu m$ and $L = 1.5 \text{ mm}$. 

**Figure 6.23:** Evolution of pinion flank contact stresses at different levels of tip relief for Geometry 5.

**Figure 6.24:** Evolution of wheel flank contact stresses at different levels of tip relief for Geometry 5.
Table 6.10 summarizes the optimal parameters for all helical gear geometries.

<table>
<thead>
<tr>
<th>Geometry</th>
<th>Optimal $\delta$ [µm]</th>
<th>Optimal $L$ [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometry 3</td>
<td>20</td>
<td>1.5</td>
</tr>
<tr>
<td>Geometry 4</td>
<td>15</td>
<td>1.5</td>
</tr>
<tr>
<td>Geometry 5</td>
<td>15</td>
<td>1.5</td>
</tr>
<tr>
<td>Geometry 6</td>
<td>15</td>
<td>1.5</td>
</tr>
</tbody>
</table>

**Table 6.10**: Optimal tip relief for Geometries 3, 4, 5, and 6.
6.3 Sensitivity to manufacturing errors for helical gears with tip relief

In helical gears, the contact lines along the gear teeth are not parallel to the center axis of the gears. One consequence of this is that the tooth will maintain contact at the tip throughout nearly the entire cycle of meshing. Since contact at the tip is maintained throughout the entire cycle of meshing, tip relief is critical for helical gears. A comparison of the contact pattern of a helical gear drive with and without tip relief is illustrated in Figure 6.27.

**Figure 6.27:** Comparison of the contact pattern of a Geometry 3 with and without tip relief. The top image shows the case of no tip relief, where the contact pattern reaches the edges of the teeth. In the bottom image, with tip relief, the contact pattern takes the form of a smooth elliptical curve.
Chapter 6. Helical Gears and Their Sensitivity to Manufacturing Errors

For this study, the baseline sensitivity for helical gears was established from the case of tip relief as the only tooth modification. For each trial in the DOE, the optimal level of tip relief as identified in Table 6.10 was applied to both the wheel and pinion of the simulated gear drive.

From the DOE, the influence of each class of manufacturing error on peak contact stress can be found individually. In the remainder of this section, plots of fitted means will be shown, which compare the average peak contact stress at each level for each factor. A sample plot illustrating how to interpret these plots can be found in Section 4.2, Figure 4.2.

6.3.1 Geometry 3

The obtained effects plots for peak contact stress experienced by the pinion and wheel for Geometry 3 are shown in Figures 6.28 and 6.29 respectively.

![Pareto Chart of the Effects](image)

**FIGURE 6.28:** Pareto chart of the effects of manufacturing errors on peak stress in the pinion flank for Geometry 3 with tip relief.

For peak pinion contact stress of Geometry 3, the most significant factors were pitch deviation, hand of helix deviation, and profile deviation direction. The most significant combined factors were profile deviation and profile deviation direction, profile deviation direction and hand of helix deviation.
6.3. Sensitivity to manufacturing errors for helical gears with tip relief

For peak wheel contact stress of Geometry 3, the most significant factors were pitch deviation, helix deviation, and hand of helix deviation. The most significant combined factors were profile deviation direction and hand of helix deviation, and helix deviation and hand of helix deviation.

The obtained sensitivity plots for peak contact stress experienced by the pinion and wheel for Geometry 3 are shown in Figures 6.30 and 6.31 respectively.
Figure 6.31: Sensitivity of peak stress in the wheel flank to manufacturing errors for Geometry 3 with tip relief.

The mean peak pinion flank stress amongst all trials was 1976 MPa. For wheel peak contact stresses, the average amongst all trials was 1914 MPa. Table 6.11 summarizes all sensitivity results for Geometry 3 with only tip relief of $\delta = 20 \, \mu m$ and $L = 1.5 \, mm$.

