Modeling the Radar Return of Powerlines Using an Incremental Length Diffraction Coefficient Approach

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Modeling the Radar Return of Powerlines Using an Incremental Length Diffraction Coefficient Approach

by

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B.S. Virginia Military Institute, 2005
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A dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the Chester F. Carlson Center for Imaging Science
College of Science
Rochester Institute of Technology

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Submitted to the
Chester F. Carlson Center for Imaging Science
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for the Doctor of Philosophy Degree
at the Rochester Institute of Technology

Abstract

A method for modeling the signal from cables and powerlines in Synthetic Aperture Radar (SAR) imagery is presented. Powerline detection using radar is an active area of research. Accurately identifying the location of powerlines in a scene can be used to aid pilots of low flying aircraft in collision avoidance, or map the electrical infrastructure of an area. The focus of this research was on the forward modeling problem of generating the powerline SAR signal from first principles. Previous work on simulating SAR imagery involved methods that ranged from efficient but insufficiently accurate, depending on the application, to more exact but computationally complex. A brief survey of the numerous ways to model the scattering of electromagnetic radiation is provided. A popular tool that uses the geometric optics approximation for modeling imagery for remote sensing applications across a wide range of modalities is the Digital Imaging and Remote Sensing Image Generation (DIRSIG) tool. This research shows the way in which DIRSIG generates the SAR phase history is unique compared to other methods used. In particular, DIRSIG uses the geometric optics approximation for the scattering of electromagnetic radiation and builds the phase history in the time domain on a pulse-by-pulse basis. This enables an efficient generation of the phase history of complex scenes. The drawback to this method is the inability to account for diffraction. Since the characteristic diameter of many communication cables and powerlines is on the order of the wavelength of the incident radiation, diffraction is the dominant mechanism by which the radiation gets scattered for these targets. Comparison of DIRSIG imagery to field data shows good scene-wide qualitative agreement as well as Rayleigh distributed noise in the amplitude data, as expected for coherent imaging with speckle. A closer inspection of the Radar Cross Sections of canonical targets such as trihedrals and dihedrals, however, shows DIRSIG consistently underestimated the scattered return, especially away from specular observation angles. This underestimation was
particularly pronounced for the dihedral targets which have a low acceptance angle in elevation, probably caused by the lack of a physical optics capability in DIRSIG. Powerlines were not apparent in the simulated data.

For modeling powerlines outside of DIRSIG using a standalone approach, an Incremental Length Diffraction Coefficient (ILDC) method was used. Traditionally, this method is used to model the scattered radiation from the edge of a wedge, for example the edges on the wings of a stealth aircraft. The Physical Theory of Diffraction provides the 2D diffraction coefficient and the ILDC method performs an integral along the edge to extend this solution to three dimensions. This research takes the ILDC approach but instead of using the wedge diffraction coefficient, the exact far-field diffraction coefficient for scattering from a finite length cylinder is used. Wavenumber-diameter products are limited to less than or about 10 ($k \cdot a \lesssim 10$). For typical powerline diameters, this translates to X-band frequencies and lower. The advantage of this method is it allows exact 2D solutions to be extended to powerline geometries where sag is present and it is shown to be more accurate than a pure physical optics approach for frequencies lower than millimeter wave. The Radar Cross Sections produced by this method were accurate to within the experimental uncertainty of measured RF anechoic chamber data for both X and C-band frequencies across an 80° arc for 5 different target types and diameters. For the X-band data, the mean error was 6.0% for data with 9.5% measurement uncertainty. For the C-band data, the mean error was 11.8% for data with 14.3% measurement uncertainty. The best results were obtained for X-band data in the HH polarization channel within a 20° arc about normal incidence. For this configuration, a mean error of 3.0% for data with a measurement uncertainty of 5.2% was obtained. The least accurate results were obtained for X-band data in the VV polarization channel within a 20° arc about normal incidence. For this configuration, a mean error of 8.9% for data with a measurement uncertainty of 5.9% was obtained. This error likely arose from making the smooth cylinder assumption, which neglects the semi-open waveguide TE contribution from the grooves in the helically wound powerline. For field data in an actual X-band circular SAR collection, a mean error of 3.3% for data with a measurement uncertainty of 3.3% was obtained in the HH channel. For the VV channel, a mean error of 9.9% was obtained for data with a measurement uncertainty of 3.4%. Future work for improving this method would likely entail adding a far-field semi-open waveguide contribution to the 2D diffraction coefficient for TE polarized radiation. Accounting for second order diffractions between closely spaced powerlines would also lead to improved accuracy for simulated field data.
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Finally to my family. For every step of my journey, they have been by my side with unconditional love and support. They mean the world to me.
# Contents

1 Introduction .................................................. 15

2 Background ................................................ 18
   2.1 GRECOSAR ........................................... 21
   2.2 SARAS ................................................. 22
   2.3 POV-Ray ............................................... 25
   2.4 DIRSIG ............................................... 26
   2.5 FDTD ................................................. 28
   2.6 Millimeter-wave Physical Optics Model ............... 30
   2.7 Xpatch ................................................ 32
   2.8 Summary ............................................... 33

3 Theory ....................................................... 34
   3.1 Radar Cross Section Definition ....................... 35
   3.2 Modeling Electromagnetic Propagation ................. 36
      3.2.1 Maxwell’s Equations .............................. 39
      3.2.2 Geometric Optics ................................. 41
      3.2.3 Geometric Theory of Diffraction ................. 43
      3.2.4 Scalar Diffraction Theory ....................... 45
      3.2.5 Physical Optics ................................. 49
      3.2.6 Modified Equivalent Current Approximation .... 52
      3.2.7 Physical Theory of Diffraction .................. 56
   3.3 Power Line Modeling Using ILDC’s ...................... 57
      3.3.1 Physical model .................................... 58
      3.3.2 2D Diffraction Coefficient ....................... 60
      3.3.3 Incremental Length Diffraction Coefficient (ILDC) 62
      3.3.4 SAR Phase History Simulation ................... 62
   3.4 Summary ............................................... 64
CONTENTS

4 Methods 66
4.1 Baseline DIRSIG Assessment ........................................... 67
4.2 Power Line Modeling ...................................................... 75
  4.2.1 Anechoic Chamber Measurements .................................. 75
  4.2.2 Gotcha Power Line RCS Comparison ............................... 80

5 Results 84
5.1 DIRSIG Results ............................................................ 85
5.2 Power Line Modeling Results ........................................... 90
  5.2.1 Anechoic Chamber Data .............................................. 90
  5.2.2 AFRL Gotcha Data .................................................... 105
5.3 Summary ................................................................. 111

6 Conclusion and Future Work 113

Appendices 125
A ILDC Module 126
B RCS Simulation Code 136
C Phase History Simulation Code 140
List of Figures

2.1 Image of two buildings with different material properties generated using SARAS. Higher order wall-ground reflections are apparent for both buildings [1]. ................................................................. 24

2.2 (Left) actual and (Right) simulated images of the Wynn Hotel. Special features in each image are labeled with corresponding letters [2]. .......... 25

2.3 Simulated amplitude of field reflected from both a smooth and rough plate using FDTD [3]. ................................................................. 29

2.4 Geometric powerline model used by Sarabandi, K. [4]. ....................... 30

2.5 Simulated versus experimental RCS’s at 94 GHz for 4 different powerlines in the VV channel. Bragg scattering peaks were accurately modeled by a PO approach at millimeter wavelengths [4]. ......................... 32

3.1 Fermat’s Principle for Edge Diffraction. ........................................... 44

3.2 Geometry for Scalar Diffraction (Kirchoff) Integral. .......................... 46

3.3 Plot of a catenary. ................................................................. 59

3.4 Plot showing the normalized 2D RCS of a circular cylinder for various radii at normal incidence. The noisy signature at the end was caused by machine precision errors which arose when too many terms were used for the calculation. Physical optics would be the preferable method in this high-frequency regime. The |||| and ⊥⊥ subscripts refer to incident and outgoing TM and TE modes, respectively. ........................................... 61

3.5 Depiction of the orientations of the polarization basis vectors from the aircraft’s frame of reference (H and V polarizations) and the powerline’s frame of reference (TE and TM polarizations). The definitions shown above were used to convert the polarization amplitudes from one frame to another. .... 63
4.1 Process imagery for Gotcha collection. (a) Total power image of AFRL data produced employing RITSAR using all 360° of azimuth for backprojection processing. Canonical targets appears as point sources in the top-left portion of the image. (b) Image of view for which powerlines were apparent in the upper-right portion of image. 68
4.2 Locations of canonical targets in the scene provided by AFRL/SNA [5]. 69
4.3 Plot showing the calibration offsets calculated using each of the canonical targets. Although the targets were of different sizes and types and placed at different orientations, the calculated offsets were very similar. The range of the plot matches the range of the intensity scale for the raw image. The mean value indicated by the blue bar represented the actual offset used. 71
4.4 Flight paths used to collect the AFRL data for all 8 passes. 73
4.5 Screen shot of Blender scene used for DIRSIG simulation. 74
4.6 Depiction of the anechoic chamber setup from the Michigan experiment [6]. 76
4.7 Image showing calibration sphere placed on top of mount [6]. 77
4.8 Image showing powerline placed on top of mount [6]. 78
4.9 Images depicting the geometric cross-section of each object for which RCS measurement were made: (a) 1.27 cm cylinder (b) 167.8 MCM Copper (c) 556.5 MCM Aluminum (d) 954 MCM Aluminum & Steel and (e) 1431 MCM Aluminum & Steel. All cables were ≈ 1 ft in length [6]. 79
4.10 Image chip of isolated transmission lines. 80
4.11 Picture taken of a representative telephone pole configuration. The actual telephone pole only had three transmission lines on top, configured similarly to what is shown in the picture, as well as a thick bundle of communication cables halfway up. Images of the actual telephone poles could not be obtained. 81
4.12 Physical model images. (a) Sketch of powerline in CAD software (b) 3D powerline plot (c) Side view of the top transmission line. 82
5.1 Qualitative Image Comparisons: (top) AFRL Gotcha, (bottom) simulated. 85
5.2 Intensity histograms for grassy field showing noise characteristics of data. A good fit using a Rayleigh distribution was obtained for both the simulated DIRSIG data and the Gotcha field data. This is consistent with the noise distribution expected for coherent imaging modalities. (left) DIRSIG simulated, (right) Gotcha. 86
5.3 RCS plots for trihedral targets: (a) TR1 (b) TR2 (c) TR3 and (d) TR4. The target labels TR1-TR4 correspond to those in figure 4.2. The DIRSIG derived RCS’s consistently underestimate the observed return, likely due to the absence of a physical optics capability. 88
5.4 RCS plots for trihedral and dihedral targets: (a) TR5 (b) TR6 (c) DR2 and (d) DR6. The target labels TR5-DR6 correspond to those in figure 4.2. The DIRSIG derived RCS’s consistently underestimate the observed return, likely due to the absence of a physical optics capability. This effect is especially pronounced for the dihedral targets which have small acceptance angle in elevation. Also absent in the simulated dihedral RCS’s are sidelobes, which require physical optics to be modeled.

5.5 View of finite cylinder at different aspects, as measured from plane normal to axial direction: (a) 0° (b) 20° (c) 40° (d) 80°. Note the increasing view of the endcap and decreasing view of the cylindrical portion.

5.6 Modeled X-band power line RCS compared to anechoic chamber measurements for 1.27 cm diameter (ka = 2.5) smooth cylinder: (top-pair) using Physical Optics and Physical Theory of Diffraction exclusively (bottom-pair) using exact smooth cylinder 2-D diffraction coefficient for cylindrical portion and PO/PTD for endcaps (left-pair) HH channel (right-pair) VV channel. Error bars on the measured data are scaled by ± 1 standard deviation. For the simulated data, error bounds depict the $O(k \cdot a)^{-1}$ error associated with using a stationary phase technique to evaluate the contour integral along the edge adjoining the endcap and cylinder.

5.7 Modeled X-band power line RCS compared to anechoic chamber measurements for 167.8 MCM (ka = 2.4) copper power line: (top-pair) using Physical Optics and Physical Theory of Diffraction exclusively (bottom-pair) using exact smooth cylinder 2-D diffraction coefficient for cylindrical portion and PO/PTD for endcaps (left-pair) HH channel (right-pair) VV channel. Error bars on the measured data are scaled by ± 1 standard deviation. For the simulated data, error bounds depict the $O(k \cdot a)^{-1}$ error associated with using a stationary phase technique to evaluate the contour integral along the edge adjoining the endcap and cylinder.

5.8 Modeled X-band power line RCS compared to anechoic chamber measurements for 556.5 MCM (ka = 4.4) aluminum power line: (top-pair) using Physical Optics and Physical Theory of Diffraction exclusively (bottom-pair) using exact smooth cylinder 2-D diffraction coefficient for cylindrical portion and PO/PTD for endcaps (left-pair) HH channel (right-pair) VV channel. Error bars on the measured data are scaled by ± 1 standard deviation. For the simulated data, error bounds depict the $O(k \cdot a)^{-1}$ error associated with using a stationary phase technique to evaluate the contour integral along the edge adjoining the endcap and cylinder.
5.9 Modeled X-band power line RCS compared to anechoic chamber measurements for 954 MCM ($ka = 6.1$) steel & aluminum power line: (top-pair) using Physical Optics and Physical Theory of Diffraction exclusively (bottom-pair) using exact smooth cylinder 2-D diffraction coefficient for cylindrical portion and PO/PTD for endcaps (left-pair) HH channel (right-pair) VV channel. Error bars on the measured data are scaled by $\pm 1$ standard deviation. For the simulated data, error bounds depict the $O(k \cdot a)^{-1}$ error associated with using a stationary phase technique to evaluate the contour integral along the edge adjoining the endcap and cylinder.

5.10 Modeled X-band power line RCS compared to anechoic chamber measurements for 1431 MCM ($ka = 7.0$) steel & aluminum power line: (top-pair) using Physical Optics and Physical Theory of Diffraction exclusively (bottom-pair) using exact smooth cylinder 2-D diffraction coefficient for cylindrical portion and PO/PTD for endcaps (left-pair) HH channel (right-pair) VV channel. Error bars on the measured data are scaled by $\pm 1$ standard deviation. For the simulated data, error bounds depict the $O(k \cdot a)^{-1}$ error associated with using a stationary phase technique to evaluate the contour integral along the edge adjoining the endcap and cylinder.

5.11 Modeled C-band power line RCS compared to anechoic chamber measurements for 1.27 cm diameter ($ka = 1.3$) smooth cylinder: (top-pair) using Physical Optics and Physical Theory of Diffraction exclusively (bottom-pair) using exact smooth cylinder 2-D diffraction coefficient for cylindrical portion and PO/PTD for endcaps (left-pair) HH channel (right-pair) VV channel. Error bars on the measured data are scaled by $\pm 1$ standard deviation. For the simulated data, error bounds depict the $O(k \cdot a)^{-1}$ error associated with using a stationary phase technique to evaluate the contour integral along the edge adjoining the endcap and cylinder.

5.12 Modeled C-band power line RCS compared to anechoic chamber measurements for 167.8 MCM ($ka = 1.2$) copper power line: (top-pair) using Physical Optics and Physical Theory of Diffraction exclusively (bottom-pair) using exact smooth cylinder 2-D diffraction coefficient for cylindrical portion and PO/PTD for endcaps (left-pair) HH channel (right-pair) VV channel. Error bars on the measured data are scaled by $\pm 1$ standard deviation. For the simulated data, error bounds depict the $O(k \cdot a)^{-1}$ error associated with using a stationary phase technique to evaluate the contour integral along the edge adjoining the endcap and cylinder.
5.13 Modeled C-band power line RCS compared to anechoic chamber measurements for 556.5 MCM ($ka = 2.2$) aluminum power line: (top-pair) using Physical Optics and Physical Theory of Diffraction exclusively (bottom-pair) using exact smooth cylinder 2-D diffraction coefficient for cylindrical portion and PO/PTD for endcaps (left-pair) HH channel (right-pair) VV channel. Error bars on the measured data are scaled by $\pm 1$ standard deviation. For the simulated data, error bounds depict the $O(k \cdot a)^{-1}$ error associated with using a stationary phase technique to evaluate the contour integral along the edge adjoining the endcap and cylinder. . . . . . . . . . . 101

5.14 Modeled C-band power line RCS compared to anechoic chamber measurements for 954 MCM ($ka = 3.0$) steel & aluminum power line: (top-pair) using Physical Optics and Physical Theory of Diffraction exclusively (bottom-pair) using exact smooth cylinder 2-D diffraction coefficient for cylindrical portion and PO/PTD for endcaps (left-pair) HH channel (right-pair) VV channel. Error bars on the measured data are scaled by $\pm 1$ standard deviation. For the simulated data, error bounds depict the $O(k \cdot a)^{-1}$ error associated with using a stationary phase technique to evaluate the contour integral along the edge adjoining the endcap and cylinder. . . . . . . . . . . 102

5.15 Modeled C-band power line RCS compared to anechoic chamber measurements for 1431 MCM ($ka = 3.5$) steel & aluminum power line: (top-pair) using Physical Optics and Physical Theory of Diffraction exclusively (bottom-pair) using exact smooth cylinder 2-D diffraction coefficient for cylindrical portion and PO/PTD for endcaps (left-pair) HH channel (right-pair) VV channel. Error bars on the measured data are scaled by $\pm 1$ standard deviation. For the simulated data, error bounds depict the $O(k \cdot a)^{-1}$ error associated with using a stationary phase technique to evaluate the contour integral along the edge adjoining the endcap and cylinder. . . . . . . . . . . 103

5.16 Qualitative Image Comparisons: (left) AFRL Gotcha (right) simulated (top) platform at top-right (bottom) platform at bottom-left. . . . . . . . . . . 105

5.17 Quantitative RCS Comparisons: (left) HH channel (right) VV channel (top) platform at top-right (bottom) platform at bottom-left. . . . . . . . . . . 107

5.18 Image chip of isolated transmission lines when the platform was at (a) top-right and (b) bottom-left of image. Four distinct signals are apparent in the latter image even though only three transmission lines were observed on the telephone pole (plus the already accounted for single bundle of communication wires). This was observed for all 8 passes. . . . . . . . . . . 109

5.19 RCS Comparisons assuming no sag: (left) HH channel (right) VV channel (top) platform at top-right (bottom) platform at bottom-left. . . . . . . . . 110
List of Tables

4.1 AFRL platform parameters used for the DIRSIG simulation. . . . . . . . . 72
4.2 Material reflectivity parameters used for the DIRSIG simulation. . . . . . . 75

5.1 X-band results summary. . . . . . . . . . . . . . . . . . . . . . . . . . . . . 98
5.2 C-band results summary. . . . . . . . . . . . . . . . . . . . . . . . . . . . . 104
5.3 AFRL Gotcha results summary. . . . . . . . . . . . . . . . . . . . . . . . . 108
List of Major Symbols

\( a \) Powerline diameter.

\( \mathbf{d} \) 2D diffraction coefficient. For backscattering from an infinitely long, perfectly conducting 1D object, this quantity is also equal to the Incremental Length Diffraction Coefficient.

\( \mathbf{D} \) 3D diffraction coefficient.

\( \mathbf{E}^i \) Incident field, oriented along the polarization direction.

\( \mathbf{E}^s \) Scattered field, oriented along the polarization direction.

\( k \) Wavenumber.

\( \mathbf{k} \) Wavevector.

\( k \cdot a \) Wavenumber-diameter product.

\( \Psi \) Obliquity angle of incident/scattered radiation, measured from plane perpendicular to the powerline’s axial direction.

\( \beta \) Obliquity angle of incident/scattered radiation, measured from the powerline’s axial direction.

\( R \) Distance from receiver to target.

\( \mathbf{R} \) Distance from receiver to target, oriented along propagation direction.

\( r \) Distance from a point on the powerline to a scene reference point.

\( \sigma \) Radar Cross Section.
Chapter 1

Introduction

Detecting powerlines is an important remote sensing application. The information gleaned from power line detection can be used to characterize an area’s power infrastructure. It can also be used in hazard avoidance applications for low flying aircraft. In both cases, a potential application can be found in humanitarian relief for natural disasters. Specifically, having the ability to remotely sense powerlines can give decision-makers a quick survey of the damage to a power grid over a wide area. It can also aid pilots of low flying search-and-rescue aircraft in mission planning. In the optical regime, powerlines do not often exhibit a strong return due to their smallness and the radiometry of the problem. In the radar regime, however, powerlines can be a very prominent feature in the processed imagery for certain viewing geometries. Since the diameter of a typical power line is often on the order of only a few wavelengths for radar, a significant portion of the return is due to diffraction.

This research focused on modeling the radar return from power lines. There are numerous benefits that result from having an accurate model for powerline returns. One benefit is the ability to determine if a conceptual system will be able to perform powerline detection given the system’s operating parameters, such as carrier frequency and flight path. This benefit gives program managers the ability to predict if a given system design can meet certain mission requirements with regards to powerline detection, avoiding significant hardware expense. Similarly, a second benefit would be to determine if an existing system can perform the mission given its operating parameters. The enhanced capability of a space-borne SAR system, for example, would illustrate this benefit. If the model predicts powerline detection will be more effective on one pass than another, this information can be used by mission planners to optimize the sensor’s tasking. Finally, a model of the powerline return for an existing system and operating parameters can be used to determine an \textit{a priori} matched filter for post-processing on actual data. This is a
common application for forward modeled data.

There has been much work recently on the development of software which simulates the SAR signal for large urban scenes. An overview of the research in this area performed to date is provided in Chapter 2. In particular, an experimental capability has been added to the Digital Imaging and Remote Sensing Image Generation (DIRISIG) tool for the radar imaging modality [7]. DIRISIG is a tool widely used in the Electro-optic/Infra-red (EO/IR) modalities which produces radiometrically accurate, physically based imagery [8]. DIRISIG is shown to simulate the SAR phase history for a given scene and operational parameters and is unique to other software tools that have the same focus on modeling the SAR signal for complex scenes. There is also a plethora of tools available for modeling the Radar Cross Section of an object. One popular example is Xpatch [9], a tool used by the Air Force research labs which uses the Shooting and Bouncing of Rays (SBR) technique. Unfortunately, as will be explained later, SBR does not perform well for optically thin objects such as powerlines at typical radar wavelengths. One effort made by Sarabandi, K. [4] resulted in a Method of Moments (MoM) approach for the specific purpose of modeling powerline RCS’s. At millimeter-wave frequencies, evaluation of the source current integral was simplified by making a physical optics approximation. However, this approximation breaks down for wavenumber-diameter products $\lesssim 1$. All previous work considered, a gap is apparent for efficiently yet accurately modeling the radar return of powerlines in this regime.

A brief overview of the methods used to model the scattering of Electromagnetic (EM) waves is provided in Chapter 3. This chapter begins by reviewing how a RCS is defined. The RCS of an object describes how efficiently it scatters radiation given its shape, material properties, the viewing angle, and the angle of incidence of the incoming radiation. The remainder of Chapter 3 reviews the different ways to calculate how an object scatters EM radiation, with a concentration on how diffraction is modeled. As mentioned earlier, the dominant scattering mechanism by powerlines is due to diffraction. Diffraction is a phenomenon that describes how electromagnetic radiation scatters off obstructions. This phenomenon is most readily observed when the characteristic length of the scattering object is on the order of a few wavelengths. In the case of powerlines, typical diameters are on the order of a few to tens of centimeters. Since the wavelength of radiation in the radar regime is also typically on the order of a few centimeters, diffraction becomes the dominant scattering mechanism.

In order to provide an efficient, yet accurate, way of measuring powerlines, an Incremental Length Diffraction Coefficient (ILDC) approach was used in this research. The ILDC approach is traditionally used to model how edges scatter radiation in three dimensions [10]. The approach has been adapted in this research for powerline modeling. The effort began with creating a physical model that accounted for both material and
geometric properties. The powerline was assumed to be a perfectly conducting smooth cylinder with sag described by a catenary. The two dimensional problem for scattering into the forward cone was then solved to yield a 2D diffraction coefficient. A 3D diffraction coefficient was then derived using the ILDC approach developed by Mitzner. This 3D coefficient was then used to generate both powerline RCS’s that were later compared to experimentally measured RCS’s. The 3D diffraction coefficient was also used to generate a SAR phase history. The scene resulting from this phase history was then compared to an actual circular SAR collection from the Air Force Research Labs (AFRL) [5].

The methods used to validate this ILDC approach are described in Chapter 4. First, a generic assessment of DIRSIG’s radar modality was performed. The scene used for the DIRSIG simulation roughly matched that of the AFRL field collection and included powerlines and canonical targets such as dihedrals and trihedrals. The parameters used for the simulation closely matched what was provided in the AFRL auxiliary data. The assessment included not just an investigation of whether or not DIRSIG could model powerlines, but also how well it performed against the canonical targets. Additionally, DIRSIG’s ability to simulate the noise present in a SAR image was assessed. Next, the ILDC approach for generating 3D diffraction coefficients for powerlines was validated against data taken in an anechoic chamber by the University of Michigan [6]. Included in this discussion is a description of how the experiment was setup and calibrated. Finally, the simulated powerline phase history was compared to that observed in the field-measured AFRL data [5]. The method used to perform this comparison is also provided.

The results for each validation effort are provided in Chapter 5. For the DIRSIG assessment, good scene-wide qualitative agreement between the simulated and field extracted imagery was observed. The simulated imagery also exhibited Rayleigh distributed noise, as expected. Upon closer inspection, however, it was shown that DIRSIG was not capable of accurately modeling the RCS’s of the smaller canonical targets since it does not have a physical optics capability. Powerlines were also absent from the imagery due DIRSIG’s use of a geometric optics approximation which is inadequate for optically thin objects. For the experimentally measured RCS’s from the University of Michigan, overall agreement between the simulated and measured data was observed, with errors falling within the experimental uncertainty. A few significant discrepancies were noted however, especially in the VV channel. A discussion is provided on why the smooth cylinder approximation used in this research would lead to errors in the VV channel. Similar results were obtained for the AFRL data. Chapter 5 concludes with a discussion on sources of error and how they may be mitigated in future work. Suggested follow-on efforts would include a correction for the 2D diffraction coefficient in the VV channel as well as additional field data with precisely known ground truth that focuses on the powerline modeling problem.
Chapter 2

Background

The methods used to simulate SAR imagery are significantly different from those used to simulate EO/IR imagery. For those most familiar with how a traditional optical system works, the most striking difference is the way in which an image is formed. In conventional optics, the spatial aperture collecting the signal is two dimensional. For SAR, the spatial aperture is the one-dimensional flight path of the platform. The temporal length of the received signal forms the second dimension. The time domain signal, before demodulation, is a measurement of the phase of the electric field. This is unlike conventional optics where a detector element usually measures power, which is proportional to the time average of the electric field squared. Since SAR systems measure and record the phase of the electric field, signal processing needs to be applied to form an interpretable image. This signal processing is usually done at the hardware level or in post-processing. This is in contrast to how an image is formed in optics, where the physics of light propagation does all of the signal processing. For example, in a single lens system, the phase of the electric field across the aperture is mixed with the lens thickness function and is then matched filtered by the Fresnel convolution kernel. All of this signal processing is transparent to users of traditional optical imaging systems.

