5-12-2015

Allocation of Workers Utilizing Models with Learning, Forgetting, and Various Work Structures

Austin T. Chacosky

Rochester Institute of Technology

Allocation of Workers Utilizing Models with Learning, Forgetting, and Various Work Structures

A Thesis

Submitted in partial fulfillment of the requirements for the degree of Master of Science in Industrial Engineering

in the

Department of Industrial & Systems Engineering
Kate Gleason College of Engineering

by
Austin T Chacosky

May 12, 2015

DEPARTMENT OF INDUSTRIAL AND SYSTEMS ENGINEERING
KATE GLEASON COLLEGE OF ENGINEERING
ROCHESTER INSTITUTE OF TECHNOLOGY
ROCHESTER, NEW YORK

CERTIFICATE OF APPROVAL

M.S. DEGREE THESIS

The M.S. Degree Thesis of Austin Chacosky
has been examined and approved by the
thesis committee as satisfactory for the
thesis requirements for the
Master of Science degree

Approved by:

____

Dr. Scott E. Grasman, Thesis Advisor Date

AND

____

Dr. Michael E. Kuhl, Committee Member Date

Dedication

This thesis is dedicated to my friends and family. Their accomplishments have continuously been an inspiration. I will always appreciate everything they have done to support me.

Acknowledgements

This thesis would not have been possible without the support of many people. I offer many thanks to my advisor, Dr. Scott Grasman, for reading my numerous revisions and for his guidance in preparation of this research work. I would not have undertaken this work without the encouragement and support of Dr. Mike Hewit to whom I am very grateful. I would also like to thank my committee member, Dr. Michael Kuhl, for his guiding service and support. Additional thanks go to Dr. Ruben Proano for sparking my interest in Operations Research and for sharing his computing resources used for this work. Thanks to the Rochester Institute of Technology for providing me an environment to grow and succeed.

Allocation of Workers Utilizing Models with Learning, Forgetting, and Various Work Structures

Abstract

Much of the literature on cross-training and worker assignment problems focus on simulating production systems under cross-training methods. Many have found that for specific systems some methods of allocating workers are better performing than others in terms of overall productivity and ability to deal with change. This has lead researchers to create mathematical programming models with a goal of finding optimal levels of cross-training by changing worker allocations. Learning and forgetting curves have been a key method to improve the solutions produced by the optimization models, but learning curves are often nonlinear causing increased solving times. Because of this, most works have been restricted to modeling small, simple production systems.

This thesis studies the expansion of worker allocation models with human learning and forgetting to include variable work structures, thus allowing the models to be used to address a larger set of problems than previously possible. A worker assignment model with flexible inventory constraints capable of representing different production structures is constructed to demonstrate the expansion. Utilizing a reformulation technique to counteract the increased solve times of learning curve incorporation, the scale of the production systems modeled in this work is larger than in similar works and closer to the scale of systems seen in industry. Production systems with multiple products and corresponding due dates are modeled to better represent the production environment in industry. Investigative tests including a 2^4 factorial experiment are included to understand the performance of the model. The output of the optimization model is a schedule of worker assignments for the planning horizon over all of the tasks in the modeled system. Production managers could apply the schedule to their existing lines or run what-if scenarios on line structure to better understand how alternative structures may affect worker training and line productivity over the planning horizon.

Contents

1 Introduction

Faisal Hoque once wrote,"Constant change is the new dynamic of the global economy, making agility more necessary than ever" [Hoque, 2009]. In production systems, change often means variable workloads, new products, untrained workers, and new production structures. Dealing with these changes is an ever important problem for production managers. A common solution is to reallocate workers to where they are needed. Transferring labor between areas with a surplus of workers and those with a deficit can provide relief when demands shift. The problem of deciding which workers will cover which tasks grows quickly with the number of workers and tasks. Soon the possible allocations become too large for production managers to evaluate and choose the solution which best satisfies their objectives. Because of the often vast number of combinations of workers and tasks, worker allocation problems are great applications of mathematical programming optimization. The objective of such programs is commonly to minimize the gap between the need for workers and supply of labor by allocating the workforce to where it is needed. When there are many workers to reallocate and the impacts of those decisions are vast, using mathematical modeling can make a world of difference.

Worker allocation models are not the complete answer though. Sometimes there are needs that cannot be filled with the current workforce. Even when the capital and labor are available, the problem can be training. Forethought and planning are often key to success in business. Cross-training is one way that production managers can prepare for changes to their business. Cross-training involves teaching workers to perform multiple tasks. This simple concept has many benefits. First, when workers know more than one or two tasks they can be reallocated to fill in when other workers are sick, are out on vacation, or when additional labor is needed elsewhere. Workers have a better chance of filling gaps. Second, cross-trained workers are mentally stimulated when asked to learn new tasks and get less bored. Workers are happy to be challenged. Without cross-training, workers are specialized in specific tasks and can become discontent [Nembhard and Osothsilp, 2004]. In the past, before labor reform, this was not uncommon [Smith, 1776].