<table>
<thead>
<tr>
<th>Manufacturing Error Class</th>
<th>Range of Error</th>
<th>Pinion Sensitivity [MPa]</th>
<th>Wheel Sensitivity [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch Deviation</td>
<td>0.007 to 0.028 [\mu m]</td>
<td>$\pm 228.1$</td>
<td>$\pm 188.6$</td>
</tr>
<tr>
<td>Profile Deviation</td>
<td>0.0034 to 0.0130 [\mu m]</td>
<td>$\pm 84.66$</td>
<td>$\pm 87.41$</td>
</tr>
<tr>
<td>Profile Deviation Direction</td>
<td>Positive or Negative</td>
<td>$\pm 173.8$</td>
<td>$\pm 102.5$</td>
</tr>
<tr>
<td>Helix Deviation</td>
<td>0.003 to 0.012 [\mu m]</td>
<td>$\pm 42.36$</td>
<td>$\pm 166.9$</td>
</tr>
<tr>
<td>Hand of Helix Deviation</td>
<td>Right or Left</td>
<td>$\pm 194.8$</td>
<td>$\pm 162.8$</td>
</tr>
</tbody>
</table>

Table 6.11: Summary of peak stress sensitivities to manufacturing error for Geometry 3 with tip relief.

For all factors except helix deviations, the pinion was more sensitive than the wheel. Also interesting is that a positive profile deviation reduced the stresses in both the pinion and the wheel, while a negative deviation increased stresses in both gears. Additionally, a right handed helix deviation reduced the stresses in both the pinion and the wheel, while a left handed deviation increased stresses in both gears. These two factors combined had a large influence on the
peak contact stress for both the wheel and the pinion, being the fifth most significant factor for
the pinion and the fourth most significant for the wheel.

6.3.2 Geometry 4

The obtained effects plots for peak contact stress experienced by the pinion and wheel for
Geometry 4 are shown in Figures 6.32 and 6.33 respectively.

For peak pinion contact stress of Geometry 4, the three most significant factors were hand of
helix deviation, profile deviation direction, and pitch deviation, in that order. The most signif-
icant combined factors were profile deviation direction and hand of helix deviation, followed
by profile deviation and profile deviation direction.

For peak wheel contact stress of Geometry 4, the most significant factor was the combined
influence of profile deviation direction and hand of helix deviation, with an influence of 641
MPa. The next two factors with the most influence were pitch deviation and helix deviation.
Chapter 6. Helical Gears and Their Sensitivity to Manufacturing Errors

Figure 6.33: Pareto chart of the effects of manufacturing errors on peak stress in the wheel flank for Geometry 4 with tip relief.

The sensitivity plots for Geometry 4 are shown in Figures 6.34 and 6.35.

Figure 6.34: Sensitivity of peak stress in the pinion flank to manufacturing errors for Geometry 4 with tip relief.
6.3. Sensitivity to manufacturing errors for helical gears with tip relief

The mean peak pinion flank stress amongst all trials was 2041 MPa. For wheel peak contact stresses, the average amongst all trials was 2093 MPa. Table 6.12 summarizes the results of the influence of the individual factors.

<table>
<thead>
<tr>
<th>Manufacturing Error Class</th>
<th>Range of Error</th>
<th>Pinion Sensitivity [MPa]</th>
<th>Wheel Sensitivity [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch Deviation</td>
<td>0.007 to 0.028 [µm]</td>
<td>±242.8</td>
<td>±184.1</td>
</tr>
<tr>
<td>Profile Deviation</td>
<td>0.0034 to 0.0130 [µm]</td>
<td>±79.57</td>
<td>±161.4</td>
</tr>
<tr>
<td>Profile Deviation Direction</td>
<td>Positive or Negative</td>
<td>±263.2</td>
<td>±102.5</td>
</tr>
<tr>
<td>Helix Deviation</td>
<td>0.003 to 0.012 [µm]</td>
<td>±94.74</td>
<td>±175.7</td>
</tr>
<tr>
<td>Hand of Helix Deviation</td>
<td>Right or Left</td>
<td>±279.8</td>
<td>±84.40</td>
</tr>
</tbody>
</table>

Table 6.12: Summary of peak stress sensitivities to manufacturing error for Geometry 4 with tip relief.

