General Relativistic Gas Dynamics in the Central Cavity of Binary Black Holes

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Author: Dennis B. Bowen
Advisor: Manuela Campanelli

A Dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Astrophysical Sciences and Technology in the School of Physics and Astronomy

September 13, 2017
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A Dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Astrophysical Sciences and Technology in the School of Physics and Astronomy

Approved by

Prof. Joel Kastner

Date

Director, Astrophysical Sciences and Technology
The Ph.D. Dissertation of Dennis B. Bowen has been approved by the undersigned members of the dissertation committee as satisfactory for the degree of Doctor of Philosophy in Astrophysical Sciences and Technology.
To the memory of my grandfather, Terry Allison, who started me down the road of mathematical and scientific reasoning so many years ago.
Supermassive binary black holes (SMBBHs) represent an excellent candidate for future combined gravitational wave and electromagnetic astrophysics, commonly referred to as multimessenger astrophysics. While much is known about the gravitational wave signal of merging BBHs, little is known about the electromagnetic emission. Modeling the electromagnetic emission coincident with gravitational waves requires simulations of SMBBHs coupled to their astrophysical environment, particularly during the late stages of inspiral and merger. These simulations necessitate a broad range of physics including general relativity, magnetohydrodynamics, and radiation physics.

In this Dissertation we present simulations of SMBBHs coupled to their astrophysical environment. We explore, for the first time, the gas dynamics in a relativistic binary black hole (BBH) system in which an accretion disk (a “mini-disk”) orbits each black hole. In addition to studying the structure and dynamics of the mini-disks, we present spectra from ray-tracing calculations of SMBBH accretion including mini-disks. Due to the immense computational burden of these simulations (millions of CPU hours per binary orbit), we restrict our study to equal-mass, non-spinning SMBBHs.

Relativistic effects alter the dynamics of gas in this environment in several ways. Because the gravitational potential between the two black holes becomes shallower than in the Newtonian regime, the mini-disks stretch toward the L1 point and the amount of gas passing back and forth between the mini-disks increases sharply with decreasing binary separation. This “sloshing” is quasi-periodically modulated at 2 and 2.75 times the binary orbital frequency, corresponding to timescales of hours to days for SMBBHs. In addition, relativistic effects add an azimuthal $m = 1$ component to the tidally driven spiral waves in the disks that are purely $m = 2$ in Newtonian gravity; this component becomes dominant when the separation is $\lesssim 100$ gravitational radii. We find that the spiral structure of the mini-disks is further altered through a coupling of the mini-disks to an $m = 1$ mode in the circumbinary accretion disk via streams of gas peeled off the inner edge of the latter. This modulation in the accretion stream flux has a quasi-periodic nature of 0.74 times the binary orbital frequency. Both the sloshing and the spiral waves have the potential to create distinctive radiation features that may uniquely mark SMBBHs in the relativistic regime. Finally, we observe a broadened thermal spectrum due to the combined photospheres of the mini-disks and circumbinary disks in the range of $\approx 1 - 1000\text{eV}$, and an inverse Compton spectrum at tens to hundreds of keV dominated by the mini-disks.
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I, Dennis B. Bowen ("the Author"), declare that no part of this dissertation is substantially the same as any that has been submitted for a degree or diploma at the Rochester Institute of Technology or any other University. I further declare that this work is my own. Those who have contributed scientific or other collaborative insights are fully credited in this dissertation, and all prior work upon which this dissertation builds is cited appropriately throughout the text. This dissertation was successfully defended in Rochester, NY, USA on July 27, 2017.

Modified portions of this dissertation have previously been published by the Author in peer-reviewed papers appearing in The Astrophysical Journal (ApJ):

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\(^1\)http://aasnova.org/2017/04/03/featured-image-mini-disks-in-a-black-hole-binary/
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Physical Constants/Units

\( G \)  Newtonian gravitational constant
\( c \)  speed of light
\( M_\odot \)  solar mass
\( \dot{m}_{Edd} \)  Eddington accretion rate
\( r_g \)  gravitational radii

List of Symbols

\( x^\mu \)  coordinates
\( g_{\mu\nu} \)  spacetime metric
\( g \)  spacetime metric determinant
\( \Gamma^\gamma_{\alpha\beta} \)  Christoffel symbols
\( \nabla_\mu \)  covariant derivative
\( \Phi \)  gravitational potential
\( T_{\mu\nu} \)  stress-energy tensor
\( \rho \)  rest-mass density
\( \Sigma \)  surface density
\( \epsilon \)  specific internal energy
\( u \)  internal energy density
Nomenclature

\( p \)  pressure
\( p_m \)  magnetic pressure
\( h \)  specific enthalpy
\( \gamma \)  Lorentz factor
\( v^i \)  primitive velocity
\( u^\mu \)  four-velocity
\( B^i \)  magnetic field
\( b^\mu \)  magnetic four-vector
\( \beta \)  plasma beta
\( H/r \)  aspect ratio
\( S \)  entropy
\( \mathcal{L}_c \)  rest-frame cooling rate per unit volume
\( \nu \)  light frequency
\( M \)  total binary mass
\( a \)  binary separation
\( q \)  binary mass-ratio
\( \Omega \)  orbital frequency
\( t_{bin} \)  binary orbital period
\( r_{in} \)  inner edge of a disk
\( r_p \)  radial location of the pressure maximum
\( r_t \)  tidal truncation radius

Acronyms / Abbreviations

AGN  Active Galactic Nuclei
AMR  Adaptive Mesh Refinement
BBH  Binary Black Hole
BH   Black Hole
BH1  The Black Hole Initially on the Positive \( x \)-axis \( (q = 1) \)
<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>BH2</td>
<td>The Black Hole Initially on the Negative x-axis ($q = 1$)</td>
</tr>
<tr>
<td>BL</td>
<td>Boyer-Lindquist</td>
</tr>
<tr>
<td>BZ</td>
<td>Buffer Zone</td>
</tr>
<tr>
<td>CS</td>
<td>Cook-Scheel</td>
</tr>
<tr>
<td>EOM</td>
<td>Equation of Motion</td>
</tr>
<tr>
<td>GR</td>
<td>General Relativity</td>
</tr>
<tr>
<td>GRMHD</td>
<td>General Relativistic Magnetohydrodynamics</td>
</tr>
<tr>
<td>GW</td>
<td>Gravitational Wave</td>
</tr>
<tr>
<td>ISCO</td>
<td>Innermost Stable Circular Orbit</td>
</tr>
<tr>
<td>IZ</td>
<td>Inner Zone</td>
</tr>
<tr>
<td>$\Lambda$CDM</td>
<td>Lambda Cold Dark Matter</td>
</tr>
<tr>
<td>LIGO</td>
<td>Laser Interferometer Gravitational-Wave Observatory</td>
</tr>
<tr>
<td>LISA</td>
<td>Laser Interferometer Space Antenna</td>
</tr>
<tr>
<td>MHD</td>
<td>Magnetohydrodynamics</td>
</tr>
<tr>
<td>MPMD</td>
<td>Multiple Program Multiple Data</td>
</tr>
<tr>
<td>MRI</td>
<td>Magnetorotational Instability</td>
</tr>
<tr>
<td>NZ</td>
<td>Near Zone</td>
</tr>
<tr>
<td>PN</td>
<td>post-Newtonian</td>
</tr>
<tr>
<td>PNH</td>
<td>post-Newtonian Harmonic</td>
</tr>
<tr>
<td>PNHC</td>
<td>post-Newtonian Harmonic Cartesian</td>
</tr>
<tr>
<td>PNHS</td>
<td>post-Newtonian Harmonic Spherical</td>
</tr>
<tr>
<td>SMBBH</td>
<td>Supermassive Binary Black Hole</td>
</tr>
<tr>
<td>SMBH</td>
<td>Supermassive Black Hole</td>
</tr>
<tr>
<td>WARP</td>
<td>Warped Coordinate System</td>
</tr>
</tbody>
</table>
1.1 Astrophysical Binary Black Holes

Stellar-mass black hole (BH) mergers were detected for the very first time a little more than a year ago (Abbott et al., 2016c,d,b, 2017). This extraordinary discovery marks the beginning of an entirely new field of astrophysics, one in which experiments like advanced LIGO can be expected to see events similar to GW150914, GW151226 and GW170104 multiple times per year (O’Shaughnessy et al., 2017; Belczynski et al., 2016; Abbott et al., 2016a).

In contrast, mergers of supermassive binary black holes (SMBBHs), remain elusive for the time being. SMBBHs are expected to be formed during galaxy mergers (see Khan et al. (2016); Kelley et al. (2017) for recent work). In essence, the current prevailing cosmological model, Lambda cold dark matter ($\Lambda CDM$), predicts that galaxies evolve through the repeated merger events of smaller galaxies in a hierarchical structure (Coles & Lucchin, 2002; Springel et al., 2005). Additionally, there is strong evidence that galaxies containing a bulge, if not all galaxies, contain a supermassive black hole (SMBH) at their center (Gültekin et al., 2009). As the galaxies merge, the central SMBHs of the progenitor galaxies will form a bound pair in the central region of the galaxy merger remnant. Once the BHs are separated by $\sim 10^3$ gravitational radii, the SMBBH enters the
relativistic, gravitational radiation-driven regime and rapidly inspirals to merger.

However, there has long been uncertainty about how their orbits may evolve toward the relativistic regime and merger of the BHs (Begelman et al., 1980). This so-called “final parsec problem” hypothesized that the SMBBH may form a bound pair and only inspiral down to a separation of \( \sim 1 \text{pc} \) (where one parsec is the distance light travels in 3.26 years) before stalling. Thus, the binary would never reach the gravitational radiation-driven regime. However, numerous mechanisms to accomplish this have been studied in recent years. First, as the BHs pass through the galactic material, they will preferentially deflect stars and gas towards them. This will leave a relative over density of material behind the BHs rather than in front of them, resulting in a net gravitational pull “backwards”, an effect known as dynamical friction. Additionally, stars and bodies which are pulled too close to the binary may be forced onto unstable trajectories and slingshot out of the system. Both mechanisms can effectively remove angular momentum and energy from the SMBBHs, but with decreasing efficiency as approaching a binary separation of a parsec. However, N-body simulations of stellar loss cone repopulation in galactic nuclei following galactic mergers (Khan et al., 2011; Vasiliev et al., 2015; Gualandris et al., 2017) suggest comparatively rapid evolution of SMBBH orbits by these processes. Additionally, if the SMBBH forms in a region with sufficient gas, an accretion disk of material will form orbiting the SMBBH. Provided that the mass of the disk \( M_{\text{disk}}(a) \) contained within the orbit of the less massive BH (the secondary) is larger than the mass of the secondary \( m_2 \), the secondary will behave as a fluid element inspiraling onto the more massive primary on a viscous timescale

\[
    t_{\text{decay}} = \frac{M_{\text{disk}}(a) + m_2}{M_{\text{disk}}(a)} t_{\text{visc}}
\]

(see, e.g., (Dotti et al., 2012) for a review). If the mass is depleted and the binary stalls, Ivanov et al. (1999) showed that this will in turn lead to an increased mass in the circumbinary disk until the orbital compression resumes.

Upon entering the gravitational radiation-driven regime, the SMBBH will quickly circularize and
inspiral due to energy emitted in the form of gravitational waves (GWs). This GW emission should then take them to coalescence in less than a Hubble time (Milosavljević & Phinney, 2005). In fact, numerical relativity simulations have shown that SMBBHs can radiate vast amounts of gravitational radiation in short timescales, even outshining the entire photon Universe (Campanelli et al., 2010). It has also been shown that this sudden burst of radiation can be emitted anisotropically, kicking the SMBH remnant at thousands of km/s (Baker et al., 2007; Campanelli et al., 2007a,b; González et al., 2007; Herrmann et al., 2007; Koppitz et al., 2007; Baker et al., 2008; Healy et al., 2009; Lousto et al., 2010; Lousto & Zlochower, 2011; Lousto et al., 2012). Such GW signals emanating from SMBBH coalescence could allow SMBBHs to act as a standard candle (Sathyaprakash & Schutz, 2009), supplementing the cosmic distance ladder. However, the much lower frequency GW emission of SMBBHs requires detectors quite different from LIGO. Pulsar Timing Array observations may probe the early inspiral regime of the most massive SMBBHs \( (10^9 M_\odot+ ) \) (Shannon et al., 2015) within the next decade, but space missions such as LISA (Amaro-Seoane et al., 2012, 2013; Seoane et al., 2013) will be necessary to detect directly the GW radiation from the merger proper, and such missions are still very far in the future.

On the other hand, because most SMBBHs are expected to coalesce in gas-rich environments at the center of galaxies (Cuadra et al., 2009; Chapon et al., 2013; Colpi, 2014), these systems should be excellent targets for electromagnetic as well as GW observations. While information about the SMBBH is encoded in the GW emission, the electromagnetic signal will contain a wealth of information about the host environment of the binary. For instance, the gas and stellar content of the host environment will alter the properties of any electromagnetic emission. Furthermore, electromagnetic counterparts are vital in directly pinpointing the GW source on the sky. The difficulty is that our knowledge of the specific kind of electromagnetic signals to expect remains quite primitive. To the extent that the ultimate source of heat in gas near SMBBHs is gravity, the Equivalence Principle suggests that the total amount of energy available for photon radiation should be directly related to the mass of gas present during the merger, with the energy per unit mass likely greatest in the region nearest the BHs (Krolik, 2010). Further progress, though, requires
an understanding of the configuration of this gas, which likely depends on parameters such as the binary mass ratio and the BH spins, both magnitude and direction, not to mention the availability of gas from the host galaxy’s interstellar medium.

1.2 Accretion Disks

As previously mentioned, SMBBHs are expected to coalesce in gas-rich environments. As the gravitational pull of the BHs drives gas towards the center-of-mass, conservation of angular momentum will cause the material to form into a disk orbiting around the binary. Disks of gas orbiting a central mass, known as accretion disks, are among some of the most luminous and ubiquitous astrophysical phenomena; applications range from planetary and stellar systems to relativistic, compact objects (neutron stars and BHs).

A common approximation used in accretion disk theory is that the disk is “thin”. That is to say, the overall aspect ratio \( H/r \), where \( H \) is the height of the disk and \( r \) is the radial distance from the central mass, is sufficiently small that the disk is geometrically thin and radiation pressure can be neglected (the disk is “radiatively efficient”). This is because the overall thickness of the disk is intimately tied to the thermodynamics of the disk. For instance, heating events cause the disk to “puff up” in vertical extent and become thicker. When these approximations are violated, the disk is referred to as a “thick” disk and is said to be “radiatively inefficient”. Additionally, it is often assumed that the surface density is sufficiently low such that the self-gravity of the disk may be neglected. For a detailed review on accretion disk physics see Abramowicz & Fragile (2013).

We elect to simulate disks with vertical structure, but still sufficiently thin to enable the thin-disk approximation. This is consistent with the expectations that the disks will be dense, optically thick, and efficiently radiating. Doing so allows us to make the approximation that the accretion disks will not be self-gravitating and dramatically simplifies our treatment of gravity (see Sections 2.1-2.2 for an in-depth description of our disk model).

Under the thin-disk approximation, we neglect the disk’s self-gravity and back reaction on the
gravitational field. Instead, we evolve material on an analytic spacetime (see Chapter 2 for full details). An inviscid, purely hydrodynamic thin-disk built initially in an hydrostationary equilibrium solution around a BH will maintain its structure and never accrete onto the BH. However, astrophysical accretion disks do not behave in this manner. A fluid element orbiting a BH will interact with neighboring elements. This is what drives the luminosity of accretion disks. As fluid elements in the presence of a differentially rotating disk interact with their neighbors they will shear against one another as the radially outward element slows down the radially inward element. This causes the slowed down element to lose orbital angular momentum, gain a radial component to the fluid flow, and move closer to the central source. In doing so, gravitational energy is deposited into the gas to be radiated away.

While this has been understood for some time, there was a long-standing unsolved problem in how angular momentum transport was physically accomplished. Currently, it is thought that angular momentum is transported outward via turbulence in the flow induced by the magnetorotational instability (MRI) (Balbus & Hawley, 1991). Though several recent works (Ju et al., 2016; Muñoz & Lai, 2016; Ryan & MacFadyen, 2017; Bowen et al., 2017) have proposed that spiral density waves induced in the disks could contribute non-negligibly to the angular momentum budget of disks in binary systems.

In order to ascertain the electromagnetic output of accretion disks, one must have a means of prescribing the accretion onto the central mass. In this vein, Shakura & Sunyaev (1973) produced what has become one of the most heavily cited papers in astrophysics by reducing the equations of motion for a thin disk to a set of algebraic equations and deriving a vertically and time-averaged description of the accretion stresses via stresses in a viscous fluid. This was then generalized to general relativity (GR) by Novikov & Thorne (1973) with revisitations by (Page & Thorne, 1974; Riffert & Herold, 1995; Penna et al., 2012).

Shakura & Sunyaev (1973) supposed that the mechanism for producing the internal stresses could be magnetic fields, turbulence, and molecular and radiative viscosity. They argued that molecular and radiative viscosity are likely not the key causes of internal stress and instead con-
sidered the effects of magnetic fields and turbulence. When considering magnetic fields, the \( \text{div}(B) \) constraint requires alternating signs on the radial component of the field. Combining this with differential rotation leads the fields to operate on small scales. The authors then state that due to plasma instabilities and reconnection of magnetic field lines, the energy of the field will be less than the thermal energy of the matter and that the tangential stress can be written as

\[
\sigma_{r\phi} \propto -\left( \frac{\mathcal{H}^2}{4\pi \rho c_s^2} \right) \rho c_s^2, \tag{1.2}
\]

where \( \mathcal{H}, \rho, c_s, \) and \( \sigma_{r\phi} \) are the magnetic field, matter density, sound speed, and tangential stress respectively. To describe the average motion due to turbulence, the equations for laminar flows are used replacing molecular viscosity with turbulent viscosity, and it is found that the tangential stress takes a similar form to that for magnetic fields;

\[
\sigma_{r\phi} \propto -\rho c_s^2 \frac{v_t}{c_s}, \tag{1.3}
\]

where \( v_t \) is the turbulent velocity. Simply summing the two contributions to the stress one finds that it can be written as

\[
-\sigma_{r\phi} \propto \left( \frac{v_t}{c_s} + \frac{H^2}{4\pi \rho c_s^2} \right) \rho c_s^2 = \alpha \rho c_s^2. \tag{1.4}
\]

If one then assumes that the gravitational energy released is radiated away efficiently, the gas becomes, at least approximately, isothermal. In this scenario, the gas pressure becomes proportional to the matter density via

\[
p = c_s^2 \rho. \tag{1.5}
\]

In other words, the averaged tangential stresses responsible for transporting angular momentum outward in the disk are proportional to the pressure by a dimensionless quantity \( \alpha \). This is the so called \( \alpha \)-disk model. In this prescription the torque is assumed to vanish at the innermost stable
circular orbit (ISCO) and the disk thickness becomes infinitely thin and therefore \( \alpha = 0 \) in this region. Note that this assumption has been repeatedly contested (Gammie, 1999; Krolik, 1999b; Balbus, 2012). In this model, the entirety of accretion stresses can be modeled by constructing the source term for a viscous fluid with shear tensor \( \sigma_{\mu\nu} \) and kinematic viscosity parameter \( \eta = \rho \alpha \). While this allows for simplified calculations, both analytic and numeric, this is a purely phenomenological model based on dimensional arguments as true viscosity is far too weak to account for accretion rates observed.

Although the \( \alpha \)-model allows for an analytic framework to understand accretion disk theory, there are serious flaws. In actuality, the dominant accretion stresses are likely Maxwell stresses caused by turbulence present within the disk via the MRI and cannot be accurately described by a single \( \alpha \). Chiefly, the assumption that the internal stresses of the fluid may be understood in a vertically and time-averaged sense cannot accurately reproduce the turbulent flows of magnetized disks. Additionally, astrophysical mini-disks are not isothermal and have local shock-heating events that radiate away energy at non-instantaneous timescales.

The linearized stability criteria for magnetized fluids was discussed as early as 1960 (Velikhov, 1959; Chandrasekhar, 1960), but it would take 30 years for such an analysis to make an appearance in the accretion disk literature (Balbus & Hawley, 1991). The simplest case for understanding this instability is an axisymmetric disk in hydrostationary equilibrium threaded with a weak vertical magnetic field and a fluid element displaced by \( \xi \) within the equatorial plane from circular orbits. Below we follow Balbus & Hawley (1998) in deriving the behavior of the disk under these assumptions. Balbus & Hawley (1998) derived the magnetic tension force

\[
\frac{ikB}{4\pi\rho} \delta B = - (k \cdot u_A) \xi
\]  

(1.6)
and equations of motion for the displacement

\[
\ddot{\xi}_R - 2\Omega \dot{\xi}_\phi = -\left( \frac{d\Omega^2}{d\ln R} + (k \cdot u_A)^2 \right) \xi_R \tag{1.7}
\]

\[
\ddot{\xi}_\phi + 2\Omega \dot{\xi}_R = -(k \cdot u_A)^2 \xi_\phi. \tag{1.8}
\]

Here \(k\), \(u_A\), and \(\delta B\) are the wavenumber, Alvén velocity and linear amplitude of the magnetic field respectively. In this scenario, the displacement equations are analogous to a mass \(m_i\) located at some distance \(r_i\) from the central mass and \(m_o\) at some distance \(r_o\) connected by a massless spring with spring constant \((k \cdot u_A)^2\).

If \(r_i < r_o\), the orbital velocity of \(m_i\) will be greater than that of \(m_o\). In this scenario, the tension restoring force of the spring will slow down the faster \(m_i\) and pull along \(m_o\) at an increased orbital speed. The decreased angular momentum of \(m_i\) will cause the mass to fall radially inwards toward the central mass. Conversely, the increased angular momentum of \(m_o\) will move the mass radially outward. This, in turn, will increase the spring tension, further removing angular momentum from \(m_i\) and transporting it to \(m_o\) in a runaway fashion. For a diagram representation, see Figure 1.1.

In actuality, the spring and masses are replaced with fluid elements connected by magnetic field lines. An in-depth linearized stability analysis will show that the criteria for instability is (Balbus...
Chapter 1. *Introduction*


$$\frac{d\Omega^2}{d\ln R} < 0.$$  \hspace{1cm} (1.9)

As this criteria is naturally met by Keplerian rotation profiles in accretion disks, one small seed displacement will lead to many more such displacements with a maximum growth rate of $0.75\Omega$ (Balbus & Hawley, 1998). It is important to point out that this instability is independent of the overall magnetic field strength and can be generalized to more generic geometries without altering the overall interpretation. Given these qualities, the angular momentum profile of astrophysical disks, and the prevalence of astrophysical magnetic fields, Balbus & Hawley (1991) argued that this runaway MRI is the most likely candidate for angular momentum transport in accretion disks.

In addition to turbulent flows within the disk, the emitted spectrum and accretion stresses can be altered by the presence of spiral shock fronts within the disk. Such spiral waves were proposed by Lynden-Bell & Pringle (1974) as a candidate mechanism to drive *all* accretion stresses, but were later showed to likely not be the full answer (see Shu & Lubow (1981); Papaloizou & Lin (1995); Boffin (2001) and references therein). However, more recent studies in the context of binary stellar systems (Ju et al., 2016; Muñoz & Lai, 2016) and SMBBHs (Bowen et al., 2017; Ryan & MacFadyen, 2017) have demonstrated that spiral density waves in binaries may contribute significantly to accretion stresses and proposed the effectiveness may even be comparable to MRI driven stresses in disks around the individual members of the binary. The increased effectiveness of spiral shocks in driving accretion stresses in these systems is due to the strong functional dependence on temperature and high temperatures of disks in binary systems, particularly SMBBHs.