The study of industrial productivity is not something new. In the first chapter of his famous work, "On the Wealth of Nations", published in 1776, Adam Smith explains the benefits of what he calls the Division of Labor. Smith states that in a manufacturing setting when work is split into equal parts and given in equal share to workers, the productivity of each worker and the productivity of the group increases. To summarize a long and elaborate section of his work, in

Book 1, Smith states that when workers specialize in certain tasks, they get very good at those specific tasks and the systems, as a whole, can produce more products [Smith, 1776]. Productivity was an important factor to businesses then, it also is today, so Smith's writing is of value. In a later chapter, Smith notes that specialization can lead to worker dissatisfaction with work due to a separation of the laborer from the product and the daunting repetition of specialized work [Smith, 1776]. This led to the idea of job rotation and eventually cross-training.

Cross-training workers is now a heavily adopted idea in many fields, but it is not a trivial endeavor. Cross-training can be hard to implement well. Managers often rely on researchers, creating models to simulate cross-training policies, to better understand how cross-training workers will affect production. Those models often focus on parameters such as task tenure, worker multifunctionality, and cross-training level, and how they affect productivity [Nembhard and Shafer, 2008]. As cross-training methodologies in industry involve workers rotating through different tasks, task tenure is a measure of how often workers change tasks. Interruptions from learning lead to workers forgetting their training. The relationship between task tenure and forgetting will be further discussed later. The term worker multifunctionality refers to the number of different jobs on which a worker is trained. Finally, cross-training level refers to the number of workers that are training on the same job. A complete cross-training program must include levels for task tenure, worker multinationality, and cross-training level. A good cross-training program has levels for the parameters which balance cross-training benefits with its costs [Hewitt et al., 2013].

While cross-training has been shown to be useful in increasing flexibility of production systems, cross-training also has non-obvious costs that can depend on the structure of the systems. These costs include direct training costs associated with instruction and materials for training, decreased productivity during the training process, and the decreased productivity due to forgetting when workers are removed from primary task. Yang [2007], Sayin and Karabati [2007], Nembhard and Norman [2007], Bokhorst and Gaalman [2009], Molleman and Slomp [1999], Nembhard and Osothsilp [2004], Malhotra et al. [1993] have proposed that there exists an optimal level of cross-training for production systems that balances worker flexibility and system productivity.

To model the costs of cross-training, works such as Sayin and Karabati [2007], Nembhard and Norman [2007], Nembhard and Osothsilp [2004], Malhotra et al. [1993] have utilized experiential learning curves. Because job rotation involves placing workers on tasks that are new to them, there are some negative impacts to cross-training that relate to human learning. It takes workers some time to "get up to speed" on new tasks and correlates to a temporary loss of productivity. To better model the rate at which workers acquire skill, learning curves have recently been used in cross-training models. Incorporating learning into worker allocation cross-training models can lead to gains in model validity, but often greatly increase formulation complexity due to the non-linearity of useful learning curves [Sayin and Karabati, 2007, Nembhard and Norman, 2007].

Worker allocation models with learning are not currently designed to represent all systems. Decreased productivity during training and worker forgetting is increasingly difficult to include in worker allocation models with high fidelity, but has been accomplished on a small scale by incorporating functions of experiential learning into models of systems with cross-training. Modeling the learning of workers in systems with multileveled product structures, involving many workers and numerous products have largely been left unstudied in the cross-training modeling scope because of solving difficulties. Through the use of new reformulation methods, which transform the model from a nonlinear mixed integer program into a linear mixed integer program, those systems now can be explored to see if results and conclusions from works studying smaller systems can be used to explain behaviors of larger production systems of differing structures.

Specifically, in Dual Resource Constrained (DRC) systems, where both employees and capital equipment are constrained, at least some employees must be trained on more than one task to cover the tasks. Because of this, the loss of productivity due to worker learning is an issue in DRC job-shop systems. When a worker is learning to operate a machine in a DRC system, there is one less machine for a trained operator to be productive on and the cross-trained employee is taken off of a task where he could have been more productive. For an overview of DRC studies see Hottenstein and Bowman [1998]. This work focuses on expanding existing worker allocation models that include human learning and forgetting in order to make cross-training worker allocations in DRC systems with various work structures. Discussed later, this thesis introduces a worker allocation model with learning, forgetting, and a flexible work structure to model various systems which practitioners may manage in industry.

2 Literature Review

Because of its many benefits, cross-training and worker allocation problems are well represented in the literature. Works that include human learning are of a particular interest in this work, and those, while seeming to grow in recent popularity, are seen in much fewer numbers. Previous laboratory and empirical studies have identified some aspects of cross training that have been shown to affect the effectiveness of worker flexibility on a system. The aspects relating to this thesis are summarized in the following table:

Worker Multifunctionality	*The number of tasks the worker is trained on.*
Task Redundancy	*The number of workers trained on the same job.*
Cross-training Distribution	*The distribution of Worker Multifunctionality and Task Redundancy across workers.*
Task Tenure	*The length of time which a worker trains on a task before rotating to another task to train.*
Task Similarity	*The effect that learning a task has on the a worker's productivity on other analogous tasks of similar form.*
Learning Rate	*The rate at which workers acquire skill on tasks during training.*
Forgetting Rate	*The rate at which workers dispossess skill on tasks when not training.*
Staffing Level	*The ratio of workers to tasks.*
Worker Attrition	*The permanent loss of workers.*
Absenteeism	*The temporary loss of workers due to sickness or vacation.*
Due Dates	*The planned production schedule and associated deadlines.*
Multiple Products	*The number of various products the system produces.*
Worker Similarity	*The level of homogeneity between worker parameters.*

The following subsections discuss some of these aspects.