The variability of signal processing methods that can be used to process SAR data presents difficulties for methods attempting to simulate SAR imagery, using conventional Fourier optics. Measurements made by SAR systems are further up the imaging chain than that of optical systems. For a SAR system, the measurement of the phase of the electric field is made at the aperture level, whereas for traditional optics, measurements are made after “Fresnel post-processing” and measuring the time average of the electric field squared. As a result, the system transfer function of a SAR system is more complex and also much more customizable than that of a traditional optical system. Once an optical system is built, the impulse response usually remains static. For SAR systems, the
impulse response can change from one collection to another, based on a variety of factors. These include, but are not limited to, the length of the flight path, the characteristics of the transmitted signal, and the different kinds of reconstruction algorithms that were applied to the recorded phase history. Additionally, since the reflectivity of a target can vary greatly across the length of the SAR aperture, applying a shift-invariant impulse response to a reflectivity map can lead to inaccurate results. For example, the impulse response of a perfect point reflector will be much different from the point spread function (PSF) for a point reflector which has a small acceptance angle. Along azimuth, the spatial frequency region of support for the perfect reflector will be determined by the length of the full synthetic aperture, whereas the target with the low acceptance angle will be smeared in azimuth, since only a fraction of the full aperture will have contained signal.

In addition to the differences in the image formation process, there are other more subtle differences between conventional EO/IR and radar imaging modalities. Since SAR involves coherent illumination of the target area, speckle is the dominant source of noise. Speckle is observed when the amplitude of the surface roughness is on the order, or less than, a wavelength and the correlation length across the surface is much less than a resolution cell. When these conditions are met, the observed intensity of a single resolution cell is determined by a sum of impulse responses with random phases [11].

Effects due to diffraction are also more pronounced in SAR since radar wavelengths are longer than EO/IR wavelengths. Early SAR simulators modeled diffraction effects in SAR imagery in a statistical sense, similar to how speckle is modeled [3]. This worked fairly accurately at low resolution [3]. As the resolution of SAR systems has increased in recent years, the need to use more accurate diffraction models has increased [3]. This is especially important in urban scenery where narrow streets can act as wave guides, rectangular structures can produce unattended diffractions, and strong scattering from man-made surfaces that have edges or corners can dominate the return signal [3].

The methods for simulating SAR imagery or RCS’s presented in the following sections each have their own specialty. As of yet, there is no general purpose radar simulator. One of the more widely used SAR simulators in the community is GRECOSAR [12, 13, 14, 15]. GRECOSAR uses high-frequency approximation techniques such as geometric optics, physical optics, physical theory of diffraction, and incremental length diffraction coefficients to generate the SAR signal. This is done on a pulse-by-pulse basis. GRECOSAR has been shown to produce radiometrically accurate results for naval vessels. Research on how well this tool models complex urban scenery is on-going but initial results are promising [15]. Another well known SAR simulator is SARAS [16, 1]. This tool builds up the SAR signal in the frequency domain using a surface model that is discretized into rectangular facets. The scattering contribution from each facet is determined by the surface properties of the material and the radiation diagram predicted by physical optics.
Later versions of the tool could model multiple scattering by using geometric optics for all but the final bounce back to the receiver. Physical optics was then used to calculate the scattered field on the final bounce.

GRECOSAR and SARAS are examples of code that have been specifically written to simulate the SAR phase history for a given scene. Some examples of software tools traditionally used for modeling in the optical regime that have been adapted for SAR simulation are POV-RAY [2] and DIRSIG [7, 8]. One of the effects commonly seen in urban imagery is displacement of a signal due to multiple bounce returns. This effect manifests itself when a pulse bounces multiple times before finally coming back to the receiver. Since the range of the pulse is determined by the time of flight, the signal is often located farther downrange than the actual location of the last position the pulse was scattered from. This can lead to reduced image interpretability. Ray-tracers such as POV-RAY have been used to create efficiently images which replicate the effect of multiple bounces, but are otherwise radiometrically inaccurate. The intended use of this program is to aid analysts in interpreting SAR imagery which contain large, complex structures. Another tool adapted to simulating SAR imagery is DIRSIG. While DIRSIG is a mature and widely used tool for producing radiometrically accurate imagery in the EO/IR regime, its radar modality is still experimental at this stage. DIRSIG takes the geometric optics approach to modeling the return signal and does not incorporate physical optics or any other high-frequency technique. The signal is built up on a pulse-by-pulse basis by convolving the transmitted pulse with delta functions centered at the time of flight of each ray shot into the scene. The result is a tool which is efficient, models multiple scattering, and accounts for targets with dynamic, aspect-dependent reflectivities, but is limited by the geometric optics approximation.

For achieving a very high degree of accuracy, the computationally intensive Finite Difference Time Delay (FDTD) [3] method has also been used. Due to the complexity of this approach and the stringent hardware requirements needed for implementation, simulations have been limited to simple objects commonly found in urban scenery. While this method would be too complicated to use for modeling a complex scene given current hardware capabilities, it has value in obtaining high fidelity results for simple, common objects.

Another accurate but computationally intense method found in the literature uses the Method of Moments (MoM) [17]. More specifically, work done by Sarabandi, K. and Moonsoo, P. demonstrated how MoM could be used to model the source currents on the surface of a powerline [4]. In the special case of millimeter-wave radiation, they simplified the source current integral to a perturbation series that did not require the use of the computationally intensive MoM. The source currents in the millimeter-wave regime were calculated using physical optics for the 0th-order term in the perturbation series. The
problem was further simplified for periodic structures, such as helically wound cables found in powerlines. In such cases, a periodic Green’s function could be used to calculate the scattered field from the source currents. This requires only one period (or helix pitch for powerlines) for the analysis rather than the entire structure. It was demonstrated that this technique could accurately model the Bragg scattering peaks observed in measured data at a frequency of 94 GHz.

Finally, the Xpatch\textsuperscript{®} toolkit is worth mentioning. Xpatch is a ubiquitous radar modeling tool that has a number of capabilities. For estimating source currents, it uses the Shooting and Bouncing of Rays (SBR) technique. This technique requires a high-frequency assumption. Consequently, it will not provide accurate results for surfaces whose wavenumber-diameter product is \( \lesssim 10 \). This includes typical powerlines at X-band frequencies and lower.

2.1 GRECOSAR

The Graphical Electromagnetic Computing (GRECO) tool is a graphical modeling tool which computes the RCS of a target \cite{12} using a variety of high-frequency techniques. These include Physical Optics, Method of Equivalent Currents, Physical Theory of Diffraction, and Impedance Boundary Condition (IBC) techniques.

Processing the RCS of a target begins with creating a 3D CAD model of the target. An image of this target, as viewed from a given observation point, is rendered onto a workstation’s screen by the graphics hardware. Assuming a monostatic collection, each pixel on the screen corresponds to a point illuminated by the incident radiation. All other target points are assumed to be in the shadow region and, according to high-frequency approximation techniques, do not contribute to the RCS. Typically for RCS modeling, the target’s surface is broken up into a series of facets and wedges. GRECO is unique in that it describes a target using parametric surfaces. For each target pixel rendered, the position \( x, y, z \) and surface normal \( n_x, n_y, n_z \) components are derived by using non-uniform rational B-splines (NURBS) for interpolation \cite{18}. Once the surface parameters are derived for each illuminated point in the image, the CPU performs the electromagnetic scattering computation.

The physical optics integral is computed by treating each pixel as a rectangular aperture whose far-field radiation pattern is dependent on the corresponding surface normal and then summing the contributions for every pixel in the image. Impedance Boundary Conditions are then applied to the result. This is done by separating the incident field into components parallel and normal to the plane of incidence and multiplying by the respective Fresnel reflection coefficients. Finally the scattering contributions due to edge currents are added using the physical theory of diffraction. Edge pixels are identified by
discontinuities in the surface normal. For a given wedge angle, incidence angle, and polarization, there is a corresponding diffraction coefficient. Diffraction coefficients act in much the same way as reflection coefficients as they relate the incident field amplitude to the scattered field amplitude for a given geometry and polarization. A more detailed discussion of diffraction coefficients is provided in 3.2.7. The line integral along the edges is then computed by summing the contributions from each edge pixel. Results shown in [12] demonstrated a high degree of accuracy when compared to numerical solutions. Using only physical optics, accurate RCS’s for non-stealth targets could be obtained.

GRECOSAR is a code that uses GRECO to compute the RCS of objects in a given scene and simulates the SAR signal [13]. For each azimuth position, an image of the scene from the viewpoint of the platform is rendered. GRECO computes the frequency dependent RCS in the frequency domain. This response is applied to the time domain signal by means of an inverse Fast Fourier Transform along the range dimension.

The GRECO code has been exhaustively validated for both canonical and complex targets [14]. GRECOSAR is relatively experimental but has been used in studies to evaluate SAR images of fisheries [13] and individual naval vessels [14]. In the literature [13, 14], much of the phenomenology observed in real world scenarios was captured by the simulator [15]. Recently, GRECOSAR’s ability to model urban scenes has also been evaluated. The study was mostly limited to an image of a box of gypsum on top of a perfectly conducting plane. While examples of GRECOSAR’s ability to model more complex urban scenes has not been found in the literature, initial results for easily validated simple scenes were promising [15].

2.2 SARAS

The Synthetic Aperture Advanced Simulator (SARAS) is code that generates the raw SAR signal using a physical optics approach [16]. The form of the physical optics integral for backscattering, discussed later, is given by [16]:

\[
E_s(R) = \frac{ik \exp(-ikR)}{4\pi R} E_0 \left( \mathbf{I} - k\hat{k} \right) \cdot \mathbf{F}(\hat{n}) \int_A \exp(2ik \cdot \rho) dA
\]  

(2.1)

where \( R \) is the distance from the barycenter of a facet on the scattering surface and the platform, \( \mathbf{I} \) is a 2x2 identity matrix, \( \mathbf{F} \) is a function related to the Fresnel reflection coefficients, and the integral term is the re-irradiation diagram of the facet. For perfectly smooth rectangular facets, this irradiation diagram results in a 2-D sinc function. For rough surfaces, the irradiation diagram is broadened. If the dimensions of the facet are much less than the correlation length, this broadening can be ignored. Usually, however,
this method results in strict sampling requirements. One way to avoid this is to create facets whose characteristic length is much larger than the correlation length and to use an approximation for the re-irradiation diagram. Possible approximations include $\cos \delta$, $\cos^2 \delta$, and $\exp\left[-\left(\frac{\delta}{\delta_0}\right)^4\right]$ where $\delta$ is the angle between the line-of-sight and surface normal and $\delta_0$ is given by the user. More accurate, numerical solutions can also be used for the irradiation diagram. For the $F(\hat{n})$ term, the height of the facet vertices is approximated by assuming a normal distribution about the mean height and a deviation from the nominal surface normal is derived using these modified vertex positions. This results in speckle in the observed image, as expected for coherent illumination of stochastic surfaces.

Once $E_s$ has been computed for each facet, a derived parameter referred to as the reflectivity map $\gamma(x,y)$ is calculated for each facet location. The value of $\gamma(x,y)$ is equal to the amplitude of the field scattered by a small area centered on $(x,y)$, divided by that area and the incident field amplitude [1]. The quantity $E_s$ can be calculated based on a single bounce assumption [16] or account for multiple bounces [1]. As shown in the literature [1], accurate results can be obtained by using geometric optics for all but the last bounce, at which point physical optics (eq. 2.1) is used for the final propagation step to compute the field scattered back to the receiver.

After calculating the reflectivity map $\gamma(x,y)$, the raw signal is built up assuming a transmitted linear FM signal. Defining a temporal variable $t' = t - t_n - 2R_0c$ where $t$ is the fast time and $t_n$ is the slow time, the heterodyne signal is computed using the following formula:

$$h\left(x' = vt, r' = \frac{ct'}{2}\right) = \int \int \gamma(x,y) \cdot g(x' - x, r' - r; x, r)$$

(2.2)

where

$$g(x' - x, r' - r; x, r) = w^2 \left(\frac{x' - x}{X}\right) \text{rect}\left[\frac{r' - r}{c\tau/2}\right] \exp(i\phi)$$

(2.3)

$$\phi = -\frac{4\pi}{\lambda} \Delta R + \frac{\alpha}{2} \left(t' - \frac{2r}{c} - \frac{2\Delta R}{c}\right)^2$$

(2.4)

and $w$ is the illumination footprint. Noting that equation 2.2 represents a convolution, efficient calculation of the raw signal can be performed via the Fast Fourier Transform and filter theorem:

$$H(\xi, \eta) = G(\xi, \eta)\Gamma(\xi, \eta)$$

(2.5)

where the capital letters represent the Fourier transform of the respective lowercase terms in 2.2 and $(\xi, \eta)$ represent the spatial frequencies of the azimuthal and range coordinates, $(x, r)$, respectively.

SARAS has been used in a number of studies on modeling the signal from urban structures. One of the most frequently encountered canonical targets in urban scenes is
the dihedral. Returns from dihedral structures, such as where a wall intersects with the ground, appear bright in SAR imagery due to multiple bouncing of rays. For wall-ground intersections, the wall can be approximated as a smooth surface with a strong specular component predicted by physical optics, whereas the ground can be approximated to first order as having a random surface height about some mean [19]. In work done by Franceschetti, these approximations were used to model the multiple bouncing of rays from urban structures [1]. The resulting SAR imagery, shown below, exhibited many effects one would expect from a dihedral return such as a bright return from the corner and higher order returns farther along the range direction.

![Image of two buildings with different material properties generated using SARAS. Higher order wall-ground reflections are apparent for both buildings [1].](image)

Figure 2.1: Image of two buildings with different material properties generated using SARAS. Higher order wall-ground reflections are apparent for both buildings [1].

A comparison of the geometric optics/physical optics (GO-PO) predicted RCS’s for dihedrals was made to experimental data, as well as to a relatively more exact numerical method that used PO for more bounces and PTD for the edges [20]. Results indicated that the GO-PO method performed reasonably well for dihedral angles of around 90°.
2.3 POV-Ray

In a paper by Stefan Auer et. al. [2], a distinction is made between SAR simulators that focus on radiometric quality and those that focus on geometric quality. The previously mentioned simulation tools, GRECOSAR and SARAS, are examples of algorithms that focus on radiometric quality. These computationally intensive algorithms are limited by the level of detail allowed for the 3D models [2]. For scenes that contain complex geometrical structures, using an approach that sacrifices radiometric accuracy for geometric accuracy may be preferable. The software package used in the work by Auer et. al. [2] was POV-Ray, an open-source ray tracing algorithm. Examples given in the literature [2] include modeling the scattering from the Wynn Hotel and the Eiffel Tower, shown below:

![Figure 2.2: (Left) actual and (Right) simulated images of the Wynn Hotel. Special features in each image are labeled with corresponding letters [2].](image)

Modeling with POV-Ray begins by creating a model of the scene. Illumination is simulated with a cylindrical light source. This approximation does not account for $1/r^4$ attenuation, wavefront curvature, or the transmitting antenna’s gain pattern. An orthographic camera is placed at the source location and oriented along the slant range vector. Scene sampling is handled by shooting rays from the camera and letting them bounce about the objects in the scene. The first ray is called the primary ray and subsequent rays...
are called secondary rays. The contribution from each bounce is weighted by a reflection coefficient. For the sake of simplicity and to reduce computational complexity, the surface reflectance was modeled as lambertian with a specular lobe. Next, the contributions from each ray are mapped to an azimuth and range coordinate. For each secondary ray, an additional ray is created that is parallel to the primary ray but travels in the opposite direction from the intersection point back towards the camera. The intersection of this additional ray with the camera is labeled the ray origin. The azimuth coordinate in the image plane is calculated by taking the mean value of the azimuth coordinate for the ray’s origin and the azimuth coordinate for the ray’s pixel location. Range is derived from the depth value of the intersection point. The final output of the system is a reflectivity map of the scene in azimuth and range coordinates. This output is obtained by superimposing a rectangular grid over the irregularly spaced returns from each ray and performing an interpolation step.

Simulations were made on a skyscraper, the Wynn Hotel, and the Eiffel Tower. In many cases the brightness of the reflectivity map did not match well with the actual data. This was especially the case along building edges where a significant diffracted return would be expected, or at locations on the building’s surface that contained dihedrals/trihedrals not modeled by the smoothed version in the ray tracing simulator. The main utility, however, was to be able to classify features in the reflectivity map by the number of bounces and the macroscopic structures from which the signal was scattered. This improved image interpretability, as it identified why some features present in the SAR imagery, were absent from traditional overhead imagery taken by a camera.

2.4 DIRSIG

The Digital Imaging and Remote Sensing Image Generation (DIRSIG) tool is a physics-based ray-tracing algorithm used to model imagery across the electromagnetic spectrum for remote sensing applications [8]. Its ability to produce radiometrically accurate images for single-band, multi-spectral, and hyperspectral modalities is very well established. DIRSIG also has a mature LIDAR modeling capability [8]. DIRSIG’s ability to model SAR imagery is still experimental at this stage. No work has been done that validates DIRSIG’s SAR simulation capability.

Being a ray-tracing algorithm, DIRSIG’s ability to model electromagnetic scattering is limited to high-frequency approximation techniques. As will be shown in sections 3.2.3, 3.2.6, and 3.2.7, ray-tracers, although treating light as a particle, can still model diffraction using high-frequency techniques. Most imaging modalities traditionally modeled by DIRSIG at shorter wavelengths do not exhibit strong diffraction features and subsequently, no significant effort has been made to account for diffraction within DIRSIG. The addi-
tion of these techniques adds another layer of computational complexity, especially when imperfectly conducting surfaces or higher-order bounces are involved.

Modeling in DIRSIG starts with the creation of a scene. Scenes are usually built in the ray tracing software Blender and a .ODB file is created. For each object ID in the scene, the material properties are specified in a .mat (material) file. The next step is to specify the pertinent system parameters and viewing geometry. For a traditional DIRSIG simulation, this can usually be done through a Graphical User Interface (GUI). To date, all of these parameters have to be provided in xml tables as specified in the DIRSIG SAR Modality Handbook [7]. DIRSIG then models the SAR return on a pulse-by-pulse basis. For a given pulse, particles are shot into the scene from the transmitter using the LIDAR package. In single-pass mode, the receive computations are performed in the same pulse simulation as the transmitted one. In two-pass mode, a hit map is created from the intersections, stored to file, and a separate simulation is run for the receive computations. Using two-pass mode lessens numerical noise [7]. For each return, the transmitted waveform is convolved with a delta function delayed by the time-of-flight. The fast-time signal for a given pulse is then created by summing each of the returned waveforms. The amplitude of each return is also scaled by the Fresnel reflection coefficients. This is repeated for each pulse until the entire time domain phase history for the user-defined collection is obtained.

There are a number of capabilities absent from DIRSIG normally found in a physically based SAR simulator. As mentioned earlier, DIRSIG uses the geometric optics approximation and has no physical optics or edge diffraction modeling capability. Consequently, scattering is modeled using a Bi-directional Reflectance Distribution Function (BRDF), which is independent of target shape. This is adequate for rough-textured distributed targets but not for smaller targets with smooth reflecting surfaces whose RCS is shape-dependent. An example would be the RCS for a small plate, which results in a 2D sinc pattern that does not conform to a standard BRDF model. Options for the antenna gain pattern are also limited to a few simple cases. Finally, transmitted waveforms are limited to linear FM signals. Despite these limitations, and in some respects because of them, DIRSIG is still able to efficiently reproduce imagery in the radar modality with good scene-wide qualitative agreement. This will be shown later in section 5.1. Additionally, with the way DIRSIG treats textured objects, Rayleigh distributed noise can also be observed in the amplitude data, as expected. This will also be shown later in section 5.1.

The final output of DIRSIG is the complex phase history stored in an Envi .img file. This data can then be processed with user-defined code. It can also be processed using RITSAR, a basic SAR image processing toolbox developed at RIT that is free and open-source and can be downloaded at www.github.com/dm6718/RITSAR.
2.5 FDTD

Many of the approaches discussed thus far involve a high-frequency approximation for the incident field. In geometrical optics, this approximation is applied to the Luneberg-Kline series. As shown in section 3.2.2, only the first term of this series is taken as a solution to Maxwell’s equations in the high-frequency limit. This results in an eikonal equation which leads to the well known laws of reflection and refraction, as well as an additional equation that accounts for amplitude loss with propagation and polarization effects. The Geometrical Theory of Diffraction and Uniform Theory of Diffraction are extensions of Geometrical Optics that describe how rays interact with edges, curved surfaces, and with themselves, for example at a caustic. In physical optics, the high-frequency approximation leads to an assumption that non-uniform currents near an edge or shadow boundary can be neglected. In both physical optics and the physical theory of diffraction, asymptotic forms of diffraction integrals are acquired by making a high-frequency approximation. This permits form solutions for diffraction coefficients without having to compute an integral for each differential diffracting element.

All of the high frequency techniques above are only approximate solutions to Maxwell’s equations. All of these approximations become less accurate at low frequencies. Many of them also do not take into account internal propagations [3] such as physical optics, which assume source currents are constrained to the surface. Obtaining an exact solution to Maxwell’s equations can be practically impossible for arbitrary surfaces where the boundary conditions are difficult to implement in a natural coordinate system. In such cases, where high-frequency techniques are insufficient, and exact closed form solutions cannot be obtained, numerical techniques are frequently used. The two most common techniques are the Method of Moments [17] and Finite Different Time Delay (FDTD) [21].

The FDTD algorithm works by sampling a scene with a Yee lattice [21], creating a source, then modeling propagation by differencing the field in adjacent cells for a given time step, as specified by the scalar rectilinear components of Maxwell’s curl equations. Boundary conditions are imposed by specifying the behavior of certain cells. For example, for a point on the surface of a conductor, the tangential E-field components are always 0. The main drawback to this algorithm is the sampling requirements. In [3], the spatial sample spacing used was $\frac{\lambda}{10}$. Even for manageable scene sizes, the temporal resolution requirements often result in thousands of time steps [3].

The utility of the FDTD algorithm for modeling urban scenes was investigated by Delièrè et. al. [3]. The scene sizes were generally on the order of 10 $m^3$ to 25 $m^3$. Rather than model the signal from a complex scene, the paper focused on modeling how individual canonical surfaces scatter light. This enabled simulations to be run in a reasonable time frame while still providing results that describe the backscattered signal of
common objects in an urban scene. One of the canonical problems investigated was how perfectly conducting plates, with smooth or rough surfaces, scattered radiation. For the smooth surface, the backscattered signal was dominated by diffraction peaks along the edges, whereas for rough surfaces, the signal was dominated by a speckle pattern which appeared across the surface. The statistics of the speckle pattern followed a Rayleigh distribution, as expected [3]. Below is a plot showing the field amplitude for both a smooth and rough plate:

![Figure 2.3: Simulated amplitude of field reflected from both a smooth and rough plate using FDTD [3].](image)

The signal from a wall-ground interface was also investigated. The return showed multiple peaks that correlated with the different orders of reflection that occur. The location of the peaks from this last simulation matched well with what is predicted by geometrical optics.
2.6 Millimeter-wave Physical Optics Model

At Ka-band frequencies and higher, Bragg scattering features can be observed in a powerline’s RCS [22]. This is due to the fact that powerlines are normally comprised of a number of cable strands which are tightly wound in a helical structure, which are periodic in nature. At certain aspect angles, adjacent rays reflected from this periodic surface constructively interfere, leading to multiple orders of peaks in the RCS signature. The locations of these peaks are given by [4]:

$$\theta_n = \sin^{-1} \frac{n\lambda}{2L}$$  \hspace{1cm} (2.6)

Where $n$ is an integer, $\theta$ is the aspect angle measured from a plane normal to the powerline’s axial direction, $\lambda$ and $2L$ is the length of one period. The phenomenon that causes these peaks is the same as the one that gives rise to the Bragg diffraction pattern observed when x-rays are scattered from a periodic crystal lattice. Sarabandi, K. and Moonsoo, P. developed a way to compute the scattered field from powerlines using a method that accounts for Bragg scattering [4]. This method uses an accurate powerline geometrical model, shown below:

Figure 2.4: Geometric powerline model used by Sarabandi, K. [4].
The tangential component of the total magnetic field on the surface of the powerline induces source currents. These source currents can be computed from the incident magnetic field using an integral which can be evaluated numerically via the Method of Moments (MoM). The MoM is a technique which provides accurate results but is computationally intensive at higher frequencies [4]. In order to efficiently evaluate the source current integral without having to use MoM, a high-frequency approximation was made. This allowed the source currents to be computed using a perturbation series instead of MoM. The $0^{th}$ order term in the series was the usual physical optics current given by $J_0(r) = 2(n \times H_i)$. Higher order terms were computed from this $0^{th}$ order approximation using the perturbation series approach. The resulting $n^{th}$ order physical optics current was then used to compute the scattered field using Green’s theorem. For periodic structures, a periodic Green’s function was used and only one period of the powerline was needed for calculating the scattered field. For non-periodic structures, such as powerlines with a significant amount of sag, the normal free space Green’s function was used and the entire lit portion of the powerline was needed for calculation instead of just one period.

This method was able to accurately simulate the RCS’s for a number of cables with various diameters and pitches within $20^\circ$ about normal incidence for measurements taken at 94 GHz. The Bragg peaks in the simulated data lined up with those observed in the measured data. Accuracy was degraded, beyond $20^\circ$ about normal incidence, partly due to surface irregularities not modeled in Figure 2.4. Additionally, the physical optics approach did not take into account the VV contribution from the grooves in the cable [23], resulting in increased error away from normal incidence for the VV channel. Below are plots depicting how the simulated data compared to experimentally measured RCS’s for a variety of powerlines:
Figure 2.5: Simulated versus experimental RCS’s at 94 GHz for 4 different powerlines in the VV channel. Bragg scattering peaks were accurately modeled by a PO approach at millimeter wavelengths [4].

One of the primary drawbacks to using a physical optics approach is that the high frequency approximation breaks down at X-band frequencies and lower for typical powerline diameters. Additionally, physical optics does not account for the VV contribution from the grooves observed in experiment [4, 6] and predicted by theory [23]. While the use of MoM becomes more tractable for modeling powerline RCS’s at lower frequencies, it would still be difficult to implement this for a SAR application involving thousands of pulses. For non-periodic powerline geometries with an appreciable amount of sag, the MoM approach would require calculations to be performed across the entire surface of the powerline as opposed to just one period. This would include the non-lit regions for lower frequencies. Consequently, hardware requirements could be hard to fulfill for existing commercial off-the-shelf desktops.

2.7 Xpatch

A well known RCS modeling toolkit used by the Air Force Research Labs (AFRL) and the Defense Advanced Research Projects Agency (DARPA) is Xpatch® [9]. Among Xpatch’s advertised capabilities are modeling near-field and far-field scattering, rough surface scattering, and diffraction from an edge or point. According to the Xpatch website, diffraction
is modeled using a Shooting and Bouncing of Rays (SBR) technique. As will be discussed later in section 3.2.6, SBR is a high-frequency method which uses ray tracing to predict source currents on a facet-by-facet basis. These source currents are then used to calculate the contribution to the scattered field from the surface of a facet using physical optics (assuming far-field observation) and the contribution from an edge using PTD. In order for SBR to work, either the size of the facet has to be large enough or the bundle of rays has to be dense enough for the incident field to be adequately sampled. This may not be practical for optically thin wires or powerlines. Being a high-frequency technique, SBR is also susceptible to errors in cases where the radius of curvature of the wire is too small. Generally, this is considered to be the case when the wavenumber-diameter product, $k \cdot a$, is $\lesssim 1$ which is where this research is focused.