Spiral density waves can be excited through several mechanisms; such as non-axisymmetric perturbations in the gravitational field (Savonije et al., 1994) or instabilities in the disk (Papaloizou & Pringle, 1984). Accretion disks in binary systems will experience periodic, non-axisymmetric perturbations by the gravitational field in which they are embedded. These perturbations by the gravitational field launch spiral density waves near Lindblad resonances (Goldreich & Tremaine,
In Newtonian terms, we can describe such a gravitational field as

\[ \Phi = \Phi_{\text{background}} + \Phi_{\text{pert}}(t), \quad (1.10) \]

where \( \Phi_{\text{background}} \) and \( \Phi_{\text{pert}} \) are the portions of the gravitational potential of the primary source and any time-dependent perturbations. The nature of the spiral density waves induced can be understood through a Fourier series expansion of the perturbing potential \( \Phi_{\text{pert}} \).

\[ \Phi_{\text{pert}}^m = a_0 + \sum_m a_m \cos(m\phi) + b_m \sin(m\phi), \quad (1.11) \]

where the coefficients are

\[ a_0 \propto \int \Phi_{\text{pert}} \sqrt{-g} \, d^3x \quad (1.12) \]

\[ a_m \propto \int \Phi_{\text{pert}} \cos(m\phi) \sqrt{-g} \, d^3x \quad (1.13) \]

\[ b_m \propto \int \Phi_{\text{pert}} \sin(m\phi) \sqrt{-g} \, d^3x \quad (1.14) \]

and \( \phi \) is the azimuthal coordinate over which the function is periodic. The strength of the \( m \)-th mode can then be calculated as

\[ S_m = \sqrt{a_m^2 + b_m^2}. \quad (1.15) \]

The modal structure of this potential determines the radial locations of the Lindblad resonances and modal structure of the spiral density waves, calculated in precisely the same manner replacing \( \Phi_{\text{pert}} \) with \( \rho \), excited within the disk. If fluid is contained near the vicinity of the Lindblad resonances, a spiral density wave will be launched in the disk that orbits at the periodic frequency of the perturber. In Figure 1.2 we plot a schematic for such a system with \( m = 2 \) spiral shock fronts in a frame corotating with the disk where the spiral shocks appear fixed.
In essence, this mechanism requires a spiral density wave which orbits the central mass slower than the local orbital frequency of gas to transport angular momentum. As the spiral density wave will orbit with the frequency of the time-dependent perturbation, we require that $\Omega_{\text{pert}} < \Omega(r)$. Additionally, the spiral front must steepen into a shock in order to couple to the fluid and transport angular momentum via dissipation (Papaloizou & Lin, 1995; Goodman & Rafikov, 2001; Heinemann & Papaloizou, 2012; Rafikov, 2016). In these conditions, the shock front would appear to be orbiting “backwards” in the fluids frame, effectively carrying negative angular momentum. As the orbiting fluid comes into contact with the shock front, the orbital velocity will be slowed. This in turn removes angular momentum from the fluid and introduces a radial component to the flow, thus facilitating the accretion of material inwards and the transportation of angular momentum outwards through the shock front.

1.3 Supermassive Binary Black Hole Accretion

Early work suggested that little gas would actually reach the vicinity of merging black holes despite mass accreting toward it through a circumbinary disk. The reasoning was based on two arguments. The first was that even when gravitational radiation losses are too weak to force orbital evolution, the binary exerts torques on nearby gas strong enough to clear out a large cavity in the region within $\approx 2a$ of the binary (here $a$ is the binary semi-major axis) when the binary mass-ratio
is not too far from unity (Artymowicz & Lubow, 1994, 1996; D’Orazio et al., 2016). It was therefore argued that these same torques would prevent any mass from proceeding closer to the binary than the outer edge of that gap (Pringle, 1991). However, a few years ago multi-dimensional numerical simulations of circumbinary disks with internal stresses showed that streams of gas are, in fact, readily peeled off the inner edges of such disks (MacFadyen & Milosavljević, 2008; Shi et al., 2012; Noble et al., 2012; D’Orazio et al., 2013). More recently, simulations with carefully defined external accretion rates have shown that essentially all the mass passing through the circumbinary disk is conveyed to the binary (Farris et al., 2014; Shi & Krolik, 2015). Shi & Krolik (2015) showed in detail how binary torques acting on streams in the gap can drive gas back out to the circumbinary disk, where a portion of the streams’ mass loses enough of its angular momentum by shock deflection that it then falls directly to the binary. The second argument was that once the orbit was tight enough for GW emission to drain energy from the orbit faster than stresses within the circumbinary disk could drive inflow, the binary would “decouple” from the external accretion flow, accepting no further gas (Milosavljević & Phinney, 2005). This too has been undermined by actual simulations. Noble et al. (2012) and Farris et al. (2015a) showed that, at least for the time required for the binary to shrink by a factor of a few, accretion can continue at more or less the same rate because the very fact that orbital evolution is more rapid than stress-driven inflow means that mass for accretion does not need to be brought in from very far out in the circumbinary disk. Thus, recent work has shown that the prospects for finding significant amounts of gas near merging SMBBHs are much more favorable than previous efforts indicated.

It then remains to ask what happens to the gas delivered to the SMBBH. Initial work focused on matter accreting directly from a circumbinary disk to the BHs during the few binary orbits immediately preceding merger. These investigations were “proof of principle” calculations, designed to show that gas dynamics and full solutions of the Einstein Field Equations could be done in tandem (Bode et al., 2010; Palenzuela et al., 2010; Farris et al., 2011; Bode et al., 2012; Farris et al., 2012; Giacomazzo et al., 2012; Gold et al., 2014). However, because of their brief duration, the total mass transferred from the circumbinary disk to the central cavity was relatively small.
and no formation of individual disks around the BHs was observed. In contrast, recent Newtonian simulations (see, e.g. (Farris et al., 2014; Muñoz & Lai, 2016)) have clearly demonstrated that individual “mini-disks” form around each BH over many binary orbital periods.

Therefore, in our approach, we begin with the supposition that the persistent feeding of material into the domain of the BHs leads to the formation of individual mini-disks around each BH. The mini-disks grow in extent until tidal forces exerted by the companion destabilize larger orbits. This limiting size is often called the “tidal truncation radius” $r_t$. Much work has been done studying such systems in the Newtonian regime (Paczynski, 1977; Papaloizou & Pringle, 1977; Lin & Papaloizou, 1979; Artymowicz & Lubow, 1994; Mayama et al., 2010; de Val-Borro et al., 2011; Nelson & Marzari, 2016). The circular orbit data of Paczynski (1977), for example, can be fit reasonably well by the expression $r_t = 0.27q^{-0.3}a$ where $q$ corresponds to the primary and secondary BH respectively, and $q \leq 1$ is the binary mass ratio (Roedig et al., 2014). As previously mentioned, other Newtonian work has shown that tidal torques from the companion can excite spiral waves that steepen into shocks (Lynden-Bell & Pringle, 1974; Spruit et al., 1987; Papaloizou & Lin, 1995; Ju et al., 2016; Rafikov, 2016), supplementing the internal stresses due to correlated MHD turbulence. Similar conclusions were recently obtained in the context of a binary system where the mini-disks are placed around non-spinning BHs, but the binary separation is taken to be in the Newtonian regime (Ryan & MacFadyen, 2017) and in the relativistic inspiral (Bowen et al., 2017).

In this Dissertation, we present the first simulations of mini-disks during both the quasi-Newtonian and GW-dominated inspiral regime of BBHs ($a \lesssim 100M$). In our approach, we use an approximate general relativistic spacetime (Mundim et al., 2014; Ireland et al., 2015; Zlochower et al., 2016; Nakano et al., 2016) that accurately describes the dynamics of BBHs during the inspiral phase. The inspiral itself is described through the post-Newtonian (PN) equations of motion (Blanchet, 2014). The PN approximation accurately describes spacetime when fields are weak ($r_g/r = GM/(rc^2) \ll 1$) and the BH motions are slow $((v/c)^2 \ll 1)$. In PN theory, spacetime is described as Newtonian gravity plus an asymptotic series of relativistic corrections in powers of these small quantities; the PN-order is characterized by the order of the expansion (i.e., 3PN con-
Chapter 1. *Introduction*

tains terms up to \((r_g/r)^3\) and \((v/c)^6\). Although our implementation of general relativistic effects is valid for any BH mass-ratio \((q \leq 1)\) and can accommodate spins, the work reported here focuses on equal-mass, non-spinning BHs. We start by performing a suite of simulations at binary separations ranging from the quasi-Newtonian regime \((a \approx 100M)\) down to well within the relativistic, PN regime of SMBBH inspiral \((a \leq 20M)\) in 2D, inviscid hydrodynamics in the equatorial plane of the binary (see Chapter 3 for full details). Hydrodynamics is initially treated in 2D to study how general relativistic effects alter mini-disk dynamics, including tidal truncation, interactions between the mini-disks, and spiral shocks. While not incorporating important effects (continuing accretion from the circumbinary disk, vertical structure, and magnetic fields), this work complements our 3D general relativistic magnetohydrodynamic (GRMHD) simulation and other existing simulations of these systems. This vital stepping stone towards 3D GRMHD facilitated the first study of PN effects on the mini-disks and resulted in discussion of several previously unmentioned, relativistically enhanced effects.

Next, we extended this study to full 3D GRMHD at a binary separation of \(20M\) (see Chapter 4 for full details). While we are unable to perform a suite of simulations as in the 2D hydrodynamic case due to the enormous computational cost of these 3D simulations, this simulation includes the mini-disks coupled to the circumbinary accretion disk. This simulation is the first BBH simulation to include mini-disks coupled to the circumbinary in either MHD or GR. In doing so, we are able to reincorporate the previously neglected effects of continuing accretion from the circumbinary disk, vertical structure, and magnetic fields. The primary goal of this simulation is to address open questions regarding the full 3D structure of the mini-disks and the interplay between the mini-disks and the circumbinary accretion disk. By extending to full 3D MHD, we are able to begin addressing whether accretion stresses from spiral shocks within the disk are significant when compared to magnetic stresses. Additionally, the vertical structure of spiral waves within the disk is of importance. This is because the spiral density wave must steepen into a shock within the bulk of the disk and not be channeled to the edges in order for angular momentum transport to be efficient. Unlike in the 2D case, where the mini-disks are completely isolated, we find that accretion
onto the mini-disks from the circumbinary (which has an $m = 1$ mode) can force the mini-disks to appear slightly asymmetric with respect to one another (even in the equal-mass case). Finally, moving to 3D GRMHD allows our simulation to be accessible by post-processing radiative transfer codes, such as BOTHROS (Noble et al., 2007, 2009, 2011), to make direct predictions of light curves and spectra from SMBBHs in the relativistic regime.

As we will show in detail in this Dissertation, we find that relativistic effects can create qualitative changes to the mini-disks when the binary separation shrinks to several tens of gravitational radii or less (hereafter, we quote all distances in gravitational units: $r_g \equiv GM/c^2 = M$ when $G = c = 1$). The spiral waves found in so many Newtonian studies take on an entirely new symmetry. The shape of the potential through the L1 region changes, allowing substantially greater mass to find its way out of the mini-disks and into that region. This gas, liberated from the mini-disks, continually passes back and forth from the domain dominated by one BH to that dominated by the other BH and back again, shocking against the outer edge of the mini-disks at each passage. In principle, enough mass can be injected into this “sloshing” region to create observable photon signals; in systems with unequal BH masses, the sloshing could even result in net mass-transfer from one disk to the other.

1.4 Observational Considerations

While much is known about the electromagnetic signal of accretion disks around an individual SMBH (Active Galactic Nuclei or AGN), little is known at this point about accretion disks in the presence of SMBBHs. Given that there is little hope of ever being able to spatially resolve the individual members of a relativistic SMBBH, there is great need for direct predictions of the electromagnetic signature. In this Dissertation, we present the first simulation capable of being ray-traced to produce light curves and spectra from the mini-disks coupled to the circumbinary disk. Additionally, there has been some debate within the community as to which modifications to standard AGN should be present and detectable. There are some candidate detections of SMBBHs
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(Graham et al., 2015; D'Orazio et al., 2015) or more convincingly (Bansal et al., 2017), but the definitive detection of a SMBBH remains illusive. Despite this, there have been many proposed signatures and possible means of detecting SMBBHs via their electromagnetic signature emanating from the accretion disks and jets.

One possible distinction between standard AGN and SMBBH accretion is the time variability in the emission caused by the orbit of the binary or unique features in the disks. One such candidate object is PG 1302-102, which has an optical periodicity of approximately 5.2 years (Graham et al., 2015). While Graham et al. (2015) admit that the physical cause of the observed optical periodicity is uncertain, they attribute the sinusoidal behavior to a kinematic origin. They provide possible origins such as precessing jets in a binary, a lump at the inner edge of the circumbinary (though claiming this to be improbable because of a derived mass of $10^{11.4-12.2}M_\odot$), or a warp induced by a SMBBH. Ultimately, Graham et al. (2015) propose that PG 1302-102 could be a binary of total mass $10^{8.5}M_\odot$ with a separation of approximately 0.01 pc. Alternatively, D'Orazio et al. (2015) proposed that the optical periodicity could in fact be due to the lump and arrive at a binary separation two to four times smaller. In mass-ratios near unity, the dominant variability in the spectrum may result from the accretion disks rather than motion of individual members the SMBBH (Shi et al., 2012; Noble et al., 2012; D'Orazio et al., 2013; Farris et al., 2014, 2015a; Bowen et al., 2017). However, it is important to note that the lump has been reported to orbit at much higher frequencies in MHD (Shi et al., 2012; Noble et al., 2012) than in viscous hydrodynamics (D'Orazio et al., 2013; Farris et al., 2014, 2015a). This could substantially impact the predictions of D'Orazio et al. (2015). While nearly all studies have focused on quasi-periodic modulation of the spectrum due to the circumbinary disk, Bowen et al. (2017) proposed that sloshing of material back and forth between the individual mini-disks could introduce a quasi-periodic signature in the X-ray portion of the spectrum in the mini-disks (see Chapter 3 for more details).

Given that periodicities are not exclusive to binaries, standard AGN have been shown to have periodic emission (see (Mushotzky et al., 1993; Ulrich et al., 1997) for reviews), some work has focused on alternative methods of detecting SMBBHs. One key feature of SMBBHs over standard
AGN is the splitting of gas into three distinct accretion disks rather than a single disk. This can have direct consequences on the spectrum of the disk. Roedig et al. (2014) predicted the existence of a “notch” in the thermal spectrum of the disk. In essence, the spectrum of the circumbinary disk will closely mirror that of an accretion disk around a SMBH of the total mass of the binary. However, because the disk will be truncated at approximately $2a$, the thermal emission at higher energies associated with closer radial distances from the BH will be absent. The mini-disks, on the other hand, will be quite dense and hot. They will therefore radiate at the higher energies truncated out of the circumbinary’s spectrum. The region between these disks will be mostly empty of material with the exception of accretion streams. These streams will have relatively little emissivity and will therefore not likely contribute significantly to the thermal spectrum directly. However, they will continually pile onto the mini-disks and shock heat the edges. This, enhanced further in the relativistic regime by sloshing, will serve to further enhance the high energy portions of the SMBBH thermal spectrum. Ultimately, one will be left with a thermal spectrum of an accretion disk orbiting a SMBH of the total mass of the binary for energies corresponding to radii larger than $2a$, a large bump in the high energy portions of the spectrum, and relatively little emission at energies between. Note that no such notch was found in a recent viscous, hydrodynamic calculation (Farris et al., 2015b) or in our MHD simulation (see Chapter 4).

In addition to electromagnetic signatures emanating from inspiraling SMBBHs, the merger proper may leave a footprint on the remnant’s emission as well. Depending on the spin configuration of the progenitor SMBHs, the remnant SMBH may be given a strong enough kick to partially or completely disrupt the circumbinary disk. This could reignite AGN-like accretion, even if the binary had previously decoupled. Additionally, the AGN could appear to have a central SMBH off-set from the center of the galaxy (e.g. (Batcheldor et al., 2010; Lena et al., 2014; Chiaberge et al., 2017)). For more details see (Schnittman, 2011; Lousto et al., 2017) and references therein.

In order for any SMBBH direct detections to occur definitively via variability or spectral energy distribution signatures, in-depth predictions from radiative transfer calculations must be made. The GRMHD simulation presented in Chapter 4 is the first simulation including the mini-disks
coupled to the circumbinary disk capable of such predictions. We present direct predictions of
the time-averaged spectra of inspiraling SMBBHs without reliance on purely analytic accretion
disk models, fully including non-linear disk-disk and disk-stream interactions. As expected, given
the high temperatures and strong gravitational fields in the mini-disks, we find a highly luminous
central cavity with strong emission emanating from the mini-disks.

1.5 Chapter Layout

Throughout this Dissertation, unless otherwise noted, we use geometrized units in which \( G = c = 1 \). When used as tensorial indices, we reserve Greek letters (e.g., \( \alpha, \beta, \gamma, \ldots \)) for spacetime
indices (0 to 3) and Roman letters (e.g., \( i, j, k, \ldots \)) as indices spanning spatial dimensions (1 to 3).

In Chapter 2 we describe our physical models for SMBBHs used in the simulations. Here
we provide an overview of our analytic spacetime used to describe the SMBBH, the equations of
motion and thermodynamic models governing our simulations, and the construction of initial data
for individual mini-disks around each component of the SMBBH.

In Chapter 3 we present the first ever simulations of mini-disks during the quasi-Newtonian
and inspiral regime of SMBBHs. In these simulations, we discuss the findings for the case of
2D, inviscid hydrodynamics where the mini-disks are confined to the equatorial plane and isolated
from the circumbinary disk. We find relativistic modifications to the overall size of the mini-disks,
structure of spiral density waves, and a previously unknown phenomena we call “sloshing” which
may induce periodic modulations to the electromagnetic signal.

In Chapter 4 we present the first ever simulation of mini-disks coupled to the circumbinary
accretion disk in general relativity or MHD. We consistently include vertical structure, magnetic
fields, and continuing accretion from the circumbinary disk. We find the presence of relativistically
modified spiral density waves and a potential mechanism for mini-disk coupling to an \( m = 1 \) mode,
or lump, present within the circumbinary accretion disk. This coupling further alters the spiral
density waves in the mini-disks and could potentially enhance any quasi-periodic electromagnetic
signals associated with the lump. Finally, we present time-averaged spectra from the first ever ray-tracing of SMBBH accretion including mini-disks coupled to the circumbinary disk. Finally, in Chapter 5 we provide a brief summary of the findings and conclusions contained within this Dissertation separated by physical mechanism. We conclude by presenting possible future directions of this work.
2.1 Gravitation

The study of gravitational interactions between bodies dates back to Newton, who sought to explain the gravitational attraction of matter via instantaneously felt forces. Newtonian gravity describes the gravitational field via a potential ($\Phi$) that accelerates bodies according to

$$\ddot{a} = \nabla \Phi.$$  \hspace{1cm} (2.1)

However, with the publication of Einstein’s special theory of relativity, there appeared to be inconsistencies. If forces are felt instantaneously, then the force carrier must move faster than the speed of light. This would mean that all events in all frames would have to remain simultaneous despite the loss of simultaneity in all frames predicted by special relativity.

Motivated by this, Einstein abandoned this notion of gravity and first published his theory of GR just over 100 years ago. This theory radically changed our understanding of gravity, introducing gravitational interaction as a consequence of spacetime curvature rather than an instantaneous
force. In deriving the Einstein Field Equations,

\[ G_{\mu\nu} = 8\pi T_{\mu\nu}. \]  

(2.2)

Einstein found that spacetime \((G_{\mu\nu})\) tells matter \((T_{\mu\nu})\) how to move and matter tells spacetime how to bend. More explicitly, \(G_{\mu\nu}\) can be written in terms of the Ricci tensor \((R_{\mu\nu})\) and Ricci Scalar \((R = g^{\delta\lambda}R_{\delta\lambda})\) as

\[ G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R. \]  

(2.3)

The Ricci tensor in turn is a contraction of the Riemann tensor

\[ R_{\mu\nu} = R^{\lambda}_{\mu\lambda\nu}, \]  

(2.4)

which can be expressed in terms of the Christoffel symbols or connection coefficients as

\[ R^{\delta}_{\alpha\beta\gamma} = \partial_\alpha \Gamma^\delta_{\beta\gamma} - \partial_\beta \Gamma^\delta_{\alpha\gamma} + \Gamma^\delta_{\alpha\mu} \Gamma^{\mu}_{\beta\gamma} - \Gamma^\delta_{\beta\mu} \Gamma^{\mu}_{\alpha\gamma}. \]  

(2.5)

These Christoffel symbols are in turn related to the 4-metric \((g_{\mu\nu})\) as

\[ \Gamma^\alpha_{\mu\nu} = \frac{1}{2}g^{\alpha\lambda}(\partial_\mu g_{\lambda\nu} + \partial_\nu g_{\mu\lambda} - \partial_\lambda g_{\mu\nu}). \]  

(2.6)

In other words, the Einstein Field Equations amount to ten second order, non-linear, coupled, partial differential equations for the metric tensor \((g_{\mu\nu})\) which describes the distances between points \(x^\mu\) as

\[ ds^2 = g_{\mu\nu} dx^\mu dx^\nu \]  

(2.7)

on a pseudo-Reimannian manifold \(\mathcal{M}(x^\mu, g_{\mu\nu})\) given a matter field \(T_{\mu\nu}\). By writing the Field
Equations in this covariant form, gravitation can be geometrically expressed in any coordinate system without changing the underlying equations.

In GR, how relativistic an object is will scale with the compactness factor $M/R$ where $M$ is the mass and $R$ is the physical length scale. The most compact objects in the Universe are BHs which can be described entirely by their mass, spin, and charge (Misner et al., 1973). However, astrophysical BHs are thought to have no charge. The Kerr metric for a spinning BH was found by (Kerr, 1963) to be

$$\begin{align*}
\text{ds}^2 &= - \left(1 - \frac{2Mr}{\rho^2}\right) dt^2 - \frac{2Mar \sin^2 \theta}{\rho^2} (dtd\phi + d\phi dt) \\
&+ \frac{\rho^2}{\Delta} dr^2 + \rho^2 d\theta^2 + \frac{\sin^2 \theta}{\rho^2} \left[(r^2 + a^2)^2 - a^2 \Delta \sin^2 \theta\right] d\phi^2
\end{align*}$$

(2.8)

where

$$\Delta = r^2 - 2Mr + a^2$$

(2.9)

and

$$\rho = \sqrt{r^2 + a^2 \cos^2 \theta}.$$  

(2.10)

In this Dissertation, we will discuss the flow of gas in SMBBH spacetimes. However, due to the non-linear nature of GR, no analytic solution exists and one cannot simply add together two Kerr spacetimes. Numerically integrating the Einstein Field Equations is computationally expensive. Fortunately, by making assumptions about the spacetime one can make use of approximations such as PN and BH perturbation theory.

Due to the expense of integrating the Einstein Field Equations, an important part of our model is the use of such approximations to construct an analytic BBH spacetime (Mundim et al., 2014). Because it is analytic, numerically integrating the Einstein Field Equations forward in time is unnecessary. By avoiding numerically integrating the Einstein Field Equations, we gain several
computational advantages. Firstly, open source numerical relativity codes capable of evolving BBHs, as currently implemented, use adaptive mesh refinement (AMR) in Cartesian coordinates. However, it is known that Cartesian grids lead to excessive numerical diffusion and require far more cells to accurately model spherical flows than spherical grids. Additionally, AMR grids have the potential to introduce gridding effects with reflections off boundary conditions. By constructing an analytic spacetime, we are able to evolve our fluid in spherical coordinates and can therefore dramatically reduce the number of cells required to preserve angular momentum (note that the computational cost scales as $\Pi N_i$ where $N_i$ is the number of cells in each dimension). Combining the increased physical space of each cell and the reduced characteristic velocity of the fluid equations (the characteristic velocity of the Einstein Field Equations is the speed of light), we are also able to use larger timesteps with our analytic spacetime. Avoiding these computational loads of numerical relativity allows us to follow inspiraling SMBBHs over the timescales on which gas accumulates (hundreds of binary orbits).

The spacetime is broken into 4 regions: (i) Inner Zone for BH1 (IZ1), (ii) Inner Zone for BH2 (IZ2), (iii) Near Zone (NZ), and (iv) Far Zone (FZ). For a schematic representation of the spacetime see Figure 2.1.

In regions sufficiently far from the BHs, the metric is described by the NZ metric. This metric encompasses the domain where the separation from an individual BH is much greater than the BH’s mass but still much less than the gravitational wavelength, $2\pi c/\Omega_{bin}$ in the general case, $\pi c/\Omega_{bin}$ when $q = 1$. In this domain the metric can be described by PN theory (Blanchet, 2014) in PN harmonic coordinates.

![Figure 2.1: Schematic representation of our spacetime. We denote the BZs using gray shells and the BHs with black circles. Note that these zones are not drawn to scale, and that in practice the zones are not perfectly circular as drawn. The full metric is stitched together in a weighted sum whose weights are determined using transition functions (see (Mundim et al., 2014; Ireland et al., 2015) for more details).](image-url)
Chapter 2. *Our Model*

The PN approximation accurately describes spacetime when fields are weak \((r_g/r = GM/(rc^2) \ll 1)\) and the BH motions are slow \(((v/c)^2 \ll 1)\). In PN theory, spacetime is described as Newtonian gravity plus an asymptotic series of relativistic corrections in powers of these small quantities; the PN-order is characterized by the order of the expansion (i.e., 3PN contains terms up to \((r_g/r)^3\) and \((v/c)^6\)).

The IZ is defined as the region where the distance to an individual BH is much less than the binary separation. In these regions the metric is approximately Schwarzschild, but boosted into the binary center-of-mass frame and perturbed by the gravity of the other BH. In this region we write the metric as

\[
g_{\mu\nu} = \Lambda^\delta_{\mu} \Lambda^\epsilon_{\nu} \left( g_{\delta\epsilon}^{\text{Schwarzschild}} + h_{\delta\epsilon} \right),
\]

where \(h_{\delta\epsilon}\) contains the perturbations of the binary companion and \(\Lambda\) is the transformation tensor containing the boost relating the center-of-mass frame to the frame comoving with the BH. The velocity of the boost is set to be the instantaneous velocity of the BH, as found by the PN equations of motion under the quasi-circular approximation (Blanchet, 2014). The equations of motion are accurate to 3.5PN-order and include gravitational radiation losses, making the binary’s inspiral rate consistent with Einstein’s equations for the separations explored in this Dissertation. We note that the spacetime construction is consistent in that the same PN equations of motion are used in the NZ spacetime calculation. The metric in the IZ ensures that our BHs have true horizons by being constructed via perturbation theory around the Schwarzschild BH solution; it is written in horizon-penetrating Cook-Scheel harmonic coordinates to avoid coordinate singularities (Cook & Scheel, 1997).