The pinion was more sensitive than the wheel to the factors of pitch deviation, profile deviation direction, and hand of helix deviation. Unlike Geometry 3, altering the directionality of the profile and helix deviation loaded one gear, and reduced stresses in the other. Recall that Geometry 4 was characterized by a pressure angle of 25°, and the same helix angle as Geometry 3.
6.3.3 Geometry 5

The obtained effects plots for peak contact stress experienced by the pinion and wheel for Geometry 5 are shown in Figures 6.36 and 6.37 respectively.

![Pareto Chart of the Effects](image)

Figure 6.36: Pareto chart of the effects of manufacturing errors on peak stress in the pinion flank for Geometry 5 with tip relief.

For peak pinion contact stress of Geometry 5, the most significant factors the combined influence of profile deviation direction and hand of helix deviation. The most significant combined factors were helix deviation and hand of helix deviation, and profile deviation and profile deviation direction.

For peak wheel contact stress of Geometry 5, the most significant factor was the combined influence of profile deviation direction and hand of helix deviation. The next three factors with the most influence were pitch deviation, profile deviation, and helix deviation.
6.3. Sensitivity to manufacturing errors for helical gears with tip relief

**Figure 6.37:** Pareto chart of the effects of manufacturing errors on peak stress in the wheel flank for Geometry 5 with tip relief.

The sensitivity plots for Geometry 5 are shown in Figures 6.38 and 6.39.

**Main Effects Plot for Pinion Peak Contact Stress**

![Main Effects Plot for Pinion Peak Contact Stress](image)

**Figure 6.38:** Sensitivity of peak stress in the pinion flank to manufacturing errors for Geometry 5 with tip relief.
The mean peak pinion flank stress amongst all trials was 2195 MPa. Within the pinion, the difference between the average of profile deviation and helix deviation at grades ISO 2 and ISO 6 were so small, it was within the margins of error of the finite element simulation. For wheel peak contact stresses, the average amongst all trials was 1907 MPa. For the wheel, sensitivity to any individual factor never exceeded 120 MPa.

<table>
<thead>
<tr>
<th>Manufacturing Error Class</th>
<th>Range of Error</th>
<th>Pinion Sensitivity [MPa]</th>
<th>Wheel Sensitivity [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch Deviation</td>
<td>0.007 to 0.028 [µm]</td>
<td>±216.1</td>
<td>±112.4</td>
</tr>
<tr>
<td>Profile Deviation</td>
<td>0.0034 to 0.0130 [µm]</td>
<td>±29.96</td>
<td>±100.4</td>
</tr>
<tr>
<td>Profile Deviation Direction</td>
<td>Positive or Negative</td>
<td>±316.3</td>
<td>±78.09</td>
</tr>
<tr>
<td>Helix Deviation</td>
<td>0.003 to 0.012 [µm]</td>
<td>±0.4566</td>
<td>±85.44</td>
</tr>
<tr>
<td>Hand of Helix Deviation</td>
<td>Right or Left</td>
<td>±308.3</td>
<td>±94.66</td>
</tr>
</tbody>
</table>

TABLE 6.13: Summary of peak stress sensitivities to manufacturing error for Geometry 5 with tip relief.

Like seen in Geometry 3, for Geometry 5, a positive profile deviation reduced the stresses in both the pinion and the wheel, while a negative deviation increased stresses in both gears.
6.3. Sensitivity to manufacturing errors for helical gears with tip relief

Additionally, a right handed helix deviation reduced the stresses in both the pinion and the wheel, while a left handed deviation increased stresses in both gears.

6.3.4 Geometry 6

The obtained effects plots for peak contact stress experienced by the pinion and wheel for Geometry 6 are shown in Figures 6.40 and 6.41 respectively.

![Pareto Chart of the Effects](image)

**Figure 6.40:** Pareto chart of the effects of manufacturing errors on peak stress in the pinion flank for Geometry 6 with tip relief.

For peak pinion contact stress of Geometry 6, the most significant factors were pitch deviation, the combined effect of profile deviation direction and hand of helix deviation, and hand of helix deviation.