2.8 Summary

The techniques for simulating SAR imagery and RCS’s presented here each have unique capabilities. Examples of code that have been developed for the express purpose of being a SAR simulator are GRECOSAR and SARAS. Software traditionally used for modeling imagery in the EO/IR regime such as DIRSIG and POV-RAY have also been adapted for efficient modeling in the radar modality. While computationally intensive, FDTD has also been used to calculate the scattering from canonical targets for simple scenes, materials, and geometries. With respect to modeling powerlines, a physical optics based approach was established for millimeter-wave radiation. Xpatch is another tool commonly used for radar modeling. This tool relies on SBR, a high-frequency approximation technique. Unfortunately, SBR breaks down for wavenumber-diameter products which are $\lesssim 10$.

Absent from many of the aforementioned methods are ways to incorporate the return from thin cylindrical objects such as cables and power lines at X-band frequencies and lower. While GRECOSAR uses wedge ILDC’s to calculate the scattering from edges, there is no mention of a capability that models the diffracted return from thin cylindrical objects. The original paper by Mitzner that derived the ILDC for wedges of various angles also contained the solution for thin cylindrical objects [10]. However, this does not appear to be incorporated into any current modeling tool. Work done by Sarabandi, K. and Moonsoo, P. did result in a numerical MoM based approach that could theoretically be applied across all wavelength regimes, but would be practically difficult to implement for real world powerline geometries given the current computational resources of commercial off-the-shelf hardware.
Chapter 3

Theory

In this chapter, a discussion that focuses on the different ways to model electromagnetic interactions with matter are presented. At a top-level, an object’s RCS is often used to describe how well the given object scatters power incident on its surface. The RCS is a function of wavelength, angle of incidence, angle of observation, and the object’s material and geometric properties. It can be derived from the radar range equation, which relates the power transmitted by an antenna to the power received at an antenna after the incident radiation has scattered off an object. The RCS can also be expressed in terms of a ratio of the squared magnitude of the fields incident on the surface and scattered from the surface, as measured in the far-field. This latter definition enables the RCS to be computed from first principles. This chapter begins with a formal definition of the RCS and then moves to a discussion of the different ways to model how objects scatter electromagnetic radiation.

Once traditional methods for modeling electromagnetic scattering have been reviewed and a common terminology established, the method used in this research to model power-line scattering is presented. This effort began with the creation of a physical model which defines the geometric and material properties of the powerline. Once these were established, the 2D problem of scattering into the forward cone (defined by Fermat’s principle for Edge Diffraction, section 3.2.3) was solved. Work done by Mitzner [10] relates the 2D solution to the 3D scattering problem. It was shown that the 3D diffraction coefficient can be expressed as a function of an Incremental Length Diffraction Coefficient (ILDC). For the special case of backscattering, this ILDC is equal to the 2D diffraction coefficient. Once the ILDC-based method for calculating the 3D diffraction coefficient was established, a number of follow-on activities were enabled. This included the simulation of powerline RCS’s as well as the quad-polarization phase history for a SAR collection.
3.1 Radar Cross Section Definition

The radar range equation is commonly used to assess how well a given set of operating parameters for a radar system can meet mission requirements. It relates the transmitted power to the received power through various loss and gain terms. The radar range equation, as defined in Ruck’s RCS Handbook, is given below [24]:

\[
P_r = \left( \frac{P_t G_t}{L_t} \right) \left( \frac{1}{4\pi r_t^2 L_{mt}} \right) \sigma \left( \frac{1}{4\pi r^2 L_{mr}} \right) \left( \frac{G_r \lambda_0^2}{4\pi L_r} \right) \left( \frac{1}{L_p} \right)
\]  (3.1)

where [24],

- \(P_t\) = transmitter power in watts
- \(G_t\) = Gain of the transmitting antenna in the direction of the target
- \(L_t\) = numerical factor to account for the losses in the transmitting system
- \(L_r\) = a similar factor for the receiving system
- \(r_t\) = range between the transmitting antenna and the target
- \(\sigma\) = radar cross section
- \(L_{mt}, L_{mr}\) = numerical factors which allow the propagating medium to have loss
- \(r\) = range between the target and receiving antenna
- \(G_r\) = gain of the receiving antenna in the direction of the target
- \(\lambda_0\) = radar wavelength
- \(L_p\) = numerical factor to account for polarization losses

When the transmitter and receiver are located on separate platforms, the collection geometry is termed bi-static. When the transmitter and receiver are co-located \((r = r_t)\), the collection is termed mono-static. Of particular interest for this research is the \(\sigma\) term in equation 3.1. This is the RCS of the target. The RCS is a commonly used metric that describes how efficiently an object scatters radiation. Rearranging the range equation and solving for \(\sigma\) yields the following expression [24]:

\[
\sigma = 4\pi r^2 \frac{4\pi P_r L_{mr} L_p}{\lambda_0^2 G_r} \left/ \frac{P_t G_t}{4\pi r_t^2 L_t L_{mt}} \right.
\]  (3.2)
The equation above is useful for measuring the RCS of a target in a lab. When calculating the RCS of a target from first principles, however, it is often more convenient to express $\sigma$ as a function of the incident and scattered fields rather than power. The power density of electromagnetic radiation, $W$, is given by the following expression [25]:

$$W = Z_0 \frac{(E \cdot E^*)}{2}$$  \hspace{1cm} (3.3)

Where $Z_0$ is the impedance of free space and $E$ is the electric field vector (oriented along the polarization axis). Substituting this expression for the braced terms in equation 3.2, an alternate way of calculating $\sigma$ is as follows:

$$\sigma = 4\pi r^2 \frac{(E_s \cdot E^s_*)}{(E_i \cdot E^i_*)} = 4\pi r^2 \frac{(H_s \cdot H^s_*)}{(H_i \cdot H^i_*)}$$  \hspace{1cm} (3.4)

where the $i$ and $s$ superscripts refer to incident and scattered fields, respectively. At first glance, the equation above implies that the RCS is a function of distance. This is often undesirable since a more useful target metric would be one that is independent of the receiver distance and is solely dependent on target properties. One can achieve such a metric by first noting the quantity $(E^s \cdot E^{s*})$ falls off as $1/r^2$ in the far-field [25]. Consequently, the more widely used definition of the RCS, which does not depend on receiver distance, is given as follows:

$$\sigma = 4\pi \lim_{r \to \infty} r^2 \frac{(E^s \cdot E^{s*})}{(E^i \cdot E^{i*})} = 4\pi \lim_{r \to \infty} r^2 \frac{(H^s \cdot H^{s*})}{(H^i \cdot H^{i*})}$$  \hspace{1cm} (3.5)

One way of interpreting this result is that $\sigma$ can be thought of as “the area intercepting the target that, when scattered isotropically, produces at the receiver a density that is equal to the density scattered by the actual target” [24]. Importantly, $\sigma$ describes how efficiently a target scatters power but does not provide any insight into how a target affects the phase of the scattered field. Since SAR systems measure the phase of the scattered field, this will become important later on when methods for modeling how radiation is scattered from a power line are discussed.

### 3.2 Modeling Electromagnetic Propagation

There are a number of ways to model electromagnetic propagation. A recent paper by Kravtsov, Y. and Zhu, N. provides a useful survey of the most common approaches used in contemporary research [26]. Provided in this section is a more in-depth look at many of these approaches. First are Maxwell’s equations and some solutions that can be derived directly from them. The case of perfectly conducting cylinders falls into this category. The
modeling of scattering from more complex objects using Maxwell’s equations, however, is often too difficult to do analytically. The main issue with complex objects is the difficulty one has in applying boundary conditions to the scattering surface in a natural coordinate system. Numerical methods such as Finite Difference Time Delay (FDTD) [21] and Method of Moments (MoM) [17] can be used for more complex surfaces. The drawback to these methods is the spatial and temporal sampling requirements become onerous if the scene becomes too large or too complex [3]. Usually, approximations for the solutions to Maxwell’s equations need to be made. One such approximation is geometric optics (GO), the basis for essentially all ray tracing models. While GO does not fully take into account the wave nature of light, one can attach to each ray a “payload” that keeps track of polarization, amplitude, and phase information as the ray travels through space and interacts with objects. In this way, one can account for some of the effects associated with the wave nature of light such as interference and polarization dependent reflection coefficients. GO works well when the wavelength of light is much smaller than the characteristic length of the object. This approximation does not work well for describing how rays scatter off of features such as edges, tips, curved surfaces with small radii of curvature, and caustics. A solution is to use the Geometrical Theory of Diffraction (GTD) developed by Keller [27]. GTD essentially uses exact methods to determine ray amplitudes and what Keller describes as Fermat’s Principle for Edge Diffraction to determine all the possible ray paths (and consequently, phases) for a given geometry.

In addition to geometric-based approaches for modeling electromagnetic propagation, there are also integral-based approaches. One such approach is physical optics (PO). The first step in this approach is to estimate the surface currents on an object. Once the surface currents are known, a vector potential at the surface can be computed. This vector potential can then be used to calculate the observed scattered field at some observation point. This method is closely related to the Kirchhoff diffraction integral in scalar diffraction theory. In both cases the Kirchhoff Approximation is used, which essentially assumes the total field in the shadow region of a scattering object is zero whereas the total field in the illuminated region is explicitly due to that of the incident field. This method is usually more accurate than GO since (1) it’s dependent on the frequency of radiation and (2) it partially accounts for diffraction, the effect obstructions have on radiation, by filtering the angular spectrum of the incident radiation. PO does not, however, take into account perturbations of the source field caused by the edges themselves. In this sense, the Kirchhoff approach is not a complete method for modeling diffraction since it does not accurately model the edge waves. PO works well when: the size of the object is sufficiently large such that the edge-diffracted waves are negligible when compared to the uniform component waves; the observation point is located in the far-field; and the surface of the object is convex with a minimum radius of curvature much larger than a few wavelengths so that
the surface can be segmented into local tangent planes.

In both GTD and PO, a singularity arises when computing the scattered field from shadow boundaries for certain surfaces. Ufimtsev’s Physical Theory of Diffraction (PTD) [28] corrects this by subtracting the scattered field due to uniform currents (those predicted by PO) from the fields predicted by more exact solutions (GTD and the Sommerfeld solution for scattering from a half plane), to obtain the scattered field solely due to non-uniform currents. The difference in the GTD and PO singularities is finite [25]. The main limitation with PTD is it only predicts the scattered field in what is termed the Keller cone [25], defined by Fermat’s Principle for edge diffraction [27]. For scattering directions outside the Keller cone, an extension of PTD developed by Mitzner which uses Incremental Length Diffraction Coefficients (ILDC’s) can be used [25, 10].

The approximations to Maxwell’s equations described here are termed high frequency approximations. They are labeled as such since a high frequency approximation is made to either neglect higher order terms in the Luneberg-Klein series solution to Maxwell’s equations (Geometric Optics) or to obtain analytic approximations for diffraction integrals using a steepest descent or stationary phase method. While the wavelength of radar is much longer than that at optical regimes, these high frequency techniques still yield accurate results for many applications [25]. The main drawback to these techniques, however, is their inability to account for surface traveling (creeping) waves easily [25]. While GTD makes allowance for these creeping waves, it does so only for simple shapes. Even with simple shapes, however, it is still possible for a creeping wave to make many revolutions around a closed surface, emitting radiation as it travels along. These higher order emissions drastically increase the computational complexity. Efficiently modeling the scattering due to creeping rays is an area of ongoing research [25].

While it may seem at first that the integral and geometric based approaches are exclusive to each other, much literature indicates otherwise. One example already covered is GTD, which uses exact solutions derived for canonical targets to determine the amplitude of rays and Fermat’s Principle for Edge Diffraction to determine their phase. Another example is the Modified Equivalent Current Approximation or MECA [29]. Presented in the literature [30] is an extension of PO that is well suited for modeling the scattering of dielectric materials using surface models that segment the object into triangular facets. Basically, GO (a ray tracer) is used to estimate the field incident on a facet by assuming each intersection represented a plane wave having been incident on the surface. The Kirchhoff Approximation is then used to derive the source currents. These in turn can then be used to derive a vector potential, at which point the Physical Optics integral can be used to calculate the scattered field. For a triangular surface or facet, this integral has a closed form solution and the scattered field can be efficiently computed knowing only the angles of incidence and observation, material properties (specifically the Fresnel reflection coeffi-
cients), and wavelength of radiation. A very similar approach is taken by the Shooting and Bouncing of Rays (SBR) technique [31, 32]. In this method, rays incident on a surface can come from both the source as well as from secondary reflections from other objects in the scene. Once the rays have propagated, the surface currents are approximated assuming plane wave incidence and a PO integral is evaluated to yield the final result.

3.2.1 Maxwell’s Equations

With very few exceptions, solutions to Maxwell’s equations represent the most accurate model for how electromagnetic radiation interacts with matter. The more accurate theory of Quantum Electro-Dynamics is usually more applicable to light-matter interaction problems such as scattering from single atoms at resonant frequencies, the photo-electric effect, and in the case of very strong fields, vacuum polarization resulting from electron-positron pair creation [33]. This research will investigate how light interacts with relatively macroscopic objects whose work functions are much greater than the energy contained in a radar-wavelength photon. Consequently, the solutions to Maxwell’s equations will be considered the most accurate for the purposes of this research. Maxwell’s equations are given as follows:

\[ \nabla \cdot E = \frac{\rho}{\epsilon_0} \]  \hspace{1cm} (3.6)
\[ \nabla \cdot B = 0 \] \hspace{1cm} (3.7)
\[ \nabla \times E = -\frac{\partial B}{\partial t} \] \hspace{1cm} (3.8)
\[ \nabla \times B = \mu_0 j + \frac{1}{c^2} \frac{\partial E}{\partial t}. \] \hspace{1cm} (3.9)

While Maxwell himself did not derive the four equations above, he is credited with having recognized that they result in a wave equation:

\[ \nabla^2 E - \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} = \frac{1}{\epsilon_0} \nabla \rho + \mu_0 \frac{\partial j}{\partial t} \] \hspace{1cm} (3.10)
\[ \nabla^2 B - \frac{1}{c^2} \frac{\partial^2 B}{\partial t^2} = \mu_0 \nabla \times j. \] \hspace{1cm} (3.11)

In the absence of free charge and current (\( \rho = j = 0 \)), equations 3.10 and 3.11 transform into the vector Helmholtz equation:

\[ \nabla^2 E + k_0^2 E = 0 \] \hspace{1cm} (3.12)
\[ \nabla^2 B + k_0^2 B = 0. \] \hspace{1cm} (3.13)
with \( k_0^2 = \omega^2/c^2 \). The equations above can be solved by resolving the fields into their vector components and solving the resultant scalar equations. Under certain constraints, the resulting scalar equations result in the scalar Helmholtz equation:

\[
[\nabla^2 + k_0^2] \psi(\mathbf{r}) = 0 \tag{3.14}
\]

where an \( e^{i\omega t} \) is suppressed and \( \psi(\mathbf{r}) \) is a scalar field that can be related to the fields \( \mathbf{E}, \mathbf{B} \), or as shown later a vector potential \( \mathbf{A} \), by a vector \( \mathbf{a} \) such that \( \mathbf{E}, \mathbf{B}, \mathbf{A} = \mathbf{a} \psi(\mathbf{r}) \). One of the constraints required to go from the vector Helmholtz equation to the scalar Helmholtz equation is propagation through isotropic media, a condition automatically satisfied in free space. The other constraint is on the choice of the direction of \( \mathbf{a} \) with respect to the coordinate system used for the Laplacian operator \( \nabla^2 \). In rectangular coordinates, \( \mathbf{a} \) can be in any arbitrary direction [34]. In a spherical coordinate system, \( \mathbf{a} \) must point in the radial direction [34]. For a cylindrical coordinate system, \( \mathbf{a} \) must lie along the z-axis [34]. In other words, analysis is restricted to TE\( z \) and TM\( z \) modes. This may seem restrictive at first glance. However, given that the TE and TM modes form an orthogonal basis and the incident and scattered fields in radar applications are usually linearly polarized, solutions can be obtained by decomposition of the fields into the two basis vectors. When \( \mathbf{a} \) cannot be chosen to lie along one of the system’s coordinates, solutions to the scalar Helmholtz equation can still be used but specifying the boundary conditions becomes exceedingly difficult. One example of scattering by a surface specified in prolate spheroidal coordinates is given in [35]. In that paper, \( \mathbf{a} \) was chosen to lie along one of the rectangular coordinates (TE\( x \), for example). The incident field, originally specified in rectangular coordinates, had to be decomposed into a sum of spheroidal wavefunctions so that the boundary conditions in that coordinate system could then be specified and the solution for the scattered field obtained.

On the assumption the problem can be reduced to solving the scalar Helmholtz equation, Green’s theorem can be used to transform the problem into an integral equation. A well known example of this in the field of optics is shown later in section 3.2.4. Another approach is to assume the solution is separable into three orthogonal coordinates \( \xi_1, \xi_2, \xi_3 \):

\[
\psi(\xi_1, \xi_2, \xi_3) = \psi_1(\xi_1)\psi_2(\xi_2)\psi_2(\xi_2) \tag{3.15}
\]

For propagation through free-space, the solution is given as follows:

\[
\psi(x, y, z) = \psi_0 e^{i(k_0 x + k_0 y + k_0 z)} \tag{3.16}
\]

Another example is propagation through a rectangular waveguide. In this case, boundary conditions need to be imposed, mainly that the field goes to 0 along the walls of the rectangular prism. The result is a solution which has discrete wave numbers:

\[
\psi_{p,q}(x, y, z) = \psi_0 e^{i\left(p k_0 x + q k_0 y + \sqrt{1-p^2-q^2} z\right)} \tag{3.17}
\]
where p and q are integers.

For infinitely long cylinders, the problem can be reduced to two dimensions by examining the solution at some point along the cylinder’s axis. In polar coordinates, the two-dimensional Helmholtz equation can be written as: 

\[
\nabla^2 f + k^2 f = 0.
\] 

(3.18)

Since we are working in cylindrical coordinates, \( f \) represents solutions for TE\(_z\) or TM\(_z\) modes. After separation of variables with \( f = R(r)\Theta(\theta) \), and choosing \( -m^2 \) to be the separation constant for the \( \theta \) coordinates, solutions to the polar two-dimensional Helmholtz equation are of the form:

\[
\psi_m(r, \theta) = \sum_{m=0}^{\infty} [a_m \cos(m\theta) + b_m \sin(m\theta)] Z_m(kr) \] 

(3.19)

where \( Z_m \) is any function that satisfies Bessel’s ODE:

\[
r^2 \frac{d^2}{dr^2} Z_m(kr) + r \frac{d}{dr} Z_m(kr) + (k^2 r^2 - m^2) Z_m(kr) = 0
\] 

(3.20)

Functions that satisfy the above ODE are Bessel functions of the first kind \( J_m(kr) \), Neumann functions \( N_m(kr) \), and Hankel functions \( H_m(kr) \) [36]. Any linear combination of these functions that satisfies the boundary conditions for a given problem may be used. The boundary conditions also specify the coefficients \( a_m \) and \( b_m \). This will become important later in section 3.3.2 when the scattering of electromagnetic waves from cylinders is examined in detail.

### 3.2.2 Geometric Optics

One of the most common high frequency approximations to Maxwell’s Equations is that of Geometrical Optics (GO). This approximation is also referred to as ray optics. Using this method, light propagation is modeled using bundles of rays that emanate from a source and interact with objects. The direction of a ray is governed by Fermat’s principle. Applying conservation of energy to a tube of rays yields amplitude information. Mathematically, GO is derived from the Luneberg-Kline series solution to the Helmholtz equation [34]:

\[
E(R, \omega) = e^{-ik\psi(R)} \sum_{m=0}^{\infty} \frac{E_m(R)}{(i\omega)^m}
\] 

(3.21)

Substituting this into the vector Helmholtz equation 3.12, applying the constraint \( \nabla \cdot E = 0 \), and equating like terms yields the following sets of equations [34]:

1. Eikonal Equation

\[ \| \nabla \psi \|^2 = n^2 \]  

(3.22)

2. Transport Equations

\[
\frac{\partial E_0}{\partial s} + \frac{1}{2} \left\{ \frac{\nabla^2 \psi}{n} \right\} E_0 = 0 \quad (3.23)
\]

\[
\frac{\partial E_m}{\partial s} + \frac{1}{2} \left\{ \frac{\nabla^2 \psi}{n} \right\} E_m = \frac{v_p}{2} \nabla^2 E_{m-1} 
\]

\[
m = 1, 2, 3 \ldots \quad (3.24)
\]

3. Conditional Equations

\[
\hat{s} \cdot E_0 = 0 \quad (3.26)
\]

\[
\hat{s} \cdot E_m = v_p \nabla \cdot E_{m-1} \quad (3.27)
\]

\[
m = 1, 2, 3 \ldots \quad (3.28)
\]

where \( \psi \) is a surface which describes the wavefront, \( n \) is the index of refraction, \( s \) is the ray distance, and \( \hat{s} \) is normal to the wavefront \( \psi \). The geometric optics approximation is represented by the 0th order term. This is called a high-frequency approximation since higher order terms in the Luneberg-Kline series, equation 3.21, are assumed to be negligible. Equations 3.22 and 3.26 together are a representation of Fermat’s Principle, or more specifically, Snell’s law and the law of reflection. Taking the first term of equation 3.21 and substituting into the transport equation 3.23 yields the following result [34]:

\[
E(s) = E'_0(0)e^{i\phi_0(0)} \sqrt{\frac{\rho_1\rho_2}{(\rho_1 + s)(\rho_2 + s)}} e^{iks} \quad (3.29)
\]

where \( \rho_1 \) and \( \rho_2 \) are the radii of curvature of the wavefront at some reference \( s = 0 \) and \( \phi_0 \) is the initial phase at the reference point. For a small spherical source located near \( s=0 \) and choosing the location of the observation point to be far removed, the field amplitude is proportional to \( 1/s \). This conforms to the well known fact in radiometry that the power from a point source is proportional to \( 1/s^2 \). For scattering from an arbitrary surface and
incorporating Fresnel reflection coefficients, the resulting equation becomes [34]:

\[
E^r(s) = E^i(Q_r) \cdot \overline{R} \sqrt{\frac{\rho^r_1 \rho^r_2}{(\rho^r_1 + s)(\rho^r_2 + s)}} e^{iks}
\]

where \( Q_R \) is a point on the surface of the scattering object and \( \rho^r \) is the radius of curvature of the wavefront at the point \( Q_r \). For a spherical mirror, it can be shown that if the incident field is a plane wave, \( \rho^r = R/2 \) where \( R \) is the radius of curvature of the mirror [34]. When \( s = -R/2 = f \), where \( f \) is the focal length of the mirror, a singularity occurs in the spatial attenuation factor. This highlights a limitation of geometric optics for calculating the field at a caustic.

There are several limitations to using the geometric optics approach. For one, the spatial attenuation factor is independent of frequency. For some wavefronts, such as a Gaussian, the spatial attenuation or divergence factor is strongly dependent on frequency. Using higher order terms in the Luneberg-Kline series will result in a more accurate solution. However, since it is usually difficult to work with these higher order terms, and the resulting solutions still retain sharp discontinuities at shadow boundaries and do not account for diffraction, they are of limited use. Additionally, singularities occur at caustics such as when \( s = -\rho \). Nonetheless, geometric optics produce accurate results for many applications [34].

### 3.2.3 Geometric Theory of Diffraction

As shown, one of the main drawbacks to using GO is its inability to account for diffracted fields. The Geometrical Theory of Diffraction [27] extends GO by adding rays that diffract from an edge in addition to the normal rays that are reflected or transmitted from a surface. Much like how GO uses reflection and transmission coefficients to describe how the amplitude of a ray is attenuated by some surface interaction, GTD introduces diffraction coefficients which describes how ray amplitudes are affected by an edge or by shadow boundaries. The direction rays travel is governed by Fermat’s principle for edge diffraction. This principle states that the path a ray travels from one point to another, with an intermediate point bound to the interacting surface, is that which has a stationary optical path. For scattering from surfaces, Fermat’s principle results in the law of reflection and Snell’s law. For diffraction, this results in a cone of rays rather than a single incident and scattered ray pair. This is because, unlike surfaces, edges are one-dimensional. This cone is sometimes termed the “Keller cone.” Fermat’s principle for edge diffraction agrees well
with other theories. In the Maggi-Rubinowitz approach to solving the Kirkchoff diffraction integral, the scattered field is expressed as the sum of a geometric optics field and a diffracted field [37]. The diffracted field is computed by taking a line integral around the diffracting aperture. Angles specified by Fermat’s principle for edge diffraction are those which correspond to the stationary phase points that contribute most to the integral. Experimentally, this can be observed as a spot along the axis of the shadow of a circular disk (the Spot of Arago). Fermat’s principle for edge diffraction has also been verified for elliptical disks and oblique incidences [27]. A graphical depiction of Fermat’s principle for edge diffraction is given below:

![Figure 3.1: Fermat’s Principle for Edge Diffraction.](image)

For two-dimensional scattering, the diffracted field is given by the following equation:

$$u_{\text{edge}} = Du_i r^{-\frac{1}{2}} e^{ikr}$$

(3.31)

where D is the diffraction coefficient. The equation above is very similar to 3.30 only with the diffraction coefficient D replacing the dyadic Fresnel reflection coefficient $\mathbf{R}$. The value of D can be obtained from exact solutions to canonical problems. For example, comparing equation 3.31 with Sommerfield’s exact solution for diffraction from an edge, the following
expression for $D$ is obtained:

$$D = -\frac{e^{i\pi/4}}{2(2\pi k)^{1/2} \sin \beta} \left[ \sec \frac{1}{2}(\theta - \alpha) \pm \csc \frac{1}{2}(\theta + \alpha) \right]$$ (3.32)

where $\beta$ is the angle between the incident ray and the edge and $\theta$ and $\alpha$ are the angles the incident and diffracted rays make with the surface normal, respectively. Note that when $\lambda \to 0 \ (k \to \infty)$, $D$ goes to 0, as expected. Exact solutions can also be used to derive correction coefficients for fields at a caustic [27].

In addition to a straight edge, the diffraction coefficient for scattering from other canonical surfaces can be derived in a similar manner. These include scattering from the tips of cones, pyramids, and from points on a curved surface tangential to the incident rays. In the latter case, creeping rays result. This is because optical paths for tangential rays which travel along a geodesic before heading in the scattered direction are stationary. In fact, a ray can make a number of revolutions around a surface resulting in higher order creeping rays, although the amplitude decays exponentially with distance traveled [27]. Calculating the contribution from creeping rays can be difficult for canonical surfaces addressed by GTD. For arbitrary surfaces, creeping ray calculations are an ongoing area of research [25].

While GTD is more accurate than GO since it accounts for diffraction, there are still a number of limitations. For one, GTD still relies on a high-frequency approximation. The Sommerfeld solution computes the scattered field by integrating the contributions from a spectrum of plane waves along the surface near the edge. In order to express Sommerfeld’s solution in the form of equation 3.31, a steepest descent method assuming $kr \to \infty$ had to be applied [37]. Additionally, the diffraction coefficients become singular along geometric shadow boundaries (such as when $\theta = \alpha + \pi$ in equation 3.32). Although application of the residue theorem to the integral in Sommerfeld’s formulation leads to a finite result [37], the diffraction coefficients in GTD use the wide angle solution [25] which retains the singularities. This is addressed later by Ufimtsev’s Physical Theory of Diffraction (section 3.2.7) and also by the Uniform Theory of Diffraction [38]. Finally, GTD only predicts the amplitudes of rays that lie in the Keller cone. For finite surfaces, a spreading of energy outside this cone can occur. Calculating the scattered field outside this cone can be accomplished using the closely related Incremental Length Diffraction Coefficients, discussed later in section 3.3.3.