Worker Multifunctionality, Task Redundancy, and Cross-training Distribution

There has been recent research regarding the subject of creating effective worker assignments. Much of that research has used the assumption of constant worker productivity levels (e.g. Molleman and Slomp [1999], Bokhorst and Gaalman [2009], and Campbell [2010]). Many of those models are descriptive in nature as opposed to *pre*scriptive in nature and seek to show how system output performance indicators are affected by certain cross-training methods such as Skill-Chaining, a method which workers have overlapping task responsibilities [Hopp and Oyen, 2004]. Molleman and Slomp [1999] show the effects of worker absenteeism, task redundancy, and worker multifunctionality on system performance metrics in a descriptive manor. They identify task redundancy as an effective way of countering absenteeism under varying levels of worker multifunctionality. Cross-training with heterogeneously productive workers is shown to have differing effects on DRC systems from those seen in Single Resource Constrained (SRC) Systems [Bokhorst and Gaalman, 2009]. Yang [2007] identifies that certain levels of cross-training distribution across the workforce are well performing. They show that in resource constrained systems small increases in cross-training, specifically cross-training one or two workers per department, can yield a large portion of the total cross-training benefits possible. This recommendation is in conflict with the managerial suggestions put forth by Molleman and Slomp [1999] who recommend a uniform level of cross-training across all workers. The conflict is in light of the similarity in how the authors chose to model worker efficiencies. In particular, both Molleman and Slomp [1999] and Yang [2007] utilize constant worker efficiencies where experienced workers operate better than standard and inexperienced workers operate at less than standard efficiency for the duration of the simulation. Both papers do, however, cite research done by Malhotra et al. [1993] which models variable worker efficiency as a function of past experience as a possible option. Neither paper uses learning or forgetting functions.

Learning and Forgetting

Malhotra et al. [1993] addresses arguments that in the acquisition of a cross-trained workforce, firms will face decreased productivity during times of training due to human learning. Nembhard and Norman [2007] point out that incorporating individual learning and forgetting into cross-training worker allocation decisions is an effective way to advise managers on addressing important skill mix, job sequencing, and job rotation challenges.

There are a handful of papers in the literature which have used learning models in studies on production and workforce planning. Biskup and Simons [2004] and Corominas et al. [2010] utilize models of learning to better schedule and sequence machining jobs. Jaber and Bonney [1996] demonstrate the usefulness of including forgetting in manufacturing decisions and how it relates to human learning. Mazzola et al. [1998] include forgetting along with learning of a workforce to solve the “multiproduct production planning problem in the presence of work-force learning”. Sayin and

Karabati [2007] note the distinct complexity challenge that worker assignment models face when choosing to include learning curves.

Nembhard and Uzumeri [2000b,a], Nembhard and Osothsilp [2001], Jaber [2006], Jaber et al. [2003], Jaber and Sikström [2004] utilize worker performance data to formulate and analyze learning and forgetting functions for use workforce allocation modeling efforts. They also compare and contrast models of learning and forgetting. The literature points out two main competing learning curves: the Learn Forget Curve Model (LFCM) discussed by Jaber and Bonney [1996] and the Hyperbolic Recency Model introduced by Nembhard and Uzumeri [2000a]. Both the LFCM and the HRM track the improvement of a worker's productivity. The LFCM estimates a worker's time to complete replications of a task. The HRM estimates the output of a worker over a standard time bucket. Because many workforce planning models separate time into discrete time buckets the HRM can be easily included in period based worker allocation models. Jaber and Sikström [2004] compares the LFCM and the HRM directly and shows slight differences in favor of the LFCM. Jaber and Sikström [2004] uses seven characteristics, identified in Jaber et al. [2003], that models of learning and forgetting should have to compare the LFCM and the HRM. The seven characteristics follow:

1. Levels of forgetting depend on the amount of prior learning;
2. Levels of forgetting depend on the length of time away from a task;
3. Rate of learning after interruption is the same as original learning rate;
4. Power-function can represent forgetting;
5. Learning and forgetting are mirror images of each other;
6. There is a positive relationship between forgetting rate and learning rate; and
7. Model distinguishes between types of tasks: cognitive or manual.

Jaber and Sikström [2004] notes that the LFCM conforms to characteristics (1) through (6) while noting that the HRM has some issues. Particularly, they note that the HRM violates Jost's Law, a finding in the literature on human memory which suggests that older memories last longer than fresher memories. Despite this, Nembhard and Osothsilp [2001] test both the LFCM and HRM against empirical data and found that the HRM performed best. Additionally, Nembhard and Norman [2007] introduce an exponential model based on the HRM which is very suited to use in optimization models.

Attrition

The work of Malhotra et al. [1993] represents the first work to discuss the effects of worker learning and attrition on DRC systems. The work is a descriptive simulation of workers being placed in 6 functionally different departments to acquire worker flexibility. By including worker learning into the evaluation of cross-training schemes, the costs of those schemes due to decreased productivity during training can be modeled. Worker learning is modeled with the log-linear model learning curve which requires two parameter inputs, a learning rate factor and the time required to produce the first unit of output. See Nembhard and Uzumeri [2000b] for an in-depth discussion of the log-linear learning curve.