The mini-disks are contained entirely within the NZ and IZ domains. No problems are created at the zone boundaries because each zone defining the boundary, by construction, shares a region of common validity with its partners (labeled BZ (Buffer Zone) in Figure 2.1.). In these regions the
metrics are stitched together through asymptotic matching. In essence, the coordinate mappings between the IZs and NZ spacetimes are derived by requiring the metrics be equal order-by-order in an asymptotic series. We then smoothly map from one approximation to another using a transition function. The resultant global spacetime has been shown to satisfy the Einstein Field Equations to the expected level of accuracy (Mundim et al., 2014). The BH trajectories are updated according to equations of motion accurate to 3.5PN-order; in our largest separation run ($a = 100\,M$), the separation barely changes over the course of the simulation, while in our smallest separation case ($a = 20\,M$), the orbit shrinks by almost 20% in only 14 orbital periods (see Figure 2.2). Finally, we note that this spacetime was extended to aligned spins by Ireland et al. (2015).

### 2.2 General Relativistic Magnetohydrodynamics

Given a means of prescribing the gravitation of SMBBHs, we elect to use HARM3D to solve the equations of general relativistic magnetohydrodynamics on this background spacetime in flux-conservative form. We use the thin-disk approximation and neglect self-gravity. This is justified for plausible SMBBHs, where the gas mass is a tiny fraction of the binary mass, so its contribution to gravity should always be negligible. HARM3D was originally written and used to study magnetized accretion disks around fixed, single BH spacetimes (Gammie et al., 2003; Noble et al., 2009). More recently, HARM3D was extended to use arbitrary dynamic spacetimes and coordinate systems. This

![Figure 2.2: Binary separation normalized to the initial binary separation as a function of binary orbital periods. Note that as the separation shrinks the rate of inspiral increases.](image)
extension facilitated studies of magnetized accretion around inspiraling SMBBHs (Noble et al., 2012; Zilhão & Noble, 2014; Zilhão et al., 2015; Zlochower et al., 2016). The use of spherical-polar coordinates minimizes artificial diffusion in disk problems, increasing our confidence that the measured accretion is due to physical accretion stresses, because the gas flow is nearly normal to azimuthal cell faces. This is not the case for Cartesian grids, which are known to lead to numeric diffusion and unphysical angular momentum transport. Finally, HARM3D has been shown to have strong scaling to tens of thousands of cores. The strong scaling of HARM3D allows the use of petascale systems such as the BlueWaters supercomputer.

2.2.1 Equations of Motion

The equations of motion (EOM) for GRMHD amount to conservation of baryon number density, conservation of stress-energy, the Maxwell induction equations, and divergence free constraint on the magnetic field (see Noble et al. (2009) for more details). Taken together, they can be written as

\[ \partial_t U(P) = -\partial_i F^i(P) + S(P), \]  \hspace{1cm} (2.12)

where \( P \) are the “primitive” variables, \( U \) the “conserved” variables, \( F^i \) the fluxes, and \( S \) the source terms. In terms of the primitive variables and metric functions they can be expressed as

\[ U(P) = \sqrt{-g} \left[ \rho u^t, T^t_t + \rho u^t, T^t_j, B^k \right]^T, \]  \hspace{1cm} (2.13)

\[ F^i(P) = \sqrt{-g} \left[ \rho u^i, T^t_t + \rho u^i, T^t_j, \left( b^i u^k - b^k u^i \right) \right]^T, \]  \hspace{1cm} (2.14)

\[ S(P) = \sqrt{-g} \left[ 0, T^k \chi \Gamma^l_{tk} - \mathcal{F}_t, T^k \chi \Gamma^l_{jk} - \mathcal{F}_j, 0 \right]^T, \]  \hspace{1cm} (2.15)

where \( g \) is the determinant of the metric, \( \chi \) are the Christoffel symbols, \( b^\alpha = \left( 1/u^t \right) \left( \delta^\alpha_\nu + u^\alpha u_\nu \right) B^\nu \) is the magnetic 4-vector projected into the fluid’s comoving reference frame, and \( u^\alpha \) are the com-
ponents of the fluid’s 4-velocity. The stress-energy tensor can be written as

\[ T_{\alpha\beta} = (\rho h + 2p_m) u_\alpha u_\beta + (p + p_m) g_{\alpha\beta} - b_\alpha b_\beta, \]  

(2.16)

where \( h = 1 + \epsilon + p/\rho \) is the specific enthalpy, \( \epsilon \) is the specific internal energy, \( p \) is the gas pressure, \( p_m = \frac{1}{2}b^2 \) is the magnetic pressure, and \( \rho \) is the rest-mass density. We ensure that the constraint, \( \partial_i \sqrt{-g} B^i = 0 \), is maintained throughout the simulation by use of FluxCT (Tóth, 2000). In essence, FluxCT uses constrained transport (CT), a method of maintaining the divergence constraint on a particular grid discretization to round-off via boundary conditions, on the numerical fluxes calculated when solving the Reimann problem. Provided the initial state of the constraint is sufficiently small, the error will neither increase nor propagate away.

The gas’s thermodynamics are governed by an adiabatic equation of state with index \( \Gamma = 5/3 \) and local cooling. The cooling is introduced via an explicit source term in the stress-energy conservation equation: \( \nabla_\lambda T^{\lambda}_{\beta} = -\mathcal{L}_c u_\beta \). The fluid rest-frame cooling rate per unit volume \( \mathcal{L}_c \) is determined via the prescription of Noble et al. (2012) in which the gas is cooled at a rate

\[ \mathcal{L}_c = \frac{\rho \epsilon}{t_{cool}} \left( \frac{\Delta S}{S_0} + \frac{|\Delta S|}{S_0} \right), \]  

(2.17)

where \( \Delta S \equiv S - S_0 \), \( S = p/\rho^\Gamma \) is the local entropy, and \( t_{cool} \) is the cooling timescale. This prescription cools the gas to the initial entropy \( S_0 = 0.01 \) in regions of increased entropy on a timescale \( t_{cool} \) usually set to the orbital period of the fluid element. This is consistent with our radiative efficiency approximation, and releases any local increases in entropy. To ensure that our cooling function doesn’t excessively release energy from the floor state, we cool only bound material \( (u_t (\rho h + 2p_m) \geq -\rho) \) (Noble et al., 2012). The cooling time is set differently in each of four distinct regions; one for the circumbinary region, one for each mini-disk, and one for the cavity between the mini-disk and circumbinary regions. We define the circumbinary region as the full azimuthal extent of an annulus extending in radius from \( r = 1.5a \) out to the end of our numerical domain, where \( r \) is the PN Harmonic (PNH) radial coordinate (Blanchet, 2014) with origin at the binary
center-of-mass and $a$ is the binary separation. In this region the cooling timescale is the local Keplerian orbital period about the total mass, $t_{\text{cool}} = 2\pi (r + M)^{3/2} / \sqrt{M}$, where $M$ is the total BH mass. The mini-disk regions are defined by the areas within which $r_i \leq 0.45a$, where $r_i$ is the PNH radial distance from the $i^{\text{th}}$ BH. In these regions, $t_{\text{cool}}$ is again the local orbital period, but now it is calculated in terms of the local Boyer-Lindquist (BL) coordinates with respect to the closest BH (see Section 2.3 for how these are calculated), i.e., $t_{\text{cool}} = 2\pi r_{BL}^{3/2} / \sqrt{m_i}$. Finally, in the cavity region between the mini-disks and circumbinary region where there are no quasi-stable circular orbits, $t_{\text{cool}}$ is set to be the (constant) period of a local Keplerian orbit at $r = 1.5a$.

2.2.2 Evolution Scheme

Having prescribed the spacetime and matter content of our SMBBH systems, we must numerically evolve the equations presented in Section 2.2.1. A key feature of fluid flows is the existence of shocks, or discontinuities, in the solutions. In essence, merely finite differencing to update the evolution variables over a shock would fail to preserve the physical discontinuity. This is effectively due to an inability of finite differencing approximations to calculate derivatives over a discontinuity. Because of this, a standard hyperbolic solver alone will fail to yield the correct solution.

We must therefore update the EOM in flux conservative form as presented in Section 2.2.1 and solve the resulting Riemann problem. That is, one must ensure that the flux entering a given cell face is the same as the flux leaving the same face of the adjacent cell. To achieve this, we employ a second order piece-wise parabolic reconstruction step to calculate the variables at each cell face with a monotonized central slope limiter and Lax-Friedrichs flux. Additionally, the metric and Christoffel symbols are needed at all substeps of the update procedure. We analytically calculate the metric tensor $g_{\mu\nu}$ at each time level, numerically invert for $g^{\mu\nu}$, and numerically construct $\Gamma^\alpha_{\mu\nu}$ by fourth order finite differencing of the metric. Below we roughly sketch the GRMHD update procedure.

First, we reconstruct in each spatial dimension of the simulation to obtain the primitive variable’s values at the cell faces. For instance, that means calculating the variable’s state at $(i - 1/2, j, k)$
and \((i + 1/2, j, k)\) for all \((i, j, k)\), given the appropriate cell centered values when reconstructing in the \(i^{th}\) direction. After completing the reconstruction of the primitive variables we label the “left” (the cell face belonging to \((i - 1, j, k)\)) and “right” (the cell face belonging to \((i, j, k)\)) faces of each cell interface. From here, we calculate the left and right states of the conserved quantities and fluxes of equations 2.13 and 2.14. This procedure is then repeated for the “\(j\)” and “\(k\)” directions.

This now allows us to solve for the Lax-Friedrichs flux at each cell in each spatial direction (Gammie et al., 2003),

\[
F^i = \frac{1}{2} \left[ F^i_R + F^i_L - c_{\text{max}} (U^i_R - U^i_L) \right],
\] (2.18)

where \(c_{\text{max}}\) is the maximum characteristic speed and \(i\) denotes the direction in which reconstruction was performed. We calculate the maximum characteristic speeds in each dimension via the magnetosonic speed,

\[
c^2_{\text{ms}} = \frac{\Gamma p + b^2}{\rho h},
\] (2.19)

as measured by an observer with four-velocity \(u^i\). We take \(c_{\text{max}} = \max (|v_+|, |v_-|)\), where \(v_\pm\) are the quadratic roots to \(Av^2 + Bv + C = 0\) with coefficients

\[
A = u^i u^i - c^2_{\text{ms}} (g^{ii} + u^i u^i) \\
2B = u^i u^i - c^2_{\text{ms}} (g^{ii} + u^i u^i) \\
C = u^i u^i - c^2_{\text{ms}} (g^{ii} + u^i u^i). \] (2.20)

The obtained fluxes are then constrained by FluxCT (see (Tóth, 2000) for full details). Having obtained the fluxes, we can now update the conserved quantities via 2nd order Runge-Kutta in the predictor and corrector steps as

\[
U_{\text{updated}} = U_{\text{current}} + \frac{dt}{2} \left( - \partial_t F^i + S \right),
\] (2.21)
given timestep $dt$.

Given the updated form of the conserved variables, we need to obtain the updated primitive variables at each substep of the update. While the closed-form mapping from primitive variables to conserved variables exists, no such inverse is known. For this reason, the primitive variables must be calculated numerically from the updated conserved quantities. Recalling the magnetic field components are both primitive and conserved variables, this amounts to inverting five equations $\mathbf{U}(\mathbf{P})$ for $\mathbf{P}(\mathbf{U})$. However, this can be reduced to two equations as outlined in Noble et al. (2006). To begin, the authors define the conserved variables as

\begin{align*}
D &= \gamma \rho, \\
Q_\mu &= \gamma \left( \rho h + b^2 \right) u_\mu - \left( p + b^2/2 \right) n_\mu + (n_\nu b^\nu) b_\mu = \alpha T^t_\mu, \quad (2.22) \\
\end{align*}

and

\begin{align*}
\mathcal{B}^i &= \alpha B^i; \quad (2.23) \\
\end{align*}

where $\gamma = \alpha u^t$, $n_\mu = - (\alpha, 0, 0, 0)$, and $\alpha = \sqrt{-1/g^{tt}}$. Using the projection tensor, $j_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu$, a new variable is defined as

\begin{align*}
\tilde{Q}^\nu &= j^\nu_\mu Q^\mu. \quad (2.25) \\
\end{align*}

Given this, Noble et al. (2006) showed that $\mathbf{P}(\mathbf{U})$ can be obtained by simultaneously solving

\begin{align*}
\tilde{Q}^2 &= v^2 \left( B^2 + W \right)^2 - \frac{(Q_\mu B^\mu)^2}{W^2} \left( B^2 + 2W \right) \quad (2.26) \\
Q_\mu n^\mu &= - \frac{B^2}{2} \left( 1 + v^2 \right) + \frac{(Q_\mu B^\mu)^2}{2W^2} - W + p \quad (2.27) \\
\end{align*}
Chapter 2. Our Model

for $W = \gamma \rho h$ and $v^2$. Explicitly, the order in which the primitive variables are back-calculated are

$$\rho = \frac{1}{\gamma} D$$

$$\rho h = W (1 - v^2)$$

$$p = \frac{\Gamma - 1}{\Gamma} (h - 1) \rho$$

$$u = (h - 1) \rho + p$$

$$v^i = \frac{\gamma}{W + B^2} \left( \tilde{Q}^i + Q_\mu B^\mu \frac{B^i}{W} \right)$$

$$B^i = \frac{1}{\alpha} B^\phi.$$  (2.31)

Finally, the newly obtained primitive variables can be fed into the next cell update.

2.3 Mini-Disk Initial Data Construction

The solution for an individual mini-disk in hydrostatic equilibrium in a binary spacetime is not known. However, solutions for hydrostationary torii orbiting a central mass are known (Chakrabarti, 1985; De Villiers & Hawley, 2003). These solutions suppose that the metric has Killing symmetries, $(\partial_t)^\alpha$ and $(\partial_\phi)^\alpha$, and that the metric can be expressed in a spherical coordinate system whose only non-zero diagonal metric component is $g_{tt\phi}$. Although this is only a crude approximation to the actual spacetime, and tidal forces do create significant departures from stationary behavior, it is a feasible means of constructing an initial state. Following Noble et al. (2012), we construct the solution for an initially isentropic accretion disk in hydrostatic equilibrium around an individual BH as a function of local BL coordinates $(r_{BL}, \theta_{BL})$, neglecting the presence of the binary companion. We specify the radial location of the inner edge of the disk ($r_{in}$) and pressure maximum ($r_p$) for a desired $(H/r)$ at the pressure maximum.

Once the hydrodynamic quantities are set, we calculate the magnetic vector potential

$$A_{\phi{BL}} = \max \left( \tilde{\rho} - \frac{\rho_{max}}{4}, 0 \right).$$

(2.32)
where $\bar{\rho}$ is the average density of a cell and its neighbors and $\rho_{\text{max}}$ is the maximum density of the mini-disk. All other components of the vector potential are set to zero. This corresponds, physically, to an initially poloidal, dipole field. As we show later, the lack of exact hydrostatic balance leads to a transient, but it decays in 1–3 orbits.

We relate the local BL coordinates in which the mini-disk initial data are constructed to the PNH coordinates describing the NZ through an intermediate Cook-Scheel (CS) harmonic coordinate (Cook & Scheel, 1997). For any location specified in Cartesian PNH coordinates, we calculate the corresponding CS coordinates ($X_{\text{CS}}$) and Jacobian $\frac{\partial X_{\text{PNH}}}{\partial X_{\text{CS}}}$ as in Appendix B of Mundim et al. (2014). Going the other way, we can express the CS coordinates and Jacobian $\frac{\partial X_{\text{CS}}}{\partial X_{\text{BL}}}$ in terms of the local BL coordinate system (Gallouin et al., 2012) through

$$
T = t_{\text{BL}} + \frac{r_+^2 + \chi^2}{r_+ - r_-} \ln \left| \frac{r_{BL} - r_+}{r_{BL} - r_-} \right|
$$

$$
X + iY = (r_{BL} - m + i\chi) e^{i\phi_{IK}} \sin \theta_{BL}
$$

$$
Z = (r_{BL} - m) \cos \theta_{BL}
$$

$$
\phi_{IK} = \phi_{BL} + \frac{\chi}{r_+ - r_-} \ln \left| \frac{r_{BL} - r_+}{r_{BL} - r_-} \right|
$$

(2.33)

where $\chi$ is the dimensional spin parameter (zero for this work), $r_\pm = m \pm \sqrt{m^2 - \chi^2}$, $m$ is the mass of the individual BH, and the initial BH orbital phase ($\phi_{IK}$) is assumed to be zero. Combining these two coordinate transformations completes the rule for transforming quantities between the BL and PNH systems.

The actual simulation is done in a different coordinate system we call “warped coordinates” derived from a spherical coordinate system (see Section 3.2.3). The last stage in initial condition preparation is therefore to transform the data, originally prepared in BL coordinates, from Cartesian PNH coordinates (as described above) to spherical PNH coordinates, and finally to the warped coordinates, i.e.,

$$
X_{\text{BL}} \rightarrow X_{\text{CS}} \rightarrow X_{\text{PNHC}} \rightarrow X_{\text{PNHS}} \rightarrow X_{\text{WARP}}
$$

(2.34)
where $X_{PNHC}$, $X_{PNHS}$, and $X_{WARP}$ are the Cartesian, spherical, and warped representation of PNH coordinates, respectively. The stored BL solution is then bi-linearly interpolated onto the numerical grid. To construct the magnetic field from the vector potential, we calculate the curl in numerical coordinates to get $B' = \nabla \times A$ and set

$$B^i = \frac{B'^i}{\sqrt{\beta}} \sqrt{\frac{2}{\Gamma - 1}} \sqrt{\Gamma - 1} \sqrt{\frac{u \sqrt{-g}}{\int b^2 \sqrt{-g} d^3x} \int b^2 \sqrt{-g} d^3x},$$

(2.35)

where $\beta = 0.01$ and $b^2 = b^\lambda b_\lambda$. 
3.1 Introduction

In this chapter, we present the first hydrodynamic simulations of mini-disks during both the quasi-Newtonian and GW-dominated inspiral regime of BBHs ($a \lesssim 100M$). In our approach, we use an approximate general relativistic spacetime (Mundim et al., 2014; Ireland et al., 2015), which accurately describes the dynamics of BBHs during the inspiral phase. The inspiral itself is described through the PN equations of motion (Blanchet, 2014). Although our implementation of general relativistic effects is valid for any BH mass-ratio and can accommodate spins, the work reported here focuses on equal-mass, non-spinning BHs. Hydrodynamics is treated in 2D because our goal is to study how general relativistic effects alter mini-disk dynamics, including tidal truncation, interactions between the mini-disks, and spiral shocks. While not incorporating important effects (continuing accretion from the circumbinary disk, vertical structure, and magnetic fields), the work presented here complements existing simulations of these systems and is a stepping stone towards the goal of performing 3D general relativistic MHD simulations of the entire inspiral to merger of BBHs with circumbinary disks.

As we will show in detail in this Chapter, we find that relativistic effects can create qualitative
changes to the mini-disks when the binary separation shrinks to several tens of gravitational radii or less (hereafter, we quote all distances in in gravitational units: \( r_g \equiv GM/c^2 = M \) when \( G = c = 1 \)).

The spiral waves found in so many Newtonian studies take on an entirely new symmetry. The shape of the potential through the L1 region changes, allowing substantially greater mass to find its way out of the mini-disks and into that region. This gas, liberated from the mini-disks, continually passes back and forth from the domain dominated by one BH to that dominated by the other BH and back again, shocking against the outer edge of the mini-disks at each passage. In principle, enough mass can be injected into this “sloshing” region to create observable photon signals; in systems with unequal BH masses, the sloshing could even result in net mass-transfer from one disk to the other.

### 3.2 Simulation Details

#### 3.2.1 Hydrodynamic Prescription

We use HARM3D (Noble et al., 2009) to solve the equations of general relativistic hydrodynamics on a background spacetime in flux-conservative form restricted to the equatorial plane. In this limit, the magnetic field equations are solved trivially and only conservation of baryon number density and conservation of stress-energy remain (see Noble et al. (2009) for more details). Recall, they can be written as

\[
\partial_t U(P) = -\partial_i F^i(P) + S(P) ,
\]  

(3.1)
where \( \mathbf{P} \) are the “primitive” variables, \( \mathbf{U} \) the “conserved” variables, \( \mathbf{F}^i \) the fluxes, and \( \mathbf{S} \) the source terms. In terms of the primitive variables and metric functions they can be expressed as

\[
\mathbf{U}(\mathbf{P}) = \sqrt{-g} \left[ \rho u^i, T^i_{\ t} + \rho u^i, T^i_{\ j} \right]^T,
\]

\[
\mathbf{F}^i(\mathbf{P}) = \sqrt{-g} \left[ \rho u^i, T^i_{\ t} + \rho u^i, T^i_{\ j} \right]^T,
\]

\[
\mathbf{S}(\mathbf{P}) = \sqrt{-g} \left[ 0, T^\kappa_{\ \lambda} \Gamma^\lambda_{\ t\kappa} - F^t_{\ \lambda}, T^\kappa_{\ \lambda} \Gamma^\lambda_{\ j\kappa} - F^j_{\ \lambda} \right]^T,
\]

where \( g \) is the determinant of the metric, \( \Gamma^\lambda_{\ \alpha\beta} \) are the Christoffel symbols, and \( u^\alpha \) are the components of the fluid’s 4-velocity. The hydrodynamic stress-energy tensor can be written simply as

\[
T_{\alpha\beta} = \rho h u_\alpha u_\beta + p g_{\alpha\beta},
\]

where \( h = 1 + \epsilon + p/\rho \) is the specific enthalpy, \( \epsilon \) is the specific internal energy, \( p \) is the gas pressure, and \( \rho \) is the rest-mass density. The gas’s thermodynamics are governed by an adiabatic equation of state with index \( \Gamma = 5/3 \) and local cooling as described in Section 2.2.

### 3.2.2 Initial Data Parameters

We performed seven different hydrodynamic evolutions, differing primarily by initial binary separation. We tabulate their parameters in Table 3.1. For the “small”, 30\( M \), and 20\( M \) runs, the mini-disks’ initial outer radii were approximately the Newtonian truncation radius (0.3\( a \)); for the “large” and 40\( M \) runs, the initial radii were larger (0.4\( a \)). This distinction permitted us to explore whether the quasi-steady state obtained is independent of the initial mini-disk size. The binary separations range from 100\( M \) (found to be quasi-Newtonian by Zilhão et al. (2015)) to 20\( M \), at which relativistic effects are substantial. In Figure 3.1 we plot the initial density contours around BH1 (the BH initially on the positive x axis) for the 20\( M \) binary separation and “small” 50\( M \) and 100\( M \) binary separation runs. For reference, we mark the black hole horizon, ISCO, the inner edge of the IZ-NZ BZ (the outer edge of the IZ-NZ BZ lies well outside the mini-disk), and the Newtonian
Chapter 3. *Hydrodynamic Mini-Disks*

Table 3.1. Initial Data Parameters

<table>
<thead>
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<th>Run name</th>
<th>Initial Separation</th>
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</tbody>
</table>

Note. — Initial data parameters used for the hydrodynamic runs. Radial coordinates are in the local BL system centered on the individual BH. $M$ is the total mass of the binary in all entries. $r_{\text{in}}$ and $r_p$ denote the locations of the inner edge and pressure maximum of the mini-disk.

estimate for the tidal truncation radius. The majority of the disk mass is located within the IZ-NZ BZ, well outside the ISCO. For the 50M and 100M runs, we put in place a circumbinary disk following the prescription of Noble et al. (2012) with $r_{\text{in}} = 3a_0$ and $r_p = 5a_0$, with only floor values of density and pressure between the circumbinary disk and the mini-disks. We did not observe any substantial inflow from the circumbinary into the mini-disks in these runs because, in the absence of MHD accretion stresses, there is no mechanism to drive accretion into the central cavity aside from numerical diffusion, and that is kept small by our spherical grid. Having seen this behavior in the larger separation runs, in the 20M, 30M, and 40M runs, we set the initial gas density and pressure everywhere outside the mini-disks and in the cavity to floor values, eliminating any circumbinary disk. While astrophysical mini-disks will be influenced by accretion streams from the circumbinary disk, removing such streams allows us to focus on purely gravitational effects of GR on the mini-disk structure.
3.2.3 Grid and Boundary Conditions

Our hydrodynamic simulations are performed in the equatorial plane of a dynamic, double fish-eye (warped) spherical coordinate system whose origin is at the binary center-of-mass (Zilhão & Noble, 2014). Cells within this coordinate system are spaced uniformly in numerical spatial coordinates \( \{ x^i \} \). By this means, we are able to focus resolution in the vicinity of the BHs and rarefy resolution in the dynamically less interesting portions of the cavity. Near the BHs the grid is approximately Cartesian, while farther out the grid is spherical.