### 3.2.4 Scalar Diffraction Theory

In addition to geometric or ray optics approaches to modeling electromagnetic scattering, there are also integral-based methods. Instead of starting with the Luneberg-Kline series,
these integral-based approaches start with the Helmholtz equation. Solutions to this equation are obtained with the help of Green's functions.

In this section, a brief review of scalar diffraction theory applied to a planar aperture is presented. This discussion will be useful later on when the closely related but more general physical optics approach is discussed. As noted in section 3.2.1, there are certain coordinate systems where the vector Helmholtz equation can be transformed into the scalar Helmholtz equation. The rectangular coordinate system is one of them. Fortunately, the problem of a planar aperture can be treated naturally in this coordinate system. Below is a figure depicting an aperture located in the plane $z = 0$ and some of the variables that will be used in the remainder of this section:

![Figure 3.2: Geometry for Scalar Diffraction (Kirchoff) Integral.](image)

A common way of solving inhomogeneous linear differential equations in the form
\[
\mathcal{L}y(x) = f(x)
\]
(3.33)

where $\mathcal{L}$ is a linear operator, is to first find a Green’s function which satisfies the equation $\mathcal{L}G(x, t) = \delta(x - t)$ and has the same boundary behavior as the solution $y(x)$ [36]. Using
the sifting property of the Dirac delta function, the solution is given by:

\[ y(x) = \int_a^b G(x,t)f(t)dt \]  

(3.34)

where \( a \) and \( b \) define the boundary [36]. This can easily be verified by applying \( \mathcal{L} \) to both sides and using the sifting property of the Dirac delta function. Equation 3.34 leads to an important interpretation of the Green’s function \( G \), mainly that it is the impulse response of the operator \( \mathcal{L} \). For example, let the source term in equation 3.33 be a delta function centered at \( x_0 \): \( f(x) = \delta(x - x_0) \). Plugging this impulse into equation 3.34 yields the solution \( y(x) = G(x,x_0) \). The example just shown is one-dimensional, but the concept can be extended to three dimensions also. While the scalar Helmholtz equation is homogeneous (in other words, there is no source term from a mathematical standpoint), the Green’s function for the linear operator \( \mathcal{L} = \nabla^2 + k_0^2 \) can still be used to obtain a solution.

Let \( u(r) \) be the solution to the Helmholtz equation which satisfies the appropriate boundary conditions. For the present moment, let \( G(r,r') \) be any scalar function. From Green’s theorem, the following condition holds [33]:

\[
\int_\Omega \left[ u(r') \nabla^2 G(r',r) - G(r',r) \nabla^2 u(r') \right] \, d^3r = 
\int_\Sigma \left[ u(r') \nabla' G(r',r) - G(r',r) \nabla' u(r') \right] \cdot dS' \]  

(3.35)

Now, let \( G(r,r') \) be a Green’s function which satisfies the following equation [33]:

\[
[\nabla^2 + k_0^2] \ G(r,r') = -\delta(r - r'). \]  

(3.36)

After some algebraic manipulations and noting that \( [\nabla^2 + k_0^2] \ u(r) = 0 \), the left hand side of equation 3.35 simplifies [33]:

\[
u(r) = \int_\Sigma \left[ u(r') \nabla' G(r',r) - G(r',r) \nabla' u(r') \right] \cdot dS'. \]  

(3.37)

One function which satisfies 3.36 is given by [33]:

\[
G_0(r',r) = \frac{\exp(ik_0|r - r'|)}{4\pi|\mathbf{r} - \mathbf{r}'|}. \]  

(3.38)
Equation 3.37 can be further simplified if $G(r, r') = 0$ over the surface $\Sigma$ in figure 3.2. This is also known as a Dirichlet boundary condition. Another way of expressing this boundary condition is with the following equations [33]:

$$G(r', r) = 0 \quad \text{when } z' = 0^+ \quad (3.39)$$

$$r' \left( \partial G/\partial r' - ik_0 G \right) = 0 \quad \text{when } r' \to \infty. \quad (3.40)$$

When these two equations are satisfied, and assuming $u(r')$ also equals 0 when $r' \to \infty$, the integral in 3.37 reduces to the following [33]:

$$u(r) = \int_{z' = 0^+} u(r') \frac{\partial}{\partial z} G(r, r') dS'. \quad (3.41)$$

Here, we have used the fact that $dS'$ points in the $\hat{z}$ direction. Equation 3.38 satisfies equation 3.39 but not 3.40. Using the method of images, an alternate Green’s function can be constructed that satisfies both 3.36 and the Dirichlet boundary condition. This alternate Green’s function is given by [33]:

$$G(r', r) = \frac{\exp(ik_0|\mathbf{r} - \mathbf{r}'|)}{4\pi|\mathbf{r} - \mathbf{r}'|} - \frac{\exp(ik_0|\mathbf{r}^* - \mathbf{r}'|)}{4\pi|\mathbf{r}^* - \mathbf{r}'|}. \quad (3.42)$$

Now, the second term in the integrand of equation 3.37 involving $\nabla' u(r')$ is eliminated and the solution simplifies even further to yield the Rayleigh-Sommerfeld diffraction integral of the first kind [33]:

$$u(r) = -\frac{1}{2\pi} \int_{z' = 0^+} u(r') \frac{\partial}{\partial z} \left[ \frac{\exp(ik_0 s)}{s} \right] dS'. \quad (3.43)$$

where $s = \mathbf{r} - \mathbf{r}'$ and $s^2 = (x - x'^2) + (y - y'^2) + z^2$. At this point, a few approximations are normally made. The first is the Kirchoff approximation, which assumes the field in the aperture region is exactly equal to the incident field, $u(x, y, 0) = u_0(x, y, 0)$ [33]. Using this approximation, the integral over the entire $z' = 0^+$ plane reduces to an integral over just the aperture $\Sigma$. This approximation is valid when $\lambda/a \ll 1$ where $a$ is the size of the aperture [33]. It neglects, however, perturbations to $u_0$ caused by the aperture itself. Another familiar result is the Fresnel diffraction integral, which can be obtained from equation 3.43 by approximating $s$ to second order. An even further simplification can be made if the following criterion is met: $k_0 r \gg 1$. If this condition is met, then the observation point at $\mathbf{r}$ is said to be in the far-field. The far-field diffraction integral, also known as the Fraunhofer diffraction integral is given as follows [33]:

$$u(r) \approx \frac{ik_0 \exp(ik_0 r)}{2\pi r} \int_{\text{aperture}} u_0(r')\exp(-i\mathbf{p} \cdot \mathbf{r}')dS'. \quad (3.44)$$
Scalar diffraction theory cannot be applied to all scattering problems. As mentioned in section 3.2.1, there are few coordinate systems that support use of the scalar Helmholtz equation. For the planar aperture example given, it was straightforward to apply boundary conditions to the Green’s function in rectangular coordinates. For arbitrarily shaped surfaces, it is often more difficult to define the boundary conditions. Additionally, the Kirchoff approximation neglects contributions from the edge of the aperture. These edge contributions arise from what Ufimtsev terms non-uniform sources [39]. Neglecting these contributions can lead to large errors when the condition $\lambda/a \ll 1$ is not met or when the observation point is far from the geometrically illuminated region [25].

3.2.5 Physical Optics

In many applications, there is no easy way to describe the scattering surface in an orthogonal coordinate system. Even when this is possible, there is no guarantee that the vector Helmholtz equation will result in at least one de-coupled scalar differential equation for the coordinate system being used. For this, we will turn back to the wave equation and examine what simplifications and approximations can be made. For convenience, the electromagnetic wave equations are restated below:

\begin{equation}
\nabla^2 E - \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} = \frac{1}{\epsilon_0} \nabla \rho + \mu_0 \frac{\partial j}{\partial t} \tag{3.45}
\end{equation}

\begin{equation}
\nabla^2 B - \frac{1}{c^2} \frac{\partial^2 B}{\partial t^2} = \mu_0 \nabla \times j \tag{3.46}
\end{equation}

Previously, it was assumed there was no free charge or current. This assumption is mostly valid behind an opaque aperture, excluding points very close to the edges. Here, assume that the radiation incident on the scattering object, which before was an aperture, produces some surface current in the illuminated region but does not induce any current in the shadowed region. Letting $\rho \to 0$ and keeping the $j$ term, equations 3.46 and 3.45 become coupled in $j$. We can decouple these equations via a Lorentz gauge transformation. Defining a vector potential $A$ for the magnetic field, the following equations can be derived:

\begin{equation}
E = -\nabla \phi - \frac{\partial A}{\partial t} \quad \text{and} \quad B = \nabla \times A. \tag{3.47}
\end{equation}

After substitution of 3.47 into the wave equations 3.45 and 3.46 with $\rho = 0$, we have

\begin{equation}
\nabla^2 A - \frac{1}{c^2} \frac{\partial^2 A}{\partial t^2} = -\mu_0 j \tag{3.48}
\end{equation}

Now, the problem is simplified to merely finding solutions for equation 3.48, which is the vector analog to the scalar inhomogeneous wave equation. Equation 3.48 is very...
distinct from the scalar Helmholtz equation in the previous section, since now there is a source term, \( j \), making the differential equation inhomogeneous. Before, the Green’s function was simply a mathematical tool used to obtain a solution to the homogeneous Helmholtz equation via Green’s theorem. Dirichlet Boundary conditions were imposed on the Green’s function to simplify the problem and limit the assumptions made about the boundary conditions for \( u(r) \). The physical meaning of the resulting Green’s function (two point sources that were mirror images of each other) was abstract. Now with a source term in equation 3.48, the forthcoming Green’s function has more physical significance. It represents the contribution to the magnetic vector potential \( A \) due to a point of current density \( j \).

The objective at this point is to find an expression for the vector potential \( A \). Using the technique introduced in 3.2.1, this vector equation can be solved by finding solutions to its scalar equivalent, provided an appropriate coordinate system is used. Without loss of generality, a rectangular coordinate system will be assumed and analysis will proceed, using the scalar form of equation 3.48. For now, the only boundary condition to be applied will be the radiative boundary condition, mainly that the field goes to 0 as \( r \to \infty \). Once this boundary condition has been imposed, we will return to the vector equations.

The scalar inhomogeneous wave equation is given by:

\[
\left[ \nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right] \psi(r, t) = -f(r, t) \tag{3.49}
\]

Solutions for \( \psi(r, t) \) can be obtained by finding a Green’s function which satisfies the following equation [33]:

\[
\left[ \nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right] G(r, t | r', t') = -\delta(r - r')\delta(t - t') \tag{3.50}
\]

It can be shown that the following function satisfies the equation above [33]:

\[
G_{\pm}(r, t | r', t') = \frac{1}{4\pi|r - r'|} \delta(t - t' \pm |r - r'|/c) \tag{3.51}
\]

where the primed coordinates indicate the location of the source and the time the source exists, and the unprimed coordinates indicate the location of the observation point and the time of observation. The Green’s functions \( G_+ \) and \( G_- \) represent advanced and retarded solutions, respectively. When using Green’s theorem to solve for \( \psi \), the volume of integration is often evaluated in the limit \( V \to \infty \). This makes the advanced solution, representing an incoming wave from some external source, irrelevant. Using the retarded solution and applying Green’s theorem, the following solution for \( \psi \) is obtained [33]:

\[
\psi(r, t) = \int_V \left[ \int V G(r, t | r', t') f(r', t')d^3r' \right] d^3r = \frac{1}{4\pi} \int_V \frac{f(r', t - |r - r'|/c)}{|r - r'|} d^3r' \tag{3.52}
\]
where the sifting property of the Dirac delta has been used for the integral over time. Reverting to the vector representation of the problem, the following solution for $A$ is as follows [33]:

$$A(r, t) = \frac{\mu_0}{4\pi} \int \frac{j(r', t - |r - r'|/c)}{|r - r'|} d^3r'. \quad (3.53)$$

Now, an assumption will be made that the observation point is in the far field, $r' \ll r$. With this assumption, and knowing that $A$ should go as $1/r$ in order to match the behavior of the Green’s function, $|r - r'|$ is approximated in the denominator as just $r$. For the argument of $j$, a binomial expansion of $|r - r'|$ is performed. Taking terms up to second order, equation 3.53 can be simplified even further [33]:

$$A(r, t) = \frac{\mu_0}{4\pi r} \int j(r', t - r/c + \hat{r} \cdot \hat{r}'/c) d^3r' \quad (3.54)$$

In most cases this work addresses time-harmonic fields. Taking the Fourier transform of 3.54 with respect to time, letting $k = (\omega/c) r$, and letting frequency be a parameter, the following time harmonic potential can be derived [33]:

$$A(r|\omega) = \frac{\mu_0}{4\pi r} \int j(r'|\omega)e^{-ikr'}d^3r' \equiv \frac{\mu_0}{4\pi} e^{ikr} \hat{j}(k|\omega) \quad (3.55)$$

where $\hat{j}(k|\omega)$ is the three dimensional spatial Fourier transform of the function $j(r'|\omega)$. For perfectly conducting objects, the $j(r'|\omega)$ term is only non-zero on the surface. Consequently, the volume integral becomes a surface integral and the final result on the right-hand side of 3.55 is the vector analog of the Fraunhofer diffraction integral.

The form of equation 3.55 given by Zangwill is especially useful when the scattering surface is illuminated by non-coherent illumination. It allows the vector potential $A(r|\omega)$ to be computed through a Fourier decomposition of source terms. Essentially, the spectrum for $A(r|\omega)$ can be built up from the monochromatic solutions on a frequency-by-frequency basis. This is a widely used approach in optics [40]. Knowing that the solution for $A(r, t)$ can be built up in the temporal frequency $\omega$ domain from the decomposed monochromatic source terms, the parameter $\omega$ will be implicit for the remainder of this paper.

Thus far, the only approximation made to arrive at equation 3.55 is that the observation point is in the far-field. Now, two additional approximations will be made. The first is the tangent plane approximation. Assuming plane wave incidence on the surface and assuming the surface can be locally approximated as planar (in other words, the radius of curvature is large compared to the wavelength), then the surface current at the point $r'$ is given by $\mu_0 j(r', t) = 2\hat{n} \times B_{mc}(r', t)$ [33]. Additionally, the Kirchoff approximation will be made so
that there are no surface currents in the shadow region. Summarizing,

$$\mu_0 j(r) = \begin{cases} 
2\hat{n} \times B_{inc}(r) & \text{in illuminated region} \\
0 & \text{in shadow region.} 
\end{cases} \quad (3.56)$$

When the incident wave is a plane wave, $B_{inc} = B_0 \exp(i k_0 \cdot r)$. Using this expression for $B(r)$, collectively with equations 3.47, 3.55, and 3.56, the E field is given by:

$$E(r) \simeq \frac{i \omega}{2\pi} \frac{\exp[i kr]}{r} \hat{k} \times \left\{ \hat{k} \times \int_{S} (\hat{n}' \times B_0) \exp[i(k_0 - \hat{k} \cdot r') \cdot dS'] \right\} \quad (3.57)$$

The approximations used to arrive at this result, mainly the far-field, tangent plane, and shadow boundary approximations, are collectively termed the physical optics approximation [24].

Another derivation of equation 3.57 often found in the literature is via the Chu-Stratton integral. This is another integral form of Maxwell’s equations that does not use the Lorentz gauge transformation or the vector potential $A$. Applying the physical optics approximation leads to the same results, however (5.10 in [25]).

The most obvious downside to using the physical optics integral is that it inaccurately models currents near an edge or shadow boundary. For large structures, this usually is acceptable [24] since the contributions from non-uniform sources are greatly outweighed by the uniform components. It can also be shown that the physical optics integral, equation 3.57, does not produce any cross-polarization terms [25]. This is a result that conflicts with known experimental data [25]. Finally, the physical optics integral produces erroneous results for observation points far removed from the specular direction [25].

### 3.2.6 Modified Equivalent Current Approximation

In the previous section, it was assumed that the scattering object was perfectly conducting. The Modified Equivalent Current Approximation (MECA) [29, 30] extends physical optics by incorporating imperfectly conducting surfaces. The macroscopic forms of Maxwell’s equations for dielectric media are given by [34]:

$$\nabla \times E = -M - \mu \frac{\partial H}{\partial t} \quad (3.58)$$

$$\nabla \times H = j + \epsilon \frac{\partial E}{\partial t} \quad (3.59)$$

$$\nabla \cdot E = \frac{\rho}{\epsilon} \quad (3.60)$$

$$\nabla \cdot H = 0. \quad (3.61)$$
The constants \( \epsilon \) and \( \mu \) are introduced to account for the material’s response to external electric and magnetic fields. The quantity \( H \) is an auxiliary field related to the microscopic field \( B \) by the following relation [33]:

\[
H = \frac{1}{\mu_0} B - M = \frac{1}{\mu} B
\]  

(3.62)

The term magnetization is used to describe \( M \). Physically, \( M \) represents a macroscopic magnetic dipole density arising from a collection of point magnetic dipoles [33]. This dipole density can arise when an external magnetic field induces a preferred electron spin orientation or orbital state. Mathematically, \( M \) can also represent a virtual magnetic current that gives rise to a vector potential for \( E \) in much the same way that the current density \( j \) gives rise to the magnetic vector potential \( A \). Consequently, \( M \) is often called the magnetic current density [34]. The term virtual is used to describe this current since the second of Maxwell’s equations, \( \nabla \cdot B = 0 \), does not support the existence of magnetic charge.

Assuming no free electric charge, the wave equations for dielectric materials are [34]:

\[
\nabla^2 E + k^2 E = \nabla \times M + \mu \frac{\partial J}{\partial t}
\]

(3.63)

\[
\nabla^2 H + k^2 H = -\nabla \times j + \epsilon \frac{\partial M}{\partial t}.
\]

(3.64)

Similar to what was done in the previous section, a vector potential for the field \( H \) will be given by \( A \). Additionally, a new vector potential will be introduced for the field \( E \) and will be denoted by the quantity \( F \). The fields can be derived from the vector potentials using the following equations [34]:

\[
H_A = \frac{1}{\mu} \nabla \times A \quad E_A = -\frac{\partial A}{\partial t}
\]

(3.65)

\[
H_F = -\frac{\partial F}{\partial t} \quad E_F = \frac{1}{\epsilon} \nabla \times F
\]

(3.66)

Plugging these expressions into the dielectric wave equations results in the following inhomogeneous differential wave equations [34]:

\[
\nabla^2 A + k^2 A = -\mu j
\]

(3.67)

\[
\nabla^2 F + k^2 F = -\epsilon M.
\]

(3.68)

Using the same method presented in section 3.2.5, the following solutions for the potentials
A and M can be obtained from the source currents \( j \) and \( M \) [29]:

\[
A(r) = \frac{\mu}{4\pi} \frac{e^{ikr}}{r} \int j(r')e^{-ikr'}dS'
\]  
(3.69)

\[
F(r) = \frac{\epsilon}{4\pi} \frac{e^{ikr}}{r} \int M(r')e^{-ikr'}dS'.
\]  
(3.70)

It is important to note that the equations above assume the source currents reside on the surface. This assumption is valid if the material is thin and opaque such as for radar-absorbing paint. This is not the case in general for imperfectly conducting materials which can have bound currents throughout its volume.

\( E \) and \( H \) can now be calculated by using equations 3.65 and 3.66 which relate the fields to the potentials as well as equations 3.69 and 3.70 which relate the potentials to the sources. After some manipulation, the following equations provide the simplified relation between the fields and the vector potentials [34]:

\[
E = E_A + E_F \simeq i\omega \left[ -A + \eta \hat{k} \times F \right]
\]  
(3.71)

\[
H = H_A + H_F \simeq i\omega \left[ \frac{1}{\eta} \hat{k} \times A - F \right].
\]  
(3.72)

The source terms in equations 3.69 and 3.70 can be approximated by using a physical optics approximation and incorporating Fresnel reflection coefficients [30]:

\[
M = E_{TE}^{inc}(1 + R_{TE})(\hat{e}_{TE} \times \hat{n}) + E_{TM}^{inc}(1 + R_{TM})(\hat{k}_0 \cdot \hat{n})\hat{e}_{TE}
\]  
(3.73)

\[
j = \frac{1}{\eta} \left[ E_{TE}^{inc}(1 - R_{TE})(\hat{k}_0 \cdot \hat{n})\hat{e}_{TE} + E_{TM}^{inc}(1 - R_{TM})(\hat{n} \times \hat{e}_{TE}) \right]
\]  
(3.74)

where \( \hat{e}_{TE} \) is a unit vector perpendicular to the plane containing \( \hat{k}_0 \) and the surface normal, \( \hat{n} \). For plane wave incidence, \( E_{TE}^{inc} \) is the product of an amplitude term and a phase term \( e^{ik_0r'} \) that can be factored out of both the TE and TM terms. Using this knowledge, \( M \) can be re-written in the following form:

\[
M = M_0 e^{ik_0r'}
\]  
(3.75)

where \( M_0 \) is a vector which includes all the amplitude information. A similar process can be applied to the current \( j \). Concentrating on just \( M \), the vector potential \( F \) can be expressed in an alternate form to equation 3.70:

\[
F(r) = \frac{\epsilon}{4\pi} \frac{e^{ikr}}{r}M_0I(r)
\]  
(3.76)
where,

\[ I(\mathbf{r}) = \int e^{i(k_0 - k) \cdot \mathbf{r}'} dS'. \]  

(3.77)

The term \( I \) is sometimes referred to as the re-irradiation diagram [16].

This final expression is useful when a closed form for \( I \) can be obtained. This is true when the surface is a triangular facet. Introducing new coordinates \( \mathbf{r}' = v_{01} + f v_{12} + g v_{13} \) where \( v_{mn} \) is a vector which points from vertex \( m \) to vertex \( n \), the integral \( I \) can now be re-written [30]:

\[ I(\hat{\mathbf{r}}) = 2A e^{-i(a+b/3)} \int_0^1 \int_0^{1-f} e^{i(a f + b g)} dg df \]  

(3.78)

where \( A \) is the area of the facet and:

\[ a = k v_{12} \cdot (\hat{\mathbf{r}} - \hat{k}_0) \]  

(3.79)

\[ b = k v_{13} \cdot (\hat{\mathbf{r}} - \hat{k}_0). \]  

(3.80)

The advantage of expressing the integral in this manner is that there are 5 possible closed form solutions to the integral \( I \) that correspond to 5 combinations of values for \( a \) and \( b \) [30]. Restricting \( a \) and \( b \) to be non-zero results in only two combinations [30]:

\[ I(\hat{\mathbf{r}}) = \begin{cases} 
2A \exp \left\{-i \frac{a+b}{3}\right\} \left[ \frac{a \exp\{ib\} - b \exp\{ia\} + b - a}{(a-b)ab} \right] & a = b \\
2A \exp \left\{-i \frac{2a}{3}\right\} \left[ \frac{\exp\{ia\}(1-ia)-a}{a^2} \right] & a \neq b 
\end{cases} \]  

(3.81)

An important consequence of this result is it allows ray tracing software to be modified to efficiently calculate either the physical optics, or more generally, MECA solutions [29, 30, 41]. This can be seen by first considering a single ray representing an incident plane wave with some polarization. For a single intersection on the scattering surface with normal \( \hat{\mathbf{n}} \) and Fresnel reflection coefficients specified by the material properties, all other required parameters to compute the source currents, mainly \( \hat{k}_0, \hat{\mathbf{r}}, \hat{\mathbf{e}}_{TE}, E_{inc}^{TE}, \) and \( E_{inc}^{TM} \), are immediately given. When the surface is broken up into triangular facets, these parameters can be used to compute efficiently the vector potentials via 3.81 and, subsequently, the fields scattered in the direction \( \hat{\mathbf{r}} \).

This approach is frequently found in the recent literature. It has been used as the basis for a software tool that efficiently calculates the RCS’s of arbitrary objects [41]. It has also been used when assessing multiple scattering. Multiple scattering is handled by using ray tracing software for all paths except the final one to the observer. Computation for this final path is computed by using MECA for all intersections. This is referred to as the Shooting and Bouncing of Rays (SBR) technique and has been used to investigate
the RCS of cavities with a good degree of accuracy [31]. An implementation using the NVIDIA CUDA toolkit has also been implemented [32]. A software tool which uses MECA to simulate radar imagery [42] has also been developed by the companies ONERA and OKTAL. Information on the performance of this last tool is sparse in open literature but the developers claim excellent agreement with real-world data [42].

Significantly, MECA similar limitations to those for the physical optics approximation. No allowance has been made here for edge currents. Consequently, there will be significant errors in the scattered field away from specular observation angles just as with physical optics. Additionally, bound currents throughout the scatterer’s volume may exist for imperfectly conducting materials that would be unaccounted for in the surface integral. Unlike in physical optics, however, cross-polarization terms can be obtained from MECA.

3.2.7 Physical Theory of Diffraction

One of the main drawbacks with physical optics is the inaccurate results obtained for observation points which are far from specular viewing angles [25]. For the case of scattering from a semi-infinite plane, the physical optics solution also contains singularities at the geometric shadow boundaries. These singularities are similarly observed in the GTD diffraction coefficients for geometric shadow transition regions [25]. Ufimtsev’s Physical Theory of Diffraction (PTD) seeks to correct this [28].

The most illustrative canonical problem in PTD is scattering from a semi-infinite half plane or wedge. Ufimtsev hypothesized that the exact solutions given by Sommerfeld represent contributions from both uniform currents across the surface and non-uniform currents near an edge. The physical optics solution represents contributions from just the uniform components. By subtracting the physical optics solution of scattering from a semi-infinite plane or wedge, from the exact solution, the only term remaining is that due to non-uniform currents along the edge itself. Mathematically, this is expressed as [39]:

\[ u^{(1)} = u - u^{(0)} \]  

(3.82)

where \( u^{(1)} \) is the field due to edge (non-uniform) currents and is equal to the difference between \( u \), the exact solution, and \( u^{(0)} \), the field due to uniform currents predicted by physical optics. While reference is made to non-uniform currents, no expression for them is ever provided in PTD. Expressions are only provided for the far-field scattered fields that result from these currents. This is not relevant since it is the scattered field rather than the surface field that is of interest [25].

It turns out the singularities in the physical optics solution counterbalance those observed in the GTD diffraction coefficients which were derived from Sommerfeld’s solution. The result is diffraction coefficients that are non-singular, even along shadow boundary regions, and agree well with experimental observation [25].
While the example given only addresses scattering from a semi-infinite plane or wedge, Ufimtsev also derived diffraction coefficients for other canonical shapes such as cones, disks, strips, ruled surfaces, and finite length cylinders [39]. In many cases, the semi-infinite half-plane problem was used as a starting point for deriving the diffraction coefficients for other objects. The approach is the same in all cases: mainly subtract the physical optics solution from the exact solution to obtain the edge diffraction coefficients.