Malhotra et al. [1993] study the effect of labor attrition on a system in that manor. Losing a skilled laborer in a system, in which labor is already constrained, can be a major set back [Malhotra et al., 1993]. Worker attrition is modeled in the work by setting an annual percentage of workers who leave at an assigned time. Workers who leave the systems are replaced instantly by workers who are completely untrained. The effect of attrition of trained workers was compared to that of attrition of less flexible workers in order to understand the performance impacts of losing multi-skilled workers in a learning environment. Malhotra et al. [1993] show that small levels of worker multifunctionality can counter-act the effects of labor attrition even in instances of slow learning workers, but in such cases the production loss due to worker learning is much greater. While they model individual learning using the log-linear learning curve to better estimate the effects of acquiring new skills, the authors note that future research should focus on better understand the effects of forgetting and relearning on worker skill acquisition.

Task Similarity

Olivella [2007] identifies task similarity as an important factor in learning tasks. The task similarity effect, for this thesis, is defined as the induced increase in productivity on a second, related task caused by learning a task with which it shares similar characteristics. As an example, consider a student who is learning a certain programming language; some of the training the student receives on the first programming language will be applicable to learning a second programming language. The task similarity effect would dictate that the student's training on either language will benefit his training on both. Corominas et al. [2010] suggest that the inclusion of task similarity in worker assignment models including individual learning has been absent because of the mathematical complexity it creates. With the assumption of no task similarity effects, works like Malhotra et al. [1993] that do not include task similarity take on the risk of underestimating the productivity of workers in training whom may have carry over skills from other tasks.

Absenteeism

Many researchers have found worker absenteeism to be a very useful factor to consider in cross-training models as absence of workers puts additional stress on the remaining workers and is a main driver of cross-training efforts [Yang, 2007, Nembhard et al., 2005, Brusco, 2008, Molleman and Slomp, 1999]. Absenteeism is shown to be a decrement to system performance in almost all system performance metrics [Molleman and Slomp, 1999]. It is thus, when facing uncertain worker attendance, important to consider when making staffing decisions.

Literature Summary Table

Table 1 highlights how previous literature has addressed the aspects of interest:

Table 1: Research Focus in the Literature

Supporting Works	Aspects Discussed												
	Worker Multifunctionality	***Task Redundancy***	***Cross-training Distribution***	***Task Tenure***	***Task Similarity***	***Learning Rate***	***Forgetting Rate***	***Staffing Level***	***Worker Attrition***	***Absenteeism***	***Due Dates***	***Multiple Products***	***Worker Similarity***
Agnihothri et al. [2003]	X	X	X										
Biskup and Simons [2004]						X					X		
Bokhorst and Gaalman [2009]	X	X						X					X
Brusco [2008]			X					X		X			
Campbell [2010]	X		X										
Corominas et al. [2010]					X	X					X		
Hopp and Oyen [2004]	X	X	X	X									
Iravani et al. [2007]	X	X						X					
Jaber and Bonney [1996]				X		X	X						
Jaber et al. [2003]	X			X		X	X						
Jaber and Sikström [2004]						X	X						
Jaber [2006]						X	X						
Malhotra et al. [1993]	X	X		X		X			X				
Mazzola et al. [1998]						X		X				X	
McDonald [2004]	X			X		X							
Molleman and Slomp [1999]	X	X	X					X		X			X
Nembhard [2001]						X	X				X		X
Nembhard and Osothsilp [2004]	X	X		X		X	X						
Nembhard and Shafer [2008]						X	X		X				X
Sayin and Karabati [2007]	X	X	X	X		X	X	X			X		
Yang [2007]	X	X	X					X		X			

Research Gap

The previous section has identified various aspects of cross-training that have should be considered when working with worker allocation problems. Most previous studies that include learning are simulation studies which describe how systems are affected by changes in cross-training policies. The simulation papers have shown that certain characteristics in how workers are cross-trained can have significant impacts on system performance. While some of the works in the literature describe the differences in methodologies, this thesis will take a prescriptive approach to the problem. The goal of this research is to design a modeling approach that is robust and can be applied to various work structures that can allow managers to make better staffing decisions. While the incorporation of learning curves has been shown to be helpful in understanding the costs and benefits of cross-training, the increase in solve times associated with of doing so has restricted models to small scale examples of simple work structures. Table 2 shows how the scope of this work compares to four closely related works from the literature.