For peak wheel contact stress of Geometry 6, the most significant factor was again the combined influence of profile deviation direction and hand of helix deviation. The next two factors with the most influence were pitch deviation and helix deviation. The ranking of influence for Geometry 6 was the same as for Geometry 4.
Chapter 6. Helical Gears and Their Sensitivity to Manufacturing Errors

Figure 6.41: Pareto chart of the effects of manufacturing errors on peak stress in the wheel flank for Geometry 6 with tip relief.

The sensitivity plots for Geometry 6 are shown in Figures 6.42 and 6.43.

Figure 6.42: Sensitivity of peak stress in the pinion flank to manufacturing errors for Geometry 6 with tip relief.
6.3. Sensitivity to manufacturing errors for helical gears with tip relief

The mean peak pinion flank stress amongst all trials was 2195 MPa. For wheel peak contact stresses, the average amongst all trials was 2056 MPa.

<table>
<thead>
<tr>
<th>Manufacturing Error Class</th>
<th>Range of Error</th>
<th>Pinion Sensitivity [MPa]</th>
<th>Wheel Sensitivity [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch Deviation</td>
<td>0.007 to 0.028 [µm]</td>
<td>±290.9</td>
<td>±168.3</td>
</tr>
<tr>
<td>Profile Deviation</td>
<td>0.0034 to 0.0130 [µm]</td>
<td>±106.2</td>
<td>±127.5</td>
</tr>
<tr>
<td>Profile Deviation Direction</td>
<td>Positive or Negative</td>
<td>±161.4</td>
<td>±81.46</td>
</tr>
<tr>
<td>Helix Deviation</td>
<td>0.003 to 0.012 [µm]</td>
<td>±156.2</td>
<td>±165.4</td>
</tr>
<tr>
<td>Hand of Helix Deviation</td>
<td>Right or Left</td>
<td>±171.9</td>
<td>±54.22</td>
</tr>
</tbody>
</table>

Like seen in Geometry 4, for Geometry 6, altering the directionality of the profile and helix deviation loaded one gear, and reduced stresses in the other. In this geometry, the pinion was highly sensitive to pitch deviations, but showed sensitivity of over 100MPa to all factors. The wheel was most sensitive to pitch deviation and amount of helix deviation.
6.4 Further results on hand of helix deviation and profile deviation direction for helical gear geometries

In section 6.3, it was seen that Geometries 3 and 5 exhibited an unusual response to hand of helix deviation and profile deviation direction. Typically, a positive profile deviation direction is expected to result in increased loading on the wheel, and decreased loading on the pinion. A negative profile deviation direction typically will result in the opposite behavior. Geometries 4 and 6 exhibited this behavior (as did Geometries 1 and 2). However, in Geometries 3 and 5, a positive profile deviation direction resulted in decreased stresses among both the pinion and the wheel. Furthermore, this behavior was also seen with regards to hand of helix deviation, where a right handed helix deviation was noted to be preferable to a left handed deviation for Geometries 3 and 5.

To further investigate this, additional simulations were performed with Geometry 3, varying only hand of helix deviation and profile deviation direction at grade ISO 6. The generated gears were then simulated both counter-clockwise (similarly to the DOE) and clockwise, to determine if direction of rotation was responsible for this observation. The resulting evolutions of peak contact stress are shown in Figures 6.44 and 6.45 for counter-clockwise rotation, and Figures 6.46 and 6.47 for clockwise rotation.

![Figure 6.44: Evolution of peak stress in the pinion flank for Geometry 3 at ISO grade 6 and counter-clockwise rotation with different error directions.](image-url)
6.4. Further results on hand of helix deviation and profile deviation direction...

Figure 6.45: Evolution of peak stress in the wheel flank for Geometry 3 at ISO grade 6 and counter-clockwise rotation with different error directions.

Figure 6.46: Evolution of peak stress in the pinion flank for Geometry 3 at ISO grade 6 and clockwise rotation with different manufacturing error directions.