Given physical PNHS coordinates \((r, \phi)\), Zilhão & Noble (2014) considered three regions of warping: one for each BH and one for the region between the BHs. The coordinate transformations from PNHS to WARP take a different form in each region and are smoothly interpolated (analytically) to match each other at the regions’ boundaries. The transformations are designed so that \( \Delta r(r) \) has a local minimum at \( r = a/2 \) and \( \Delta \phi(\phi) \) has a local minimum at the instantaneous azimuthal positions of the BHs. Approximately 32 cells span each black hole horizon in each dimension, a resolution chosen to ensure that the near-horizon spacetime is well resolved. Parameters control...
various aspects of the transformations. In particular, it is important to achieve a smooth transition from nearly-Cartesian cells near the BH horizons to nearly-spherical cells far from the BHs. The values we used are stated in Table 3.2. Quantitative expressions for our transformations and the parameter definitions may be found in Eqs. (29-32) of Zilhão & Noble (2014). For convenience, we also include in Table 3.2 the number of cells within the Newtonian tidal truncation radius for each binary separation.

The warped coordinates are derived from spherical coordinates, and therefore possess a coordinate singularity at the origin. For this reason we excise a sphere of radius \( M \) from the computational domain. In Figure 3.2 we plot the grid for the inner 1.5\( a \) of the 20\( M \), 50\( M \), and 100\( M \) runs. Throughout the length of our simulations, \( \lesssim 1\% \) of the initial gas is lost through the origin cutout.

We consistently impose outflow boundary conditions at the radial \( x^1 \) boundaries, requiring \( u^r \) to be oriented out of the domain there. If at any time the velocity points inward, the value is set to zero, and we recalculate the primitive velocity. To enforce our restriction to 2D, we specify all \( x^2 \) ghost zones as copies of the physical cell in the equatorial plane. This effectively amounts to considering all primitive variables as vertically averaged quantities that are constant w.r.t. \( \theta \). Finally, we apply periodic boundary conditions in the azimuthal \( x^3 \) direction and cover all 2\( \pi \).

3.2.4 Geodesics

To differentiate between hydrodynamic and purely gravitational effects we also simulated ensembles of particles orbiting a single BH in the BBH system. We performed these calculations in BOTHROS (Noble et al., 2007, 2009, 2011), which solves the geodesic equations of motion

\[
\partial_\lambda x^\alpha = N^\alpha \tag{3.6}
\]

\[
\partial_\lambda N_\alpha = \Gamma^\kappa_{\alpha\eta}N_\kappa N^\eta. \tag{3.7}
\]

Here \( N^\alpha \) is the particle’s 4-velocity, and \( \lambda \) is the affine parameter, or in the case of time-like paths, the proper time for the particle. We terminate the geodesic if it exits the central cavity or falls
Table 3.2. Warped Grid Parameters

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<td>20.</td>
<td>20.</td>
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<td>0.01</td>
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</tr>
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<td>0.01</td>
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<tr>
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<td>4.</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
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<td>10.</td>
<td>6.5</td>
<td>6.5</td>
</tr>
<tr>
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<td>10.</td>
<td>10.</td>
<td>6.5</td>
<td>6.5</td>
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<tr>
<td>$b_3$</td>
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<td>40.</td>
<td>40.</td>
<td>40.</td>
<td>6.5</td>
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<tr>
<td>$R_{out}$</td>
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<td>400 X 400</td>
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<td>300 X 320</td>
<td>600 X 640</td>
</tr>
<tr>
<td>Cells per Mini-Disk</td>
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<td>19258</td>
<td>14782</td>
<td>14798</td>
<td>48532</td>
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</tbody>
</table>

Note. — Parameters of the warped grid used for each initial binary separation. Please see Eqs. (29-32) of Zilhão & Noble (2014) for the explicit expressions defining the warped system and the significance of these parameters. For all runs the inner radial cutout is set to $1M$. 
Figure 3.2: (Left to right) Grids used for the 100M, 50M, and 20M binary separation runs. We plot every tenth grid line within the innermost $3a_0 \times 3a_0$ region of the domain, where $a_0$ is the initial binary separation. We show blue circles at $r = 0.3a_0$ to illustrate the approximate location of the Newtonian tidal truncation radius and the outer edge of mini-disk initial data for the “small” runs.

within an ISCO. Bothros was originally written for stationary spacetimes, so support for handling dynamic spacetimes was added for this investigation. Spatial and temporal derivatives needed for evaluation of the Christoffel symbols are computed by centered fourth-order finite differences, using a resolution of $10^{-6} M$ in space and time for all runs. The Christoffel symbols are calculated for every sub-step of the multiple sub-step Bulirsch-Stoer procedure (Press et al., 1992) used to integrate the geodesics in time.

For a given particle orbiting the non-spinning BH we specify the initial 4-velocity in local BL coordinates as

$$u^\alpha = \gamma (1, 0, 0, \eta \Omega_K)$$  \hspace{1cm} (3.8)

where $\Omega_K$ is the Keplerian orbital frequency and $\eta$ is a parameter selected with uniform probability density within $[0.9, 1.1]$ in order to sample local tidal effects. This 4-velocity is then transformed to the $X_{PNHC}$ coordinate system and evolved in Bothros using the spacetime of Section 2.1. We perform three runs of 5,376 test particles each at 100$M$, 50$M$, and 20$M$ binary separations. The
geodesics are launched from 16 × 16 uniformly spaced points in \((r_{BL}, \phi_{BL})\) space. We use different ranges of \(r_{BL}\) depending on the initial separation: \(r_{BL} \in [15, 30] M\) for 100\(M\), \(r_{BL} \in [7.5, 15] M\) for 50\(M\), and \(r_{BL} \in [3.00, 6.75] M\) for 20\(M\) binary separations. The full \(\phi_{BL} \in [0, 2\pi]\) extent is used for all runs. At each location, 21 geodesics are each launched with a different value of \(\eta\).

3.3 Results

3.3.1 Overview

Our simulations span a range of binary separations from \(a \approx 100 M\), where one might expect physics to be quasi-Newtonian, to \(a \approx 20 M\), where relativistic effects, including binary inspiral, become very important. Their general character during the first few binary orbits is illustrated by the snapshots of the mini-disk around BH1 shown in Figure 3.3. A casual look suggests there is little variation, either as a function of time for fixed separation or as a function of separation; a closer look reveals both significant variability and important trends with separation.

The first snapshot, at 0.5 orbits, shows the disks at their greatest extent, as the disks undergo an expansion due to the departures from hydrostatic balance in our initial conditions caused by the omission of tidal forces in our approximate hydrostatic balance equations. However, one or two more orbits suffice for this transient to decay, letting the mini-disks achieve their approximate long-term structure. Once equilibration has completed, we find that both the hydrodynamic and test particle mini-disks have settled down to similar stable configurations.

Similarly, although all three separations exhibited show similar initial transients, they also show a dependence on separation. In particular, note the greater gas density on the side toward BH2 (the left side) at smaller separations. Although not visible in these snapshots, gas is readily shared back and forth between the two mini-disks in a bar-like region centered on the L1 point. The fraction of all available gas finding itself in this region grows sharply with decreasing separation, rising more than an order of magnitude by \(a = 20 M\) (see Section 3.3.4 for further details).

Finally, another effect better seen in other figures (see below) is the development of spiral shocks.
Figure 3.3: Logarithmic density contours for the 100M Large (top row), 50M Large (middle row), and 20M (bottom row) simulations shown underneath particles (black dots) from the test particle runs. Only the region around BH1 is shown to better view the evolution of the gas and particles. BH2 is located off-frame to the left. Columns, from left to right, correspond to snapshots taken after 0.5, 1, 1.5, and 2 binary orbits. The scale of each figure is a region of $a_0 \times a_0$ around each BH with the ticks corresponding to distances of 20M (top row), 10M (middle row), and 5M (bottom row). We note that the decreased test particle count of the 100M run is simply due to a limit on the number of geodesic timesteps that were allowed.

within the mini-disks. Qualitatively similar spiral features have been observed in simulations of accretion disks in cataclysmic variables (Ju et al., 2016) and BBHs (Ryan & MacFadyen, 2017).
However, we find that relativistic effects drive the formation of an \( m = 1 \) mode in addition to the \( m = 2 \) mode predicted by Newtonian gravity (see Section 3.3.5 for further details).

### 3.3.2 Relativistic Effects in the Potential

As discussed in the Introduction, our principal goal is to explore how mini-disk dynamics may change as relativistic effects become more important. The simplest way to highlight how they enter is to study the lowest-order PN corrections to the metric. At this level of approximation, one can isolate the gravitational potential \( \Phi \) in a frame corotating with the binary through the relation

\[
g_{tt} = -(1 + 2\Phi). \tag{3.9}
\]

This potential combines what in Newtonian language would be called the genuine gravitational potential with the centrifugal contribution of the corotating frame. In Figure 3.4 we plot \( \Phi \), scaled by the binary separation, along the line connecting the BHs for all separations we simulated. In Newtonian gravity all the rescaled binary potentials would be identical. The fact that they differ, and in a fashion that is monotonic with binary separation, illustrates the relativistic modifications to Newtonian expectations. The net effect of the PN corrections is to create shallower potential wells at closer binary separations, particularly in the secondary’s Roche lobe. Here, when we say shallower potential we mean a decrease in the difference of the potential at the outer edge of the mini-disk on one side of the BH to the other. There, this effect is, in relative terms, stronger for mass-ratios farther from unity (see Figure 3.5).

Figure 3.4: Binary potential (\( \Phi \)) scaled by binary separation as a function of distance from BH1 along the line connecting the BHs. Distance is measured in units of binary separation and BH2 is located at \(-0.5a\).
For example, for a separation of $20M$, when $q = 1$, the PN potential difference between the L1 point and the nearest edge of the mini-disk is $0.75 \times$ the Newtonian potential difference; the corresponding ratios (for the secondary) when $q = 0.33$ and $q = 0.1$ are 0.53 and 0.38, respectively. In the primary’s Roche lobe, the trend with mass-ratio is rather weaker, with the ratio of PN to Newtonian potential difference $\simeq 0.75–1$ and varying little with $q$ (see Figure 3.6). These PN changes in the potential will be a recurring theme in our description of the simulations’ behavior.

Figure 3.5: Binary potential ($\Phi$) scaled by binary separation as a function of distance from the secondary along the line connecting the BHs. The primary is located at $+0.5a$. All curves are for $a = 20M$; distance is measured in units of this separation. Solid lines correspond to the spacetime described in Section 2.1 and dashed lines denote the Newtonian potential. Mass ratio is indicated in the legend.

Figure 3.6: Binary potential ($\Phi$) scaled by binary separation as a function of distance from the primary along the line connecting the BHs. The secondary is located at $-0.5a$. All curves are for $a = 20M$; distance is measured in units of this separation. Solid lines correspond to the spacetime described in Section 2.1 and dashed lines denote the Newtonian potential. Mass ratio is indicated in the legend.
3.3.3 Density Distribution and Tidal Truncation of Mini-Disks

One of our primary goals is to see whether relativistic effects alter the structure of mini-disks. To accomplish this, the first step is to demonstrate that the structures examined are not influenced by transients due to our initial conditions. We do so in two ways. The first is by contrasting simulations in which the initial conditions differ, and confirming that after the decay of transients they reach similar states. The second makes use of time-averaging over a period later than the $\simeq 2t_{\text{bin}}$ required for transient decay.

Contrasting Initial Conditions

To test our results’ sensitivity to initial conditions, we ran two versions of each of the larger separation cases (100M and 50M). In the “small” runs, the disks were initially filled with matter to a point just inside our analytic estimate of the tidal truncation radius, $r_t \simeq 0.3a$, an estimate appropriate to circular-orbit equal-mass Newtonian binaries (Paczynski, 1977; Papaloizou & Pringle, 1977; Artymowicz & Lubow, 1994). In the “large” runs, in the initial state the disks extended to $\simeq (0.4–0.45)a$.

Although it hides departures from axisymmetry, we have chosen the distribution of mass enclosed within a given radius as a diagnostic of internal structure. In terms of the conserved

---

Figure 3.7: Mass-enclosed for the 100M and 50M binary separation runs. The solid lines denote the time-averaged data while dashed lines correspond to the initial data. The shaded vertical gray area denotes the Newtonian prediction range for $r_t$, $(0.27–0.33)a$ (Paczynski, 1977; Papaloizou & Pringle, 1977; Artymowicz & Lubow, 1994; Roedig et al., 2014).
rest-mass, this quantity is

$$M(< r_{BL}) = \int_{r_{min}}^{r} dr' d\phi_{BL} d\theta \rho v_{BL} t \sqrt{-g_{BL}},$$  \hspace{1cm} (3.10)$$

where $r_{min} = 2M_1$. Because the code data are defined with respect to warped PNH coordinates, to obtain the data necessary for the integral we first transformed the code data into the BL system, resolving 4-vector components with respect to the local BL basis, and then interpolated onto a regular grid in BL coordinates.

In Figure 3.7 we show how well the initial conditions relax to essentially the same structure by contrasting the large and small initial mass-enclosed distributions for 100$M$ and 50$M$ separation with the distribution averaged over the interval $[2t_{bin}, 3t_{bin}]$. Although the 100$M$ Small case differs somewhat from 100$M$ Large in the inner quarter of mass even in the later state of the disk, the remainder of the two 100$M$ mass-enclosed distributions are very close, as are the entire distributions for the Large and Small 50$M$ cases. Because our prescription for the initial condition omits tidal forces, and tidal forces partially cancel the nearby BH’s gravity, the Small cases are over-pressured; as a result, they expand. However, tidal forces also prevent matter from staying near the disk when it lies at distances beyond $r_t$. For this reason, the Small cases cease expanding when they reach the tidal truncation limit, while in Large cases gas beyond $r_t$ does not stay attached to the disk.

On the basis of this success in quickly reaching a state almost entirely independent of initial conditions, we ran only one case for the $a = 20M$, 30$M$, and 40$M$ separations. For $a = 40M$, the initial disk extended to $\simeq 0.4a$; for the two smaller separations, it was filled to only $\simeq 0.3a$.

**Time-Averaging**

Before continuing on to other measures of disk structure, it is necessary to elaborate on our methods of time-averaging. We chose to begin time-averaging for all runs at $2t_{bin}$ because the results shown in Figure 3.3 demonstrate that initial transients have almost entirely decayed by this time. We ended it at $3t_{bin}$ for all cases except $a = 20M$ because the 100$M$ simulation stopped at about...
this time, and we wished to enforce a consistent procedure on all our cases. The smallest separation case demanded special treatment because in it the binary’s separation shrinks appreciably over a single binary orbital period. In this case, we therefore average successive intervals of duration $t_{\text{bin}}$ beginning at roughly 2, 4, 6, 8, 10, 12, 14, and 15.5 binary orbits. These correspond to the times at which the binary separation is $19.5M$, $19.0M$, $18.5M$, $18.0M$, $17.5M$, $17.0M$, $16.5M$, and $16.0M$.

Figure 3.8: (Left to right) Time-averaged density contours on a linear scale normalized to the peak value (top) and on an unnormalized log-scale (bottom) for the 100M Large, 50M Large, and 20M runs. Time-averaging was done over the third binary orbit for each run. Dashed lines overlaid on top of the color density contours show the binary’s potential, $\Phi$, in the frame corotating with the binary.
In Figure 3.8 we plot the time-averaged density contours both on a linear and a logarithmic scale, overlaid with contours of the binary potential evaluated in the frame corotating with the binary. Several features can be clearly seen in these two representations. In a linear scale it is apparent that, rather than being axisymmetric around its BH, the density on the side of each mini-disk nearer the L1 point is higher than on the opposite side. Interestingly, the orientation of the density gradient is rotated by $\approx \pi/5$ relative to the line between the two BHs when $a = 100M$, but this rotation diminishes with smaller binary separation. On a logarithmic scale, we see that as the binary separation shrinks, the relative amount of mass in the zone between the disks rises, hinting that the density cut-off at the edge of the disk facing the center-of-mass is becoming less sharp.

For a more quantitative view of the time-averaged structure, we study the surface density,

$$\Sigma(r_{BL},\phi_{BL}) = \int_{\phi_{BL}} d\theta \rho \sqrt{-g_{BL}} \sqrt{g_{\phi_{BL}\phi_{BL}}} (\theta = \pi/2),$$

in local BL coordinates. The surface density profile of all the BH1 disks is displayed in Figure 3.9. To be more precise, this figure shows the surface density averaged within a pair of wedges, each centered on BH1 and stretching $45^\circ$ on either side of the line between the two BHs. Several qualitative facts are apparent in this figure. First, the location of the near-side (nearest to BH2) surface density peak varies little between $a = 100M$ and $a = 30M$, but it moves sharply outward when $a = 20M$. On the other hand, the far side peak position changes...
little with separation, but the net sense is to move inward as the separation becomes smaller. In addition, although at $a = 100M$ the two surface density peaks, near-side and far-side (farthest from BH2), are very nearly equal in height, the near-side peak becomes increasingly dominant as $a$ decreases.

Quantitative Measures of Tidal Truncation

The time-averaged surface density profile can be used to determine quantitatively the edge of the mini-disks in two different ways. In the first, we define it as the point where the radial gradient of the time- and azimuthally-averaged surface density has the greatest magnitude, both on the side nearest ($0.75\pi \leq \phi_{BL} \leq 1.25\pi$) to and farthest ($-0.25\pi \leq \phi_{BL} \leq 0.25\pi$) from the binary companion (see Figure 3.10). In the second, we search for an outer limit on bound streamlines of the fluid element’s velocity field in snapshots taken during the quasi-steady state (see Figure 3.11).

![Figure 3.10:](image)

Figure 3.10: (Left to right) Time-averaged surface density contours, normalized to the peak value, for the 100M Large, 50M Large, and 20M runs. The time-averaging periods used is the same as those used in Figure 3.8. The near-side and far-side truncation estimates are represented as dashed wedges.

In Table 3.3 we tabulate our estimates for the edge of the mini-disk around BH1 using both methods on both the side nearest ($\phi = \pi$) and farthest from the binary companion ($\phi = 0$). Echoing
Figure 3.11: (Left to right) Logscale density contours for the 100M Large, 50M Large, and 20M runs at the start of the quasi-steady period with velocity streamlines of the fluid flow in black. The start of a streamline is denoted by a black circle.

the behavior of the near-side surface density peak, for separations down to \( a \gtrsim 30M \), the gradient definition yields near-side truncation radii approximately \( 0.23a \), while for \( a \simeq 20M \), it moves to \( \simeq (0.3\sim 0.35)a \). On the far-side, the radii fluctuate between \( 0.23a - 0.30a \) with little trend as a function of separation. The streamline truncation radii tell a different story. They vary more irregularly with binary separation, ranging between \( 0.3a - 0.4a \) (near-side) and \( 0.19a - 0.24a \) (far-side). These contrasting results may be due, in part, to a larger measurement error in determining the truncation radius from streamlines: contrasting BH1 and BH2 by this measure at the same time, the radius can differ by as much as \( 0.05a \). In a fashion qualitatively similar to Newtonian gravity, the disks appear to be significantly larger on the near-side than on the far-side, particularly when defined by streamlines, but also (for \( a \lesssim 20M \)) when defined by surface density gradients; in other words, the truncation “surface” is asymmetric, resembling a Roche-lobe shape more than a sphere. In addition, particularly as judged by the surface density gradient, the disks extend farther on the near-side as the binary separation shrinks. For larger separations, then, these results are in qualitative agreement with the Newtonian \( r_t \simeq 0.3a \) (Paczynski, 1977; Papaloizou & Pringle, 1977; Artymowicz & Lubow, 1994), but even in that regime it is worth noting the non-circularity of the
Chapter 3. *Hydrodynamic Mini-Disks*

Table 3.3. Truncation Measurements

<table>
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<tr>
<th>$a[M]$</th>
<th>$\nabla (\Sigma(r, \phi = \pi))$</th>
<th>$\nabla (\Sigma(r, \phi = 0))$</th>
<th>Streamlines($\phi = \pi$)</th>
<th>Streamlines($\phi = 0$)</th>
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<td>0.17</td>
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<td>0.40</td>
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<tr>
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<td>0.33</td>
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<td>0.30</td>
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<td>0.29</td>
<td>0.33</td>
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<td>0.29</td>
<td>0.27</td>
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<td>0.23</td>
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</table>

Note. — Measured radial extent of the mini-disks on the NEAR($\phi_{BL} = \pi$) and FAR($\phi_{BL} = 0$) sides of the disk as fractions of the binary separation for the surface density and streamline method.

disks.
3.3.4 Sloshing

As previously remarked in Section 3.3.1, matter sloshes back and forth within a bar-like region centered on the binary center-of-mass. Its mass during early times of our simulations is largely due to a transient whose origin lies in our only approximately hydrostatic initial condition for the mini-disks. That this should be so is demonstrated most dramatically by the 50\(M\) and 100\(M\) binary separation runs, in which the ratio of sloshing mass to total mass quickly drops from its initial peak as gas settles back onto the mini-disks (see Figure 3.12). However, if we consider the mass in this region only after decay of the transient, we find a remarkable increase as the separation diminishes below \(\simeq 50M\). To quantify this statement, we must first define this region more precisely: for our purposes, it is a rectangle in the corotating frame with dimensions \((\tilde{x} \times \tilde{y}) = (0.4a(t) \times 0.6a(t))\), centered on the center-of-mass. Because the separation \(a\) in the initially 20\(M\) separation run decreases with time, we adjust this box size as a function of time so that its dimensions are always the same fraction of \(a(t)\). The motivation for choosing these dimensions can be seen by inspection of Figure 3.13.

Employing this definition, we can calculate the ratio \(M_{\text{slosh}}/M_{\text{cav}}\) as a function of time measured in binary orbital periods for the 20\(M\), 30\(M\), 40\(M\), 50\(M\) Large, and 100\(M\) Large hydrodynamic runs. Here \(M_{\text{slosh}}\) is the mass within the box just defined, while \(M_{\text{cav}}\) is the mass within a circle of radius 1.5\(a(t)\) from the center-of-mass. This quantity is plotted in Figure 3.12. All the different separation runs begin with similar values of this ratio, \(\simeq 5-10 \times 10^{-3}\). In the quasi-Newtonian 100\(M\) and 50\(M\) separation runs, this ratio drops to \(\simeq 2-8 \times 10^{-4}\) within three binary orbits. In sharp contrast to this behavior, as the simulations become progressively more relativistic (decreasing binary separation), the mean fractional gas content of the sloshing region increases monotonically. In the 20\(M\) separation run, this ratio maintains its initial value–\(\simeq 5 \times 10^{-3}\)–albeit with factor of several fluctuations, for the entire 14 orbit duration of the simulation. In other words, a decrease in separation by about a factor of 2 leads to an increase in \(M_{\text{slosh}}/M_{\text{cav}}\) of an order of magnitude. The most natural explanation for this dramatic change is also the progressively shallower gravitational potential found as the system becomes increasingly relativistic (see Figure 3.4).
Figure 3.12: The mass within the “sloshing region” normalized to the total mass of the cavity for the 100M large, 50M large, 40M, 30M, and 20M hydrodynamic runs (solid lines). We plot horizontal dashed lines for the mean value of each run for $t \geq 2t_{\text{bin}}$. 
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Figure 3.13: Logscale density contours of the 20M run to demonstrate sloshing at various times: $5.55t_{\text{bin}}$ (top left), $5.66t_{\text{bin}}$ (top right), $5.91t_{\text{bin}}$ (bottom left), and $6.03t_{\text{bin}}$ (bottom right). We denote the “sloshing region” with the dashed black rectangle. At $5.55t_{\text{bin}}$ we see two distinct arms connecting the BHs which then collapses to a single arm at $5.66t_{\text{bin}}$. We then see the formation of a double armed stream pattern again at $5.91t_{\text{bin}}$ and $6.03t_{\text{bin}}$ corresponding to time lapses of $0.36t_{\text{bin}}$ and $0.48t_{\text{bin}}$. 
As a result, gas can more easily flow out of the potential well of a single BH into the center-of-mass region. However, it cannot remain there in a stationary state because the L1 region is dynamically unstable. Once in the sloshing region, gas can only move back and forth between the relatively stable regions near the mini-disks. Upon reaching the truncation radius of a disk, the streams shock.

To gain further insight into the properties of the sloshing region, in Figure 3.13 we plot density contours at four selected times. This figure demonstrates that the back-and-forth motions are concentrated into discrete streams, but sometimes there are two streams at once, while at other times there is only one. Due to the $q = 1$ symmetry of our system, the internal motions of the dual stream features are equal in magnitude and opposite one another. We speculate that the number of streams is related to interaction with the spiral structure (see Sec. 3.3.5), which exhibits both $m = 1$ and $m = 2$ components.