Table 2: Research Gap

Supporting Works	Model Attributes							
	Prescriptive	***Multiple Products***	***Due Dates***	***Learning***	***Forgetting***	***Task Similarity***	***Heuristic vs. Optimal Solution***	***Work Structure***
Corominas et al. [2010]	X	X		X	X	X	Approximation	Sequentially Scheduled Tasks
Malhotra et al. [1993]			X	X			Exact	Random Sequence
Nembhard and Norman [2007]	X			X	X		Exact	Single Flow
Sayin and Karabati [2007]	X			X	X		Approximation	Independent Tasks
This Work	X	X	X	X	X		Exact	Flexible Work Structure

Corominas et al. [2010] address the problem of assigning workers to tasks with the goal of minimizing the time it takes to complete all tasks. A Learning and Forgetting function controls the time it takes workers to complete a task. An interesting concept implemented by Corominas et al. [2010] is induced learning through similar tasks. Corominas et al. [2010] assume a concave learning function and approximate constraints with linear expressions to address the computational complexity of the model of including Learning and Forgetting with Task Similarity. Task Similarity is not considered in this work, but is of substantial interest for future work. Corominas et al. [2010] do not include inventory tracking constraints or due dates, but does allow for setting precedence constraints between tasks. The precedence relationships control when a task can be started in a Finish-to-Start manor, tasks can only be started once their preceding task is completed. The ability to set precedence relationships will be included in this work, allowing the creation of various work structures, but will take the form of inventory constraints instead of Finish-to-Start task ordering.

Malhotra et al. [1993] study the effects of worker attrition on a DRC job-shop under various simulated cross-training policies. Workers gain experience following learning curves as they rotate through various tasks. While learning is considered, forgetting is not. Forgetting is a large concern when workers are rotated through multiple jobs and is considered in this thesis. In the simulation of Malhotra et al. [1993], jobs (orders) are routed through six departments where workers have been allocated. Each job is randomly assigned a sequence through the system and a due date. Order due dates cause stress in DRC systems and how they effect the cross-training decisions is of interest in this work. Product due dates will be included in this work to investigate that effect.

Nembhard and Norman [2007] demonstrate a worker allocation model of a single serial production line with inventory buffers between tasks with worker learning and forgetting. The modeled system is a single serial flow production line with one product. The simple structure and small scale of the modeled production line are seen in industry, but this work focuses on larger more stratified systems with multiple products and due dates. Nembhard and Norman [2007] introduce options for a objective function, but the objective chosen is to maximize the production out of the last task in the line over the planning horizon. Using random assignments to represent worker allocations based on worker preferences, they show how managers can benefit from using worker allocation models to make important staffing decisions. Nembhard and Norman [2007] demonstrate the benefits of introducing learning and forgetting curves and show that there is a optimal level of cross-training. Their work uses an interesting Learning and Forgetting function based on the HRM which is also used in this thesis. That experiential learning model is used in this work as well as a form of the inventory buffer constraints used in that model.

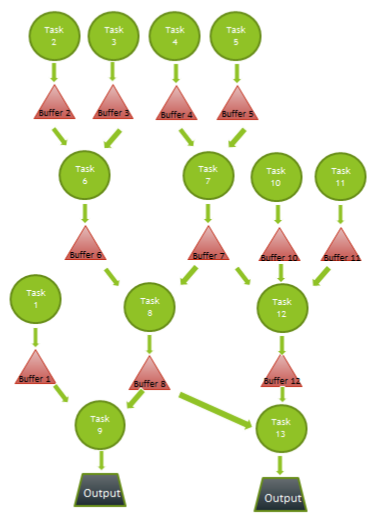
Sayin and Karabati [2007] take an interesting approach to the problem of assigning workers. Sayin and Karabati [2007] introduce a general framework for allocating workers to departments based on labor requirements, department utility weight, and worker skill levels utilizing sequential optimization objectives of minimizing departmental labor shortage and maximizing worker learning. The approach tries to build a better performing set of workers through intelligent work assignments. The model, designed for use in both production and service settings, uses the hyperbolic learning curve to estimate worker skill acquisition through assignments, but they do not consider systems with multiple products or due dates. Also, due to computational challenges Sayin and Karabati [2007] elect to use a single-period model solved repetitively as opposed to a multi-period model utilized in this thesis. A single-period approach is unable to make decisions that consider long-term challenges.

Using a recent exact reformulation technique [Hewitt et al., 2013] to combat solve time limitations, a key contribution of this work is to create a flexible model framework that is capable of modeling systems in which tasks can be done in parallel and are larger than the single flow processes found in works such as Corominas et al. [2010], Sayin and Karabati [2007], Nembhard and Norman [2007], and Malhotra et al. [1993] that are common to many prescriptive models of this kind. Unlike Nembhard and Norman [2007] and Sayin and Karabati [2007], multiple products with separate due dates and quantities are included to represent the environment in which DRC job-shops operate. The following section describes the model which will sit at the backbone of the work. Specific proposed modeling additions will be discussed in the Research Strategy section. The experiments will explore how the model reacts to changes in due dates, demand, and work structure have on the levels of worker flexibility, system output, and solve times. A description of the specific models used to address the challenges mentioned will follow.

3 Mathematical Programming Formulation of Worker Allocation Models with Learning and Forgetting

Consider a worker-task assignment problem for a DRC within a multi-tiered manufacturing environment. The various production lines have different configurations through which inventory flows. Assume that the machines on which the tasks are performed are connected by WIP buffers. Many lines share workers and their production is planned together. Also, many of the products are produced on many of the same machines and share components which means that workers may pull inventory from the same buffer for different products. Products are often only differentiated by tasks at the end of the lines. An example system structure for the problem considered is shown in Figure 1.

Figure 1: Example Work Breakdown Structure Model Product Flow



Consider that the multiple products may have different due dates that should be met or else risk the loss of sales. Larger facilities with many production lines and products often utilize models to allocate workers within large departments to meet due dates for the products.