Figure 6.47: Evolution of peak stress in the wheel flank for Geometry 3 at ISO grade 6 and clockwise rotation with different manufacturing error directions.
As seen in the plots, the evolutions of stress are nearly identical regardless of rotation direction. In both cases, a positive profile deviation and a right-handed helix deviation result in the lowest peak contact stress in the pinion and wheel. Cases with either a negative profile deviation or a left-handed helix deviation experienced minimally increased stress. However, cases with both a negative profile deviation and left-handed helix deviation experienced significantly increased stress at contact position 16. Examination of the simulations in the Abaqus viewer demonstrated that although the contacting position of peak stress was the same in both cases, the tooth in contact at that time was not the same.
6.5. Longitudinal crowning and front/back relief

For both clockwise and counter-clockwise rotation, the peak stress was experienced when one of the pinion teeth began contact. With a left-handed deviation, this leading edge of the tooth gained interference with the wheel tooth. Additionally, a negative profile deviation compounded this effect, also contributing towards increased interference between contacting teeth. The combined effects of these two errors resulted in a much higher contact stress experienced by the tooth.

6.5 Longitudinal crowning and front/back relief

Unlike in spur gears, helical gears do not have contact lines across the full width of the tooth during meshing. Because of this, the benefits of longitudinal crowning are typically less apparent in helical gears than spur gears, since pulling the contact towards the center of the tooth can also substantially delay contact between teeth during meshing, increasing the period of time where there are fewer contacting teeth. In an attempt to further reduce sensitivity to manufacturing errors, longitudinal crowning, as well as front and back relief were simulated for Geometry 3 with gears of grade ISO 2 and ISO 6, for consistency with the DOE results. Figures 6.51, 6.52, 6.53, and 6.52 illustrate these results.
Chapter 6. Helical Gears and Their Sensitivity to Manufacturing Errors

**Figure 6.51:** Longitudinal crowning vs front/back relief pinion flank stresses for Geometry 3.

**Figure 6.52:** Longitudinal crowning vs front/back relief wheel flank stresses for Geometry 3.
6.6 Influence of manufacturing errors on the loaded function of transmission errors

As mentioned in Chapter 5, the function of loaded transmission errors (LTEs) can be challenging to calculate since it accounts for the deformation of the teeth throughout the cycle of meshing. The function of loaded transmission errors can be calculated by combining the results of finite element analysis with unloaded transmission error results, which can be obtained from the undeformed gear models. The function of transmission errors for helical gears tends to be smoother than that of spur gears, in general. Figure 6.55 compares the function of transmission errors for helical gears with different manufacturing error configurations. Figure 6.53 and Figure 6.54 show the comparison for pinion and wheel fillet stresses, respectively, for Geometry 3.
errors through two cycles of meshing for Geometries 1 (spur gear) and 3 and 5 (helical gears) with positive profile deviation. All three of these geometries share a pressure angle of $20^\circ$.

Since multiple teeth are in contact at any given time, the helical gear teeth experience less deflection than their spur gear counterpart (with the same pressure angle, module, face width, and number of teeth). As seen in Figure 6.55, the two helical gear geometries follow a gentle curve, and the spur gear drive has steep drop-offs at contacting positions where a tooth comes out of contact, and a steep incline when a tooth comes into contact. As a general statement, the total contacting area between all teeth remains relatively constant throughout the cycle of meshing for helical gear drives, but spur gear drives have a large change in contacting area when a tooth comes into or out of contact. By comparison, these helical gears have a much smoother function of transmission errors, which should result in less excitation of the meshing frequencies.

Figures 6.56 through 6.58 demonstrate the sensitivity of the excitation of the first three meshing frequencies to each class of manufacturing error for Geometry 3.
6.6. Influence of manufacturing errors on the loaded function of transmission errors

**Figure 6.56:** Sensitivity of the excitation of the first meshing frequency (350Hz) to manufacturing errors with $\delta = 20 \, \mu m$ and $L = 1.5 \, mm$ tip relief for Geometry 3.