The stream pairs carrying mass between the mini-disks appear to form in a quasi-periodic manner. To characterize this behavior quantitatively, we first pre-whiten the $a = 20M$ data by removing any secular linear trends in $\tilde{M}_{\text{slosh}}(t) \equiv M_{\text{slosh}}(t)/M_{\text{cav}}(t)$. We calculate this pre-whitened function ($\psi$) as

$$\psi(t) = \frac{\tilde{M}_{\text{slosh}}(t) - \tilde{M}_{\text{fit}}(t)}{|M_0|}$$

Figure 3.14: Fourier power density of $\tilde{M}_{\text{slosh}}(t)/M_{\text{cav}}(t)$ for the sloshing region in the 20M run. The “Large Box” is centered on the center-of-mass and has dimensions $0.4a \times 0.6a$; it extends from the near-side of the BH1 mini-disk to the near-side of the BH2 mini-disk. The “Small Box” is likewise centered on the center-of-mass, but has dimensions $0.26a \times 0.6a$. Angular frequency is measured in units of the mean binary orbital angular frequency during the time period covered by the Fourier analysis.
where \( M_0 \) is the largest value of \( \tilde{M}_{\text{slosh}} \), \( \tilde{M}_{\text{fit}}(t) \) is a linear fit for \( \tilde{M}_{\text{slosh}}(t) \), and we use only data for \( 2000 \leq t/M \leq 6000 \). This corresponds roughly to the time between binary orbits 3.5–11.

We then compute \( \Psi \), the Fourier transform of \( \psi \). In Figure 3.14 we plot \( |\Psi|^2 \), the Fourier power density, as a function of frequency. It possesses two distinct peaks, with angular frequencies roughly 2 and 2.75 times the mean orbital frequency of the binary (\( \tilde{\omega}_{\text{bin}} \)). The width of these peaks is comparable to the range of binary orbital frequencies during the period included in the Fourier analysis, from \( \simeq 1.05 \times \) the initial frequency to \( \simeq 1.23 \times \) that frequency.

### 3.3.5 Spiral Density Waves

Figure 3.15: (Left to right) Time-averaged local sound speed at binary separations of 100M, 50M, and 19.5M. The white circle denotes the BH horizons while the dashed black line approximately corresponds to the tidal truncation radii of 0.3\( a \), 0.3\( a \), and 0.33\( a \) respectively.

Within the main body of the mini-disks, we observe the formation of spiral density waves. They can be seen in density plots (Figures 3.3, 3.8, and 3.13) and stand out even more clearly in a plot showing the local sound speed (Figure 3.15) because a strong shock raises the local temperature by a much larger factor than it increases the density. These spiral density waves are of special interest, in part because they have the potential to alter the locally-emitted spectrum, but also because they may contribute significantly to accretion stresses. In fact, spiral density waves were
put forward very early on as a candidate mechanism to explain all accretion stresses (Lynden-Bell & Pringle, 1974), although subsequent studies determined that they were unlikely to be a general answer to the problem (see (Shu & Lubow, 1981; Papaloizou & Lin, 1995; Boffin, 2001) and references therein). In regions where the spiral density wave pattern speed is less than the local fluid orbital frequency of the disk, the spiral density wave orbits backwards through the disk, thereby carrying negative angular momentum with respect to the fluid in the disk. Once the spiral density wave amplitude becomes nonlinear, it can steepen into shocks, allowing it to couple to the disk fluid via dissipation (Papaloizou & Lin, 1995; Goodman & Rafikov, 2001; Heinemann & Papaloizou, 2012; Rafikov, 2016). The resulting loss in the matter’s angular momentum introduces a radial component to the flow and transports angular momentum outwards in the disk, thereby facilitating accretion onto the BH.

In accretion disks in binary systems, these spiral density waves are the result of the perturbing non-axisymmetric gravitational potential of the binary companion (see e.g. Savonije et al. (1994) and references therein). The density waves in the disk due to the perturbing gravitational potential are launched at Lindblad resonances (Goldreich & Tremaine, 1979; Papaloizou & Lin, 1984). In Newtonian gravity, the perturbing potential (in a coordinate system centered on the central mass surrounded by the disk) due to the binary companion on a circular orbit is given by:

\[
\Phi_{\text{pert}} = -\frac{M_2}{|\mathbf{r} - \mathbf{r}_2|} + \frac{M_2(\mathbf{r} \cdot \mathbf{r}_2)}{r_2^3}, \quad (3.13)
\]

where \( M_2 \) is the mass of the perturber, \( \mathbf{r} \) is the position vector and \( \mathbf{r}_2 \) is the position vector of the secondary (see, e.g. (Binney & Tremaine, 1987; Savonije et al., 1994)). The first term in Eq. (3.13) is the Newtonian potential of the secondary; the second term accounts for centrifugal forces due to the fact that the corotating frame is non-inertial.

To analyze the effects of the perturbing potential, it is useful to expand it as a Fourier series in the azimuthal angle. The \( m \)-th component of the perturbing potential excites a spiral density wave with \( m \) arms. If the perturber’s orbit is not too close to the outer edge of the disk, the dominant
azimuthal Fourier component of the Newtonian perturbing potential is the $m = 2$ mode (Savonije et al., 1994). Consequently, in Newtonian simulations of these systems, a two-armed spiral density wave is often found to develop in the disk (see e.g. the simulations of (Savonije et al., 1994; Makita et al., 2000; Ju et al., 2016) or the early evolution of the disks studied in Kley et al. (2008)).

Because the tidal perturbation remains constant in the frame corotating with the binary, the induced spiral density waves’ pattern speed is the binary orbital frequency, which is slower than the orbital velocity of the disk fluid. In the fluid frame, a mode of order $m$ creates a spiral wave whose phase speed is $m$ times the binary orbital frequency, as the fluid encounters $m$ spiral arms in each orbit.

Although Newtonian simulations find the development of clearly visible two-armed spiral density waves as the result of the dominant $m = 2$ mode in the perturbing potential, our simulations show a more complicated flow morphology in the mini-disk for all binary separations studied. To study the structure of the spiral waves in our relativistic simulations, we compute the amplitude of different $m$ modes in the disk surface density as a function of time (see, e.g. (Zurek & Benz, 1986; Heemskerk et al., 1992)):

$$D_m(t) = \int_{r_{\text{min}}}^{r_{\text{max}}} dr_{BL} d\phi_{BL} \Sigma(r_{BL}, \phi_{BL}; t) e^{-im\phi_{BL}} \sqrt{-g_{BL}},$$

(3.14)

where $r_{\text{min}} = 0.13a$ and $r_{\text{max}} = 0.38a$. The results are insensitive to the selection of $r_{\text{min}}$. As a check, we performed this azimuthal mode analysis in two frames, a frame co-moving with BH1, and a frame rotating at the binary orbital period. Both calculations give the same mode strengths, as expected.

The time-dependence of these mode amplitudes for initial binary separation 20$M$, 50$M$, and 100$M$ are shown in Figure 3.16. The $m = 1$ amplitude is larger than the $m = 2$ amplitude in all cases, although $D_1/D_2$ appears to diminish over time for the larger separations. At late times in the $a = 50M$ run, $D_1/D_2 \simeq 3$. When the separation is as small as 20$M$, $D_1/D_2$ oscillates around a constant mean value $\simeq 4$. Although the 100$M$ case did not run long enough to reach “late
Figure 3.16: **Time-dependence of the $m = 1$ and $m = 2$ modes of the surface density in the mini-disks.** (Left) Initial binary separation 100\(M\) shown by solid curves, 50\(M\) by dashed curves. (Right) Initial binary separation 20\(M\).

...times", $D_1/D_2$ appears to be roughly $\sim 2$, suggesting a slow increase in this ratio with decreasing separation.

The origin of this departure from Newtonian experience appears to lie in the different structure of the tidal potential in the PN regime. To demonstrate this, we computed the amplitudes with respect to $m$ in the potential in much the same fashion as we did for the surface density; we call them $S_m$. In order to calculate them, we used the same PN approximation for the perturbing potential as we did when discussing the tidal potential itself, i.e., $\Phi = -(1 + g_{tt})/2$. However, before computing the mode integrals, we subtracted off the isotropic Newtonian potential due to the individual BH. The Newtonian ratio $S_1/S_2 = 0.143$, irrespective of binary separation, because Newtonian gravity is scale-free. However, relativistic gravity is scale-dependent (see Table 3.4). Even at $a = 100M$, $S_1/S_2$ is close to triple the Newtonian value, and it grows monotonically with decreasing binary separation, exceeding unity for separations slightly less than 30\(M\). Relativistic contributions to $m = 1$ modes can be significant at separations as large as 100\(M\) because they
first appear in the PN expansion in the lowest-order non-Newtonian terms (see, e.g., the explicit expression for the PN Lagrangian governing a test-particle in a binary system in Ratkovic et al. (2005)). In addition, the same low-order terms also create new contributions to \( m = 2 \) modulation, altering the Newtonian component in both magnitude and phase.

As a final point, we note that we have performed a test simulation of a BBH system in which initially only BH1 was surrounded by a mini-disk, in order to test if shocks induced by the sloshing might produce spiral density waves in the mini-disks. The morphology of the fluid flow and mode strengths were very similar to those from simulations with two mini-disks. We therefore conclude that the main driving factor of the spiral waves in our mini-disks is indeed the tidal field of the binary companion.
### Table 3.4. Azimuthal Potential Modes

<table>
<thead>
<tr>
<th>$a[M]$</th>
<th>$S_1/S_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Newtonian</td>
<td>0.143</td>
</tr>
<tr>
<td>100</td>
<td>0.412</td>
</tr>
<tr>
<td>50</td>
<td>0.665</td>
</tr>
<tr>
<td>40</td>
<td>0.825</td>
</tr>
<tr>
<td>30</td>
<td>0.990</td>
</tr>
<tr>
<td>19.5</td>
<td>1.209</td>
</tr>
<tr>
<td>19.0</td>
<td>1.212</td>
</tr>
<tr>
<td>18.5</td>
<td>1.239</td>
</tr>
<tr>
<td>18.0</td>
<td>1.276</td>
</tr>
<tr>
<td>17.5</td>
<td>1.298</td>
</tr>
<tr>
<td>17.0</td>
<td>1.335</td>
</tr>
<tr>
<td>16.5</td>
<td>1.359</td>
</tr>
<tr>
<td>16.0</td>
<td>1.382</td>
</tr>
</tbody>
</table>

Note. — Strength of the $m = 1/m = 2$ modes of the perturbing potential as a function of separation.

### 3.4 Discussion

#### 3.4.1 Tidally Truncated Mini-Disks

Previous discussions of the shapes of disks in binary systems have been almost exclusively posed in terms of Newtonian gravity. In addition, when describing their size, it has generally been customary to specify only a mean radius from the mass at the center of such a disk, often without a precise operational definition of what exactly makes that radius the “size” of the disk. In the course of our investigation of PN effects on the tidal truncation of disks in binaries, we have found that quantitative description demands such a definition: these disks are, at the $\sim 20\%$ level, non-circular even in the Newtonian limit, and their sizes (when more precisely defined) do begin to change as
In principle, there could be many definitions of the “size” of a disk in a binary system. Here we explored the implications of two choices. One, a definition focusing on the surface mass density profile, is suited to answering questions such as “if an external fluid stream strikes the disk, where does it encounter the full inertia of the disk?” The other, a definition focusing on the shapes of disk material’s orbits, is designed to discriminate between fluid elements that repeatedly orbit the central mass of a disk from those that may pass near the disk, but then swing away from it.

Even in the Newtonian limit (exemplified by our 100M separation simulation), the disks appear to be asymmetric in terms of both these measures. The far side (away from the other mass in the binary system) extent defined by the surface density gradient agrees with the near side extent defined by streamlines, and both are consistent with the usual Newtonian estimate for equal-mass binaries ($\approx 0.30a$), but the near side surface density definition and the far side streamline definition both give extents $\sim 20\%$ smaller. In other words, by the surface density definition, disks are cut off more tightly on the near side than the far, while by the streamline definition they extend farther from the central mass on the near side. Thus, at this level of precision, it’s clear that a single “truncation radius” fails to be an adequate description: the disks are neither round nor admit a single definition appropriate to all intuitive meanings of “disk edge”.

Relativistic effects also begin to alter mini-disk shapes once $a$ is a few tens of $M$ or less, particularly with respect to the near-side surface density definition. On the near-side, the disk extends to a progressively larger and larger fraction of $a$ as the binary separation moves farther and farther into the relativistic regime. When $a \approx 20M$, the (surface density) near edge is $\approx 35\%$ farther from the central mass than in the Newtonian limit. As we have already remarked, this effect can be attributed directly to the progressively shallower potential gradient in the L1 region produced by increasingly relativistic gravity (as illustrated in Figure 3.4). When the binary mass-ratio is less than unity, the relativistic effects are, if anything, stronger (Figure 3.5). A similar result, also in the PN formulation but posed in terms of the volume occupied by the secondary’s Roche lobe, was found by Ratkovic et al. (2005). It is worth noting, however, that for their parameters (separations
of $5M$ and $10M$), the Roche lobe volume decreases with greater relativistic effects when $q < 0.7$.

On the basis of our results, we might speculate that at still smaller separations the mini-disks continue to exist until the truncation radius (defined in any of these senses) becomes only somewhat larger than the absolute minimum scale set by the ISCO. However, once the inspiral becomes more rapid than the internal accretion rate within such a disk, matter from the outer edge, which now extends beyond the truncation radius, must lose its binding to an individual BH, and instead travel through both its original Roche lobe and the partner’s. Such effects are closely related to the topic of the next subsection, the “sloshing region”.

### 3.4.2 Sloshing

Gas extending across the L1 region in a system with binary disks has been previously observed in stellar systems (Mayama et al., 2010); it has also appeared in Newtonian hydrodynamics simulations of binary stellar systems (de Val-Borro et al., 2011; Nelson & Marzari, 2016) and SMBBHs (Farris et al., 2014; D’Orazio et al., 2016), although without comment in the latter work. However, we have found a number of new properties of this gas. It is in a constant state of back-and-forth motion, and, as shown in Figure 3.14, the variations in this motion are strongly modulated at two frequencies, one very close to $2\bar{\omega}_{bin}$, the other at $2.75\bar{\omega}_{bin}$. Second, and most surprisingly, the fraction of disk mass in this sloshing region rises steeply as relativistic effects become important.

The quantity of gas in the sloshing region is determined through a complex mechanism. When sloshing gas shocks against the edge of a mini-disk, it “spalls” off gas parcels, giving them enough energy to travel across the L1 region. The amount required is comparatively small. Although Figures 3.4, 3.5, and 3.6 show the effective gravitational potential in the rotating frame of the binary, they do not include the contribution to the effective potential associated with the orbital motion of a fluid element around one of the BHs. Even in the Newtonian limit, this contribution causes the total effective potential to be rather shallower than the curves shown in this figure; when PN effects further decrease the change in effective potential from the edge of the mini-disk to the L1 point, the energy delivery threshold for liberation of gas becomes only a small fraction of $GM/a$. 

In agreement with this potential picture, the mean fraction of available gas in the sloshing region 
\( \frac{M_{\text{slosh}}}{M_{\text{cavity}}} \) increases monotonically with decreasing binary separation, with a particularly
sharp increase for \( a \lesssim 30M \). In addition, animations of our simulations show correlations between
fluctuations in spiral wave structure and ejection of matter into the sloshing region. It is possible
that when accretion streams from a circumbinary disk enter the picture, their impact may also
influence gas injection into the sloshing region.

These new properties may lead to a significant new observable. Every time a sloshing stream
shocks against the edge of a mini-disk, the associated energy dissipation is available for photon
radiation. Moreover, the periodic character of these motions means that if the cooling time post-
shock is shorter than the dynamical period, the radiation may be strongly modulated. Because
the characteristic speed of this motion is comparable to the orbital speed, and therefore increases
\( \propto a^{-1/2} \), and the amount of mass involved increases much more rapidly with decreasing binary
separation when \( a \lesssim 30M \), the energy available for this potentially periodic radiation signal should
increase sharply as the binary inspirals through the last few tens of \( M \) in separation.

More quantitatively, for BBH systems with mass-ratios of order unity, we may estimate the
time-averaged heat release in the sloshing region by

\[
L_{\text{slosh}} \sim \left( \frac{M_{\text{slosh}}}{M_{\text{cav}}} \right) \left( \frac{\dot{M} t_{\text{in}}}{v_{\text{orb}} \omega_{\text{slosh}}} \right),
\]

(3.15)

where the total mass of the mini-disks \( M_{\text{cav}} \) is determined by the accretion rate \( \dot{M} \) and the inflow
time through an individual mini-disk \( t_{\text{in}} \). We further estimate the speed of the sloshing to be
comparable to the binary orbital speed \( v_{\text{orb}} \). The repetition rate of the sloshing cycle is \( \omega_{\text{slosh}} \),
which we have already determined to be \( \simeq 2\omega_{\text{bin}} \). For greater insight, this luminosity estimate may
be rewritten as

\[
L_{\text{slosh}} \sim \left( \frac{M_{\text{slosh}}}{M_{\text{cav}}} \right) (r_g/a) \left[ \left( \frac{H}{r} \right)^2 \alpha' \right]^{-1} L_{\text{acc}},
\]

(3.16)
in which \( L_{\text{acc}} \) is the luminosity released by accretion onto the BHs, \( H/r \) is the aspect ratio of the
mini-disks, and $\alpha'$ is the ratio between vertically-integrated and time-averaged accretion stresses and the similarly vertically-integrated and time-averaged disk pressure. We write it as $\alpha'$ rather than the conventional $\alpha$ as a reminder that stresses associated with spiral waves can add to the usual correlated MHD turbulence. We have already found that when $a = 20M$, $M_{\text{slosh}}/M_{\text{cav}} \simeq 3 \times 10^{-3}$. Although the disk aspect ratio must certainly depend on such parameters as the accretion rate, for the time being we simply note that if the accretion rate is not far below Eddington, when the system is in the PN regime, $H/r \sim 10^{-2}$ and $\alpha' \sim 10^{-1}$ should be reasonably conservative estimates. If so, $L_{\text{slosh}}$ might be somewhere in the neighborhood of $\sim 10^{-1}L_{\text{acc}}$. Further work will be required to make sensible predictions about its spectrum.

The optical depth in the sloshing region can be estimated in similar fashion:

$$\tau_{\text{slosh}} \sim 4 (M_{\text{slosh}}/M_{\text{cav}})(r_g/a)^{1/2} \left[(H/r)^2 \alpha'\right]^{-1}(\dot{m}/\eta),$$

where $\dot{m}$ is the ratio of the accretion rate to the Eddington rate and $\eta$ is the rest-mass efficiency of energy release by accretion. Here, to be consistent with the definition of $M_{\text{slosh}}$ found in Sec. 3.3.4, we estimate the area occupied by the sloshing region as $\simeq a^2/4$. Repeating the estimate we just made for the mini-disk internal inflow rate, this expression suggests that the sloshing region should generally be optically thin to Thomson scattering because $(r_g/a)^{1/2} \lesssim 1$ while $\dot{m}/\eta$ is likely to be bounded above by $\sim 10$, but could be considerably less. Thus, in many instances the $\sim 10\%$ addition to the time-averaged bolometric luminosity should, in fact, be modulated with the period of the sloshing, half the binary orbital period.

The sloshing may also lead to a wholly new kind of mass-transfer. In our simulation, with its unity mass-ratio, the sloshing is, on average, perfectly symmetric. However, the general case for binaries is a mass-ratio different from unity; when this is true, the sloshing is likely to lose its symmetry, so that there is a net mass-flow from one mini-disk to the other. As a result, mass that leaves the inner edge of the circumbinary disk and arrives at the outer edge of the mini-disk around, for example, the secondary, may, through the sloshing mechanism, find its way to the mini-disk
around the primary. This will be an interesting phenomenon for future simulations to explore.

3.4.3 Spiral Density Waves

Perhaps somewhat surprisingly, although spiral waves in disks within binaries were first studied in the context of cataclysmic variables (Lynden-Bell & Pringle, 1974), disks in SMBBHs whose separations are small enough to be in the relativistic regime may be the environment in which their effects are strongest. The reason is that the angular momentum transport that can be accomplished by spiral shocks increases with disk temperature, in part because a higher ratio of disk sound speed to orbital speed makes the spiral opening angle larger, leading to stronger shocks (Savonije et al., 1994; Ju et al., 2016); disks in SMBBHs are especially hot in this sense. In such disks, if the accretion rate in Eddington units $\dot{m} \gtrsim 0.01$, the local pressure is dominated by radiation out to $\sim 100M$, and the ratio of the effective sound speed (including radiation pressure) to orbital speed is $\gtrsim 0.1$ for all radii within $\sim 15(\dot{m}/0.1)M$ (Shakura & Sunyaev, 1973). Thus, it is possible that spiral shocks, driven by some combination of tidal forces and accretion stream shocks, may be of special interest in relativistic SMBBHs. We caution, however, that there are 3-dimensional effects which might complicate the situation (see Sec. 3.4.4 for further discussion of this point). We also remark that our simulations described disk thermodynamics in an extremely simplistic fashion, so the details of spiral shock behavior in them should not be taken as predictive of real systems.

The spiral shocks in relativistic SMBBHs may be enhanced in another way as well. As the binary peels streams of gas off the inner edge of the circumbinary disk, a portion of the streams falls into the central cavity and shocks against the outer edges of the mini-disks. These shocks could also contribute to spiral shocks in two ways. First, the accretion stream shocks may heat the mini-disk and therefore enhance the effectiveness of angular momentum transport in spiral shocks. The degree to which the temperature throughout the mini-disk is raised by the accretion shocks will depend on how rapidly the heat they generate is radiated. Some estimates (Roedig et al., 2014) suggest that this reradiation is quite rapid, but the immediate post-shock temperature is so high that the ultimate post-shock temperature might still exceed the temperature expected from
ordinary disk dissipation processes. Secondly, other simulations (Shi et al., 2012; Noble et al., 2012; D’Orazio et al., 2013; Farris et al., 2014) have shown that the rate at which matter is delivered from a circumbinary disk to mini-disks exhibits strong periodic modulation, with a period comparable to the binary orbital period. This periodicity, which does not affect ordinary mass transfer through Roche lobe overflow, could drive the formation of additional spiral shocks. In Chapter 4, we observe a possible coupling between the mini-disks and the circumbinary disk via accretion streams which serves to alter the spiral shock fronts.

It is also of interest that spiral shocks in relativistic SMBBHs have a different symmetry from those in Newtonian systems. Previous Newtonian simulations (see, e.g. (Savonije et al., 1994; Makita et al., 2000; Ju et al., 2016)), as well as analytic theory (Spruit et al., 1987; Savonije et al., 1994; Rafikov, 2016), predict the dominant spiral shock mode should be \( m = 2 \). So, too, does the recent work of Ryan & MacFadyen (2017), in which mini-disk dynamics driven by the BH at the center of the disk were treated in full general relativity, but the tidal field due to a BH companion was added as a perturbation. By contrast, our work found that the amplitude of surface density modulation associated with \( m = 1 \) modes was always a few times greater than that due to \( m = 2 \) modes. This discrepancy is almost certainly explained by the fact that in all these previous efforts the companion’s tidal field was described in terms of Newtonian gravity, as in Eqn. 3.13. In this context, it should be noted that the Newtonian approximation was actually appropriate to the work of Ryan & MacFadyen (2017) because the binary separation in their simulations was fixed at 1000M.

In fact, not only do the \( m = 1 \) modes dominate \( m = 2 \) for separations as large as 100M, the ratio of \( m = 1 \) to \( m = 2 \) grows as the binary separation decreases and general relativistic effects become even more prominent. The ratio might increase still further at very small separation because inspiral, whose timescale is proportional to \( a^4 \), is another source of \( m = 1 \) behavior. The inspiral rate due to the loss of energy to GW emission depends on the mass ratio of the binary, and is highest for equal-mass binaries. Therefore, the two BHs in the binary approach one another faster than their respective mini-disks; on their own, the mini-disks would inspiral much more
slowly due to less efficient GW emission. As a result, at any given time the BHs are slightly offset from the centers of their own disks, with the magnitude of this offset comparable to the orbital shrinkage during a mini-disk fluid dynamical time. Relative to the size of the mini-disk, this offset can become sizable in the late stages of inspiral:

$$\frac{\Delta x}{r_t} \sim 0.1 \frac{q^{1/2}}{(1+q)^{3/2}} (r_t/0.3a)^{1/2} (a/10M)^{-5/2}. \quad (3.18)$$

Analogs to this effect have been seen in other kinds of systems. For example, an inverse version can be observed in self-gravitating disks around single masses: in this case, the central mass is forced onto a spiral trajectory by the gravitational potential of an $m = 1$ mode in the disk (Adams et al., 1989; Heemskerk et al., 1992; Korobkin et al., 2011; Mewes et al., 2016a,b). Similarly, $m = 1$ spiral shocks can be created in accretion disks around black holes recoiling after a merger (Corrales et al., 2010; Ponce et al., 2012).