Nembhard and Norman [2007] introduce a worker allocation model which utilizes a human learning and forgetting curve. Hewitt et al. [2013] later reformulate that model to decrease solve times, potentially allowing for larger models to be solved. The model formulation discussed in the following sections is largely based on the reformulation technique developed by Hewitt et al. [2013].

The following definitions are made to construct the worker-task assignment model:

- There are I workers available to perform tasks
- There are J sequential tasks to be completed within the system, and each task is done at a separate workstation
- Time is separated into T periods, where a worker performs only one task during each time period
- There is buffer storage between the workstations. However, the size of this buffer storage can be constrained
- The rate of learning and forgetting is a function of the time a worker has spent performing a particular task, the worker's learning rate and the worker's forgetting rate
- Workers are heterogeneous with respect to learning and forgetting rates and initial productivity

3.1 Sets, Parameters, and Variables

The purpose of the model is to help managers allocate workers to tasks in an effective manor. The model takes data related to the tasks that need to be completed such as the number of tasks, how product flows between the tasks, and the standard output expected per planning period on each task as parameters. Also taken as parameters, are data related to the workers that are available such as the number of workers and their learning parameters. The learning parameters which include the workers' initial productivity on the tasks, their learning rates, their forgetting rates, and maximum productivity are estimated using data collected from the workers' past performance. The model utilizes the worker parameters and task parameters to allocate workers in such a way as to maximize the output of the system with consideration of workers' learning and forgetting parameters when making decisions on worker cross-training.

Sets:

- $\mathcal{I} = \{1, ..., I\}$ - set of all workers
- $\mathcal{J} = \{1, ..., J\}$ - set of all tasks
- $\mathcal{T} = \{1, ..., T\}$ - set of all periods
- $EndTasks \subset \mathcal{J}$ - set of tasks which are at the end on the line which have no parents
- $Parents_{j \in \mathcal{J}}$ - set of parent tasks which task j feeds

Variables:

- X_{ijt} - binary - indicates if worker i is assigned to produce at task j in period t
- P_{ijt} - productivity of worker i at task j in period t
- O_{ijt} - output of worker i at task j in period t
- B_{jt} - buffer (inventory) of task j in period t which task j fed
- W_{jt} - binary - indicates if task j has met its demand in period t

Parameters:

- BI_j - initial buffer at task j
- BE_j - ending inventory in buffer at task j
- L_{ij} - learning rate of worker i at task j
- F_{ij} - forgetting rate of worker i at task j
- I_{ij} - initial productivity of worker i at task j
- K_{ij} - steady state productivity rate of worker i at task j
- D_{jt} - demand to be met at task j by the end of period t
- S_j - standard single period output of a worker on task j
- $C_{j,j' \in \text{ Parents} j}$ - units of j output required for one output of j'
- δ - parameter for weighting the importance of meeting due dates in the objective function
- U - minimum allowed utilization of any assigned worker in a given period

3.2 Worker Allocation Model with Learning and Forgetting

$$Maximize \qquad \sum_{i \in \mathcal{I}} \sum_{j \in EndTasks} \sum_{t \in \mathcal{T}} O_{ijt} + \sum_{t \in \mathcal{T}} \sum_{j \in \mathcal{J}} \delta * W_{jt} \tag{1}$$

$$O_{ijt} \leq P_{ijt} * X_{ijt} * S_j \quad \forall i \in \mathcal{I} \; j \in \mathcal{J} \; t \in \mathcal{T} \tag{2}$$

$$O_{ijt} \geq P_{ijt} * X_{ijt} * S_j * U \quad \forall i \in \mathcal{I} \; j \in \mathcal{J} \; t \in \mathcal{T} \tag{3}$$

$$B_{j1} = BI_j - \sum_{i \in \mathcal{I}} \sum_{j' \in \text{Parents}_\text{j}} C_{jj'} * O_{ij'1} + \sum_{i \in \mathcal{I}} O_{ij1} \qquad \forall j \in \mathcal{J}, j \notin \text{ EndTasks} \tag{4}$$

$$B_{jt} = B_{j(t-1)} - \sum_{i \in \mathcal{I}} \sum_{j' \in \text{Parents}_{\text{j}}} C_{jj'} * O_{ij't} + \sum_{i \in \mathcal{I}} O_{ijt} \qquad \forall j \notin \text{ EndTasks}, t \in \mathcal{T},\ t \neq 1 \tag{5}$$

$$B_{jT} \geq BE_j \ \ \forall j \in \mathcal{J}, j \notin \text{ EndTasks} \tag{6}$$

$$\sum_{j \in \mathcal{J}} X_{ijt} \leq 1 \quad \forall i \in \mathcal{I}, t \in \mathcal{T} \tag{7}$$

$$\sum_{i \in \mathcal{I}} X_{ijt} \leq 1 \quad \forall j \in \mathcal{J}, t \in \mathcal{T} \tag{8}$$

$$\sum_{i \in \mathcal{I}} \sum_{q=1}^{t} O_{ijq} \geq D_{jt} * W_{jt} \quad \forall j \in \text{ EndTasks}, t \in \mathcal{T} \tag{9}$$