**Figure 6.57:** Sensitivity of the excitation of the second meshing frequency (700Hz) to manufacturing errors with $\delta = 20 \, \mu m$ and $L = 1.5 \, mm$ tip relief for Geometry 3.

**Figure 6.58:** Sensitivity of the excitation of the third meshing frequency (1050Hz) to manufacturing errors with $\delta = 20 \, \mu m$ and $L = 1.5 \, mm$ tip relief for Geometry 3.
Figures 6.59 through 6.61 demonstrate the sensitivity of the excitation of the first three meshing frequencies to each class of manufacturing error for Geometry 4.

**Figure 6.59:** Sensitivity of the excitation of the first meshing frequency (350Hz) to manufacturing errors with $\delta = 20 \mu m$ and $L = 1.5$ mm tip relief for Geometry 4.

**Figure 6.60:** Sensitivity of the excitation of the second meshing frequency (700Hz) to manufacturing errors with $\delta = 20 \mu m$ and $L = 1.5$ mm tip relief for Geometry 4.

**Figure 6.61:** Sensitivity of the excitation of the third meshing frequency (1050Hz) to manufacturing errors with $\delta = 20 \mu m$ and $L = 1.5$ mm tip relief for Geometry 4.
6.7 Summary of results

The excitation of the first and second meshing frequencies for Geometries 3 and 4 demonstrated moderate sensitivity to all factors. The major distinction between the two Geometries is the sensitivity of the excitation of the first meshing frequency to profile deviation direction, where Geometry 3 showed less sensitivity than Geometry 4. However, the excitation of the third meshing frequency was mostly sensitive to only pitch deviations. It is worthy to note that the overall performance of Geometries 3 and 4 (helical gear drives) was distinctly better than that of Geometry 1 (a spur gear drive). This aligns with expectations set by previous works [6].

6.7 Summary of results

In this Chapter, four helical gear geometries were considered as outlined in Tables 6.1 and 6.2. With the application of tip relief, the sensitivity of the peak contact stress experienced in the pinion flank by these four geometries to manufacturing errors was much greater than that of Geometries 1 and 2 (after tip relief). However, the pressure angle of the gear drive had a significant impact on how much sensitivity there was to each factor. Geometries 3, 4, 5, and 6 had similar mean pinion peak contact stress among all trials, ranging from 1976 MPa to 2195 MPa. For all geometries, the pinion was most sensitive to amount of pitch deviation, hand of helix deviation, and profile deviation direction. Geometries 3 and 5 (with a pressure angle of 20°) showed less sensitivity to amount of profile deviation and amount of helix deviation than Geometries 4 and 6 (with a pressure angle of 25°). The peak stress experienced by the wheel was most sensitive to amount of pitch deviation and amount of helix deviation. Geometries 3 and 5 demonstrated increased sensitivity to helix deviation direction than geometries 4 and 6. This observation was further examined in Section 6.4. All four geometries demonstrated a high level of sensitivity to pitch deviations. Unlike the case of spur gears, longitudinal crowning was not able to further absolve the sensitivity of the gear drive to pitch or helix deviations. In fact, longitudinal crowning and front/back relief ultimately increased the peak stresses experienced by the gear drives throughout the cycle of meshing.

As mentioned previously, Geometries 3 and 5, with a pressure angle of 20°, was highly sensitive to profile deviation direction and hand of helix deviation. Interestingly, for both geometries, it was preferable to have a positive profile deviation and a right-handed helix deviation. This
did not change when the gear drive was simulated rotating in the opposite direction, or when
the helix direction of the simulated gear was changed. Unlike what was observed in Geometries
1 and 2, the sensitivity of Geometry 3 was not further reduced by the application of additional
tooth modifications. Neither longitudinal crowning nor front/back relief could improve the
performance of the gear drive significantly.