Ray tracing software and SBR is often used in conjunction with PTD (as well as GTD). Edges are identified as those lines which adjoin two adjacent facets whose difference in facet surface normals exceed some user defined threshold [12]. The facets adjoining the edge perform ray sampling for the edge. Directions for the outgoing rays are specified by Fermat’s Principle for Edge Diffraction and their amplitudes are given by the PTD diffraction coefficients for the given wedge angle.

Thus far, no allowance has been made for imperfectly conducting material. Ufimtsev provides a short discussion on how to approach calculating diffraction coefficients for imperfectly conducting surfaces [39] but these coefficients are not explicitly given. Additionally, Ufimtsev’s theory only provides corrections for Keller’s GTD coefficients. Consequently, the PTD diffraction coefficients only describe the scattering of rays contained in the Keller cone and provide no information on the rays scattered outside the Keller cone. For this, one can use the closely related Incremental Line Diffraction Coefficient (ILDC).

3.3 Power Line Modeling Using ILDC’s

Now that traditional methods for modeling electromagnetic scattering in the radar regime have been reviewed, the approach formulated in the current research for modeling the radar return of powerlines will now be presented. This approach used a blend of using the exact solution to Maxwell’s equations for scattering from a perfectly conducting smooth cylinder and an extension of the Physical Theory of Diffraction that involves the use of Incremental Length Diffraction Coefficients (ILDC’s). To begin, a physical model for the powerline being modeled was created. This physical model described the shape as well as the material properties of the powerline. These parameters were then used to calculate the 2D solution for the scattered field. An ILDC approach was then used to extend this solution to 3D, into the backscattering cone. Work done by Mitzner detailed how ILDC’s can be used to extend 2D solutions to 3D [10]. In Mitzner’s report, a discussion was provided on circular cylinder diffraction although an analytic form for the ILDC was never given [10]. Instead, this discussion on circular cylinder diffraction was used as a starting point to lead into edge diffraction. The most common application for ILDC’s is to compute how edges on a wedge scatter incident radiation. It was shown that the ILDC is equal to the 2D diffraction coefficient for the case of backscattering. In this research, a
2D ILDC was derived from the exact solution for scattering from an infinite length circular cylinder at oblique incidence. A 3D diffraction coefficient was then computed from this ILDC. The relationship between this 3D diffraction coefficient and the target RCS was then derived. This simulated RCS was later compared to experimental measurements for validation. The 3D diffraction coefficient was also used to simulate the SAR phase history for a powerline target. These results were compared to field data for a circular SAR collection. The sections below provide more details on the work entailed for each step.

3.3.1 Physical model

Before diffraction coefficients were calculated, a physical model needed to be established which took into account material and geometrical properties. To ensure efficiency was achieved, the following assumptions were made:

1. The powerline insulation was transparent for the wavelengths being studied.
2. The material underneath the insulation was Perfectly Electrically Conducting (PEC).
3. The helically wound cables comprising the powerline were approximated as a single smooth cylinder for $k \cdot a \ll 1$.

Since the powerline was not perfectly rigid, there was also be sag present in the geometry. Assuming a uniform mass density, the equation that described the shape of a suspended cable was straightforward to derive. This was accomplished by finding the stationary value for the potential energy of the cable [43]. The potential energy $V$ is given by:

$$V = \int dm \ g y,$$

where $dm$ is a differential mass, $y$ is the height of the differential mass, and $g$ is gravitational acceleration, assumed to be constant. Assuming a uniform mass density $\lambda$, a differential element of mass is related to a differential length of cable by $dm = \lambda ds$. plugging this result into 3.83 leads to the following sets of equations:

$$= g \lambda \int y ds$$

$$= g \lambda \int y \sqrt{dx^2 + dy^2}$$

$$= g \lambda \int y \sqrt{1 + \dot{x}^2} dy.$$
Incidentally, this result matches the canonical problem of a minimum surface of revolution in the calculus of variations. The infinitesimal variation of $V$, $\delta V$, is stationary when the integrand satisfies the Euler-Lagrange equation [43]:

$$\frac{\partial f}{\partial x} - \frac{d}{dy} \left( \frac{\partial f}{\partial \dot{x}} \right) = 0 \quad (3.87)$$

Where $f = y\sqrt{1 + \dot{x}^2}$. Recognizing $\frac{\partial f}{\partial x} = 0$, and integrating with respect to $y$ leads to the following differential equation:

$$\frac{y\dot{x}}{\sqrt{1 + \dot{x}^2}} = a \quad (3.88)$$

$$\frac{dx}{dy} = \frac{a}{\sqrt{a^2 - y^2}} \quad (3.89)$$

The solution to this differential equation is that of a catenary, given by:

$$y = a \cosh \frac{x - b}{a} \quad (3.90)$$

where $a$ and $b$ are constants determined by the two endpoints. Below is a plot depicting a typical catenary curve:

Figure 3.3: Plot of a catenary.
3.3.2 2D Diffraction Coefficient

As shown earlier in section 3.2.1, the radial component of the solution to Maxwell’s equations in cylindrical coordinates consists of Bessel functions. Imposing boundary conditions for a perfectly conducting cylinder and making the large argument approximation for Bessel functions (the observation point is in the far-field), the particular solution at oblique incidence is given by [24]:

$$E^s = \sqrt{\frac{2}{\pi k_0 R}} E^i \cos \Psi \exp \left[ i (k_0 z \sin \Psi + k_0 R \cos \Psi - \pi/4) \right] \times \sum_{n=-\infty}^{\infty} (-1)^n C_n$$ (3.91)

Where $k_0$ is the carrier wavenumber, $R$ is the distance from the cylinder to the observation point; $\Psi$ is the obliquity angle; $C_n$ is a quantity which depends upon polarization, cylinder size, and wavelength of radiation; and $z$ is the distance along the cylinder from some reference point. The value $z$ is assumed to be 0 for this research with no loss of generality. This is a 2D solution in that it only gives the field scattered into the forward cone $\Psi_s = \Psi$. Assuming a Perfectly Electrically Conducting (PEC) material, the constant $C_n$ is determined by the wavelength of radiation and the diameter of the cylinder [24]:

$$C_n^{TM} = - \frac{J_n (k_0 a_0 \cos \Psi)}{H_n^{(1)} (k_0 a_0 \cos \Psi)}$$ (3.92)
$$C_n^{TE} = - \frac{J'_n (k_0 a_0 \cos \Psi)}{H_n^{(1)'} (k_0 a_0 \cos \Psi)}$$ (3.93)
$$C_n^T = C_{-n}^T$$ (3.94)

Where $J_n$ is an $n^{th}$ order Bessel function of the first kind, $H_n^{(1)}$ is an $n^{th}$ order Hankel function of the first kind, and the primes indicate derivatives. Assuming PEC material, the cross-polarization terms are 0. With equation 3.91, the 2D solution for the field scattered into the forward cone can be computed. The 2D RCS, also known as the scattering width, can be computed from this solution given the definition below [25]:

$$\sigma_{2D} = \lim_{\rho \rightarrow \infty} 2\pi \rho \frac{|E^s|^2}{|E^i|^2}$$ (3.95)

Below is a plot of $\sigma_{2D}$ versus the wave-number diameter product for both TE and TM radiation.
CHAPTER 3. THEORY

Figure 3.4: Plot showing the normalized 2D RCS of a circular cylinder for various radii at normal incidence. The noisy signature at the end was caused by machine precision errors which arose when too many terms were used for the calculation. Physical optics would be the preferable method in this high-frequency regime. The $\parallel$ and $\perp$ subscripts refer to incident and outgoing TM and TE modes, respectively.

The Rayleigh regime is roughly characterized as the region where the TE scattering component functionally behaves like a small sphere and the TM component approaches infinity with decreasing frequency [24]. As a side note, there is an asymptotic limit for the TM component, for as the frequency decreases past a certain point, the observation point becomes near-field [24] and equation 3.91 no longer holds. The resonance regime (also called the Mie regime) is characterized by oscillation in the backscattering width of the TE component. The scattering width in the physical optics regime exhibits convergence between the two polarizations, with both scattering widths approaching the geometric cross-section for the case of backscattering at normal incidence.

An important quantity which can be derived from equation 3.91 is the dyadic 2D diffraction coefficient, $\bar{d}$. This quantity relates the scattered field to the incident field as
follows [10]:
\[ E^s(R) = E_0 \frac{1}{\cos \beta_s \sqrt{kR}} e^{ikR\bar{d} \cdot p}. \] (3.96)

Combined with 3.91, the 2D diffraction coefficient for scattering from an infinite length cylinder at oblique incidence can be derived [10]:
\[ \bar{d} = 2\sqrt{\frac{2}{\pi}} \exp \left( -i\pi/4 \right) \sum_{n=0}^{\infty} (-1)^{n+1} \bar{C}_n \] (3.97)

This quantity becomes important later when scattering in three dimensions is considered.

### 3.3.3 Incremental Length Diffraction Coefficient (ILDC)

Similar to the 2D diffraction coefficient in equation 3.91 which relates the field scattered into the forward cone to the incident field, a 3D diffraction coefficient can also be defined [10]:
\[ \bar{D} = \frac{1}{2\pi} \exp \left[ -i(2k \cdot r + \pi/4) \right] kT \text{sinc} (X) \bar{d} \] (3.98)

where \( T \) is the length of the cylinder, \( X \) is defined as \( \frac{kT}{\pi} \sin \beta_i \), \( r \) is the distance from a point on the powerline to a scene reference point, and \( \beta_i \) is the obliquity angle for the incident field measured with respect to the axial direction of the cylinder. The quantity \( \bar{d} \) is referred to as the Incremental Length Diffraction Coefficient and is in general a function of the angle of incidence and the angle of observation. It was shown by Mitzner that the co-polarization ILDC for backscattering from PEC smooth cylinders is exactly equal to the 2D diffraction coefficient for forward scattering [10]. Mitzner then goes on to show how this can be computed using a physical optics based approach. For this research, \( \bar{d} \) is computed using the exact solution given by equation 3.97 instead.

From the definition of the 3D RCS given by 3.4 as well as the definition of the 3D diffraction coefficient, equation 3.98, the following relationship between the two can be derived:
\[ \sigma(\theta) = \lambda^2 \left| \sum_{i}^{n_{\text{points}}} \bar{D}_i(\theta) \right|^2 \] (3.99)

This equation is important later when this research is validated against experimentally measured powerline RCS’s.

### 3.3.4 SAR Phase History Simulation

Once the method for calculating the 3D diffraction coefficient was established, the SAR phase history could be simulated. First, some conventions had to be established defining
the relationship between the polarization channels in the radar’s frame of reference and TE and TM components in the powerline’s frame of reference. The figure below depicts how these quantities were defined:

Figure 3.5: Depiction of the orientations of the polarization basis vectors from the aircraft’s frame of reference (H and V polarizations) and the powerline’s frame of reference (TE and TM polarizations). The definitions shown above were used to convert the polarization amplitudes from one frame to another.

Simulation of the phase history was performed on a pulse-by-pulse basis using RITSAR’s point simulator. The index for each pulse is denoted throughout with the index \( j \). The powerline itself was broken up into several small sections. The length of each section was chosen so there would be 2-3 sections within one resolution cell. Each section was then treated as a point and labeled with index \( i \). Simulation of the phase history then proceeded using the following equation:

\[
S_j(t) = \sum_{i}^{n_{\text{points}}} D_{i,j} \exp(i k(t) R_{i,j})
\]

(3.100)

where \( k(t) \) represents a linear FM wavenumber, \( D_{i,j} \) is the 3D diffraction coefficient calculated for each powerline section, for each view using equation 3.98, and the dyad \( S \)
represents the quad-polarization phase-history. When calculating $D_{i,j}$, the TE and TM components were scaled by the projections of $H$ and $V$ onto these basis vectors for both the incident and scattered paths. Additionally, it was assumed there was not enough sag in the powerline to induce a significant amount of cross-polarization signal, resulting in a diagonal $\bar{S}$. The attenuation factor of $1/R^2$ was not included. This factor was approximated to be constant across the scene. Since this attenuation factor would have to be compensated for later when performing RCS comparisons, it was not deemed necessary to include it in equation 3.100.

3.4 Summary

Approaches for modeling electromagnetic scattering with a focus on radar wavelengths have been presented. At a top level, the scattering problem can be described using the radar range equation, 3.1. The parameter of interest in this equation is the RCS which provides a measure of how efficiently a target scatters radiation in a given direction at a given wavelength. The RCS also depends upon target material properties and shape but is independent of observation distance, which makes this a useful metric. It can be calculated using equation 3.4, provided an expression which relates the scattered field amplitude to the incident field amplitude is given. Common methods for deriving such an expression were also presented.

The most exact approach for calculating how electromagnetic radiation is scattered from a target would be through application of Maxwell’s equations. This approach was used to calculate the two-dimensional solution for scattering from a smooth cylinder. This solution was later used in calculating a 3D backscattering diffraction coefficient. In general, however, it is computationally inefficient to directly apply Maxwell’s equations when modeling the scattered signal from large, complex scenes. The oldest and most common approximation used is that of geometric optics. It can be shown that familiar principles such as the laws of reflection, Snell’s law, and $1/r$ field attenuation can be derived using the first order approximation for the Luneberg-Kline series. Unfortunately, geometric optics does not account for diffraction. The GTD can be used to extend geometric optics by incorporating diffracted rays. This is done by creating rays that lie in what is termed the “Keller cone” and are attenuated by a diffraction coefficient in much the same way reflected rays are attenuated by a reflection coefficient. Although more accurate than geometric optics, GTD is still limited to observation angles far from the geometric shadow boundary where singularities result and even then, only for rays contained in the Keller cone. Physical optics is a slightly more exact approach which originates from the integral form of Maxwell’s equations. This form of Maxwell’s equations is also called the Chu-Stratton integral. Using various assumptions collectively termed the physical optics
approximation, this integral can be simplified down to a form that is relatively easier to evaluate. As demonstrated in the literature, ray tracing approaches can be used as inputs to physical optics to estimate the sources for the scattered fields. In cases where the physical optics integral can be evaluated analytically, diffraction coefficients can be derived. Much like the GTD diffraction coefficients, the physical optics diffraction coefficients result in singularities near geometric shadow boundaries. Ufimtsev recognized that non-singular diffraction coefficients can be obtained in this region by subtracting the physical optics solution from the exact GTD solution (which in turn was derived from Sommerfeld’s exact solution for scattering from a half-plane) to obtain the diffraction coefficient for just the non-uniform edge currents. When this is done, the singularities near the shadow transition regions offset each other in such a way as to yield a finite value for the diffraction coefficient that matches well with experiment. Since PTD is an extension of GTD, it too is limited to the modeling of rays only contained in the Keller cone. For rays outside this cone, Incremental Length Diffraction Coefficients (ILDC’s) can be used. These ILDC’s are closely related to the diffraction coefficients derived using PTD, GTD, and far-field approximations to Maxwell’s equations.

In this research, the scattering of electromagnetic radiation was modeled using an ILDC approach. First, a physical model was created which took into account geometric and material properties. Powerlines were approximated as smooth cylinders and their sag was modeled using a catenary. Once the physical model was established, the 2D solution for scattering into the forward cone was calculated using Maxwell’s equations. From this solution, an expression for the 2D diffraction coefficient was derived. It was shown that this value was equal to the backscattering ILDC for Mitzner’s 3D diffraction coefficient. An expression for the powerline RCS as a function of the 3D diffraction coefficient was derived and used later on for validation. The 3D diffraction coefficient was also used to simulate the powerline phase history for a SAR simulation.
Chapter 4

Methods

An assessment of DIRSIG’s ability to model both canonical targets as well as powerlines was undertaken. This was performed using the AFRL Gotcha collection for the experimental data. The Gotcha dataset was particularly useful in that the scene contained a number of canonical targets that could be used for calibration. Being a circular SAR collection, it was also possible to process the data incrementally into smaller apertures to extract multiple ‘frames’ that provided a 360° view of all objects in the scene, which also included powerlines. The assessment highlighted which objects in the AFRL scene could be reasonably modeled using DIRSIG’s geometric optics-based approach and which objects required a more rigorous approach.

Validation of the ILDC approach was performed using a variety of datasets. The RCS’s of powerlines of various sizes and materials were measured in an anechoic chamber by the University of Michigan. Both X and C band frequencies were used for the measurements. Since the diameter of the powerlines were on the order of a few centimeters, this placed the $k \cdot a$ value ($k$ being the wavenumber and $a$ being the diameter) squarely within the range scoped by this research. Simulated data using Ufimtsev’s PO/PTD approach for finite cylinders were compared to simulated data generated, using an ILDC approach based on the exact 2-D solution for backscattering from a finite cylinder.

In addition to the measurements taken in an anechoic chamber, powerline RCS’s were also extracted from the AFRL Gotcha data. Unlike the powerlines in the anechoic measurements, these powerlines exhibited a significant amount of sag. This enabled the ILDC method to be evaluated for a geometry that would typically be found in field data.
4.1 Baseline DIRSIG Assessment

The primary source of data that was used for this research came from the AFRL Gotcha collection [5]. This data set consisted of the phase history and auxiliary information for a circular SAR collection of an urban scene using an X-band frequency and a bandwidth of 640 MHz. This scene included a parking lot, canonical calibration targets such as trihedrals and dihedrals, a grassy field, building, and powerlines. Images from this data set were processed in 1° increments using 4° of aperture for each increment. The bulk of the modeled phase history for the Gotcha scene, minus power lines, was generated using RIT’s DIRSIG. The approach taken for DIRSIG to model the SAR phase history is purely based on geometric optics. The SAR functionality of DIRSIG is experimental at this stage.

DIRSIG’s ability to produce radiometrically accurate imagery is well validated in the optical regime. To date, there have not been any attempts to validate the radar modality. This research began with providing an initial assessment using data from the Air Force Research Labs (AFRL). This collection, labeled Gotcha, was particularly useful as it contained targets permitting calibration and it provided 360° of azimuth viewing angles, which enabled the extraction of parametric RCS’s. The goal was to demonstrate where DIRSIG was deficient in modeling for the radar modality. It was hypothesized that powerlines would not be accurately modeled since their scattering cross section is much larger than their geometric one. The assessment included both a qualitative and quantitative comparison. The qualitative comparison involved visually inspecting the similarity between the simulated and experimental imagery. The quantitative comparison involved extracting RCS’s for the canonical targets from the experimental imagery and comparing them to the RCS’s extracted from the simulated DIRSIG data.

In order to simulate the phase history for the AFRL collection, a 3-D scene was built in blender that roughly matched what was observed in the experimental data. The experimental data contained canonical targets scattered throughout a field. Additionally, there were other features in the scene that contained objects commonly observed in urban scenes such as buildings, a parking lot with cars, roads, and powerlines. Below are processed images of the AFRL SAR data using the backprojection algorithm for all 360° of azimuth: 
Figure 4.1: Process imagery for Gotcha collection. (a) Total power image of AFRL data produced employing RITSAR using all 360° of azimuth for backprojection processing. Canonical targets appears as point sources in the top-left portion of the image. (b) Image of view for which powerlines were apparent in the upper-right portion of image.
The canonical targets in the scene enabled some analysis to be performed on the data. Since the cross sections of the trihedrals and dihedrals have known theoretical values in all four polarization channels, they were used to calibrate the imagery. By isolating the signature of the canonical targets for each viewing angle, RCS’s were derived from the data. These RCS’s were then used for comparison to the RCS’s subsequently derived in DIRSIG simulations, enabling a quantitative comparison to be made. The figure below shows the sizes and orientations of each of the canonical targets in the scene provided by AFRL/SNA:

Figure 4.2: Locations of canonical targets in the scene provided by AFRL/SNA [5].
Calibration of the imagery was performed using a backscatter correlation method [44]. This method begins with the following signal model:

\[ O = T S R^T + N \]  

(4.1)

Where \( O \) is a 2x2 dyad containing the observed intensities in each of the four polarization channels, \( T \) and \( R \) are matrices that account for transmit and receive gains, respectively, \( N \) is the system noise and \( S \) is the desired calibrated scattering matrix for the scene where the value of each pixel is a measurement of the effective cross-section of all objects within the pixel. The first assumption made in the calibration process is that the transmitter and receiver gains remain constant throughout the collection. Next, it is assumed that the \( T \) and \( R \) matrices can be further broken down as follows [44]:

\[ T = t_{hh} \begin{pmatrix} 1 & \tilde{t}_{hv} \\ \tilde{t}_{vh} & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & \tilde{t}_{vv} \end{pmatrix} \equiv t_{hh} \tilde{T} \Lambda_T \]  

(4.2)

and similarly for \( R \), where \( t_{hh} \) is a term which accounts for absolute gain, \( \tilde{T} \) accounts for polarization impurity, and \( \Lambda_T \) accounts for channel imbalance. Polarization impurity, a measure of channel cross-talk (for example, if H and V are not perfectly orthogonal), was computed using the covariance matrix for a sub-image that contains scatters for which the following assumptions can be made [44]:

1. The scatterers are reciprocal: \( s_{hv} = s_{vh} \)
2. There is no linear relationship between \( s_{hh}, s_{vv}, \) and \( s_{vh} \) (the correlation coefficient between channels \( \neq 1 \))
3. The system noise is uncorrelated with the scene scattering matrix
4. The noise covariance matrix = 0.

Natural scatters often have the qualities listed above to at least some degree. It was assumed that a patch of grass in the Gotcha scene met the above criteria. Channel imbalance was then computed using the trihedral response knowing \( s_{hh}/s_{vv} = 1 \) and \( s_{hv} = s_{vh} = 0 \). Finally, absolute gain was computed using the known theoretical cross section of the canonical targets. This information was then used to compute \( T \) and \( R \) in equation 4.2 and subsequently, to invert equation 4.1, yielding calibration factors for each of the 4 polarization channels. On a decibel scale, these factors are actually offsets. Below is a figure showing the calibration offset computed for each canonical target:
Figure 4.3: Plot showing the calibration offsets calculated using each of the canonical targets. Although the targets were of different sizes and types and placed at different orientations, the calculated offsets were very similar. The range of the plot matches the range of the intensity scale for the raw image. The mean value indicated by the blue bar represented the actual offset used.
Although the targets were of various shapes, sizes, and orientations, all yielded a very similar calibration factor.

In order to simulate the SAR phase history for this scene in DIRSIG, a number of operational parameters had to be extracted. Many were directly given in the MATLAB structure that accompanied the data. Many others, however, had to be inferred, based on an assumption that the demodulated bandwidth of the de-chirped signal was approximately equal to the transmitted pulse bandwidth. Below is a table summarizing the radar system parameters used in the DIRSIG simulation:

Table 4.1: AFRL platform parameters used for the DIRSIG simulation.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_0$</td>
<td>9.6GHz</td>
</tr>
<tr>
<td>chirprate</td>
<td>$9.14 \times 10^{14}$ Hz/s</td>
</tr>
<tr>
<td>nsamples</td>
<td>424</td>
</tr>
<tr>
<td>A/D sampling rate</td>
<td>622 MHz</td>
</tr>
<tr>
<td>bandwidth*</td>
<td>622 MHz</td>
</tr>
<tr>
<td>pulse duration*</td>
<td>681 ns</td>
</tr>
<tr>
<td>range resolution*</td>
<td>0.24 m</td>
</tr>
<tr>
<td>scene size*</td>
<td>100 m</td>
</tr>
</tbody>
</table>

*note: These are approximate values only. The bandwidth given is actually the intermediate frequency bandwidth of the de-chirped signal, the pulse duration is actually the collection duration, and the range resolution is based on the intermediate frequency bandwidth. These are the values that were used for the DIRSIG .platform file. Image quality was not adversely affected by using these assumptions for processing the phase history, indicating they are valid.

It was further assumed that the antenna gain pattern was a rectangular function in angular space with a half angle sufficient enough to ensure the entire scene was illuminated. More advanced gain patterns are not supported by DIRSIG. Also included in the MATLAB structure were the positions of the platform for each of the 8 passes recorded. Below is a plot of the flight path for each pass:
Figure 4.4: Flight paths used to collect the AFRL data for all 8 passes.

Once the operational parameters had been extracted, a scene was created in Blender that was representative of the scene for the Gotcha collection. This scene included the canonical targets which were placed, scaled, and oriented as specified in figure 4.2. Also included were a handful of cars, telephone poles, and powerlines. Below is a screen capture of the Blender scene used for simulation:
CHAPTER 4. METHODS

The scene shown above is what was used for the qualitative assessment. Ground truth for the reflectivity distributions was not obtained for any targets. Since the goal was to perform a qualitative comparison between the simulated and field imagery for the scene-wide comparison, approximations for the reflectivity distributions were assumed to be adequate for this purpose. For quantitative comparisons of the canonical targets a reflectivity of one was assumed adequate. Since DIRSIG does not have a physical optics capability, the reflectance distribution used was a simple specular lobe model. Realistically, the gain pattern for a perfectly conducting square surface would result in a 2D sinc pattern whose width would depend upon the dimensions of the surface. More specifically, the reflectivity model used in DIRSIG for all targets was described by the following equation:

$$BRDF(\theta_i, \theta_0, \Delta \phi) = d + s \cdot MM(\theta_n, n, k) \cdot e^{-\tan^2 \frac{\theta}{2\sigma^2}}.$$  

(4.3)

where $s$ and $d$ are the specular and diffuse reflectivity coefficients, respectively. For all targets, $\sigma$ was set to 0.1 radians. The reflectivities for each material contained in the DIRSIG scene are provided in the table below:
Table 4.2: Material reflectivity parameters used for the DIRSIG simulation.

<table>
<thead>
<tr>
<th>material</th>
<th>s</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>grass</td>
<td>0.01</td>
<td>0.007</td>
</tr>
<tr>
<td>asphalt</td>
<td>0.01</td>
<td>0.003</td>
</tr>
<tr>
<td>canonical targets</td>
<td>1.00</td>
<td>0.00</td>
</tr>
<tr>
<td>vehicles</td>
<td>0.04</td>
<td>0.04</td>
</tr>
</tbody>
</table>

For the quantitative assessment, only the canonical targets were included in the scene. The resulting output was ‘calibrated’ using knowledge of the known input parameters such as pulse power, pulse duration, and beam solid angle. The RCS’s of the canonical targets were then extracted individually for subsequent comparison to the experimentally derived RCS’s.

4.2 Power Line Modeling

In order to assess the viability of the method presented in this research, a number of data sources were used. The first source of data was graciously provided by the University of Michigan College of Engineering [6]. It included measurements of powerline RCS’s at different frequencies for different cable diameters. These measurements were taken in a controlled environment with well know truth data. The second source was the Gotcha data described in the previous section and provided by AFRL/SNA. The Gotcha data enabled both a qualitative and quantitative comparison of field data. Powerline images extracted from the Gotcha data were used for qualitative comparison to the simulated powerline images. Powerline RCS’s extracted from the calibrated Gotcha data were also used for quantitative comparison to simulated RCS’s.

4.2.1 Anechoic Chamber Measurements

The accuracy of the ILDC approach presented in this research was first assessed using data collected in a controlled environment. This was accomplished by comparing the simulated RCS’s of various types of powerlines to RCS measurements taken in an anechoic chamber. Below is a diagram depicting the overall setup for the experiment conducted by the University of Michigan, College of Engineering [6]:
The scatterometer used to measure the backscattered return had a dynamic range of 100 dB with quad-polarization capability. The antenna for this device consisted of an orthomode transducer (OMT) and a dual polarized square horn with a cross-polarization isolation of 20 dB [6]. Targets were placed on a styrofoam mount oriented by an azimuth-over-elevation positioner. Azimuth was controlled using a stepper motor that had a resolution of $\frac{1^\circ}{10}$ and elevation was controlled using an analog positioner.