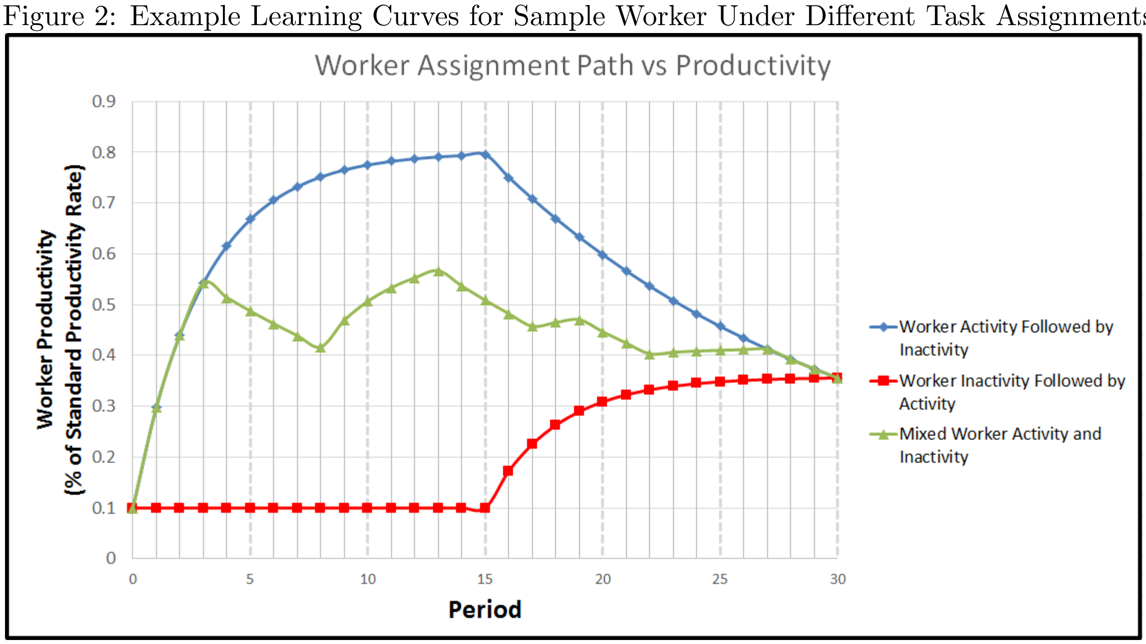
$$P_{ijt} = I_{ij} + K_{ij} \left[1 - exp\left(-\frac{1}{L_{ij}} \sum_{q=1}^{t} X_{ijq} \right) \right] exp\left[\frac{1}{F_{ij}} \left(\sum_{q=1}^{t} X_{ijq} - t \right) \right] \qquad \forall i \in \mathcal{I},\ j \in \mathcal{J},\ t \in \mathcal{T} \tag{10}$$

The objective function (1) maximizes the output of the end tasks over the production period and encourages meeting due dates by setting high values for δ. The output of each assigned worker is constrained by the worker's achieved productivity on the task and the task's standard single period output.

Constraint (5) is an inventory constraint which ensures that product flows into and out of the correct buffer without being lost. Constraint (5) allows a worker to pull inventory out of a buffer in the same period as another worker is adding to the same buffer. This constraint could be altered to reflect transfer delays from production to buffers. Constraint (4) populates the buffers with initial inventory. Due to end of horizon effects of the model, Constraint (6) is included to ensure that the buffers for the tasks are left at their targeted end inventory levels by the end of the planning horizon. The ending inventories were set as the initial inventory levels for the experimentation in this work. Without Constraint (6) inventory often flows downstream in the system leaving the line or lines barren at the end of the planning horizon.

It is assumed that workers may not switch between jobs in a single period, so Constraint (7) allows workers to only work at most 1 task per period. Similarity, it is assumed that a single job may not be performed by more than one worker. Constraint (8) represents this assumption and constrains worker assignments such that there can be at most 1 worker on any task per period. Only allowing one worker per task is representative of systems with heavy machine or other capital constraints. Parameters could be added to represent a constraint on work stations corresponding to each task. This would allow a larger number of workers to be assigned to the task. Balancing lines could be achieved in that fashion. Due date logic is represented in Constraint (9). When the sum of the output of a product is equal to the amount required by the due date, the corresponding W_jt can take a value of 1. The productivity of every worker is calculated in Constraint (10) as a function of the learning and forgetting parameters. The exponential model of human learning and forgetting used in this work is used in Nembhard and Norman [2007] and has an interesting characteristic; a worker's productivity in a given period depends only on the number of replications completed of a task. This effect is shown in Figure 2. Each curve represents a production plan where 15 replications of the task are performed during 30 periods. The convergence of the curves at period 30 indicates that the productivity in period 30 is the same regardless of when the 15 task replications were performed. Thus, at period 30, we need only know how many times the task was done in the preceding 29 periods to calculate the current productivity. The reformulation technique, discussed later, is based on this fact.

Figure 2: Example Learning Curves for Sample Worker Under Different Task Assignments



Alternative objective functions can be used which could, for instance, maximize the output at every task or minimize the time until due dates are met. Implementing the objective which attempts to maximize every task's output when in the presence of an unbalanced line and infinite buffer capacity leads to poor product flow and overproduction in upstream tasks. An alternative objective function which encourages cross-training is discussed in the next section.