The sensitivity of the excitations of the meshing frequencies in the LTE function for Geome-
tries 3 and 4 was also determined. Unlike that of Geometry 1, Geometries 3 and 4 demonstrated
relatively little sensitivity to manufacturing errors in this regard. Even when only tip relief was
considered, the sensitivity of the excitement of the first meshing frequency for Geometries 3
and 4 did not exceed 2.5 arcsec to any individual factor. By comparison, the sensitivity of the
excitement of the first meshing frequency to profile deviation direction for Geometry 1 was only
able to be reduced to slightly less than 5 arcsec.
Chapter 7

Conclusions

7.1 Conclusions and recommendations

Gear drives are subject to manufacturing errors and defects, and higher levels of precision in manufacturing to increase the quality of the gears are costly, but sometimes required to attain sufficient performance from a gear drive. However, certain micro-geometry modifications to the gear tooth surfaces can be used to reduce the sensitivity of the gear drive to manufacturing errors. Results have shown that the application of micro-geometry modifications to the gear tooth surfaces can drastically decrease the sensitivity of peak contact and bending stresses within a gear drive to manufacturing errors, reducing the manufacturing costs and enabling higher levels of transmitted power.

For spur gears, micro-geometry modifications can be expected to result in a drastic reduction in sensitivity to manufacturing errors. Tip relief modifications reduce the sensitivity of the peak pinion contact stress experienced by the gear drive to the factors of profile and pitch deviation primarily. The additional application of longitudinal crowning to a spur gear drive can reduce the influence of helix deviations to the peak contact stress experienced by the pinion, but slightly increases the sensitivity to pitch and profile deviations. In most cases, the benefits of the sensitivity reduction to helix deviations should outweigh the increased sensitivity to pitch and profile deviations. It was seen that the optimal amount of longitudinal crowning varied linearly as a function of helix deviation. Additionally, the optimal level of helix deviation for a given gear pair does not depend significantly on pressure angle. Profile crowning can be expected to reduce the sensitivity of the excitation of all meshing frequencies in the LTE function for spur
gears. Without profile crowning, the excitation of the meshing frequencies is most sensitive to the level of profile deviation, but this can be absolved with the application of profile crowning.

For helical gears, tip relief was the only modification which was able to reduce the sensitivity of the gear drive’s performance to manufacturing errors. While other works have shown that other modifications can benefit the performance of helical gear drives, they will not be able to compensate for manufacturing errors. However, sensitivity to manufacturing errors for helical gears depended significantly on the pressure angle of the gear drive. Helical gear drives with low pressure angle can be expected to be more sensitive to the direction of the deviations due to manufacturing errors, and helical gears with high pressure angles can be expected to be more sensitive to the amount of deviation which exists in the gear drive. With regards to the excitation of the meshing frequencies in the LTE function, helical gear drives show less sensitivity overall to manufacturing errors than spur gear drives. Unlike spur gear drives, the influence of any individual class of manufacturing error is mostly the same, with no class of manufacturing error accounting for the majority of the influence.

### 7.2 Opportunities for future work

In this work, manufacturing errors were simulated by considering the worst-case scenario for a given ISO grade and class of manufacturing error. This method yields results reflecting the highest levels of error possible for a given grade, but does not reflect what would be expected for gears manufactured in a real world setting. A monte-carlo analysis utilizing a statistical distribution of error levels within a given ISO grade would provide better insight on the influence of manufacturing errors on gear performance in a large-scale industrial setting. This methodology would be difficult to implement, since a very large number of finite element models would need to be generated to obtain enough results for a study looking to gauge the statistical effects of manufacturing errors. However, such a work would expand the utility of the findings in this thesis, and could help gear designers choose tolerance ranges for gear drives of each manufacturing error class individually in a wide range of industrial applications.

Another factor which could play a role in the findings of this thesis which has yet to be explored is the influence of thermal expansion on contact stresses experienced by teeth throughout
a cycle of meshing. Although thermal expansion typically results in relatively low levels of deflection at the operating temperatures of a typical gear drive, the optimal level of modification for the modifications proposed in this thesis typically is on the order of 10 µm. With the optimal level of modification this low, it is possible that thermal expansion of teeth during meshing could have an impact on the expectations of experienced stress by the gear drive.
Bibliography