In order to calibrate the measured return to units of dBm$^2$, a number of steps had to be performed. First, 10 measurements of the Complex Amplitude (CA) of a calibration sphere were taken over 10 viewing angles. Below is an image showing the sphere placed on a mount in the anechoic chamber:
Next, 10 measurements of the CA of the return with just the mount in place were taken. It was assumed that the return for both targets was isotropic. The average CA for the mount-only measurements was subtracted from the average CA of the mount-plus-sphere measurements. Dividing the magnitude squared of the result by the well known RCS of a sphere [24] yielded a calibration factor.

The CA of various powerlines were then measured over a range approximately -10° to 80° about normal incidence in 1° increments. Below is an image of the powerline on the styrofoam mount in the anechoic chamber:
As with the calibration sphere, the average CA of just the mount was subtracted from the average CA of the mount plus powerline. The calibration factor computed using the calibration sphere was then applied to the magnitude squared of the resultant data to yield measurements in units of $\text{dBm}^2$. This was done for both X- and C-band frequencies.

The targets used in the experiment consisted of a variety of cables. In order to obtain far-field measured data, the length of all cables was constrained to $\approx 1$ ft. The geometric cross section of each target as viewed along the axial direction is shown below [6]:

---

Figure 4.8: Image showing powerline placed on top of mount [6].
Figure 4.9: Images depicting the geometric cross-section of each object for which RCS measurement were made: (a) 1.27 cm cylinder (b) 167.8 MCM Copper (c) 556.5 MCM Aluminum (d) 954 MCM Aluminum & Steel and (e) 1431 MCM Aluminum & Steel. All cables were $\approx 1$ ft in length [6].
The powerline RCS’s measured as described above were used to validate the ILDC approach presented in this research. The 3D diffraction coefficient of each of the targets shown in figure 4.9 was simulated as described in section 3.3.3. These 3D diffraction coefficients were then converted to RCS’s using equation 3.99. A comparison between the simulated and measured RCS’s are presented in 5.2.1.

4.2.2 Gotcha Power Line RCS Comparison

In order to validate the method presented in this research against field data, the AFRL Gotcha scene was used. A description of the Gotcha data set and how it was processed and calibrated is described in 4.1. Once the imagery was calibrated, the RCS’s of the powerlines contained in the image were extracted. In order to extract the RCS of the powerlines contained in the Gotcha image, a number of steps were performed. First, the top 3 transmission lines were chipped from the full image for each frame. Below is an image of one of those chips:

![Image Chip for Top 3 Powerlines](image)

Figure 4.10: Image chip of isolated transmission lines.
The bottom bundle of communication wires was not included. This bottom bundle of wires was closely spaced and since the method presented in this research does not support second order diffractions, a quantitative comparison with simulated data was not performed. The RCS value for all pixels in the chipped image were summed to yield an effective RCS of the entire chip. In order to compensate for the ground contribution, a nearby patch of ground was also chipped from the image. The mean RCS of the ground chip was multiplied by the number of pixels in the power line chip, then subtracted from the power line chip. This yielded a measure of the effective RCS of just the top three transmission lines in the image.

In order to simulate the RCS of the powerlines, a physical model was needed. Although exact ground truth of the power lines contained in the Gotcha scene could not be obtained, estimates based on visual inspection were used for developing the physical model. Below is an image taken of a representative telephone pole:

![Figure 4.11: Picture taken of a representative telephone pole configuration. The actual telephone pole only had three transmission lines on top, configured similarly to what is shown in the picture, as well as a thick bundle of communication cables halfway up. Images of the actual telephone poles could not be obtained.](image)
The configuration of transmission lines along the top of the pole are representative of what was actually observed on-site. The transmission lines were approximately 2 cm in diameter and were modeled to have a 1.25 m drop over a 50 m run. Below the top three transmission lines was a single bundle of approximately five communication cables. This collection of cables was modeled for qualitative comparison purposes as a 50 cm diameter cylinder with a 50 m run and 1 m drop. The 50 m run was measured directly from the Gotcha images and the amount of sag was based on visual inspection and tweaked so that scattering width of the main lobe in the simulated data matched that of the experimental data. Below are various plots depicting the physical model used:

Figure 4.12: Physical model images. (a) Sketch of powerline in CAD software (b) 3D powerline plot (c) Side view of the top transmission line.
Once the physical model was built, the phase history for just the top three transmission lines as well as the phase history for the transmission lines with communication cables were simulated as described in section 3.3.4 using equation 3.100. The parameters listed in table 4.1 were used as input for the simulator. The phase history for a perfect point reflector was also built and processed using the same parameters. The image of the perfect reflector was used for 'algorithmic' calibration to correct the gain induced by using the backprojection algorithm as implemented in RITSAR. Once the impulse response of the powerline images was corrected, each resolution cell provided a measure of the complex 3D diffraction coefficient. Applying equation 3.99 to each resolution cell and then summing all cells in the image yielded an effective RCS of the transmission lines, measured similarly to the Gotcha data. These RCS’s were used for quantitative comparison. For qualitative comparison, the phase history of the 3 transmission lines plus the bundle of communication wires was simply added to the phase history of the DIRSIG Gotcha scene. Simulated imagery was created using RITSAR to process the phase history.
Chapter 5

Results

In order to demonstrate the current capabilities and limitations of DIRSIG’s radar modality, various comparisons were made. These comparisons provided a preliminary indication of the current capabilities in this wavelength regime and highlighted where the geometric optics approximation breaks down. Good scene-wide qualitative agreement with the Gotcha field data was achieved. The amplitude noise distribution was also observed to be Rayleigh distributed for both the simulated and field imagery, as expected. It was also demonstrated, however, that the geometric optics approximation was inadequate for simulating the scattered return from the smaller canonical targets and the powerlines.

The ILDC method was designed to augment existing SAR modeling tools to account for the scattered return from powerlines. One of the ways in which this method was evaluated was to compare predicted powerline RCS’s to those measured in an anechoic chamber. These measurements provided the opportunity to assess performance using data taken in a tightly controlled environment. A range of target types were used for the experiment and measurements were taken using both X- and C-band frequencies. Given the constraints imposed by the physical dimensions of the chamber, the length of all targets were kept to \( \approx 1 \text{ ft} \) in order to meet the far-field criterion for antennas \( \frac{d_f}{\lambda} = \frac{2L^2}{\lambda} \). This constraint did not allow for the powerlines to sag by an appreciable amount. In order to see how well the ILDC method worked for modeling powerlines as they would appear in a field-collected SAR image, comparisons were also made against the AFRL Gotcha data. As will be shown in the following sections, using the exact 2D ILDC for smooth cylinders provided an improvement over using a pure Physical Optics / Physical Theory of Diffraction (PO/PTD) approach. The ILDC approach was also able to account for sag in the powerline for field collected data. A few discrepancies were noted, however. A discussion of possible causes for these discrepancies and how they may be accounted for in future work is discussed.
5.1 DIRSIG Results

The SAR signal for the AFRL Gotcha scene was simulated as discussed in section 4.1. The resultant phase history was processed into 357 “frames.” Each frame used 4° of azimuth for processing with a starting azimuth ranging from 0° to 357° in 1° increments. Included in the DIRSIG scene was the parking lot with cars, grassy field in the top portion of the image, canonical targets, and powerlines. Below is a comparison between a simulated DIRSIG image and the corresponding actual Gotcha image:

![Comparison Image](image)

Figure 5.1: Qualitative Image Comparisons: (top) AFRL Gotcha, (bottom) simulated.

Both images appear to have good scene-wide qualitative agreement. Of note, however, is the absence of the powerline signature in the upper-right portion of the simulated image. The absence of a significant powerline signature was noted for all frames. This is a very strong validation of the statement that using a geometric optics approximation is inadequate for modeling powerlines in this wavelength regime.

Also of note is how well DIRSIG models speckle. The noise distribution for coherent imaging modalities follows a Rayleigh distribution [45]. In order to evaluate how well both the simulated and field data follow the theoretical noise distribution, a histogram for a patch of grassy area was extracted. It was assumed that this grassy area could be adequately approximated as a flat field. The normalized histograms are shown below, along with a corresponding best fit Rayleigh distribution:
CHAPTER 5. RESULTS

Figure 5.2: Intensity histograms for grassy field showing noise characteristics of data. A good fit using a Rayleigh distribution was obtained for both the simulated DIRSIG data and the Gotcha field data. This is consistent with the noise distribution expected for coherent imaging modalities. (left) DIRSIG simulated, (right) Gotcha.

The fitting algorithm in SciPy’s stats.rayleigh module was used to generate the continuous curves shown above. The results in figure 5.2 show that amplitude noise for the field data is indeed Rayleigh distributed and that DIRSIG accurately models this phenomenon. As discussed in section 4.1, there was an absence of truth data for the reflectances of the distributed targets in the scene, such as the grassy field. Since the purpose of the exercise was to assess the level of qualitative agreement between the simulated and field imagery, approximations were used for the reflectance distribution functions. Consequently, there is a mismatch between the mean and standard deviation for the observed and simulated histograms. Importantly, the results indicate that both histograms follow a Rayleigh distribution.

The RCS’s for both the simulated and field measured canonical targets were extracted as described in section 4.1. The figures on pages 93-94 (figures 5.3 and 5.4) compare the two RCS’s to each other. The peak RCS for the canonical targets, depicted by the dotted green line in those figures, are given by the following equations:

\[
\sigma_{\text{peak}} = \frac{4\pi a^4}{3\lambda^2} \quad \text{(trihedral)} \quad (5.1)
\]

\[
\sigma_{\text{peak}} = \frac{8\pi (w \cdot h)^2}{\lambda^2} \quad \text{(dihedral).} \quad (5.2)
\]

For the large 27 in. trihedral, DIRSIG came close to accurately modeling the return near specular incidence (0° phase angle in the RCS plots). Away from specular incidence, the simulated RCS diverges from the observed RCS for the 27 in. trihedral. For the smaller trihedral targets, DIRSIG consistently underestimates the scattered return for all
phase angles. This effect is especially pronounced for the dihedrals. The dihedral targets have a low acceptance angle in elevation, and the side lobes in their field-measured RCS signature indicate the need to use physical optics. For these targets, DIRSIG not only underestimates the scattered return, but also fails to account for the side lobes since the RCS was modeled using geometric optics and a simple specular lobe reflectance model. A discussion of how physical optics has been implemented in other ray tracing software using the shooting and bouncing of rays technique is provided in section 3.2.6. The focus of the current research, however, was on using an ILDC technique to model how powerlines scatter radiation.
Figure 5.3: RCS plots for trihedral targets: (a) TR1 (b) TR2 (c) TR3 and (d) TR4. The target labels TR1-TR4 correspond to those in figure 4.2. The DIRSIG derived RCS’s consistently underestimate the observed return, likely due to the absence of a physical optics capability.
Figure 5.4: RCS plots for trihedral and dihedral targets: (a) TR5 (b) TR6 (c) DR2 and (d) DR6. The target labels TR5-DR6 correspond to those in figure 4.2. The DIRSIG derived RCS’s consistently underestimate the observed return, likely due to the absence of a physical optics capability. This effect is especially pronounced for the dihedral targets which have small acceptance angle in elevation. Also absent in the simulated dihedral RCS’s are sidelobes, which require physical optics to be modeled.
5.2 Power Line Modeling Results

The ILDC approach presented in this research was validated using two primary datasets. The first dataset was provided by the University of Michigan, College of Engineering [6]. These data were collected as described in section 4.2.1 in a tightly controlled environment. This allowed the ILDC method to be evaluated against measurements with limited environmental contributions. Additionally, the configuration of target diameters and frequencies allowed an evaluation to be made with $k \cdot a$ products that ranged from 1.2 to 7, squarely within the range scoped for this research. Overall, error in the simulated RCS fell within the experimental uncertainty. The greatest discrepancy observed was for the X-band RCS of a 2.2 cm diameter cable ($k \cdot a = 4.4$) in the VV polarization channel. This was likely caused by making the smooth cylinder approximation which does not account for the TE waveguide mode stimulated by V-polarization [6, 23].

In order ensure far-field measurements were being taken for the powerlines measured in the anechoic chamber, their lengths were constrained to 1 ft. Consequently, an appreciable amount of sag was not present in the data. To see how sag affected the RCS signature, as well as to evaluate how well the ILDC method performed for a real-world scenario, powerline RCS’s extracted from the AFRL Gotcha field data were used. These RCS’s were obtained from the data using the process outlined in section 4.2.2. As with the anechoic data, significant error was observed in the VV channel. Additionally, 2nd order diffractions for the closely spaced transmission lines, which were not accounted for by the model, may have also added to the error in the simulated data.

5.2.1 Anechoic Chamber Data

One approach for modeling finite length cylinders found in the literature combines Physical Optics (PO) and the Physical Theory of Diffraction (PTD) [39]. For the cylindrical surface and the circular surfaces at each end, PO is used. For the edge adjoining the cylindrical surface and endcaps, the PTD solution for a 90$^\circ$ wedge is used. When applying PO to a finite length cylinder and the endcaps, the following assumptions are made:

1. The observation point is in the far-field, $d_f = \frac{2L^2}{\lambda} > 1$, where $L$ is the length of the cylinder and $d_f$ is the observation distance.

2. The surface is locally flat $k \cdot a >> 1$

3. No source currents are in the geometric shadow, including those which give rise to creeping waves.

4. The target is Perfectly Electrically Conducting (PEC).
Assumptions 2 and 3 in the list above are not valid for the data presented here. Additionally, the following assumptions and approximations are made when using PTD to model the edge contribution:

1. The surfaces adjoining the edge are locally flat $k \cdot a \gg 1$

2. Edge waves created by non-uniform source currents can be calculated accurately by subtracting the PO solution for scattering from an infinite half-plane from Sommerfeld’s exact solution.

3. The contour integral along the edge can be efficiently evaluated by using a stationary phase technique for only a handful of points.

Assumption 1 is once again invalid. When creeping waves are present, assumption 2 is also invalid. Finally, the error associated with the approximation made in assumption 3 is $O(ka)^{-1}$ [39]. Alternatively, the ILDC method makes the following assumptions:

1. The observation point is in the far-field

2. Bragg scattering can be neglected

3. The semi-open waveguide signature for TE radiation incident on the grooves in the helically wound cable can be neglected

4. The endcaps are not observed for real-world powerlines which will probably stretch beyond the scene.

An advantage of using this method over a pure PO/PTD approach is that no restriction is placed on $k \cdot a$, although a practical one does exist if this product starts to approach 100 when numerical machine precision is degraded, as shown in figure 3.4. Similarly, no assumptions are made about currents near a geometric shadow boundary or creeping waves. The length of the powerlines for the anechoic measurements presented here was constrained in such a way to ensure a far-field measurement was made, ensuring assumption 1 was satisfied. For the AFRL Gotcha SAR imagery, the length of a segment of powerline contained within a resolution cell was also in the far-field. Assumption 2 would have resulted in very distinct features that were not observed in the data. The literature also supports neglecting Bragg scattering for this $k \cdot a$ regime [4, 22]. Assumption 3 likely resulted in error for data in the VV channel (with V polarization nominally aligned along the TE direction). Experiments in literature show that the VV signature away from normal incidence can deviate from the smooth cylinder solution due to the TE waveguide signature [6]. Finally, assumption 4 is arguably reasonable for field data where the ends of the powerlines likely extend past the scene boundaries. Since the length of the powerlines
had to be constrained for the anechoic measurements to meet the far-field criterion, end-cap contribution dominated the signature away from normal incidence. Below are CAD generated images of a finite length cylinder as viewed from different aspect angles:

![Image](image1.png)

(a) 0° (b) 20° (c) 40° (d) 80°. Note the increasing view of the endcap and decreasing view of the cylindrical portion.

While the ILDC model does not account for endcaps, their contribution as predicted by PO/PTD was added to the ILDC signature so a reasonable comparison with the anechoic data could be made.

The following pages show how the simulated RCS’s compared to the measured data for a variety of targets at X-band frequencies:
Figure 5.6: Modeled X-band power line RCS compared to anechoic chamber measurements for 1.27 cm diameter ($ka = 2.5$) smooth cylinder: (top-pair) using Physical Optics and Physical Theory of Diffraction exclusively (bottom-pair) using exact smooth cylinder 2-D diffraction coefficient for cylindrical portion and PO/PTD for endcaps (left-pair) HH channel (right-pair) VV channel. Error bars on the measured data are scaled by ± 1 standard deviation. For the simulated data, error bounds depict the $O (k \cdot a)^{-1}$ error associated with using a stationary phase technique to evaluate the contour integral along the edge adjoining the endcap and cylinder.
Figure 5.7: Modeled X-band power line RCS compared to anechoic chamber measurements for 167.8 MCM \((ka = 2.4)\) copper power line: (top-pair) using Physical Optics and Physical Theory of Diffraction exclusively (bottom-pair) using exact smooth cylinder 2-D diffraction coefficient for cylindrical portion and PO/PTD for endcaps (left-pair) HH channel (right-pair) VV channel. Error bars on the measured data are scaled by \(\pm 1\) standard deviation. For the simulated data, error bounds depict the \(O(ka)^{-1}\) error associated with using a stationary phase technique to evaluate the contour integral along the edge adjoining the endcap and cylinder.
Figure 5.8: Modeled X-band power line RCS compared to anechoic chamber measurements for 556.5 MCM ($ka = 4.4$) aluminum power line: (top-pair) using Physical Optics and Physical Theory of Diffraction exclusively (bottom-pair) using exact smooth cylinder 2-D diffraction coefficient for cylindrical portion and PO/PTD for endcaps (left-pair) HH channel (right-pair) VV channel. Error bars on the measured data are scaled by ± 1 standard deviation. For the simulated data, error bounds depict the $O(k \cdot a)^{-1}$ error associated with using a stationary phase technique to evaluate the contour integral along the edge adjoining the endcap and cylinder.
Figure 5.9: Modeled X-band power line RCS compared to anechoic chamber measurements for 954 MCM ($ka = 6.1$) steel & aluminum power line: (top-pair) using Physical Optics and Physical Theory of Diffraction exclusively (bottom-pair) using exact smooth cylinder 2-D diffraction coefficient for cylindrical portion and PO/PTD for endcaps (left-pair) HH channel (right-pair) VV channel. Error bars on the measured data are scaled by $\pm 1$ standard deviation. For the simulated data, error bounds depict the $O(k \cdot a)^{-1}$ error associated with using a stationary phase technique to evaluate the contour integral along the edge adjoining the endcap and cylinder.
Figure 5.10: Modeled X-band power line RCS compared to anechoic chamber measurements for 1431 MCM ($ka = 7.0$) steel & aluminum power line: (top-pair) using Physical Optics and Physical Theory of Diffraction exclusively (bottom-pair) using exact smooth cylinder 2-D diffraction coefficient for cylindrical portion and PO/PTD for endcaps (left-pair) HH channel (right-pair) VV channel. Error bars on the measured data are scaled by ±1 standard deviation. For the simulated data, error bounds depict the $O(k \cdot a)^{-1}$ error associated with using a stationary phase technique to evaluate the contour integral along the edge adjoining the endcap and cylinder.
The Rayleigh far-field criteria for the measured data was:

\[ d_f = \frac{2L^2}{\lambda} = 5.9 \text{m}. \] (5.3)

Since measurements were taken at 13 m, the observation point was in the far-field. A summary of the results are provided in the table below:

Table 5.1: X-band results summary.

<table>
<thead>
<tr>
<th></th>
<th>20° half-width about normal</th>
<th>total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>% error simulated vs.</td>
<td>% uncertainty</td>
</tr>
<tr>
<td></td>
<td>measured</td>
<td>between measurements</td>
</tr>
<tr>
<td>Cylinder 1 HH</td>
<td>2.9%</td>
<td>5.2%</td>
</tr>
<tr>
<td>Cylinder 1 VV</td>
<td>0.76%</td>
<td>2.0%</td>
</tr>
<tr>
<td>Cable 1 HH</td>
<td>1.2%</td>
<td>5.8%</td>
</tr>
<tr>
<td>Cable 1 VV</td>
<td>2.4%</td>
<td>3.5%</td>
</tr>
<tr>
<td>Cable 2 HH</td>
<td>3.3%</td>
<td>4.5%</td>
</tr>
<tr>
<td>Cable 2 VV</td>
<td>29.5%</td>
<td>8.5%</td>
</tr>
<tr>
<td>Cable 3 HH</td>
<td>5.3%</td>
<td>5.0%</td>
</tr>
<tr>
<td>Cable 3 VV</td>
<td>9.5%</td>
<td>7.3%</td>
</tr>
<tr>
<td>Cable 4 HH</td>
<td>2.4%</td>
<td>5.4%</td>
</tr>
<tr>
<td>Cable 4 VV</td>
<td>2.4%</td>
<td>8.4%</td>
</tr>
<tr>
<td>mean HH</td>
<td>3.0%</td>
<td>5.2%</td>
</tr>
<tr>
<td>mean VV</td>
<td>8.9%</td>
<td>5.9%</td>
</tr>
<tr>
<td>mean</td>
<td>6.0%</td>
<td>5.6%</td>
</tr>
</tbody>
</table>

Overall, accuracy was within the experimental uncertainty of the data. In all cases, the ILDC approach performs better than the PO/PTD solution. Accuracy for the simulated VV data, especially for Cable 2, did not perform as well as the HH data. This is likely associated with error inherent in making a smooth cylinder approximation for grooved, helically wound powerlines.

The figures on the following pages demonstrate how well the simulated RCS’s matched the measured RCS’s for the C-band data:
Figure 5.11: Modeled C-band power line RCS compared to anechoic chamber measurements for 1.27 cm diameter ($ka = 1.3$) smooth cylinder: (top-pair) using Physical Optics and Physical Theory of Diffraction exclusively (bottom-pair) using exact smooth cylinder 2-D diffraction coefficient for cylindrical portion and PO/PTD for endcaps (left-pair) HH channel (right-pair) VV channel. Error bars on the measured data are scaled by ± 1 standard deviation. For the simulated data, error bounds depict the $O(k \cdot a)^{-1}$ error associated with using a stationary phase technique to evaluate the contour integral along the edge adjoining the endcap and cylinder.
Figure 5.12: Modeled C-band power line RCS compared to anechoic chamber measurements for 167.8 MCM \((ka = 1.2)\) copper power line: (top-pair) using Physical Optics and Physical Theory of Diffraction exclusively (bottom-pair) using exact smooth cylinder 2-D diffraction coefficient for cylindrical portion and PO/PTD for endcaps (left-pair) HH channel (right-pair) VV channel. Error bars on the measured data are scaled by ± 1 standard deviation. For the simulated data, error bounds depict the \(O(k \cdot a)^{-1}\) error associated with using a stationary phase technique to evaluate the contour integral along the edge adjoining the endcap and cylinder.
Figure 5.13: Modeled C-band power line RCS compared to anechoic chamber measurements for 556.5 MCM \((ka = 2.2)\) aluminum power line: (top-pair) using Physical Optics and Physical Theory of Diffraction exclusively (bottom-pair) using exact smooth cylinder 2-D diffraction coefficient for cylindrical portion and PO/PTD for endcaps (left-pair) HH channel (right-pair) VV channel. Error bars on the measured data are scaled by \(\pm 1\) standard deviation. For the simulated data, error bounds depict the \(O(ka)^{-1}\) error associated with using a stationary phase technique to evaluate the contour integral along the edge adjoining the endcap and cylinder.
Figure 5.14: Modeled C-band power line RCS compared to anechoic chamber measurements for 954 MCM ($ka = 3.0$) steel & aluminum power line: (top-pair) using Physical Optics and Physical Theory of Diffraction exclusively (bottom-pair) using exact smooth cylinder 2-D diffraction coefficient for cylindrical portion and PO/PTD for endcaps (left-pair) HH channel (right-pair) VV channel. Error bars on the measured data are scaled by ±1 standard deviation. For the simulated data, error bounds depict the $O(k \cdot a)^{-1}$ error associated with using a stationary phase technique to evaluate the contour integral along the edge adjoining the endcap and cylinder.
CHAPTER 5. RESULTS

Figure 5.15: Modeled C-band power line RCS compared to anechoic chamber measurements for 1431 MCM ($ka = 3.5$) steel & aluminum power line: (top-pair) using Physical Optics and Physical Theory of Diffraction exclusively (bottom-pair) using exact smooth cylinder 2-D diffraction coefficient for cylindrical portion and PO/PTD for endcaps (left-pair) HH channel (right-pair) VV channel. Error bars on the measured data are scaled by ± 1 standard deviation. For the simulated data, error bounds depict the $O((k \cdot a)^{-1})$ error associated with using a stationary phase technique to evaluate the contour integral along the edge adjoining the endcap and cylinder.

The Rayleigh far-field criteria for the measured data was:

$$d_f = \frac{2L^2}{\lambda} = 2.9 \text{m.}$$

(5.4)

Since measurements were taken at 13 m away, the observation point was in the far-field. A summary of the results is provided in the table below:
As before, overall accuracy fell within measurement uncertainty. Interestingly enough, there were a few HH simulations that performed worse than the VV simulations for small $k \cdot a$ configurations away from normal incidence. One possible cause for this is the error associated with using PTD for the edge contribution. PTD is a high-frequency approximation technique that breaks down for small $k \cdot a$ values. Additionally, the PTD integral around the cylinder edge is usually evaluated using a stationary phase technique. As mentioned earlier, evaluating the contour integral in this manner introduces additional error on the order of $O(k \cdot a)^{-1}$.

Table 5.2: C-band results summary.