We close this section by pointing out a curious link between spiral shocks, accretion onto the BHs, and sloshing. As shown in Figure 3.14, the mass in the sloshing region is modulated at two frequencies, $2\omega_{bin}$ and $2.75\omega_{bin}$. Surprisingly, we also find (see Figure 3.17) that the accretion rate onto BH1 (and presumably its twin, BH2) is also modulated, but exclusively at frequency $2\omega_{bin}$. We emphasize that this is not the modulation associated with accretion from the inner edge of the circumbinary disk (Shi et al., 2012; Noble et al., 2012; D’Orazio et al., 2013; Farris et al., 2014); it is the accretion rate inside a mini-disk wholly isolated from

![Figure 3.17: Fourier power density of the accretion rate onto BH1 as a function of angular frequency measured in units of the binary angular frequency.](image-url)
any external circumbinary disk.

In our purely hydrodynamic simulations, the only way matter can accrete through a mini-disk is by stresses associated with spiral shocks. However, to the extent that these spiral shocks are well described by their time-averaged structure, there is no reason for them to cause any time-dependence in the accretion rate because, on average, they are stationary in the corotating frame. It is possible, though, that their strength, and therefore the accretion rate, might be modulated due to interaction between the dominant $m = 1$ and the weaker $m = 2$ mode. We further speculate, prompted by the $2\omega_{\text{bin}}$ modulation of the sloshing mass, that this periodic variation of the spiral shock amplitude is related to periodic behavior in the sloshing region. Some of the angular momentum carried outward by the spiral shocks may be given to sloshing mass; conversely, periodic motions in the sloshing region may drive corresponding changes in amplitude in the spiral shocks.

### 3.4.4 Neglected Three-Dimensional Effects

All our calculations described in this Chapter were confined to 2D dynamics in the disk equatorial plane. Real disks are, of course, 3D objects. Although many effects we investigated (e.g., the location of the disks’ outer edges) are unlikely to be substantially affected by a change in dimensionality, others are.

Two in particular are worth comment. The first is that the thickness of the accretion streams from the circumbinary disk to the mini-disks relative to the thickness of the mini-disks themselves could be an important parameter regulating how the material in these streams joins the disk and the character of waves launched into the disk by stream impact. For example, if the streams are thicker than the disks, how far inward do the shocks wrap around the disks? When the streams shock against a disk edge, do they rise in temperature sufficiently that they are always thicker than the disk? Similar questions might also apply to the sloshing streams. In addition to vertical structure of the accretion streams, it is important to note that the calculations in this Chapter neglect any inter-play between the spiral waves within the mini-disks and accretion streams from
Chapter 3. *Hydrodynamic Mini-Disks*

the circumbinary disk.

The second is that there can be significant 3D effects in the propagation of the spiral waves within the mini-disks. Lubow & Ogilvie (1998) and Ogilvie & Lubow (1999) studied this issue for the case of waves driven by tidal gravity in a binary. When the disk temperature decreases vertically away from the midplane, they found that the spiral waves are channeled toward the surface, but also that a more gradual drop in gas density at the surface can limit the concentration of the waves there. The degree of wave focusing can, of course, have significant implications for where the waves steepen into shocks and dissipate their energy.

In Chapter 4 we describe work towards addressing these questions.

3.5 Conclusions

In this Chapter, we have presented the first exploration of how mini-disks in binary systems behave when the binary separation is small enough to make general relativistic effects in the space-time, particularly regarding tidal gravity, significant.

The gravitational potential along the line between the two masses becomes shallower, and its gradient gentler, as the system becomes more relativistic. Within the secondary’s Roche lobe, this contrast between relativistic disks and Newtonian grows with mass-ratios further from unity. One result, apparent in our $q = 1$ simulations, is that, particularly as the separation becomes $\lesssim 30M$, the disks stretch toward the L1 point. The resulting asymmetry is large enough that the outer rims of disks in a relativistic binary are significantly non-circular, so that thinking in terms of a “tidal truncation radius” for such disks can be misleading.

Newtonian studies (Mayama et al., 2010; Farris et al., 2014; D’Orazio et al., 2016) had previously shown that a small fraction of the disks’ mass can be removed from the individual disks within a binary and placed in the region stretching from one disk to the other through the L1 point. A further consequence of the shallower gradient in the relativistic regime is a sharp increase in that fraction, an order of magnitude increase between binary separations of $50M$ and $20M$. At this level,
the “sloshing mass” can play a significant role in the system. For example, when the mass-ratio is not unity, asymmetry in the sloshing may create an entirely new way for mass to pass from one part of the binary system to another.

This sloshing may also result in a striking and unique electromagnetic signal of a BBH system in the period shortly before merger. In the regime of separations in which the sloshing mass is sizable, the repeated shocks it suffers may account for $\sim 10\%$ of the bolometric luminosity; in addition, in many circumstances the region in which the heat is released may be optically thin enough for its lightcurve to follow the periodic character of the heat release. For the $q = 1$ case, there are two frequency components in the modulation, one at twice the binary orbital frequency and another at $\simeq 2.75 \times$ that frequency. For binary separation $20M$, these correspond to periods $\simeq (1/2)M_6$ hr, for $M_6$ the total binary mass in units of $10^6M_\odot$.

We have also discovered that relativistic alteration of the tidal forces leads to other contrasts with Newtonian behavior. It has long been known that tidal forces can drive spiral waves in disks within binary systems; in the Newtonian limit, these have exclusively $m = 2$ character. On the other hand, even the lowest-order relativistic corrections can introduce $m = 1$ perturbations into the binary potential while also altering the $m = 2$ component. Higher-order terms such as those associated with gravitational radiation and the orbital evolution it creates can also lead to new $m = 1$ and $m = 2$ components in disk dynamics. In consequence, the $m = 1$ component can become the dominant feature even when the separation is as large as $\sim 100M$.

Whether $m = 1$ or $m = 2$, spiral shocks can supplement the angular momentum transport produced by MHD stresses. Future work will determine the degree to which this transport is altered by the change in spiral structure in the relativistic regime. This will be a topic of particular interest because the effectiveness of angular momentum transport by spiral shocks increases with the ratio between the disk matter’s sound speed and orbital speed; when the accretion rate in an SMBBH is close enough to Eddington to make relativistic regions radiation-dominated, these relativistic disks may be particularly strongly affected by such shocks.

As inspiral progresses to yet smaller separations, both of these relativistic effects are likely to
become stronger. The “softening” of the potential can provide a channel for mass-loss from the individual disks; their dissolution is likely to be further accelerated when the orbital evolution time becomes shorter than the inflow time within the disks. To determine the consequences of these processes requires further work along these lines.
CHAPTER 4

MAGNETOHYDRODYNAMIC STRUCTURE OF THE CENTRAL CAVITY

4.1 Introduction

Encouraged by the results of Chapter 3 and the rapid equilibration of our mini-disk initial data prescription, we have extended this work to full 3D GRMHD. We use HARM3D (Noble et al., 2009) to evolve the equations of GRMHD on our background spacetime (Mundim et al., 2014) (recall Chapter 2 for full details). This represents the first simulation of SMBBH mini-disks coupled to the circumbinary disk in either GR or MHD. These simulations were performed on the BlueWaters supercomputer at the National Center for SuperComputing Applications\(^1\). Our primary simulation ran for 28 days on 19,200 processors consuming a total of 12.9 million CPU hours. Even taking measures to increase the size of the adaptive timestep, this run required just over $2.1 \times 10^6$ timestep iterations on a grid of $6.144 \times 10^7$ cells to evolve for two binary orbital periods. Given the immense computational costs of such a simulation, we limit our study to equal-mass, nonspinning BHs inspiraling from a binary separation of $20M$. We select this binary separation to further investigate disk dynamics resulting from relativistic contributions to the gravitational field presented in Chapter

\(^1\)www.ncsa.illinois.edu/enabling/bluewaters
3. We perform two evolutions with differently sized cutouts at the origin, but otherwise identical parameters to verify our results are not contaminated (see Section 4.4.2). Future works will move towards increased computational efficiency and including the center-of-mass in the computational domain (see Section 5.2 for more details).

Although we are unable to perform a large suite of binary separations, this extension incorporates previously neglected effects such as; continuing accretion from the circumbinary disk, vertical disk structure, and magnetic fields. Simulations such as the ones presented in this Chapter are capable of making direct predictions of the electromagnetic output of inspiraling SMBBHs, a necessary step for the direct observation of SMBBHs. Additionally, being the first simulation of its kind, it represents a significant milestone in the progression towards the goal of performing 3D GRMHD simulations of the entire inspiral to merger of BBHs with accretion disks and making predictions of the electromagnetic output.

4.2 Simulation Details

4.2.1 Magnetohydrodynamic Prescription

For completeness, we restate the equations of GRMHD which amount to conservation of baryon number density, conservation of stress-energy, the Maxwell induction equations, and divergence free constraint on the magnetic field (see Noble et al. (2009) for more details). Taken together, they can be written as

$$\partial_t U(P) = -\partial_i F^i(P) + S(P),$$

(4.1)
where $\mathbf{P}$ are the “primitive” variables, $\mathbf{U}$ the “conserved” variables, $\mathbf{F}^i$ the fluxes, and $\mathbf{S}$ the source terms. In terms of the primitive variables and metric functions they can be expressed as

$$
\mathbf{U}(\mathbf{P}) = \sqrt{-g} \left[ \rho u^i, T^i_t + \rho u^i, T^i_j, B^k \right]^T,
$$

(4.2)

$$
\mathbf{F}^i(\mathbf{P}) = \sqrt{-g} \left[ \rho u^i, T^i_t + \rho u^i, T^i_j, \left( b^i u^k - b^k u^i \right) \right]^T,
$$

(4.3)

$$
\mathbf{S}(\mathbf{P}) = \sqrt{-g} \left[ 0, T^\kappa \Lambda_{\lambda t \kappa} - \mathcal{F}_t, T^\kappa \Lambda_{\lambda j \kappa} - \mathcal{F}_j, 0 \right]^T,
$$

(4.4)

where $g$ is the determinant of the metric, $\Gamma^\lambda_{\alpha \beta}$ are the Christoffel symbols, $b^\alpha = \left( \frac{1}{u^t} \right) \left( \delta^\alpha_{\nu} + u^\alpha u_{\nu} \right) B^\nu$ is the magnetic 4-vector projected into the fluid’s comoving reference frame, and $u^\alpha$ are the components of the fluid’s 4-velocity. The stress-energy tensor can be written as

$$
T_{\alpha \beta} = \left( \rho h + 2p_m \right) u_\alpha u_\beta + \left( p + p_m \right) g_{\alpha \beta} - b_\alpha b_\beta,
$$

(4.5)

where $h = 1 + \epsilon + p/\rho$ is the specific enthalpy, $\epsilon$ is the specific internal energy, $p$ is the gas pressure, $p_m = \frac{1}{2} b^2$ is the magnetic pressure, and $\rho$ is the rest-mass density. We ensure that the constraint, $\partial_t \sqrt{-g} B^i = 0$, is maintained throughout the simulation by use of FluxCT (Tóth, 2000). The gas’s thermodynamics are governed by an adiabatic equation of state with index $\Gamma = 5/3$ and local cooling as described in Section 2.2.

### 4.2.2 Initialization

Here we present a detailed outline of how our MHD simulation is initialized. Before presenting the full discussion below, we provide a short outline of the general steps for reader clarity. We note that in principal the number of required divergence cleaning steps could be reduced in future runs by swapping the ordering of items 2 and 3 below.

1. We initialize the circumbinary disk with the equilibrated state of Noble et al. (2012). As this evolution was done on a different grid, we interpolate the equilibrated circumbinary onto the actual grid used in our simulations.
2. The interpolation step introduces magnetic monopoles. We therefore perform divergence cleaning on the interpolated magnetic field to remove these monopoles.

3. Because the self-consistent formation of the mini-disks would require prohibitively long simulations, we superimpose our mini-disks initial data for $r < a_0$.

4. This introduces a ring of monopoles at $r = a_0$. We therefore repeat the divergence cleaning to remove any remaining monopoles.

As previously mentioned, the computational burden of forming the mini-disks naturally in GRMHD is prohibitively high. For this reason, we make use of the mini-disk initial data prescribed in Section 2.3 for the initial state of the central cavity. This is done in exactly the same fashion as the 2D hydrodynamic simulations, except that we include solutions above and below the equatorial plane and seed an initially poloidal magnetic field. In Figure 4.1 we include an equatorial and poloidal slice of the initial density profiles. For the mini-disks in our MHD runs we set $r_m = 3.1M$, 

Figure 4.1: (Left) Equatorial slice of the initial density profile on a logarithmic scale. The domain encompasses the innermost $10a_0 \times 10a_0$. (Right) Poloidal slice of the initial density profile on a logarithmic scale at $\phi = 0$. Radial domain extends to $5a_0$ and the z-axis ranges from $-2.5a_0$ to $2.5a_0$. 

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$r_{p_{\text{max}}} = 4.6M$, $\beta = 0.01$, and $H/r = 0.09$ at the pressure maximum. The initial entropy of the mini-disks is set to 0.01. These values ensure that the inner edge of the mini-disk is exterior to the ISCO and the bulk of the mini-disk is contained within the tidal truncation radius. The values of $H/r$, entropy, and $\beta$ are set so that the thermodynamic and magnetic properties of the mini-disks resemble that of the circumbinary accretion disk.

We must also prescribe the circumbinary accretion disk in order to address accretion through streams feeding into the central cavity. In order to equilibrate the circumbinary accretion disk one must evolve the circumbinary at a fixed binary separation for many binary orbital periods. This is problematic for multiple reasons. Firstly, the approximately hydrostationary mini-disks we construct are sensitive to the BH trajectories, principally due to the coordinate transformation relating $X_{PNH}$ and $X_{CS}$ which encodes the boost between the comoving BL and center-of-mass frames. This means we cannot initialize and evolve at a fixed separation. This is because no physically consistent BH trajectory for fixed separation in GR exists at 3.5PN-order. In practice, Noble et al. (2012) equilibrated the circumbinary disk by allowing the BHs to inspiral slightly and then instantaneously setting the separation back to the initial value. While this approximation is fine for larger distances from the BHs in the circumbinary, the boost used in the mini-disk construction will be incorrect. Additionally, the timestep of the evolution in spherical grids is determined by the smallest cells near the inner cutout and scales linearly in radius. By setting the inner boundary condition at 0.75$a$, Noble et al. (2012) was able to take significantly larger timesteps with fewer cells than our simulations which sets the inner boundary near the coordinate origin. Finally, in the circumbinary disk only the NZ metric is necessary. By including the BHs in our computational domain, we must include the full matching of the NZ-IZ metrics. The calculation of this fully matched spacetime accounts for roughly half our computational expense. We therefore elect to start with a previously equilibrated state of the circumbinary disk from Noble et al. (2012) to limit the computational expense of equilibration (recall that the mini-disk initial data equilibrates quite rapidly).
Because Noble et al. (2012) evolved in a different grid discretization, we must interpolate the equilibrated circumbinary disk onto our grid. To do this, we read in the physical PNH destination coordinates \((x_{\text{dest}})\) of the warped grid and transform to the physical \((x_{\text{src}})\) and numeric \((x'_{\text{src}})\) representations of the source grid of Noble et al. (2012). We then trilinearly interpolate in the \((x'_{\text{src}})\) coordinate basis using a box of lengths equal to the numerical spacing \((dx'_{\text{src}})\). We initialize the domain to a floor state in regions where no data exists.

The approximate, interpolated values introduce some error into the initial state of order the spacing of our interpolation stencil \(dx'_{\text{src}}\). While this error is acceptable for the hydrodynamic quantities, it leads to an increase in the divergence of the magnetic field. Additionally, such violations to this constraint have characteristic velocities of zero in FluxCT. This means that once a monopole is created, it cannot propagate out of the domain and will remain for the length of the simulations.

Such constraint violations can be cleaned using a projection method (Chorin, 1968; Brackbill & Barnes, 1980; Bell et al., 1989; Noble & Zilhão, 2016). We define a new magnetic field from the interpolated field \(B\) as

\[
B' = B - \nabla \psi.
\]  
(4.6)

Taking the divergence, it trivially follows that the new field will be divergenceless if

\[
\nabla^2 \psi = \nabla \cdot B,
\]  
(4.7)
Figure 4.2: Logarithmic scale plots of the cleaning target in Equation 4.8 over the full extent of the domain in the (Left) equatorial plane and (Right) a poloidal cut along $\phi = 0$. (Top) We plot shortly into the cleaning process once $\approx 9.90\%$ of the domain is cleaned. (Bottom) We plot the final state of our simulation after all divergence cleaning with $\approx 98.81\%$ of the domain cleaned. We note that the color scale changes to emphasize the structure of the remaining error.
where the operator $\nabla$ is defined consistently with the operator used in the evolution ($\nabla \cdot \mathbf{B} = \partial_i \sqrt{-g} B^i$). We iteratively solve for the new magnetic field $\mathbf{B}'$ until we reach a target of

$$
\min \left( dx'' \right) \frac{\nabla \cdot \mathbf{B}'}{b^2} \leq 10^{-3}.
$$

In Figure 4.2 we plot snapshots for before and after the data has been cleaned in the entire computational domain. We reach our target in 98.81% of the cells in the computational domain with regions of largest violations concentrated near values of small $b^2$.

After interpolating the primitive variables from the circumbinary data and cleaning, we superimpose the mini-disk initial data solutions of Section 2.3. We replace all circumbinary data radially interior to the binary separation. This ensures that we do not have material from the circumbinary in the immediate vicinity of the mini-disks at the initial time slice. Finally, we reclean any divergence introduced by replacing circumbinary data with mini-disk and floor states.

### 4.2.3 Grid and Boundary Conditions

Our MHD simulations are also performed in the dynamic, double fish-eye (warped) spherical coordinate system whose origin is at the binary center-of-mass (Zilhão & Noble, 2014). Cells within this coordinate system are spaced uniformly in numerical spatial coordinates. By this means, we are able to focus resolution in the vicinity of the BHs and rarefy resolution in the dynamically less interesting portions of the cavity. Near the BHs the grid is approximately Cartesian in the equatorial plane, while farther out the grid is spherical. Poloidal cells are focused near the equatorial plane, limiting the number of necessary cells to resolve the vertical extent of the disks. In Figure 4.3 we plot an equatorial and poloidal slice of the grid.

We again require approximately 32 cells span each black hole horizon in each dimension, a resolution chosen to ensure that the near-horizon spacetime is well resolved. In order to include the circumbinary disk, we extend the radial domain out to $13a_0$. To achieve a smooth transition from nearly-Cartesian cells near the BH horizons to nearly-spherical cells far from the BHs and
ensure sufficient resolution in the mini-disks we altered the parameters of the grid from those presented in Chapter 3. The values we used are stated in Table 4.1. Quantitative expressions for our transformations and the parameter definitions may be found in Eqs. (29-32) of Zilhão & Noble (2014). For convenience, we also include in Table 4.1 the number of cells within the equatorial Newtonian tidal truncation radius for each binary separation. We evolve with the same cell count in the equatorial plane as the 2D hydrodynamic 20$M$ run. The number of poloidal cells is chosen to ensure a minimum of 16 cells across the vertical extent of the disk at the initial pressure maximum of the mini-disk on the side opposite the center-of-mass; this is necessary to resolve the MRI (see the appendix B of Noble et al. (2012) for more details).

We consistently impose outflow boundary conditions at the radial $x^1$ boundaries, requiring $u^r$ to be oriented out of the domain there. If at any time the velocity points inward, the value is
Table 4.1. Warped Grid Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_{x1}$</td>
<td>0.2</td>
</tr>
<tr>
<td>$\delta_{x2}$</td>
<td>0.2</td>
</tr>
<tr>
<td>$\delta_{x3}$</td>
<td>0.1</td>
</tr>
<tr>
<td>$\delta_{x4}$</td>
<td>0.1</td>
</tr>
<tr>
<td>$\delta_{y3}$</td>
<td>0.15</td>
</tr>
<tr>
<td>$\delta_{y4}$</td>
<td>0.15</td>
</tr>
<tr>
<td>$\delta_z$</td>
<td>0.4</td>
</tr>
<tr>
<td>$a_{x1}$</td>
<td>4.0</td>
</tr>
<tr>
<td>$a_{x2}$</td>
<td>4.0</td>
</tr>
<tr>
<td>$a_z$</td>
<td>4.3</td>
</tr>
<tr>
<td>$h_{x1}$</td>
<td>20.</td>
</tr>
<tr>
<td>$h_{x2}$</td>
<td>20.</td>
</tr>
<tr>
<td>$h_{x3}$</td>
<td>20.</td>
</tr>
<tr>
<td>$h_{x4}$</td>
<td>20.</td>
</tr>
<tr>
<td>$h_{y3}$</td>
<td>10.</td>
</tr>
<tr>
<td>$h_{y4}$</td>
<td>10.</td>
</tr>
<tr>
<td>$h_z$</td>
<td>20.</td>
</tr>
<tr>
<td>$s_1$</td>
<td>0.01</td>
</tr>
<tr>
<td>$s_2$</td>
<td>0.01</td>
</tr>
<tr>
<td>$s_3$</td>
<td>0.01</td>
</tr>
<tr>
<td>$b_1$</td>
<td>15.</td>
</tr>
<tr>
<td>$b_2$</td>
<td>15.</td>
</tr>
<tr>
<td>$b_3$</td>
<td>15.</td>
</tr>
<tr>
<td>$R_{out}$</td>
<td>260$M$</td>
</tr>
<tr>
<td>Cell Count</td>
<td>600 X 160 X 640</td>
</tr>
<tr>
<td>Cells per Mini-Disk ($\theta = \pi/2$)</td>
<td>44,552</td>
</tr>
</tbody>
</table>

Note. — Parameters of the warped grid used for the MHD simulations. Please see Eqs. (29-32) of Zilhão & Noble (2014) for the explicit expressions defining the warped system and the significance of these parameters. The inner radial cutout is set to 1$M$. 
set to zero, and we recalculate the primitive velocity. We apply reflective, axisymmetric boundary conditions in the poloidal $x^2$ direction. Finally, we apply periodic boundary conditions in the azimuthal $x^3$ direction and cover all $2\pi$.

The grid extends down to a sphere of radius $M$. However, the timestep of evolutions on this grid were of order $2 \times 10^{-4}M$. This makes performing multiple binary orbits of evolution computationally too expensive for our allocation as well as infeasible in terms of walltime. As the timestep of the evolution scales linearly in the radial location of the inner boundary, we extend the inner boundary to $2M$. To avoid having to reinterpolate and clean the data onto a new grid with the inner boundary at $2M$, we define an excision mask for a sphere of radius $2M$. Even with the increased cutout our evolution required over 2 million timesteps. Cells contained within this mask are not evolved and effectively act as additional ghost cells for unmasked cells exterior to the mask. We apply our radial boundary conditions at the boundary of the mask by performing a check on $u^r$ and a zeroth order radial interpolation of the boundary values for all cells inside the mask.

### 4.2.4 Ray-Tracing

To determine the electromagnetic emission of our system one would ideally wish to couple the equations of GRMHD to the equations of radiative transfer during the simulation. However, such physics is not currently implemented into our version of HARM3D. Instead, we post-process the simulation data; solving the equations of radiative transfer using the cooling function as a proxy for the bolometric luminosity. This is performed in BOTHROS (Noble et al., 2007, 2009, 2011; d’Ascoli et al., 2017), which solves the Lorentz-invariant radiative transfer equation for photons moving along geodesics through the simulation data,

\[
\partial_\lambda I = j - \alpha^a I \quad (4.9)
\]
\[
\partial_\lambda x^\mu = k^\mu \quad (4.10)
\]
\[
\partial_\lambda k_\mu = \Gamma^\kappa_{\mu\eta}k_\nu k^\eta. \quad (4.11)
\]
Here $k^\mu$ is the photon’s 4-momentum, $\lambda$ the affine parameter, $I$ the intensity, $j$ the emissivity, and $\alpha^a$ the absorption coefficient. We select $\alpha^a$ instead of the standard $\alpha$ to differentiate from the common $\alpha$ parameter of Shakura & Sunyaev (1973).

Photons are launched from a pinhole camera at some distance from the center-of-mass, $r_{\text{cam}}$, in unique directions; each direction corresponds to an individual “pixel” of the camera image. The geodesics are calculated in the physical $X_{P\text{NHC}}$ coordinate system of our analytic spacetime (Mundim et al., 2014) using a 5th order Cash-Karp algorithm (Press et al., 1992). The Cash-Karp algorithm was implemented to improve the stability of the geodesic calculations in BOTHROS (d’Ascoli et al., 2017). We integrate the photons in a “frozen” spacetime for each snapshot of the simulation data; thereby assuming that the spacetime does not evolve appreciably over the timescale of a particular geodesic calculation. Lifting this assumption is left to future studies. To solve the radiative transfer equation, we must numerically calculate $X_{WARP}(X_{P\text{NHC}})$ and interpolate the simulation data at each point along the geodesic. For the purposes of our calculations, we read and interpolate the density $\rho$, primitive velocities $v^i$, and cooling function $L_c$. In terms of physical interpretation; the density of the gas determines the absorption, the gas velocities determines the gravitational redshift, and the cooling function determines the emissivity of the data at a given point.