<table>
<thead>
<tr>
<th></th>
<th>20° half-width about normal</th>
<th>total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>% error measured vs.</td>
<td>% uncertainty between measurements</td>
</tr>
<tr>
<td>Cylinder 1 HH</td>
<td>14.5%</td>
<td>11.1%</td>
</tr>
<tr>
<td>Cylinder 1 VV</td>
<td>4.2%</td>
<td>10.5%</td>
</tr>
<tr>
<td>Cable 1 HH</td>
<td>19.5%</td>
<td>14.3%</td>
</tr>
<tr>
<td>Cable 1 VV</td>
<td>18.5%</td>
<td>15.6%</td>
</tr>
<tr>
<td>Cable 2 HH</td>
<td>4.5%</td>
<td>11.4%</td>
</tr>
<tr>
<td>Cable 2 VV</td>
<td>3.9%</td>
<td>16.8%</td>
</tr>
<tr>
<td>Cable 3 HH</td>
<td>5.8%</td>
<td>9.4%</td>
</tr>
<tr>
<td>Cable 3 VV</td>
<td>1.2%</td>
<td>12.7%</td>
</tr>
<tr>
<td>Cable 4 HH</td>
<td>7.7%</td>
<td>6.0%</td>
</tr>
<tr>
<td>Cable 4 VV</td>
<td>1.9%</td>
<td>12.1%</td>
</tr>
<tr>
<td>mean HH</td>
<td>10.4%</td>
<td>10.4%</td>
</tr>
<tr>
<td>mean VV</td>
<td>5.9%</td>
<td>13.5%</td>
</tr>
<tr>
<td>mean</td>
<td>8.2%</td>
<td>12.0%</td>
</tr>
</tbody>
</table>
5.2.2 AFRL Gotcha Data

The phase history for the powerline configuration shown in figure 4.12 was simulated using the parameters in table 4.1 and the method outlined in section 3.3. The powerline phase history was simply added to the DIRSIG-generated scene phase history. Processing was accomplished as before, in $1^\circ$ increments using $4^\circ$ of aperture for all $360^\circ$ of data. Shown below are a few of the images where the powerlines in the scene are most apparent:

Figure 5.16: Qualitative Image Comparisons: (left) AFRL Gotcha (right) simulated (top) platform at top-right (bottom) platform at bottom-left.
Qualitatively, the powerlines in the image appear similar. To quantify the performance of the ILDC method, the effective powerline cross-sections between the simulated and field data were compared. Since the bottom bundle of communication cables would have contained strong second order diffractions, they were not used for evaluation. Only the top 3 transmission lines shown in figure 4.12 were used. The method by which the RCS’s were extracted from the field data and simulated imagery was discussed in 4.2.2. Imagery containing just the powerlines was calibrated for algorithmic induced gain, and equation 3.99 was applied to each resolution cell to obtain an effective cross-section. The total effective RCS was computed by performing a sum across the entire powerline-only image. Powerlines observed in the Gotcha data were chipped from the image, excluding the bottom bundle of cables. The ground contribution was subtracted using a nearby image chip containing only ground. The total number of pixels in the powerline chip was 3764, with each pixel being 0.24 m on a side. For reference, the geometric cross-section (GCS) of this area is:

\[
GCS_{\text{dBm}^2} = 10 \cdot \log_{10} \left[ \text{number of cells} \cdot \text{cell size}^2 \cdot \cos (\text{elevation angle}) \right] \quad (5.5)
\]

\[
= 10 \cdot \log_{10} \left[ 3763 \cdot (0.24m)^2 \cdot \cos (45^\circ) \right] \quad (5.6)
\]

\[
= 22 \text{ dBA}^2 \quad (5.7)
\]

The effective total cross section of the powerlines was obtained by summing all calibrated, ground compensated pixels in the powerline chip. Comparisons between the simulated and field extracted powerline RCS’s are shown below:
Figure 5.17: Quantitative RCS Comparisons: (left) HH channel (right) VV channel (top) platform at top-right (bottom) platform at bottom-left.
A summary of these results are provided in the table below:

Table 5.3: AFRL Gotcha results summary.

<table>
<thead>
<tr>
<th></th>
<th>% error - simulated vs. measured</th>
<th>% uncertainty - between measurements</th>
</tr>
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<tbody>
<tr>
<td>Top-right HH</td>
<td>0.2%</td>
<td>3.3%</td>
</tr>
<tr>
<td>Top-right VV</td>
<td>3.8%</td>
<td>2.7%</td>
</tr>
<tr>
<td>Bottom-left HH</td>
<td>6.3%</td>
<td>3.2%</td>
</tr>
<tr>
<td>Bottom-left VV</td>
<td>16.0%</td>
<td>4.1%</td>
</tr>
<tr>
<td>mean HH</td>
<td>3.3%</td>
<td>3.3%</td>
</tr>
<tr>
<td>mean VV</td>
<td>9.9%</td>
<td>3.4%</td>
</tr>
<tr>
<td>mean</td>
<td>6.6%</td>
<td>3.3%</td>
</tr>
</tbody>
</table>

Notably, the results shown above reveal a higher amount of relative error observed when the platform was located in the bottom left of the image. Inspection of the field-measured signature in figure 5.17 shows an asymmetric sloping shape, unaccounted for in the model, and not observed for the top-right portion of the pass. Since the change in zenith angle between the two passes was small (less than a degree) the source of error was effected by small changes in optical path. This possibly points to an unaccounted for signal that may have resulted from coherent addition between two separate returns. It is possible, also, that second order diffraction between powerlines may have been the cause. Nominally, powerlines in separate resolution cells are incoherent with respect to each other unless a significant amount of energy is contained in the side lobes of the impulse response. Given the right geometry, however, radiation which scatters off one powerline onto another and then back to the receiver may coherently add with the first order diffraction from the latter powerline. This could result in a signature that is very sensitive to slight asymmetry in the powerline configuration. Imperfect knowledge of ground truth may have also been a source of error. The powerline configuration used for simulation was based on visual inspection of the scene nearly a decade after the collection. It is possible that the configuration may have changed with time. Below is an image chip of the powerlines for two different platform locations:
Four signatures can be readily observed in the imagery when the platform was at the bottom-left of the image, and only 3 when the platform was at the top-right. The bottom bundle of communication cables was already chipped out. This phenomenon is repeatable for all 8 passes. This fourth signature could be explained if a second order diffraction, whose round-trip time delay was different from the other three transmission lines, was present in the signal. One other possibility for this difference is that a fourth line may have been on the telephone pole at the time of collection, but absent when a visual inspection of the scene was later performed. This fourth line may have been foreshortened onto another line at the top-right view, but not the bottom-left, resulting in the disparity. Yet another possibility is that the sidelobes of the impulse responses for the three transmission lines may have coincidentally met a condition for constructive interference at one view but not the other, resulting in an apparent fourth signature. Finally, it is also possible that the data was mis-calibrated, which could lead to the asymmetric signature observed. Given the similarity between calibration factors computed for all 8 passes and for all trihedral targets, this would be an unlikely, although possible, explanation, leaving the fourth signature unexplained.

With regards to performance in the HH versus VV channel, the HH simulation once again performed better than the VV simulation. The reasoning for this is likely the same
as before, where the smooth cylinder approximation exhibits more error in the VV channel than the HH channel.

An interesting observation from the simulated data indicates the width of the main scattering lobe is very sensitive to the amount of sag present in the powerline. The figure below demonstrates the results when the powerline is assumed to be a straight line:

![Graphs showing RCS Comparisons assuming no sag: (left) HH channel (right) VV channel (top) platform at top-right (bottom) platform at bottom-left.]

Figure 5.19: RCS Comparisons assuming no sag: (left) HH channel (right) VV channel (top) platform at top-right (bottom) platform at bottom-left.
This finding could indicate that a method can be developed which predicts the amount of sag based on the scattering width of the main lobe. Such information would aid those who maintain powerlines to perform a large area assessment of which lines may be too taught and ready to snap in the winter, or what lines are downed during a disaster. Since the amount of sag is also tied to ambient temperature, it might also be possible, given perfect knowledge of the powerline diameter and material, to gain some insight into ambient temperature based on the width of the main scattering lobe.

5.3 Summary

The baseline assessment performed for the DIRSIG simulations shows that qualitatively realistic imagery can be generated using this software tool in the radar modality. This is especially valuable in the case for non-conducting distributed targets, where speckle noise dominates and specular lobe reflectance models likely suffice. For PEC targets that have a characteristic length on the order of 10’s of wavelengths or less, the lack of a physical optics capability leads to significant quantifiable error. This was shown to be the case in the underestimated return for all canonical targets, the lack of sidelobes for the dihedral targets, and the complete lack of any powerline signature in the observed imagery.

To account for the powerline return, an ILDC approach was developed. This method produced simulated RCS’s that exhibited an overall accuracy which fell within the uncertainty of data measured in a controlled environment. At X-band frequencies, the error associated with making the smooth cylinder approximation could be readily observed. Reasonable performance was attained at C-band, although accuracy was significantly degraded away from normal incidence. This was likely due to the use of PTD, a high-frequency approximation technique, to model the edge contribution. The edge and endcap contributions would probably be unobserved in actual field data.

For simulation against field data, reasonable qualitative and quantitative agreement was attained. Errors for the effective RCS’s extracted fell within measurement uncertainty for the HH data but not for the VV data. This was likely due errors which arose from making the smooth cylinder assumption. Additionally, the shape of the RCS curves did not match well for one of the views. This could be due to second order diffractions between the closely spaced powerlines. Imperfect ground truth knowledge regarding the exact telephone pole configuration may have also induced error. The ground truth used for the Gotcha data was based on visual inspection of the site almost a decade after the collection was performed.

While these results show promise and establish a foundation by which an ILDC approach can be implemented for powerline modeling, additional work is needed to mature this research. Derivation of a far-field 2D diffraction coefficient for incident TE radiation
on a semi-open waveguide could be used to augment the smooth cylinder ILDC derived in this research. This could improve accuracy for VV (nominally TE) polarized data. Much work has already been done by Sarabandi, K. and Natzke, J. [23] for variously shaped cavities whose sizes are smaller than the wavelength of the incident radiation. Field data tailored for the express purpose of extracting powerline signatures would also be beneficial. The AFRL Gotcha dataset was uniquely useful in that a full 360° view of the scene could be incrementally processed. Using a circular SAR collection at multiple wavelengths, with more accurate ground truth data, would be a logical task for follow-on research. Data collected from such an experiment could also be used to determine to what extent second order diffractions affect the backscattered return from a configuration of closely spaced powerlines. Finally, it was also observed that there was a connection between the width of the specular lobe of a powerline’s RCS and the amount of sag. If a method to extract this information could be developed, it would be useful for surveying electrical infrastructure or obtaining a rough approximation of ambient temperature when no other data is available.
Chapter 6

Conclusion and Future Work

There are a number of applications where a method for remotely sensing powerlines would be beneficial. In the case of natural disasters, such a method could enable a wide area survey of the electrical infrastructure of affected locales. This survey could potentially show where powerlines are down and subsequently, the areas without electricity and the critical nodes that should be prioritized for repair. Pilots of low flying search and rescue aircraft could also incorporate this information into their mission planning.

One major obstacle to be overcome with powerline detection is collecting enough signal for the problem to be tractable. At IR wavelengths and shorter, powerlines are often too thin and only occupy a small fraction of a pixel. At millimeter wavelengths and longer, however, powerlines become more apparent. Since powerlines are optically thin at these wavelengths, diffraction becomes the dominant scattering mechanism.

Empirically derived matched filters for powerline detection in quad-polarization imagery can be found in the literature [6, 46]. One way to assess the performance of these matched filters in a cost-effective way is to model the powerline signal from first principles. This can be done for a variety of systems and orbit or flight parameters.

A number of relevant research efforts for modeling electromagnetic scattering in the radar regime were reviewed. One well known tool is GRECO [12, 13, 14, 15]. This tool uses an SBR technique with Incremental Length Diffraction Coefficients for wedges to compute the RCS of an object given a CAD model. GRECOSAR uses GRECO to build up the SAR phase history for a target. The literature provides examples for maritime applications where the phase histories for individual vessels were simulated. Recent work for simulating urban areas yielded promising results against scenes containing simple objects which could be easily validated. Another example found in the literature that uses a variation of the Shooting and Bouncing of Rays technique is SARAS [16, 1]. This code is more focused on modeling urban scenery. It captures many features commonly observed in SAR imagery.
such as Rayleigh distributed amplitude noise and downrange placement of multiple-order reflections. One drawback to both GRECOSAR and SARAS is that they are relatively slower than conventional, physically-based renderers that use simple ray tracing only.

For applications where radiometric accuracy is not required, a POV-RAY-based approach can be found in the literature [2]. The way in which SAR images are formed create distortions such as foreshortening and downrange placement of multiple order reflections, which are not observed in convention optical imagery. POV-RAY accounts for these features and provides geometrically accurate imagery that can increase the interpretability of images containing large, complex structures. Another ray-tracing based tool adapted for the simulation of SAR imagery is DIRSIG [7, 8]. This tool is well validated in the optical-IR regimes and a significant amount of work has been performed for simulating LIDAR. The radar modality of DIRSIG is relatively recent and is derived from the LIDAR toolset. Simulation of the SAR phase history is built on a pulse-by-pulse basis. For each pulse, a bundle of rays is shot into a scene. The return signal is computed by centering a $\delta$-function on the time-of-flight for each ray packet, convolving the $\delta$-functions with a linear FM signal, and then adding the contribution from all packets. Currently, only linear FM signals and simple antenna gain patterns are supported. Additionally, the scattered amplitude is computed, using only a geometric optics approximation. This leads to an efficient method for simulating the phase history of large scenes, but the lack of an SBR capability in the radar modality can also lead to significant errors for certain objects, as was shown in Chapter 5, Results.

Another tool worth mentioning is Xpatch [9]. The software is proprietary but information on the website indicates it is based on an SBR approach. As was shown in this study, SBR is inadequate for modeling the return for optically thin objects such as typically sized powerlines at X-band frequencies and lower. In addition to general purpose radar simulation tools, there have also been some studies which focus on the problem of modeling how radar radiation scatters from powerlines. A notable work, led by Sarabandi, K., resulted in a MoM-based method that could be efficiently implemented at millimeter wavelengths and shorter. Efficiency was achieved by making a physical optics approximation, which breaks down when the wavenumber-diameter product is $\lesssim 1$. This is the gap identified in this study, which is to derive a way of efficiently calculating the field scattered from powerlines when the wavenumber-diameter product is $\lesssim 1$.

There are a variety of ways to model electromagnetic scattering depending on what approximations, if any, can be made for a given situation. At a top level, the RCS describes how efficiently a target scatters incident radiation. It is a metric solely dependent on target properties and aspect viewing angles. The RCS does not depend on distance to the target. One way to calculate the RCS is to use equation 3.4 which relates the squared magnitude of the incident field to the squared magnitude of the scattered far-field.
A number of ways for calculating the scattered field were presented in Chapter 3, Theory. The discussion began with Maxwell’s equations, the solutions to which provide the most accurate results. Analytically solving these equations is only tractable when boundary conditions can be specified in a natural coordinate system. The case of 2D scattering from a cylinder falls into this category. In general, however, it is difficult to impose boundary conditions for arbitrarily shaped surfaces. Numerical methods for solving Maxwell’s equations include FDTD [21] and MoM [17]. Although these approaches provide accurate results, stringent sampling requirements often lead to prohibitive computation time for current commercial-off-the-shelf computing hardware.

Approximations to Maxwell’s equations are often required to calculate efficiently the scattered field. A common approximation used is geometric optics. This approach is derived from the 0th-order term in the Luneberg-Kline series solution to Maxwell’s equations [47]. This term leads to solutions which result in the laws of reflection and refraction, as well expressions commonly found in radiometry. Ray tracing is often used for numerical computation. The geometric optics solution does not depend on frequency. It fails for longer wavelengths, at caustics, and at sharp discontinuities such as edges or shadows. To account for the scattered field from discontinuities using a geometric approach, one can use the Geometric Theory of Diffraction (GTD) [27]. This theory provides the ray directions through Fermat’s Principle for Edge Diffraction. The ray amplitudes from discontinuities are derived from exact canonical solutions such as Sommerfeld’s solution for scattering from an infinite half plane. The GTD uses a high frequency approximation so there are limits to its use. One notable limitation is in computing the scattered field near geometric shadow boundaries. In these regions, the GTD solution results in singularities.

Another high-frequency approximation technique commonly found in the literature is Physical Optics (PO) [33, 47]. The term Physical Optics has various meanings depending on the community. For radar, PO has a very specific meaning- it is the integral which results from making various simplifications to the integral form of Maxwell’s equations. These assumptions are: object is in the far-field; small differential regions on the surface are locally flat (the radius of curvature of the surface is much larger than the wavelength); and there are no source currents in the geometric shadow. This method is closely related to scalar diffraction theory and the Fraunhofer integral. Unlike scalar diffraction theory, however, PO retains polarization information and is not limited to two-dimensional apertures. In addition to breaking down when the aforementioned assumptions cannot be made, PO also exhibits singularities in geometric shadow transition regions.

For efficiently calculating the PO integral, the Modified Equivalent Current Approximation (MECA) [29] can be used. This method starts by fragmenting an object up into triangular facets and using ray-tracing software to approximate source currents. This is done by assuming each ray intersection represents a plane-wave incident on the surface of
a facet, with tilt given by the incidence angle with respect to local normal. For the specific case of triangular facets, the PO integral has an analytic form that can be efficiently calculated. Although PO is used to describe the scattering from perfect conductors, MECA can be applied to di-electric materials. When multiple ray bounces are enabled and PO calculations are performed for the final bounce only, this method becomes the Shooting and Bouncing of Rays (SBR) technique [31, 32, 42].

Avoiding the singularities observed in geometric shadow transitions regions for both GTD and PO is possible by using the Physical Theory of Diffraction (PTD) [28]. When the PO solution for scattering from an infinite half-plane is subtracted from the GTD form of Sommerfeld’s solution, the previously mentioned singularities cancel and the scattered field in geometric shadow transition regions evaluate to finite values. The resulting PTD solution represents the contribution from non-uniform currents near a discontinuity, such as an edge. PTD (as well as GTD) is often combined with SBR, where edges are identified as lines adjoining two adjacent facets that have surface normals whose difference exceed some user-defined threshold. Since one-dimensional edges cannot sample a ray bundle of finite density, the facets adjoining the edge perform the sampling. Directions of the outgoing rays are defined by Fermat’s Principle for Edge Diffraction and their amplitudes are given by the PTD diffraction coefficients. One limitation to PTD, in addition to it being a high-frequency technique, is that it only gives the contribution from an edge into what is termed the “Keller Cone,” defined by Fermat’s Principle for Edge Diffraction. For scattering outside the Keller Cone, Mitzner’s Incremental Length Diffraction Coefficient (ILDC) method is often used.

The approach developed in this study was based on the ILDC method first established by Mitzner [10]. The modeling effort started with construction of a physical model. It was assumed for this model that the powerline was a perfectly conducting smooth cylinder with sag. The curve which described the amount of sag was given by a catenary. A 2D diffraction coefficient for forward scattering was then calculated from Maxwell’s equations. The ILDC method provided a means of extending this solution to a 3D backscattering coefficient. Using the definition of the RCS, a relationship between the 3D diffraction coefficient and the powerline RCS was derived. The 3D diffraction coefficient also enabled simulation of the powerline phase history.

Multiple validation efforts were undertaken in this study. The first effort began with an assessment of DIRSIG’s radar modality to see if that tool provided any existing capability for powerline modeling. The scene used in the DIRSIG study was based off the scene in the AFRL Gotcha collection. This scene contained a parking lot with cars, a grassy field, canonical calibration targets such as dihedrals and trihedrals, and powerlines. Material properties and reflectance distributions for all objects in the scene were assumed. The reflectance distributions were based on a specular lobe model. The radar parameters and
platform positions used in the simulation matched those extracted from the auxiliary data contained in the Gotcha dataset. Since all parameters used in the simulation were user-defined, the simulated imagery was calibrated to units of effective RCS for each pixel. The Gotcha imagery was also calibrated using a backscatter correlation method [44] so like-to-like comparisons could be made. The features compared between the simulated and Gotcha imagery were the amplitude noise, canonical target RCS's, and powerline signatures. The amplitude noise in the DIRSIG generated image was Rayleigh distributed, as predicted by theory and observed in the Gotcha data. Qualitative scene-wide agreement was also observed when compared to the Gotcha imagery. For the RCS's of the canonical targets, significant discrepancies were noted. This was likely due to the lack of a physical optics capability. Finally, no powerline signal was apparent in the DIRSIG imagery. This demonstrated the inability to model powerlines using a geometric optics approximation.

The other validation efforts in this study focused on assessing the efficacy of the ILDC approach. In order to see how well RCS's could be predicted using the relation defined by equation 3.99, measurements taken in an anechoic chamber were used. These measurements were obtained in a tightly controlled environment. They consisted of observations of the scattered power for various different types of powerlines. The data was calibrated using a standard method involving the use of a calibration sphere (section 4.2.1). The observed RCS's were compared to the simulated RCS's for validation. With few notable exceptions, the accuracy of the simulated data fell within experimental uncertainty for a variety of powerline types at both X- and C-band frequencies. Errors were somewhat larger for the C-band measurements, especially away from normal incidence. This may have been due to the use of a high-frequency stationary phase technique to evaluate the scattering contribution from the endcaps. PTD coefficients for a 90° wedge were used for the endcap edges, and since PTD is also a high-frequency technique that does not account for creeping waves, the combination likely led to increased error. In both the X- and C-band frequencies, errors in the VV polarization channel were relatively higher that those observed in the HH polarization channel. As suggested by Sarabandi, K. et. al. [6], these errors could have resulted from making a smooth cylinder assumption for calculating the 2D diffraction coefficient. This approach neglects the waveguide signature that is stimulated when TE radiation (nominally VV) is incident upon the grooved helically wound cables that make up a powerline.

Field measured powerline RCS’s were also extracted from the calibrated Gotcha imagery and used for validation. These RCS’s represented a total effective RCS. In order for a like-to-like comparison to be made, powerline phase histories had to be simulated. This was done using equation 3.100 and information obtained from visual inspection of the scene. Once the phase histories were simulated, they were processed into images which were subsequently used to derive effective powerline RCS’s, measured similarly to the
Gotcha data. The powerline phase histories were also added to the DIRSIG generated phase history so a qualitative comparison could be made between the two sets of images.

Qualitatively, the powerline signatures observed in the simulated imagery appeared very similar to the those in the Gotcha imagery. There was also reasonable agreement between the simulated and extracted RCS's. It was also shown that the amount of sag present in the powerline had a significant impact on its observed signature. As with the Michigan data, greater errors were observed in the VV channel, probably for the same reason, making a smooth cylinder assumption. There was also an asymmetric sloping pattern observed in the powerlines' RCS and an additional signal in the imagery. The additional signal was observed when the platform was located to the southwest. The asymmetric sloping pattern was also more apparent when the platform was positioned to the southwest. This was observed for all 8 passes. Since the change in zenith angle between the two observation points was less than 1°, the source of error would also have been susceptible to slight changes. This is possibly indicative of an unaccounted for signal resulting from two or more coherently added returns. Such a source of error could have been caused by second-order diffractions, or the sidelobes from all returns coincidentally lining up. An additional cable, present at the time of collection and absent when visual inspection was made, could have also led to the observed error.

The results from this study indicate that an ILDC approach to modeling the radar return from powerlines provides an efficient means for calculating the scattered return. The accuracy is demonstrably better than using a pure PO/PTD approach when \( k \cdot a \lesssim 1 \) and is computationally more efficient than a MoM approach. A number of possibilities for future work are opened up by this effort. For reducing the error observed in the VV channel, a far-field 2D diffraction coefficient that takes into account the helical geometry of powerlines would increase accuracy. This diffraction coefficient could be obtained in several ways. For instance, a two-step analytic approach would be to derive a far-field, semi-open waveguide diffraction coefficient from a solution in the literature [23], and then add this as a correction term to the smooth cylinder diffraction coefficient derived in the present research. Another numerical approach may involve using MoM to obtain the source currents over a single period of an infinitely long powerline. A diffraction coefficient could then be derived from the resulting far-field solution.

Another unaccounted for feature in the field data was the sloping asymmetric RCS observed for the powerlines. Incorporating a model that accounts for 2\textsuperscript{nd}-order diffractions could improve the accuracy of the simulated data. An overall source of error for the field data was uncertainty in the ground truth. Future work involving a field collection more focused on the powerline modeling problem, and with more precise ground truth, would also help to mitigate any remaining sources of error.

A number of future applications can also be identified. It was shown that there was
some relationship between the width of the main lobe in a powerline’s RCS and its sag. If such a relationship could be quantified, it could be used by maintainers of electrical infrastructure to determine if powerlines in a certain area are in danger of snapping in winter. Given perfect knowledge of the conducting material and tension, some measure of absolute ambient temperature, or relative ambient temperature differences from one day to another, could be obtained from SAR imagery containing powerlines. Another potential application would be in the identification of buried cables for ground-penetrating UHF/VHF SAR collections. Since there is a strong mathematical relationship between the scattering of electromagnetic radiation and acoustic waves, it may also be possible to adapt this approach to model how underwater cables scatter an active sonar pulse. Pursuing any of these applications can lead to an improved capability for forward modeling powerline returns that can then be used to evaluate a given detection algorithm for a system and scene of interest.
Bibliography


Appendices
import numpy as nprom numpy import sqrt, pi, exp, sin, cos
from numpy.linalg import norm
from scipy.special import j0, j1, y0, y1, jv, hankel2, jvp, h2vp, hankel1, h1vp
import matplotlib.pyplot as plt
import mpl_toolkits.mplot3d import Axes3D
def d(x, debug=False, max_iter=2000):
    # x is the retarded wave number times a, the diameter
    # Define first few Bessel functions
    J0 = j0(x)
    J0p = jvp(0, x)
    
    # Define first few Hankel functions
    H0 = hankel1(0, x)
    H0p = h1vp(0, x)
    
    # Define first few terms
    C_TM = -J0/H0
    C_TE = -J0p/H0p
    
    # Calculate higher order terms
    thresh = 1.0e-6
    test = False
    n=1
    traceTE = []
    traceTM = []
    while not test:
        # Compute additional term
        del_TM = (-1)**(n+1)*2*jv(n, x)/hankel1(n, x)
        del_TE = (-1)**(n+1)*2*jvp(n, x)/h1vp(n, x)
        
        # Update coefficient
        C_TM += del_TM
C_TE += del_TE

# Perform test to see if we've reached the maximum number of iterations
# or see if the additional term is less than the threshold value
traceTM.append(abs(del_TM)/abs(C_TM))
traceTE.append(abs(del_TE)/abs(C_TE))
test1 = n>=max_iter
test2 = traceTM[n-2].all()<=thresh
test3 = traceTE[n-2].all()<=thresh
test = test1 | test2 | test3

# Update n
n+=1
d_TM = sqrt(2/pi)*exp(-1j*pi/4)*C_TM
d_TE = sqrt(2/pi)*exp(-1j*pi/4)*C_TE
d_out = [[d_TM,0],[0,d_TE]]
if debug:
    plt.figure()
    plt.plot(traceTE)
    plt.figure()
    plt.plot(traceTM)
    print(n)
return(d_out)
def D_endcap(platform, PL):
    # Grab relevant parameters
    a = PL['diameter']/2
    pos = platform['pos']
    f_0 = platform['f_0']
    c = 299792458.
    k = 2*pi*f_0/c
    rhat_i = np.zeros([len(pos),3])
    for i in range(len(pos)):
        rhat_i[i] = pos[i]/norm(pos[i],axis=-1)
    nhat = -PL['orientations'][-1]
    theta = np.pi - np.arccos(np.dot(-rhat_i, nhat))
    i = np.where(theta<pi/2); theta[i] = pi-theta[i]
    l = norm(PL['r_p'][-1])

    # Compute D for each point
    D = [[0,0],[0,0]]
    D[0][0] = 1j/2*a*k*np.cos(theta)/np.sin(theta)*
        *j1(2*k*a*np.sin(theta))*np.exp(1j*2*k*l*np.cos(theta))
    D[1][1] = -1j/2*a*k*np.cos(theta)/np.sin(theta)*
        *j1(2*k*a*np.sin(theta))*np.exp(1j*2*k*l*np.cos(theta))
    return(D)
def D_PO(platform, PL):
# Grab relevant parameters
a = PL['diameter']/2
pos = platform['pos']
f_0 = platform['f_0']
c = 299792458.
k = 2*pi*f_0/c
rhat_i = np.zeros([len(pos), 3])
for i in range(len(pos)):
    rhat_i[i] = pos[i]/norm(pos[i], axis=-1)

rhat = -PL['orientations'][-1]
theta = np.pi - np.arccos(np.dot(-rhat_i, rhat))
i = np.where(theta<np.pi/2); theta[i] = np.pi - theta[i]
l = norm(PL['r_p'][-1])