4.3 Preliminary Results

4.3.1 Overview

Our simulation selects a single binary separation ($a \approx 20M$), well within the relativistic inspiral regime, where binary inspiral and relativistic contributions to the gravitational field are significant (recall Section 3.3.2). We include the full vertical structure of the mini-disks, seed a poloidal magnetic field, and couple the mini-disks to the inner edge of the circumbinary. By lifting the restriction of 2D fluid flows and including MHD turbulence, we observe an increased rate of equilibration by nearly an entire binary orbital period over Chapter 3.
Initially, there is a sharp transition from the interpolated and cleaned circumbinary data to the mini-disks and floor state at $r = a_0$; this is particularly sharp in the magnetic field lines which were truncated at $r = a_0$ and altered by the cleaner to stitch together with the floor state of the cavity. In Figure 4.4, we plot the density out to the pressure maximum of the circumbinary, $r = 5a_0$, for the full length of the simulation at a sample rate of $0.25t_{\text{bin}}$. To highlight the mini-disks, we reproduce the snapshots at $t = t_{\text{bin}}, 1.5t_{\text{bin}},$ and $2t_{\text{bin}}$ out to $r = 1.5a_0$ in Figure 4.5. Several features are quickly apparent from the snapshots. By the first snapshot at $0.25t_{\text{bin}}$, the non-mini-disk portions of the cavity have already equilibrated to the quasi-steady state solution with the streams impacting the mini-disks near the L2/L3 points and relatively evacuated regions at the L4/L5 points. The streams enter the circumbinary in the form of turbulent, $m = 2$ spiral arms which are extensions of the
Chapter 4. Magnetohydrodynamic Structure of the Central Cavity

Figure 4.5: (Left to Right) Logarithmic scale density in the equatorial plane at $t \approx t_{\text{bin}}$, 1.5$t_{\text{bin}}$, and 2$t_{\text{bin}}$ for the inner 1.5$a_0$ of the simulation domain.

$m = 2$ spiral patterns within the inner edge of the circumbinary.

We find that the mini-disks undergo an expansion due to the departures from hydrostatic equilibrium in initially neglecting the presence of the binary companion. The peak of this expansion occurs at $t \approx 0.5t_{\text{bin}}$, with material extending all the way to the central cutout. By $t \approx t_{\text{bin}}$, we find that the mini-disks have been tidally truncated and enter a quasi-equilibrium state. Additionally, we note that while the mini-disks are initially significantly denser than the circumbinary disk ($\mathcal{O}(10^{1.5} - 10^2)$), much of the tidally truncated gas which would have passed through the sloshing region and resettled onto a mini-disk is lost to the central cutout or accreted onto the BHs. The resulting mini-disks are less dense than their initial values by a factor of approximately $\mathcal{O}(10)$. The quasi-equilibrium solution of the mini-disks has a density $\mathcal{O}(10^{0.5} - 10^1)$ larger than that at the inner edge of the circumbinary and streams.

We observe the formation of spiral density waves within the mini-disks. Although MHD turbulence can disrupt such density waves, as happens in the circumbinary disk, the spiral patterns persist through the entirety of the simulation. As was predicted by Bowen et al. (2017), we observe that the spiral density waves contain a strong $m = 1$ component, with an over-density near the L1 point of the binary. Strikingly, the structure of the spiral density waves in each mini-disk during
the quasi-steady state take different forms. We attribute this to a coupling to the circumbinary disk via the accretion streams. We explore this in more detail in Sections 4.3.2 and 4.4.1.

In addition to rapid equilibration and spiral density waves, we observe evidence for the quasi-periodic sloshing reported in Chapter 3. This can be seen most directly by sloshing streams leaving the edges of the mini-disk, sometimes grazing just above and below the central cutout and \((t/t_{bin} \approx 1, 1.5, 2)\) and other times extending directly towards the central cutout \((t/t_{bin} \approx 1.25)\). Unfortunately, these streams, particularly when they would collapse to a single stream connecting the mini-disks, are enveloped by the central cutout (recall that the cutout size here is twice that of Chapter 3).

Figure 4.6: (Left to Right and Top to Bottom) Snapshots of logarithmic scale plasma \(\beta\) in the equatorial plane for the innermost \(3a_0 \times 3a_0\) of the domain. The time sampling is the same as in Figure 4.4.

During the mini-disk expansion period, fluid elements within the mini-disk are largely perturbed from circular orbits. This excites the MRI, which is known to saturate on a timescale related to
the local orbital frequency. By comparing the density profiles of the first row to their counterparts in the second binary orbital period, we observe increased development of MHD turbulence. In Figure 4.6 we demonstrate the growth in magnetization and turbulence within the mini-disks. We plot equatorial slices of the plasma \( \beta = p/p_m \) parameter (see Section 4.3.2 for further details) for the inner 1.5\( a_0 \) of the domain at the same frequency rate as the density. Initially, the entirety of the central cavity is off-scale (\( \log_{10} \beta > 1.5 \)). We find that within \( \approx 0.5t_{bin} \) MHD turbulence has developed within the mini-disks. We observe that the value of \( \beta \) within the mini-disks decreases, i.e. becoming more magnetized, as the simulation progresses. Finally, we note that the \( \beta \) of the mini-disks, as with the density, forms into spiral waves.

### 4.3.2 Quasi-Steady State

#### Density Distribution

One of our primary goals is to see whether the relativistic effects on the mini-disk structure demonstrated in Chapter 3 are altered by the inclusion of vertical structure, accretion from the circumbinary, or magnetic fields. To study the quasi-steady state of the disks, we elect to time average our data in a frame corotating with the binary. Although this averaging washes out much of the turbulent appearance of the disk, it emphasizes the time-independent structure. In order to ensure that initial transients do not contaminate our results, we begin our time averaging procedure at \( t \approx t_{bin} \). This time was selected to allow for initial transient behavior to decay away while still capturing a full orbital period of the evolution.

In Figure 4.7, we plot the time-averaged density contours both on a linear and logarithmic scale in the equatorial plane of the binary. On the linear scale a few things are quickly apparent. We see an over-density in the mini-disks on the side nearest the L1 point relative to the opposite side of the mini-disk. This over-density appears to be part of a strong spiral density wave within the mini-disks continuing down to the central BH. At the edge of these over-densities, we observe that the mini-disk is tidally truncated. More strikingly, we observe that the mini-disks, despite having the same initial conditions and the \( q = 1 \) symmetry of the system, are asymmetric with respect to
Figure 4.7: Time-averaged density contours in the equatorial plane on (Top) linear and (Bottom) logarithmic scales plotted over the innermost (Left) \(10a_0 \times 10a_0\) and (Right) \(3a_0 \times 3a_0\) of the computational domain. The BHs and central cutout at the coordinate origin are denoted by black circles.

one another. While both mini-disks have a clear \(m = 1\) component of the spiral density wave in the equatorial plane, we observe the opening angles of the waves are qualitatively different. More
specifically, the spiral density wave in the disk around BH1 tightly winds back around to the $\phi = 0$ line. The spiral density wave in the disk around BH2 on the other hand appears much more openly wound, terminating at the bottom of the disk in the figure. We discuss this asymmetry in more detail in Section 4.4.1.

The logarithmic scale highlights the impact of the gravitational potential on the overall structure within the central cavity and just beyond. The logarithmic density contours closely resemble contours of the effective potential, with evacuated regions at the L4 and L5 points. The streams appear to impact the mini-disks on the sides nearest the L2 and L3 points and material is again found to be in the sloshing region between the two BHs; though the quantity is likely greatly diminished due to the increased size of the central cutout. We observe that the streams accreting onto the mini-disks form in the shape of fixed, $m = 2$ spirals in the frame corotating with the binary due to the quadrupolar nature of the gravitational field. Furthermore, these $m = 2$ spiral streams appear to be extensions of spiral density waves present within the circumbinary disk out to radii of approximately $3a$. Beyond this radius the spiral density waves are disrupted by the magnetic turbulence of the circumbinary disk (Noble et al., 2012). Finally, we note an $m = 1$ asymmetry in the streams that we attribute to an $m = 1$ mode present in the circumbinary disk. The stream impacting the mini-disk around BH1 (Right) appears as one continuous band extending from the edge of the circumbinary disk. The spiral arm giving rise to the stream impacting the mini-disk around BH2 on the other hand appears to split at the edge of the circumbinary.

In Figure 4.8 we extend the time-averaged density plots to include the fluid velocity field in a frame corotating with the binary. Several things are quickly apparent from this figure. First, as previously inferred from Figure 4.7, the accretion streams are nearly fixed patterns in the corotating frame. Second, the accretion stream asymmetry translates into different fluid flows near the outer edges of the mini-disks. The stream accreting onto the mini-disk around BH2 has a more radial flow towards the mini-disk than the stream joining the mini-disk around BH1 which appears to have a nearly azimuthal trajectory around the mini-disk. The asymmetry in the mass flux of these accretion streams near the mini-disks is discussed in more detail in Section 4.4.1.
Finally, this figure demonstrates the regions where the streams are peeled off the inner edge of the circumbinary as the fluid flow gains radial components to the flow until reaching the nearly stationary streams. This is in contrast to the orbiting flow of the circumbinary disk near the edges of the figure.

To further explore the structure of the quasi-steady state evolution, we turn our attention to Figure 4.9, where we plot poloidal slices along the line connecting the BHs at $\phi_{PNH} = 0$ and $\phi_{PNH} = \pi$. Here we observe that the $m = 1$ structure in the equatorial slices is not confined to that plane. In both mini-disks, we find that the side nearest the L1 point appears to have a more lobe-like vertical structure at smaller radii than the side nearest the L2/L3 point. The side furthest from the binary companion appears thinner and radially elongated. This is consistent with previous simulations of disks with $m = 1$ modes around single BHs (Adams et al., 1989; Heemskerk et al., 1992; Korobkin et al., 2011; Mewes et al., 2016a,b). We observe diffuse vertical structure near the L2/L3 point where streams continually impact the edge of the disk. In both mini-disks, the high density regions extend all the way down to the central BH, with the bulk of the accretion occurring near the equatorial
plane. In this slice, we again observe that the mini-disks are tidally truncated on the side nearest the binary companion. We close by noting that the overall density in the mini-disk around BH1 appears systematically higher than that of the mini-disk around BH2.

Figure 4.9: Poloidal slices \((r, z)\) of the time-averaged density distribution at \(\phi = 0\) (left) and \(\phi = \pi\) (right) on linear (top) and logarithmic (bottom) scales. In both cases, the L1 point is at \((0, 0)\) and the binary companion is off frame to the left. The BH and central cutout at the coordinate origin are both denoted by black circles.

Magnetization

Our mini-disks are initialized with a poloidal magnetic field. As the fluid evolves, the fluid elements are perturbed from circular orbits around their respective BH. Because our disks are differentially rotating and satisfy the MRI instability criteria (Balbus & Hawley, 1991, 1998)

\[
\frac{d\Omega^2}{d \ln r_i} < 0,
\]  

(4.12)
a portion of the radial field is turned into a toroidal field creating a laminar Maxwell stress (Noble et al., 2012). The MRI grows on a rate determined by the local orbital frequency of the fluid (Balbus & Hawley, 1991, 1998) until saturating, further enhancing this stress. It is this Maxwell stress which drives angular momentum transport within astrophysical accretion disks.

When speaking of the relative importance of magnetic effects, it is useful to refer to the dimensionless plasma \( \beta = p/p_m \) parameter. In other words, the lower the value of \( \beta \), the more magnetized the fluid. If the fluid is orbiting a central mass, \( \beta \) effectively measures the ratio of gas pressure to Maxwell stress. This is because, at least in the potential of a point source, the Maxwell stress is linearly proportional to the magnetic pressure (Hawley et al., 2011). While the gravitational potential the mini-disks reside in is certainly not a point mass, we nonetheless find this a useful quantity to probe magnetization and note that the binary companion enters the potential at the perturbative level within the tidal truncation radii.

In Figures 4.10 and 4.11 we plot the time-averaged values for \( \beta \) in the equatorial plane, and in poloidal slices at \( \phi_{PNH} = 0 \) and \( \phi_{PNH} = \pi \). We find that \( \beta \approx 1 - 10 \) within the equatorial plane in the mini-disks and streams, consistent with values observed in the circumbinary by Noble et al. (2012). Observing the poloidal slices, we find that the values of \( \beta \) decrease as moving above and below the equatorial plane by roughly an order of magnitude. Consistent with the density profiles, we find that the magnetization is also different between the two mini-disks. More specifically, we find lower values of \( \beta \) (more magnetized) in the mini-disk around BH2. We extend the radial extent of the poloidal slices from those in the density plots to high-

![Figure 4.10: Equatorial slice of plasma \( \beta \) for the innermost \( 3a_0 \times 3a_0 \) of the domain on a logarithmic scale.](image-url)
light the turbulent, magnetized character of the fluid in the region where the streams impact the mini-disks.

![Poloidal slices of plasma $\beta$ for BH1 (left) and BH2 (right) along the line connecting the BHs on a logarithmic scale.](image)

Figure 4.11: **Poloidal slices of plasma $\beta$ for BH1 (left) and BH2 (right) along the line connecting the BHs on a logarithmic scale.** In both frames the center-of-mass is located at $(0,0)$, the binary companion off the figure to the left, and the circumbinary off the figure to the right.

### 4.3.3 Electromagnetic Emission

We begin by stating that the results presented in this section have been generated in collaboration with and with significant effort by Stéphane d’Ascoli and will be described in full detail in an upcoming paper (d’Ascoli et al., 2017). We shoot photons through our simulation data from a distance of $r_{\text{cam}} = 1000M$. This value was chosen to ensure that the camera is always outside the simulation domain, but sufficiently close to not violate our frozen spacetime approximation. The electromagnetic emission scales with the total mass of the binary and the accretion rate. We scale our binary to a total BH mass of $10^6M_\odot$. We consider the cases of sub-Eddington accretion $\dot{m}/\dot{m}_{\text{Edd}} = 0.1$ and super-Eddington accretion $\dot{m}/\dot{m}_{\text{Edd}} \approx 5$, where $\dot{m}_{\text{Edd}} = 1.2 \times 10^{38}M_\odot/c^2$. We scale the simulation units to physical units using the accretion rate from the circumbinary into the central cavity. This assumes that the mini-disks are in inflow equilibrium, which is likely not the case (see Section 4.4.3 for more details). These accretion rates were chosen to explore possible extreme cases. Our super-Eddington accretion rate allows us to emphasize the bifurcation between optically thick and optically thin regions (see below), while the relatively lower sub-Eddington
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 accretion allows us to explore the dependence on viewing angle.

The optical depth along the line-of-sight is calculated along the geodesic as

\[ d\tau = \alpha a d\lambda, \]  \hspace{1cm} (4.13)

where the absorption coefficient at a given frequency is

\[ \alpha a = \frac{\alpha a}{\nu} = \left( \frac{\sigma T}{m_H} \right) \rho. \]  \hspace{1cm} (4.14)

\( \sigma T \) and \( m_H \) are the Thomson scattering cross section and mass of a hydrogen atom respectively.

Super-Eddington Accretion

![Figure 4.12: Total integrated logscale optical depth along the photon line-of-sight for the super-Eddington accretion rate at angles of 0 degrees. (Left to Right) Times are sampled at 1030\( M \), 1080\( M \), 1130\( M \), and 1180\( M \) covering 0.25\( t_{\text{bin}} \).](image)

In Figure 4.12 we plot the total, integrated optical depth over a quarter binary orbit in the quasi-steady state as viewed face-on and (Figure 4.13) nearly edge-on using the super-Eddington accretion rate. Viewing the binary face-on, we observe that the system bifurcates into optically thick regions within the mini-disks, streams, and circumbinary disk, and optically thin regions in the cavities near the L4/L5 points. In practice, even in the regions where \( \tau \gg 1 \), there exists a
Figure 4.13: **Total integrated logscale optical depth along the photon line-of-sight for the super-Eddington accretion rate at an angle 80 degrees.** (Left to Right) Times are sampled at 1030$M$, 1080$M$, 1130$M$, and 1180$M$ covering 0.25$\theta_{\text{bin}}$.

point in the vertical structure of the disk where the optical depth transitions from optically thin to optically thick. The point at which this occurs defines the photosphere.

To calculate the spectrum of our system we use two-emission mechanisms. We assume that the emission coming from within the photosphere can be expressed as a black body,

$$I_\nu = \frac{2h\nu^3}{c^2} \left( \exp \left( \frac{h\nu}{kT} \right) - 1 \right)^{-1},$$  

with an effective temperature ($T$) defined at the surface of the photosphere. We can naturally define the photosphere for the face-on case as the point where $\tau \equiv 1$ on a geodesic. This is because the photon geodesics move predominantly in the $\hat{z}$ direction and effectively move through the height of each individual region. We prescribe the effective temperature as the total integrated cooling function along the geodesic inside the photosphere,

$$T^4 = \frac{\int_{\tau>1} \mathcal{L}_\nu d\lambda}{2\sigma},$$  

where $\sigma$ is the Stefan-Boltzmann constant.

Given the intensity at the photosphere edge, we integrate the radiative transfer equations along
the remaining portion of the geodesic towards the camera. In this optically thin region we assume a Wien spectrum (inverse Compton scattering) (Krolik, 1999a; Roedig et al., 2014)

\[ j_\nu \propto \frac{2h}{c^2} \nu^3 e^{-\frac{h\nu}{\Theta}}, \]  

where we solve for the proportionality constant by demanding \( L_c = \int j_\nu d\nu d\Omega \) and \( \Theta = 100\text{keV} \).

In Figure 4.14 we plot the spectrum of the time-averaged state of our simulation using this model. We isolate emission due to the mini-disks and sloshing region by calculating emission coming from \( r < a \) and \( r > a \) independently. Several features can be gleaned from this Figure. First, for energies greater than 0.1keV, we find that the flux received at the camera is nearly entirely emanating from the mini-disks. Secondly, we note that there is a turning point at \( \approx 0.04\text{keV} \), under which, the circumbinary/streams are the dominant source of electromagnetic emission. We observe a significant drop off in the circumbinary/stream emission around \( \approx 0.06\text{keV} \), before picking back up and dropping off again around 1000keV. This shape is due to the two separate emission models, where the lower energy emission is thermal and the higher energy spectrum is inverse Compton scattering. The mini-disk spectrum has roughly the same shape, except that the low-energy drop off is pushed to \( \approx 0.2\text{keV} \). This shift is due to the increased temperature of the mini-disk photosphere relative to the circumbinary photosphere. Finally, we note a significant local minimum in both spectra in the tenths of keV.

Figure 4.14: Flux density for super-Eddington accretion plotted against photon energy in units of keV. We plot the contributions to the total flux from the regions interior and exterior to \( r = a \) in addition to the total flux density from the entire simulation domain. As the units of flux density here are arbitrary, we normalize to the peak value of the combined flux density.
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We turn our attention to the plots of total optical depth when viewing edge-on in Figure 4.12, where relativistic effects are amplified. We observe strong gravitational lensing when the BHs eclipse one another, producing an Einstein ring. Additionally, we note the presence of special relativistic beaming due to the kinematic motion of the BHs as the observed size of the BH changes based off whether it is moving towards or away from the camera. Noticeably, we see the breakdown of our photosphere model. Chiefly, the bifurcation in optically thick versus optical thin regions does not occur (evacuated regions still appear optically thick along the line-of-sight). This is because photons must travel through the circumbinary on their way to the mini-disks, guaranteeing that all geodesics become optically thick independent of the photosphere structure (see Section 4.4.3 for more discussion).

**Sub-Eddington Accretion**

Given the breakdown of the photosphere calculation presented for the super-Eddington accretion rates, we must consider a simpler emission model to study how the viewing angle alters the electromagnetic emission. We find that by lowering the accretion rate to ten percent of the Eddington accretion rate, the entire geodesic remains optically thin. We may therefore use the inverse Compton scattering model and radiative transfer along the entire geodesic. In Figure 4.15 we calculate the spectrum viewing the binary face-on, again isolating flux emanating from the mini-disks. We observe that when the accretion rate is diminished and we neglect the presence of the photosphere, the total flux comes entirely from the mini-disks for energies up to $\approx 1000\text{keV}$. We again find a drop off in emission at around $1000\text{keV}$, but do not observe the peaks in the spectrum at low energies as for the super-Eddington case. Recall the low energy emission is due to the thermal black body spectrum. Finally, we explore the dependence of this simpler model on viewing angle. In Figure 4.16, we place our camera at incidence angles of 0, 45, and 90 degrees as viewed perpendicular to the line connecting the BHs. We find that by moving away from face-on towards the edge-on case, the overall flux density received at the camera diminishes. At the peak of the spectrum, $\approx 200 - 300\text{keV}$, the ratio between the two flux densities is $\approx 3/2$. Finally, we
note that this is likely a strong function of both poloidal and azimuthal angle and will be a topic of discussion in (d’Ascoli et al., 2017).

Figure 4.15: Flux density for sub-Eddington accretion plotted against photon energy in units of keV. We plot the contributions to the total flux from the regions interior and exterior to $r = a$ in addition to the total flux density from the entire simulation domain. As the units of flux density here are arbitrary, we normalize to the peak value of the combined flux density.

Figure 4.16: Total flux density for sub-Eddington accretion plotted against photon energy in unit of keV. We plot the total flux density for 3 viewing angles of 0, 45, and 90 degree. Viewing angle is fixed perpendicular to the line connecting the BHs.

4.4 Discussion

4.4.1 Mini-Disk Asymmetry and a Possible Lump Mini-Disk Coupling

In the absence of external sources of asymmetry, the mini-disks of Chapter 3 were symmetric with respect to one another. This was to be expected due to the symmetry of an equal-mass binary. Despite the equal-mass symmetry of the gravitational field and being initialized with the same parameters, we find that this mini-disk symmetry vanishes once the mini-disks are coupled
to the circumbinary in full 3D GRMHD. Perhaps the most striking form of the asymmetry is the different qualitative structure of the spiral density waves in the equatorial plane of Figure 4.7. While both mini-disks have a clear $m = 1$ component to the spiral structure extending down to the central BH, the opening angles of these waves are largely different.

On the logarithmic scale of Figure 4.7, we observe potential hints to the cause of this asymmetry. Here, we observe that the two time-averaged circumbinary streams accreting into the central cavity are not identical. This is likely due to an $m = 1$ mode present within the circumbinary disk (Noble et al., 2012). We note that our time-averaging actually smooths the $m = 1$ mode out because it orbits at a different frequency than the binary orbital frequency (see below). Such $m = 1$ modes have been shown to modulate the accretion rate into the central cavity (Shi et al., 2012; Noble et al., 2012; D’Orazio et al., 2013; Farris et al., 2014). We speculate that the mini-disk asymmetry, particularly the differing spiral structure, is driven by circumbinary streams shocking against the outer edges of the mini-disks. As the streams ballistically shock against the outer edges of the mini-disks, they raise the mini-disk temperature, thus altering the opening angle of the spiral density wave (Savonije et al., 1994; Ju et al., 2016). These stream-disk shocking events are capable of driving the formation of additional spiral density waves which constructively/destructively interfere with spiral density waves excited near Lindblad resonances. We observe possible evidence for spiral patterns in the mini-disks from accretion stream impacts in Figure 4.5. Here, the stream impacting the mini-disk around BH2 appears as a direct continuation of a spiral arm extending into the mini-disk.

To further examine this, we consider the frequency at which streams enter the central cavity from the over-density in the circumbinary. The over-density, or lump, orbits at $\approx 2.4a$ away from the center-of-mass at a frequency of $\Omega_{\text{lump}} = 0.26\Omega_{\text{bin}}$ (Noble et al., 2012). A stream will depart from the lump whenever the lump comes into phase with an individual BH. This happens at a frequency of $2(\Omega_{\text{bin}} - \Omega_{\text{lump}}) = 1.48\Omega_{\text{bin}}$. Because the stream departs from an over-density in the disk, it carries excess momentum flux relative to streams departing from other portions of the circumbinary. This can be seen easily from Figure 6 of Noble et al. (2012). In the Figure, the streams peeled off
the edge of the circumbinary develop an increasing asymmetric nature as the lump grows. This is due to an increasing disproportionality of available material to be peeled off the inner edge of the disk. We note that our circumbinary initial data is taken at the start of the development of the lump in Noble et al. (2012). If we speculate our mini-disk asymmetry arises from a quasi-periodic “strengthening” of streams departing from the lump in the circumbinary, such streams will impact an individual mini-disk at the rate at which the lump comes into orbital phase with that mini-disk. For equal-mass, these lump streams will alternate from one mini-disk to the other. As the mass ratio deviates significantly from unity, all accretion streams will impact the mini-disk orbiting the secondary. Therefore, an individual mini-disk comes into phase with the lump at precisely half the rate at which such lump streams enter the central cavity, $0.74 \Omega_{\text{bin}}$. Given these frequencies, our time-averaged state includes only one such crossing. In practice, streams are continually pulled from the edge of the circumbinary. Therefore, the quasi-modulated stream-disk impacts would have a time period of strengthening associated with the overall size of the lump. However, this broadening would not alter the frequencies discussed. This quasi-periodic modulation of the stream momentum flux could help to explain the asymmetry in the mini-disks. To confirm this, one would need to perform a longer simulation and average over a longer period of time; taking care to include precisely two sets of streams departing from the lump. However, we note that the binary evolves appreciably over as little as a single binary orbital period at the separations considered in this Chapter ($\dot{a}/a \propto a^4$). Therefore, in terms of single binary separation, the system will be marked by an individual mini-disk being singled out for streams departing from the lump. Further studies will be necessary to explore the inter-play of the time periods of modulating accretion stream flux due to the lump with the orbital and inspiral periods of the binary. More specifically, how these various timescales can alter the mini-disk dynamics.