# compute D for each point
D = [[0, 0], [0, 0]]
D[0][0] = -1j/2*a*k*np.sin(theta)/np.cos(theta)*sin(2*k*l*cos(theta))
    *(j0(2*k*a*sin(theta)) - 1j*j1(2*k*a*sin(theta)))
D[1][1] = 1j/2*a*k* np.sin(theta)/np.cos(theta)*sin(2*k*l*cos(theta))
    *(j0(2*k*a*sin(theta)) - 1j*j1(2*k*a*sin(theta)))
return (D)

def D(platform, PL):
    # Grab relevant parameters
    pos = platform['pos']
f_0 = platform['f_0']
c = 299792458.
k = 2*pi*f_0/c
rhat_i = np.zeros([len(pos), 3])
for i in range(len(pos)):
    rhat_i[i] = pos[i]/norm(pos[i], axis=-1)

r_0 = PL['points']
rhat_0 = PL['orientations']
T = PL['lengths']

# For each observation angle
D = [[0, 0], [0, 0]]
D[0][0] = np.zeros([len(rhat_i), len(T)])+0j
D[1][1] = np.zeros([len(rhat_i), len(T)])+0j
for i in range(len(rhat_i)):
    print('Calculating D for look %i' %i)
    # compute D for each point
    beta_i = np.arccos(np.dot(rhat_0, rhat_i[i]) - np.pi/2)
    phase = -2*k*np.dot(r_0, rhat_i[i])
    X = 1/np.pi*k*T*abs(np.sin(beta_i))
    d2 = d(k*np.cos(beta_i)*PL['diameter'])
    D[0][0][i] = np.exp(1j*phase)*np.exp(-ij*pi/4)/(2*pi)*
        k*T*np.sinc(X)*d2[0][0]
    D[1][1][i] = np.exp(1j*phase)*np.exp(-ij*pi/4)/(2*pi)*
        k*T*np.sinc(X)*d2[1][1]
D_out = {'D' : D}

PL.update(D_out)

def Efield_create(amplitude, platform):
    # Get platform parameters
    pos = platform['pos']

    # Define viewing geometry
    r_hat = pos / np.array([norm(pos, axis=-1)]).T
    z_hat = np.array([0, 0, 1])

    # Derive H and V polarization directions
    H_hat = np.cross(r_hat, z_hat) / np.array([norm(np.cross(r_hat, z_hat), axis=-1)]).T
    V_hat = np.cross(H_hat, r_hat)

    E = {
        'H': amplitude * H_hat,
        'V': amplitude * V_hat
    }

    D_out = {'E': E}

    platform.update(D_out)

def straight_PL_create(end_points, spacing, random=True, plot=False):
    # Create baseline catenary that lies along the x-axis
    L = norm(np.array(end_points[0]) - np.array(end_points[1]))
    x = np.arange(-L/2, L/2, spacing)

    # Add randomness to point locations to avoid artificial interference
    if random:
        dx = (np.random.rand(len(x)) - 0.5) * spacing
        x = x + dx

    # Place locations into an array
    locs = np.vstack((x, np.zeros(x.shape), np.zeros(x.shape)))

    # Find how the input catenary is oriented
    e = np.array(end_points[0]) - np.array(end_points[1])
    e_hat = e / norm(e)
    theta = np.pi - np.arccos(np.dot(e_hat, np.array([1, 0, 0])))

    # Rotate the baseline catenary so it is oriented with the input catenary
    R_z = np.matrix([[np.cos(theta), -np.sin(theta), 0],
                     [np.sin(theta), np.cos(theta), 0],
                     [0, 0, 1]])
    pos = np.matrix(locs)
    pos = R_z * pos
    locs = np.array(pos).T

    # For two sequential points, replace with the mean value and
# calculate the orientation and differential lengths
n = len(x)
locs_new = np.zeros([n-1,3])
orientations = np.zeros([n-1,3])
lengths = np.zeros(n-1)
for i in range(n-1):
    locs_new[i] = (locs[i]+locs[i+1])/2
    o = locs[i+1]-locs[i]
    lengths[i] = norm(o, axis=-1)
    orientations[i] = o/lengths[i]

# Translate to match input catenary
mid = (np.array(end_points[0])+np.array(end_points[1]))/2
mid[-1] = 0
locs_new = locs_new+mid

# Compute relative positions
n = len(locs_new)
r0 = locs_new[n//2]
r_p = np.zeros([n,3])
for i in range(n):
    r_p[i]=np.array(locs_new[i])-r0

# Plot catenary for diagnostic purposes
if plot:
    pos = np.array(locs_new)
    import matplotlib.pyplot as plt
    from mpl_toolkits.mplot3d import Axes3D
    fig = plt.figure()
    ax = fig.add_subplot(111, projection='3d')
    ax.plot(pos[:,0], pos[:,1], pos[:,2])
    ax.set_xlabel('x- coordinates')
    ax.set_ylabel('y- coordinates')
    ax.set_zlabel('z- coordinates')
    def axisEqual3D(ax):
        extents = np.array([getattr(ax, 'get_{}lim'.format(dim))() for dim in 'xyz'])
        sz = extents[:,1] - extents[:,0]
        centers = np.mean(extents, axis=1)
        maxsize = max(abs(sz))
        r = maxsize/2
        for ctr, dim in zip(centers, 'xyz'):
            getattr(ax, 'set_{}lim'.format(dim))(ctr - r, ctr + r)
    axisEqual3D(ax)

# Return PL dictionary
PL = {'points' : locs_new, 'orientations' : orientations, 'lengths' : lengths, 'r_p' : r_p}

return(PL)

def PL_create(end_points, sag_factor, spacing, random=True, plot=False):

# Create baseline catenary that lies along the x-axis
L = norm(np.array(end_points[0])-np.array(end_points[1]))
x = np.arange(-L/2, L/2, spacing)

# Add randomness to point locations to avoid artificial interference
if random:
    dx = (np.random.rand(len(x))-0.5)*spacing
    x = x+dx

# Create catenary
y = sag_factor*np.cosh(x/sag_factor)

# Get height of catenary to match the height of the input points
height_offset = end_points[0][-1]-y[0]
y = y+height_offset

# Place locations into an array
locs = np.vstack((x, np.zeros(x.shape), y))

# Find how the input catenary is oriented
e = np.array(end_points[0])-np.array(end_points[1])
e_hat = e/norm(e)
theta = np.pi-np.arccos(np.dot(e_hat,np.array([1,0,0])))

# Rotate the baseline catenary so it is oriented with the input catenary
R_z = np.matrix([[np.cos(theta), -np.sin(theta),0],[np.sin(theta), np.cos(theta),0],[0,0,1]])
pos = np.matrix(locs)
pos = R_z*pos
locs = np.array(pos).T

# For two sequential points, replace with the mean value and calculate the orientation and differential lengths
n = len(x)
locs_new = np.zeros([n-1,3])
orientations = np.zeros([n-1,3])
lengths = np.zeros(n-1)
for i in range(n-1):
    locs_new[i] = (locs[i]+locs[i+1])/2
    o = locs[i+1]-locs[i]
    lengths[i] = norm(o, axis=-1)
    orientations[i] = o/lengths[i]

# Translate to match input catenary
mid = (np.array(end_points[0])+np.array(end_points[1]))/2
mid[-1] = 0
locs_new = locs_new+mid

# Compute relative positions
n = len(locs_new)
r0 = locs_new[n//2]
r_p = np.zeros([n,3])
for i in range(n):
r_p[i]=np.array(locs_new[i])-r0

# Plot catenary for diagnostic purposes
if plot:
    pos = np.array(locs_new)
    fig = plt.figure()
    ax = fig.add_subplot(111, projection='3d')
    ax.plot(pos[:,0],pos[:,1],pos[:,2])
    ax.set_xlabel('x-coordinates')
    ax.set_ylabel('y-coordinates')
    ax.set_zlabel('z-coordinates')
    def axisEqual3D(ax):
        extents = np.array([getattr(ax,'get_{}lim'.format(dim))() for dim in 'xyz'])
        sz = extents[:,1] - extents[:,0]
        centers = np.mean(extents, axis=1)
        maxsize = np.max(abs(sz))
        r = maxsize/2
        for ctr, dim in zip(centers, 'xyz'):
            setattr(ax,'set_{}lim'.format(dim))(ctr-r, ctr+r)
        axisEqual3D(ax)
    axisEqual3D(ax)

    # Return PL dictionary
    PL = {'points': locs_new,
          'orientations': orientations,
          'lengths': lengths,
          'r_p': r_p}
    return(PL)

def PL_append(*argv, plot=True):
    if plot:
        fig = plt.figure()
        ax = fig.add_subplot(111, projection='3d')
        i = 0
    for PL in argv:
        if i == 0:
            PL_out = dict(PL)
            PL_out['diameter'] = np.ones(len(PL['points']))*PL['diameter']
        else:
            PL_out['orientations'] = np.vstack((PL_out['orientations'], PL['orientations']))
            PL_out['r_p'] = np.vstack((PL_out['r_p'], PL['r_p']))
            PL_out['points'] = np.vstack((PL_out['points'], PL['points']))
            PL_out['lengths'] = np.hstack((PL_out['lengths'], PL['lengths']))
            PL_out['diameter'] = np.append(PL_out['diameter'],
                                           np.ones(len(PL['points']))*PL['diameter'])
        i+=1
    # Plot catenary for diagnostic purposes
    if plot:
```python
pos = np.array(PL['points'])
ax.plot(pos[:,0], pos[:,1], pos[:,2])
ax.set_xlabel('x-coordinates')
ax.set_ylabel('y-coordinates')
ax.set_zlabel('z-coordinates')
def axisEqual3D(ax):
    extents = np.array([getattr(ax, 'get_{}lim'.format(dim))() for dim in 'xyz'])
    sz = extents[:,1] - extents[:,0]
    centers = np.mean(extents, axis=1)
    maxsize = max(abs(sz))
    r = maxsize/2
    for ctr, dim in zip(centers, 'xyz'):
        getattr(ax, 'set_{}lim'.format(dim))(ctr - r, ctr + r)
axisEqual3D(ax)

return(PL_out)

def D_HV(platform, PL):
    # Calculate 3D diffraction coefficient using platform
    # polarization vectors
    f_0 = platform['f_0']
c = 299792458.
k = 2*pi*f_0/c
views = len(platform['pos'])
T = PL['lengths']

    # Calculate basis vectors
    t_hat = PL['orientations']
z_hat = np.array([0, 0, 1])
r_hat = -platform['pos']/np.array([norm(platform['pos'], axis=-1)]).T
    # H = platform['E']['H']
    # V = platform['E']['V']
    H = np.cross(r_hat, z_hat)/np.array([norm(np.cross(r_hat, z_hat), axis=-1)]).T
    V = np.cross(H, r_hat)

    # Initialize D
    D = [[0, 0], [0, 0]]
    D[0][0] = np.zeros([len(r_hat), len(T)])*0j
    D[0][1] = np.zeros([len(r_hat), len(T)])*0j
    D[1][0] = np.zeros([len(r_hat), len(T)])*0j
    D[1][1] = np.zeros([len(r_hat), len(T)])*0j
    for i in range(views):
        print('Calculating D_{II} for view %i' %i)
        # Calculate D for each point, for each look
        beta_i = np.arccos(np.dot(t_hat, r_hat[i]))-pi/2
        X = 1/np.pi*k*T*abs(np.sin(beta_i))
        d2 = d(k*np.cos(beta_i)*PL['diameter'])
        DTM = np.exp(-1j*pi/4)/(2*pi)*k*T*np.sinc(X)*d2[0][0] #
        DTE = np.exp(-1j*pi/4)/(2*pi)*k*T*np.sinc(X)*d2[1][1] #
```

# Calculate basis vectors for each point
TE = np.cross(t_hat, r_hat[i]); TE = TE/norm(TE, axis=-1).T
TM = np.cross(r_hat[i], TE)
TEs = 1*TE
TMs = -1*TM

# Calculate II for each point
D[0][0][i] = DTM*abs(np.dot(TM, H[i])*np.dot(TMs, H[i])) +
        DTE*abs(np.dot(TE, H[i])*np.dot(TEs, H[i]))
D[0][1][i] = DTM*abs(np.dot(TM, H[i])*np.dot(TMs, V[i])) +
        DTE*abs(np.dot(TE, H[i])*np.dot(TEs, V[i]))
D[1][0][i] = DTM*abs(np.dot(TM, V[i])*np.dot(TMs, H[i])) +
        DTE*abs(np.dot(TE, V[i])*np.dot(TEs, H[i]))
D[1][1][i] = DTM*abs(np.dot(TM, V[i])*np.dot(TMs, V[i])) +
        DTE*abs(np.dot(TE, V[i])*np.dot(TEs, V[i]))

D_out = {'D_HV': D}
PL.update(D_out)

def RECT(t, T):
f = np.zeros(len(t))
f[(t/T < 0.5) & (t/T > -0.5)] = 1
return f

def simulate_PL_phs(platform, PL, amplitude=1):
    # Retrieve relevant parameters
    c = 299792458.
gamma = platform['chirprate']
f_0 = platform['f_0']
t = platform['t']
pos = platform['pos']
views = len(platform['pos'])
nsamples = platform['nsamples']
T_p = platform['T_p']
D_HV = PL['D_HV']#PL['D'] #
points = PL['points']

    # Simulate the phase history for each pulse, for each point
    phs = np.zeros([4, views, nsamples]) + 0j
    for i in range(views):
        print('simulating pulse %i' % (i + 1))

        R_0 = norm(pos[i])
        j = 0
        for p in points:
            R_t = norm(pos[i] - p)
            dr = R_t - R_0
            phase = pi*gamma*(2*dr/c)**2 -
                    2*pi*(f_0 + gamma*t)*2*dr/c

            phs = phs +
                    D_HV[i]*abs(np.dot(TM, H[i])*np.dot(TMs, H[i]))*exp(1j-phase) +
                    D_TE[i]*abs(np.dot(TE, H[i])*np.dot(TEs, H[i]))*exp(1j-phase)

        PL.update(D_out)


```python
T_p)
phs[1,i,:] += amplitude*D_HV[0][i,j]*exp(1j*phase)*RECT((t-2*dr/c),
T_p)
phs[2,i,:] += amplitude*D_HV[1][0][i,j]*exp(1j*phase)*RECT((t-2*dr/c),
T_p)
phs[3,i,:] += amplitude*D_HV[1][1][i,j]*exp(1j*phase)*RECT((t-2*dr/c),
T_p)
```

```
j+=1

#np.save('./phase_history.npy', phs)
return(phs)
```

```python
def simulate_PL_RCS(platform, PL):
    f_0 = platform['f_0']
c = 299792458.
wvl = c/f_0
k = 2*pi/wvl
points = PL['points']
pos = platform['pos']
views = len(pos)
rhat_i = np.zeros([len(pos),3])
for i in range(len(pos)):
    rhat_i[i] = pos[i]/norm(pos[i], axis=-1)
D_HV = PL['D_HV']
index1 = [1,0,0,1]
index2 = [1,0,1,0]
for i in range(4):
    phase = -2*k*np.dot(rhat_i, points.T)
    D = D_HV[index1[i]][index2[i]]
    RCS[:,i] = 10*np.log10(wvl**2*abs(np.sum(np.exp(1j*phase)*D, axis = -1))**2)

return(RCS)
```
Appendix B

RCS Simulation Code

```python
# if __name__ == '__main__':
# Add include directories to default path list
from sys import path
from numpy import *
from matplotlib.pylab import *
path.append('..//')
import re
import numpy as np
from numpy import *
import matplotlib
import matplotlib.lines as mlines
from matplotlib import rc
rc('text', usetex=True)
rc('font', size=14)
import warnings
warnings.filterwarnings('ignore', category=RuntimeWarning)
from functions import *
from PTD_functions import *
end_points1 = ([0, 0.3048/2, 0], [0, -0.3048/2, 0])
# end_points1 = ([0, 3* wvl /2, 0], [0, -3* wvl /2, 0])
# end_points1 = ([0, 40/2, 0], [0, -40/2, 0])
obj = 'cyl1'

# Import Experimental RCS
fname = "../Experimental RCS data/xband/1 ft_"+obj+"_msav"
fname = "../Experimental RCS data/cband/"+obj+"_msav"
f = open(fname, 'r')

r = re.compile('^-?\d+\.\d+')
RCS = []
for line in f:
    m = r.findall(line)
    if m != []:
```

136
RCS.append(m)

f.close()

fname = "../Experimental RCS data/cband/"+obj+"_mser.npy"
fname = "../Experimental RCS data/xband/1ft_"+obj+"_mser.npy"
RCS_err = np.load(fname)

# Define Simulation Parameters
# **************************************************************************
# diameter = 0.0127*3*wl #
a = diameter/2
f_0 = 9.5e9
# f_0 = 4.75e9
RCS = np.array(RCS, dtype = float64)
theta = linspace(0, 90, 2000)*pi/180
theta = linspace(RCS[:,0].min(), RCS[:,0].max(), 150)*pi/180
theta = (RCS[:,0]+0)*pi/180

# Create/Import Platform Dictionary
# **************************************************************************
r0 = matrix([[ -1 ,0 ,0]]) .T# *1/ sqrt (2)
n = len(theta)
p = zeros([n,3])
for i in range(n):
    R = matrix([ [cos(theta[i]), -sin(theta[i]), 0],
                 [sin(theta[i]), cos(theta[i]), 0],
                 [0, 0, 1] ])
    p[i] = squeeze(R*r0)
platform = {'pos' : p}
platform['f_0'] = f_0

# Create Power Line Object
# **************************************************************************
n_runs = 5
sigmaHH = 0
sigmaVV = 0
for i in range(n_runs):
    print('run %i'%(i+1))
    PL = straight.PL_create(end_points1, spacing = 0.01, random=False)
    #PL = PL_create(end_points1, sag_factor = -1, spacing = 0.01, random=True, plot =False)
    PL['diameter'] = diameter

    # Compute Endcap Contribution
    # **************************************************************************
    D_end = D_endcap(platform, PL)#[[0,0],[0,0]]#

    # Compute Edge Contribution
    # **************************************************************************
D_edge = D_edge_HF(platform, PL) #, npoints = 10000)
# D_edge = D_edge_exact(platform, PL, npoints=1000)
i = np.where(theta==0)
D_edge[0][0][i] = D_edge[0][0][i[0]+1]; D_edge[1][1][i] = D_edge[1][1][i[0]+1]
D_end[0][0] += D_edge[0][0][i]
D_end[1][1] += D_edge[1][1][i]

# Update PL Object with D Values

D_HV(platform, PL)

# Compute 3D RCS

D3 = PL['D_HV']
D3 = D_PO(platform, PL)
D3 = [[0,0],[0,0]]
c = 299792458
wvl = c/f_0
k = 2*pi/wvl

sigma_d = pi*a**2*(k*a)**2

sigmaHH += wvl**2*abs(sum(D3[0][0], axis=-1)+D_end[0][0])**2
sigmaVV += wvl**2*abs(sum(D3[1][1], axis=-1)+D_end[1][1])**2

sigmaHH += wvl**2*abs(D3[0][0]+D_end[0][0])**2
sigmaVV += wvl**2*abs(D3[1][1]+D_end[1][1])**2

sigmaHH_PO = 0; sigmaVV_PO = 0
sigmaHH_end = 0; sigmaVV_end = 0
sigmaHH_end += wvl**2*abs(D_edge[0][0])**2
sigmaVV_end += wvl**2*abs(D_edge[1][1])**2

sigmaHH = sigmaHH/n_runs
sigmaVV = sigmaVV/n_runs
sigmaHH_err = 10*log10(1+wvl**2*sin(theta)/((k*a)**2*sigmaHH))
sigmaVV_err = 10*log10(1+wvl**2*sin(theta)/((k*a)**2*sigmaVV))
APPENDIX B. RCS SIMULATION CODE

```python
xlabel(r'\textit{Angle }$\vartheta$ \textit{in Degrees}')
simulated = mlines.Line2D([], [], color='red', linestyle='-', label='Simulated (Experimental)'
plt.legend(handles=[simulated, experimental])
# title ('VV $d=\lambda$, $L=3\lambda$')
# plot (theta*180/pi+90, 10*log10(sigmaVV/(sigma_d*pi)), color='black')
# plot (theta*180/pi+90, 10*log10(sigmaVV_PO/(sigma_d*pi)), linestyle='--', color='black')
# plot (theta*180/pi+90, 10*log10(sigmaVV_end/(sigma_d*pi)), linestyle=':', color='black', linewidth=2)
# legend(['PTD VV', 'PO', 'Nonuniform / Fringe Component'])
# ylim([-40,20])
# ylabel('Normalized Scattering Cross-Section')
# xlabel('Angle $\vartheta$ in Degrees')
figure()
# title ('HH RCS Using PO/PTD')
title ('HH RCS Using Exact ILDC Plus PO/PTD Endcaps')
plot (theta*180/pi, 10*log10(sigmaHH), color='red')
plot (theta*180/pi, 10*log10(sigmaHH)+sigmaHH_err/2, linestyle='--', color='red')
plot (theta*180/pi, 10*log10(sigmaHH)-sigmaHH_err/2, linestyle='--', color='red')
errorbar (RCS[:,0], RCS[:,2], yerr=RCS_err[:,2], color='black')
# plot (RCS[:,0], RCS[:,2], color='black')
ylim([-60,10])
ylabel('Scattering Cross-Section (dBm$^2$)')
xlabel('Angle $\vartheta$ in Degrees')
simulated = mlines.Line2D([], [], color='red', linestyle='-', label='Simulated')
plt.legend(handles=[simulated, experimental])
# title ('HH $d=\lambda$, $L=3\lambda$')
# plot (theta*180/pi+90, 10*log10(sigmaHH/(sigma_d*pi)), linestyle='--', color='black')
# plot (theta*180/pi+90, 10*log10(sigmaHH_PO/(sigma_d*pi)), color='black')
# plot (theta*180/pi+90, 10*log10(sigmaHH_end/(sigma_d*pi)), linestyle=':', color='black', linewidth=2)
# legend(['PTD HH', 'PO', 'Nonuniform / Fringe Component'])
# ylim([-40,20])
# ylabel('Normalized Scattering Cross-Section')
# xlabel('Angle $\vartheta$ in Degrees')

figure() plot (log10(abs(D_edge[0][0])**2))
plot (log10(abs(D_edge2[0][0])**2))
figure() plot (log10(abs(D_edge1[1][1])**2))
plot (log10(abs(D_edge2[1][1])**2))
```

Appendix C

Phase History Simulation Code

```python
# Include standard library dependencies
import numpy as np
from matplotlib import cm
cmap = cm.Greys_r
import cv2

# Include SARIT toolset
from ritsar import phsRead
from ritsar import imgTools
from ritsar import phsTools

from sys import path
path.append('..')

# Create PL PHS
# ##############################################################################
from functions import *
off_l = 1.1
imb = 1
diameter = 0.0222
end_points1 = ([[-48.3, 31.4, 13], [-1.2, 48.9, 13]])
r = array(end_points1[1]) - array(end_points1[0]); r = r/norm(r)
off = cross(r, array([0, 0, 1])) * off_l
off_l = 2 * off_l
end_points2 = ([[-48.3 - off[0], 31.4 - off[1], 13 - off_l], [-1.2 - off[0], 48.9 - off[1], 13 - imb * off_l]])
end_points3 = ([[-48.3 + off[0], 31.4 + off[1], 13 - off_l], [-1.2 + off[0], 48.9 + off[1], 13 - imb * off_l]])
end_points4 = ([[-48.3, 31.4, 6.5], [-1.2, 48.9, 6.5]])

# Create Power Line Object
# ##############################################################################
PL1 = PL_create(end_points1, sag_factor = 250, spacing = 0.1, random=False, plot=False)
```

140
PL1['diameter'] = diameter
PL2 = PL_create(end_points2, sag_factor = 250, spacing = 0.1, random=False, plot=False)
PL2['diameter'] = diameter
PL3 = PL_create(end_points3, sag_factor = 250, spacing = 0.1, random=False, plot=False)
PL3['diameter'] = diameter
PL4 = PL_create(end_points4, sag_factor = 350, spacing = 0.1, random=False, plot=False)
PL4['diameter'] = diameter
PL = PL_append(PL1, PL2, PL3, PL4, plot=True)  # PL4, plot=False  #

# Include SARIT toolset
from ritsar import phsTools
from ritsar import imgTools
from ritsar import phsRead

# Create platform dictionary
import pickle
platform = pickle.load(open('./platforms/sim_AFRL_high_sampling', 'rb'))

# Create image plane dictionary
img_plane = imgTools.img_plane_dict(platform, res_factor=1.4, aspect=1)

# Simulate phase history
wvl = 299792458./platform['f_0']
R0 = norm(platform['pos'][0])
E_field_create(sqrt(P/(4*pi*R0**2)), platform)
D_HV(platform, PL)
phs_PL = simulate_PL_phs(platform, PL)

# Apply RVP correction
for i in range(4):
    phs_PL[i] = phsTools.RVP_correct(phs_PL[i], platform)
np.save('./phase_histories/%sPL_phs' % diameter, phs_PL)

# Import DIRSIG PHS
# ##############################################################################
# Define directory containing *.au2 and *.phs files
directory = './DIRSIG_data/full_api/'
# Import phase history and create platform dictionary
[phs_DIRS, platform] = phsRead.DIRSIG(directory)

# Add two phase histories
phs = phs_PL
platform_sub = dict(platform)

ppd = len(platform['pos'])/360
dpi = 3

img = np.zeros([4, 360-dpi, 512, 512])+0j
pol = ['HH', 'HV', 'VH', 'VV']
for i in [0, 3]:


```python
img_number = 0
for j in range(0, 360 - dpi):
pulse0 = int(j*ppd)
pulse1 = int((j+dpi)*ppd-1)
pulse_index = np.arange(pulse0, pulse1)

phs_sub = phs[i, pulse_index, :]
npulses = int(phs_sub.shape[0])
platform_sub['npulses'] = npulses
platform_sub['pos'] = platform['pos'][pulse_index]

if i == 0:
    img_plane = imgTools.img_plane_dict(platform_sub, res_factor=1.4, aspect=1.0, upsample=True)

    img[i, img_number, :, :] = imgTools.backprojection(phs_sub, platform_sub, img_plane)
    img_number += 1

np.save('./imgs/%sPL%d' % (diameter, img_number), img)
t = platform['t']
T_p = t.max() - t.min()
adv = len(img[i])//4
img2 = np.roll(img, adv, axis=1)
img2 = img2.T / 69373421.298985541

# Make RCS
RCS = 10*log10(wvl**2*sum(abs(img2)**2, axis=(0,1)))/RCS/(diameter, RCS)
np.save('./RCSs/%sPL_RCS' % diameter, RCS)

# Create video writer
for i in [0, 3]:
    fname = './videos/%sPL_%s.avi' % (diameter, pol[i])
    fourcc = cv2.VideoWriter_fourcc('I','Y','U','V')
    fps = 20
    size = (512,512)
    frame = 10*np.log10(abs(img2[:,:,i])/(abs(img2[:,:,i]).max()))
    writer = cv2.VideoWriter(fname, fourcc, fps, size)
    for j in range(360-dpi):
        frame_i = frame[:,:,i]
        frame_i[frame_i < -35] = -35
        frame_i = frame_i-frame_i.min()
        frame_i = frame_i/frame_i.max()
        writer.write(frame_i)
    print(j)
writer.release()
```

APPENDIX C. PHASE HISTORY SIMULATION CODE

142