Finally, we conclude by mentioning possible electromagnetic consequences of stream-disk interactions. Noble et al. (2012) reported a modulation in the luminosity of the circumbinary at $\approx 1.47 \Omega_{\text{bin}}$. This frequency corresponded to the frequency at which portions of the accretion streams are flung back into and shock against the lump. These interactions occur with the same
frequency as streams departing from the lump. If the stream asymmetry is associated with the lump and the shock heating events are strong enough to alter the dynamics of the mini-disks, the quasi-periodic lump streams impacting the mini-disks could serve to further enhance any quasi-periodic signal at $1.47\Omega_{\text{bin}}$.

### 4.4.2 Effects of the Central Cutout

Our simulations are performed in a topologically spherical coordinate system (Zilhão & Noble, 2014). We must therefore excise the coordinate singularity located at the origin to ensure a stable evolution. To examine the effects of our central excision, we performed evolutions excising a sphere of radius $M$ (small cutout) and $2M$ (large cutout). While one would wish to limit the radius of such excisions as much as possible, it is important to note that the Courant factor in spherical grids is highly demanding. The timestep in such simulations will go as $rdx$ at the central-most cells due to a focusing of cells near the origin. Using the large cutout, our simulations required 28 days on 19,200 processes of the BlueWaters supercomputer. Evolving with the excision set to $M$ as in Chapter 3 would have roughly been twice as computationally expensive. We therefore elected to evolve using the small cutout for just over half a binary orbital period to examine the influence of increasing the central cutout size.

We begin by examining the mass loss through the central excision boundary in Figure 4.17. We normalize the mass flux and mass lost to the central excision to the initial total mass of our system. Though this can lead to numerical values that are misleadingly small (a significant fraction of the gas is in the circumbinary disk), it is a consistent means of normalization that is not subject to any mass fluxes passing through arbitrarily defined regions. We find that the largest rate at which material is exiting the domain occurs at roughly $t \approx 300M \approx 0.5t_{\text{bin}}$. This corresponds to the point where the mini-disks reach their maximal extent during the initial pressure driven expansion period. While the absolute scale of the mass flux at the central excision is dependent on the size of the cutout, the over-arching shape is similar in both runs. Finally, we note that the mass flux through the central excision settles to a mean value of $\mathcal{O}(10^{-7})M_0$ through the quasi-steady
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Figure 4.17: (Left) Mass flux integrated over the spherical excision boundary for both the small and large cutout runs normalized to the total initial mass of the system. (Right) Total integrated mass lost through the central excision boundary normalized to the total initial mass.

state. Another means of observing this is through the total integrated mass passing into the central excision. We observe noticeable upticks in the total mass lost at $t \approx 300M$ and $t \approx 500M$ resulting from the spikes in $\dot{M}/M_0$. Once the system enters the quasi-steady state, the total mass lost to the central cutout is approximately $1.72 \times 10^{-4}M_0$. After this point, the mass lost rate is nearly fixed resulting in a total mass lost through the entire simulation of approximately $2.25 \times 10^{-4}M_0$.

The most obvious impact of extending the central cutout is in the sloshing region at the center-of-mass, precisely because any mass lost to the central excision would have proceeded to the other mini-disk through this region. We define the sloshing region in the equatorial plane as a rectangle in the corotating frame with dimensions $0.4a \times 0.6a$. In these units, our central excisions have radii of $0.05a$ and $0.1a$. Because the area of the excision scales as $r^2$, we note that increasing the cutout size amounts to increasing the fraction of the equatorial sloshing region contained within the excision from $\approx 3.27\%$ to $\approx 13.1\%$. Additionally, in Figure 4.9 we observe that the central cutout is vertically extended above and below the bulk of the mini-disks. Given these considerations and that the sloshing streams pass through the L1 point at the center-of-mass, we conclude the
absence of significant sloshing in our run is due to the increased central cutout. In order to study the vertical structure of the sloshing and the potential impact on the electromagnetic emission, future studies which do not excise the center-of-mass from the domain will be required. In Section 5.2 we discuss work towards performing such simulations.

![Logarithmic density contours](image)

**Figure 4.18:** Logarithmic density contours of the innermost $3a_0 \times 3a_0$ in the equatorial plane for the (Left) small cutout and (Right) large cutout runs at $t \approx 0.5t_{\text{bin}}$. Both frames have a central mask placed for values radially interior to $2M$ so as to compare regions in both domains.

Finally, we confirm that the size of our cutout has no influence on the bulk of the mini-disks. In Figure 4.18 we plot density contours in the equatorial plane of the evolution at $0.5t_{\text{bin}}$ evolution masking values radially interior to $2M$. In both frames we note that the overall structure and density scale of the mini-disks is unaffected by the size of the central cutout. The only noticeable differences are the structure of the material directly near the central cutout. We note that the streams of material passing around the smaller central cutout appear “fuller”. This is because portions of the material in this region fall into the central cutout and exit the domain of the simulation for the large cutout. The other noticeable difference is some minor evidence for the zeroth order linear
interpolation at the boundary in the large cutout frame, which may be a visualization artifact of the contour interpolation. We therefore conclude that while we are unable to study the sloshing, all other aspects of our simulation will be unaffected by the extended central cutout.

### 4.4.3 Emission Model Limitations and Considerations

We used the energy extracted from our simulation in the cooling function as a proxy for the bolometric luminosity of the accretion disks. To calculate accurate spectra of an inspiraling SMBBH, one must describe the emission of the accretion disks and jets. As we are only able to evolve the accretion disks and do not resolve any jets, we can only speak to a portion of the total electromagnetic radiation. Our simulations do not couple the radiative equations to the GRMHD equations of the disk. We therefore must assume an emission model a posteriori. By constructing a two-part model, we can describe systems with high optical depths provided we can calculate the location of the photosphere along our geodesics. We find that such calculations are problematic at viewing angles far from zero degrees. This is chiefly because the optical depth along the line-of-sight no longer has any physical association with the photosphere. This places a restriction on our calculations, as the strongest dependence on viewing angle will likely be intimately tied to obscuration of emission from the central cavity by the surrounding circumbinary disk. Therefore, accurate predictions of the spectral dependence on viewing angle will require a more sophisticated method of photosphere calculations.

Additionally, our proxy for bolometric luminosity has several ad-hoc pieces that need be considered. Firstly, our four part cooling region model results in discontinuities in the cooling rate at the transitions. Additionally, we arbitrarily chose a cooling timescale in the central cavity between the circumbinary and mini-disks. We do not expect this will have a significant impact on the total emission. The total cooling function in this region, which is dominated mostly by voids of low density, scales linearly with the density. This, in turn, produces relatively little emission using our model as expected. Finally, we note that our test to only cool bound material is gauge dependent. In the coordinate system in which we perform the calculations, the warped center-of-mass
grid, we likely over-estimate the amount of bound material. This leads to excessive cooling in the corona, potentially overestimating the total X-ray flux. We have resolved this issue in the data by neglecting contributions below a density cutoff of \(10^{-4}\). This cutoff was selected to be orders of magnitude below the density peaks of the disks and effectively removes unphysical energy dissipation in the low density floor states. In addition to more sophisticated photosphere calculations, accurate electromagnetic emission templates will likely require a less ad-hoc cooling prescription which can accurately differentiate between bound material and hot winds, as well as smoothly transition between individual cooling regions.

We have assumed that our mini-disks are in inflow equilibrium. Given the short time-scale of our simulations, this is likely not the case. We have done this due to the complications of measuring the accretion rate directly in the warped grid and the relatively short length of our simulation. By making the inflow equilibrium assumption, we were able to make proof of principle type calculations of the spectra. This scaling of code units to physical units alters the physical emission scale of the system. Additionally, the accretion rates selected in these calculations may not directly correspond to the physics of the accretion disks. This is because the vertical structure of the accretion disks is intimately tied to the radiation pressure, which is in turn a function of the selected accretion rate.

4.5 Conclusions

In this Chapter, we have presented the first magnetohydrodynamic or general relativistic simulation of mini-disks coupled to the circumbinary in binary systems; observing for the first time the inclusion of vertical structure, magnetic accretion stresses, and continuing accretion from the circumbinary in individual mini-disks when the binary separation is small enough to include general relativistic effects in the spacetime.

We observe that the mini-disks are subject to the MRI, quickly developing turbulent magnetic flows. The overall magnetization level, \(\beta \approx 1 - 10\) in the mid-plane of the mini-disks, is consistent with simulations of circumbinary accretion disks of binary separations \(a \approx 20M\) (Noble et al.,
Moving above or below the mid-plane, we observe an increase in magnetization in the hot corona of each mini-disk by approximately half an order of magnitude.

It has long been known that tidal forces can drive spiral waves in disks within binary systems; in the Newtonian limit, these have an exclusively $m = 2$ character. On the other hand, even the lowest-order relativistic corrections can introduce $m = 1$ perturbations into the binary potential while also altering the $m = 2$ component. In consequence, Bowen et al. (2017) demonstrated that the $m = 1$ component becomes the dominant feature at the binary separation considered here. Despite the fact that magnetic turbulence may disrupt spiral density waves launched within accretion disks, such as the $m = 2$ spiral density waves in the circumbinary disk, we observe the unabated development of $m = 1$ spiral density waves in the mini-disks. Finally, we note that within the mid-plane of the disk we observe spiral waves of increased magnetization. These spiral patterns are likely intimately tied to the spiral density waves present within the mini-disks.

Most strikingly, we observe that the mini-disks in our equal-mass simulation are not symmetric with respect to one another. This is in direct conflict with previous SMBBH accretion simulations. Such simulations have either enforced $\alpha$-disk model accretion within the mini-disks (Farris et al., 2014; D’Orazio et al., 2016), evolved only a single accretion disk (Ryan & MacFadyen, 2017), or failed to include continuing accretion from the circumbinary (Bowen et al., 2017). We speculate that this mini-disk asymmetry is driven by a coupling of the individual mini-disks to an $m = 1$ mode, or lump, in the circumbinary disk via accretion streams. As a mini-disk comes into phase with the lump at a frequency of $0.74\Omega_{\text{bin}}$, the mass flux contained within the stream is quasi-periodically strengthened. This in turn, produces a quasi-periodic modulation of the mass flux imparted into the mini-disks by the accretion streams and ultimately modifies the spiral density wave structure. Simulations of circumbinary accretion disks have demonstrated that the lump secularly evolves over many binary orbital periods (Noble et al., 2012; Shi et al., 2012; Farris et al., 2014), continually growing in size and further enhancing the accretion stream asymmetry. In astrophysical systems, which have evolved through the entire Newtonian and Relativistic inspiral stages, this lump could be quite extensive. This could serve to further enhance the mini-disk-lump coupling reported here.
We presented the first ray tracing calculations of accretion disks in an inspiraling SMBBH. We calculated the time-averaged spectrum of the binary, separating contributions from the mini-disks and circumbinary/streams using a two part emission model. For a total binary mass of $10^6 M_\odot$ accreting at 5 times the Eddington accretion rate, we found a double-peaked spectrum corresponding to black body emission at the photosphere for energies of tens to hundreds of eV and inverse Compton scattering at energies of $2 \times 10^3$ to $100$ keV. The mini-disks produce a hotter photosphere and dominate the spectrum at nearly all X-ray energies with a turn-over in the UV emission where the circumbinary dominates the total spectrum. We observed that our two-emission model was only valid when viewing the binary face-on. By shifting the accretion rate to 10% the Eddington accretion rate and considering only inverse Compton scattering, we found that the X-ray emission was again dominated by the mini-disks. In this simplified model, we found that moving from face-on to edge-on produces a monotonic shifting to smaller fluxes at the camera.

Newtonian studies (Mayama et al., 2010; Farris et al., 2014; D’Orazio et al., 2016) had previously shown that a small fraction of the disks’ mass can be removed from the individual disks within a binary and placed in the region stretching from one disk to the other through the L1 point. General relativistic studies (Bowen et al., 2017) showed this effect to be greatly enhanced in the PN regime, and demonstrated a quasi-periodic behavior linked to the binary orbital frequency. Although the increased size of our central excision boundary prohibits the study of sloshing in our simulation, we observe evidence that such streams formed within our simulation near the edges of the mini-disks. As it was speculated that the sloshing may be related to the spiral density wave structure within the mini-disks (Bowen et al., 2017), the mini-disk asymmetry observed in our simulation could lead to an asymmetric sloshing of material from one mini-disk to another. Any such asymmetry in the sloshing could serve to further enhance the mini-disk asymmetry and provide a net transfer of mass from one mini-disk to another (particularly once the mass-ratio departs from unity). Future studies which include the center-of-mass on the computational domain with longer evolution lengths will be necessary to fully study the sloshing in 3D GRMHD. This will be the study of future works.
5.1 Summary and Conclusions

In this Dissertation, we presented the first simulations of mini-disks during both the quasi-
Newtonian and GW-dominated inspiral regime of BBHs ($a \lesssim 100M$). In our approach, we used
an approximate general relativistic spacetime (Mundim et al., 2014; Ireland et al., 2015; Zlochower
et al., 2016; Nakano et al., 2016), which accurately describes the dynamics of BBHs during the inspi-
ral phase by asymptotically matching BH perturbation theory to PN theory (Blanchet, 2014) and
evolving the BBH trajectory at 3.5PN-order. Although our implementation of general relativistic
effects is valid for any BH mass-ratio ($q \leq 1$) and can accommodate spins, the work reported here
focused on equal-mass, non-spinning BHs. The Dissertation is divided into two main studies, each
representing a first ever simulation of its kind. In Chapter 3, we presented a suite of simulations at
binary separations ranging from the quasi-Newtonian regime ($a \approx 100M$) down to well within the
relativistic, PN regime of SMBBH inspiral ($a \leq 20M$) in 2D, inviscid hydrodynamics. In Chapter
4, we extended this to a full 3D GRMHD simulation of an inspiraling SMBBH at $a \approx 20M$, includ-
ing mini-disks coupled to the circumbinary accretion disk. These studies both contain extensive
improvements over previous SMBBH simulations of the central cavity which either used Newtonian
gravity and ad-hoc accretion models (Farris et al., 2014, 2015a; D’Orazio et al., 2016), or simulated only a single accretion disk (Ryan & MacFadyen, 2017). Our mini-disk simulations included, for the first time, a fully consistent, general relativistic prescription of tidal gravity, and a coupling of the mini-disks to the circumbinary accretion disk including vertical structure and magnetic fields. Finally, we presented the first ray-tracing calculations of SMBBH accretion. Ultimately, we find that relativistic effects can create qualitative changes to the mini-disks when the binary separation shrinks to several tens of gravitational radii or less.

Effects of the Binary Potential and Tidally Truncated Mini-Disks

The gravitational potential along the line between the two masses becomes shallower, and its gradient gentler, as the system becomes more relativistic. Within the secondary’s Roche lobe, this contrast between relativistic disks and Newtonian grows with mass-ratios further from unity. One result, apparent in our hydrodynamic simulations, is that, particularly as the separation becomes $\lesssim 30M$, the mini-disks stretch toward the L1 point. The resulting asymmetry, with respect to the central BH, is large enough that the outer rims of mini-disks in a relativistic binary are significantly non-circular, so that thinking in terms of a “tidal truncation radius” for such mini-disks can be misleading. As inspiral progresses to yet smaller separations, relativistic effects discussed here are likely to become stronger.

Sloshing

Newtonian studies (Mayama et al., 2010; Farris et al., 2014; D’Orazio et al., 2016) had previously shown that a small fraction of the disks’ mass can be removed from the individual disks within a binary and placed in the region stretching from one disk to the other through the L1 point. A further consequence of the shallower gradient in the relativistic regime is a sharp increase in that fraction, an order of magnitude increase between binary separations of 50$M$ and 20$M$. At this level, the “sloshing mass” can play a significant role in the system. For example, when the mass-ratio is not unity, asymmetry in the sloshing may create an entirely new way for mass to pass from one
part of the binary system to another.

This sloshing may also result in a striking and unique electromagnetic signal of a BBH system in the period shortly before merger. In the regime of separations in which the sloshing mass is sizable, the repeated shocks it suffers may account for \( \sim 10\% \) of the bolometric luminosity; in addition, in many circumstances the region in which the heat is released may be optically thin enough for its lightcurve to follow the periodic character of the heat release. For the \( q = 1 \) case, there are two frequency components in the modulation, one at twice the binary orbital frequency and another at \( \simeq 2.75 \times \) that frequency. For binary separation \( 20M_6 \), these correspond to periods \( \simeq (1/2)M_6 \) hr, for \( M_6 \) the total binary mass in units of \( 10^6 M_\odot \).

**Spiral Density Waves and Mini-Disk-Lump Coupling**

We have also discovered that relativistic alteration of the tidal forces leads to other contrasts with Newtonian behavior. It has long been known that tidal forces can drive spiral waves in disks within binary systems; in the Newtonian limit, these have an exclusively \( m = 2 \) character. On the other hand, even the lowest-order relativistic corrections can introduce \( m = 1 \) perturbations into the binary potential while also altering the \( m = 2 \) component. Higher-order terms such as those associated with gravitational radiation and the orbital evolution it creates can also lead to new \( m = 1 \) and \( m = 2 \) components in disk dynamics. In consequence, the \( m = 1 \) component can become the dominant feature even when the separation is as large as \( \sim 100M_6 \).

We observe the unabated development of \( m = 1 \) spiral density waves in the magnetized mini-disks. This is despite the fact that magnetic turbulence may disrupt spiral density waves, as is the case for the \( m = 2 \) spiral density waves in the circumbinary disk. As evolution is included above and below the mid-plane, the \( m = 1 \) spiral density waves exhibit a more lobe-like structure on the side nearest the L1 point with a flatter and elongated structure near the L2/L3 point. Additionally, we note that we observe spiral waves of increased magnetization within the mid-plane of the mini-disks which are likely tied to the spiral density structure.

Finally, we observe that the \( m = 1 \) spiral density waves present in the magnetized mini-disks
may couple dynamically to an $m = 1$ mode, or lump, in the circumbinary accretion disk via the accretion streams. This coupling ultimately leads to qualitatively different spiral density wave structure within each mini-disk. As a mini-disk comes into phase with the lump, the available mass to be peeled off the inner edge of the circumbinary is quasi-periodically enhanced at a frequency of $0.74\Omega_{\text{bin}}$. This enhancing of the accretion streams ultimately increases the momentum flux imparted into the mini-disk, asymmetrically altering the spiral density wave structure. The lump present in the circumbinary accretion disk has been shown to evolve secularly over time, continually growing in size (Noble et al., 2012; Shi et al., 2012; Farris et al., 2014). This continued growth of the $m = 1$ of the circumbinary drives a continued growth of the accretion stream asymmetry. This could serve to further enhance the mini-disk-lump coupling reported here. For the equal-mass case, further studies will be necessary to study the relaxation time of the impact of the strengthened stream impacts to determine if such asymmetry persists through all orbital phases. This relaxation time becomes of particular importance as the mass-ratio deviates significantly from unity and all accretion streams impact the mini-disk orbiting the secondary. Finally, we note that this coupling has gone previously unreported by previous SMBBH accretion simulations. This is because such simulations have either enforced $\alpha$-disk model accretion within the mini-disks (Farris et al., 2014; D’Orazio et al., 2016), evolved only a single accretion disk (Ryan & MacFadyen, 2017), or failed to include continuing accretion from the circumbinary (Bowen et al., 2017).

**Ray-Tracing and Electromagnetic Spectra**

We performed the first proof of principle type ray-tracing calculations of an inspiraling SMBBH including mini-disks. By selecting two nominal accretion rates, which we call the super-Eddington and sub-Eddington models, and a total binary mass of $10^6 M_\odot$, we calculated spectra of the time-averaged state of the binary. We separated emission coming from the mini-disks and the circumbinary and found a double peaked spectrum. Once the system becomes optically thick, the mini-disks produce a hotter photosphere and dominate the spectrum at nearly all X-ray emission with a turn-over in the UV emission where the circumbinary dominates the total spectrum. The
combined photosphere emission results in a broadened thermal spectrum including contributions from the photospheres of the mini-disks and circumbinary at energies of tens to hundreds of eV. The high energy peak in the range of $2 - 1000$ keV corresponds to inverse Compton scattering in the mini-disks. Ultimately, we found that our two-emission model was only valid when viewing the system face-on where the geodesics travel approximately vertically through the disks. Shifting to the sub-Eddington accretion rate, we find that the inverse Compton scattering emission was again dominated by the mini-disks. By altering the inclination angle, we observed an obscuration of the mini-disks by the torus.

5.2 Future Work

In the immediate future, much work will be dedicated towards extending the analysis presented in Chapter 4. From this in-depth analysis shall arise two papers; one paper on the GRMHD evolution of the mini-disks when coupled to the circumbinary disk and another on the electromagnetic emission using BOTHROS (d’Ascoli et al., 2017). In these papers we intend to further explore the interplay between the mini-disks and circumbinary accretion disks, as well as further investigate the spiral shocks in the MHD mini-disks. Furthermore, in future simulations we would like to improve the physicality of our cooling function by improving the bound material test and implementing a transition between the various cooling regions. This may have a significant impact on the predictions made from post-processing calculations. Finally, we note that BOTHROS is not the only general relativistic ray-tracing tool. Others, such as Pandurata (Schnittman & Krolik, 2013), can apply different radiative models to produce spectra by integrating from the source data to the camera instead of from the camera to the source data as is done in BOTHROS. In the future, it will be useful to compare our ray-tracing results to those produced by tools such as Pandurata using the same data set.

Though the work presented in this Dissertation and these upcoming papers represents a significant milestone in the scientific goal of simulating the entire inspiral and merger of SMBBHs, much
work remains to be done. Most notably, reducing the computational expense of our current simulations. Our simulations with the large cutout required approximately six million CPU hours and two weeks walltime on the BlueWaters supercomputer to perform a single binary orbit at $a \approx 20M$.

As the timestep scales with the spacing at the smallest cell at the central cutout, this all but eliminates any hope of simulating from larger separations to merger. Additionally, the electromagnetic signal emanating from SMBBHs requires an understanding of a vast set of parameters including binary mass-ratio, BH spin, and thermodynamic and magnetic properties of the accretion disks. For instance, the warped grid used in our simulations would require a cutout in the middle of the mini-disk around the primary BH for mass-ratios far from unity.

In the future, such simulations will be performed using an on-going development called PATCHWORK (Shiokawa et al., 2017). This new code infrastructure allows for multidata, multiphysics simulations. Using PATCHWORK, one may perform a single simulation running multiple codes using different physics and different coordinate systems that communicate boundary conditions to one another through MPI under the multiple program multiple data (MPMD) paradigm. Each local system, or “patch”, need only know how to relate its local motion to an over-arching global patch that it moves on top of. In the context of SMBBH accretion; the global patch will be a spherical coordinate system with center-of-mass at the origin to resolve the circumbinary accretion disk’s symmetry, a Cartesian patch placed over the central cutout to fully resolve the sloshing region for the first time, and additional spherical patches placed around each individual BH to minimize cell count and numerical diffusion in the mini-disks. In addition to reducing the number of necessary cells and resolving the entire domain, PATCHWORK will significantly increase the timestep of the evolution by fully exploiting the symmetries of individual parts of the global system. Overall, this will result in significant reductions in the computational expense of these simulations; this could allow for many more binary orbits in our simulation and facilitating parameter space studies never before available.

PATCHWORK has been implemented and tested in hydrodynamics in a standalone version of HARM3D. Currently, work is on-going amongst collaborators at RIT and elsewhere to implement
magnetic fields into PATCHWORK and fully incorporate it into the version of HARM3D used in our simulations. We have extended our version of HARM3D to handle arbitrary spacetimes, in arbitrary coordinates, \textit{moving at an arbitrary velocity}; this is an integral part of incorporating PATCHWORK into our version of HARM3D and a necessary step to evolve moving patches in our BBH spacetime. Using PATCHWORK, we will continue our GRMHD simulation described here, evolving until the breakdown of our analytic spacetime at binary separations of $\approx 10M$ and including the entire sloshing region in the computational domain. We will explore the full vertical structure, time dependence, and electromagnetic output of sloshing predicted by (Bowen et al., 2017). Ultimately, the data of that simulation may then be passed to full numerical relativity coupled to magnetohydrodynamics to simulate the final merger proper, making predictions of the electromagnetic emission of the final moments before merger.
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