3.3 Reformulation Technique

Hewitt et al. [2013] has identified a reformulation technique and applied it to worker allocation models with learning with promising results. The following subsections discuss the addition constraints and variables that transform the model from a Mixed Integer Non-Linear Problem to a Mixed Integer Problem resulting in much more manageable solve times. For a more complete discussion of the reformulation see Hewitt et al. [2013].

The basic concept of the reformulation says that if a non-linear equation has variables with a finite and countable domain then constraints and binary variables can be added to represent that non-linear equation without approximation. While it is used to address non-linear learning and forgetting curves in this work, the general form of the reformulation can be applied to a vast array of functions.

Consider the function $f(*)$ which does not have to be non-linear but, again, non-linear functions are of particular interest. Let $g(Y)$ be a linear equation and Y be a vector of binary variables. Let $g(Y)$ have a finite and countable domain contained in Set K. Consider the function $r = f(g(Y))$ which needs to be modeled with linear constraints and binary variables. Since $g(Y)$ has a finite and countable domain, $f(g(Y))$ has a finite and countable range which can be found through enumeration. Let Z_k be a vector of binary variables that indicate which item k in Set K is chosen. The addition of the following constraints complete the remodeling:

$$r = \sum_{k \in \mathcal{K}} f(k) Z_k \tag{11}$$

$$g(Y) = \sum_{k \in \mathcal{K}} k Z_k \tag{12}$$

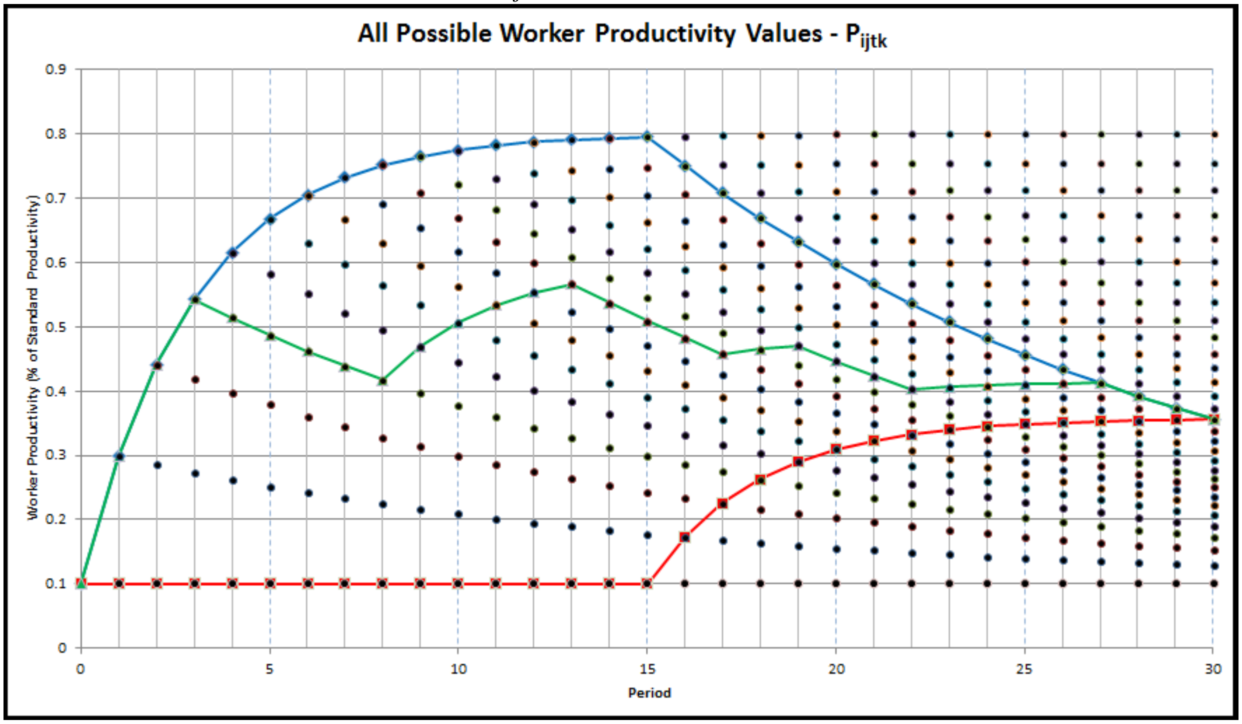
$$\sum_{k \in \mathcal{K}} Z_k = 1 \tag{13}$$

For example, consider the equation $X = (g(Y))^3$ where $g(Y) \in \{1, 2, 3\}$, a finite and countable domain. Solving for $1^3, 2^3$, and 3^3 yields $1, 8$, and 27. With the addition of three binary variables Z_1, Z_2, and Z_3 and a constraint such that $Z_1 + Z_2 + Z_3 = 1$ the equation $X = 1Z_1 + 8Z_2 + 27Z_3$ can represent the original equation $X = (g(Y))^3$. The original non-linear equation can then be replaced with the two linear equations by adding the binary variables and constraints. The use of the technique in this work to transform the model from a mixed integer nonlinear program to a mixed integer program is explained below.

The productivity of a worker in a given period is a function of the number of replications he has performed on a given task. The number of replications a worker could have performed, k, in a given period t is of finite and countable domain, specifically $k \in \mathbb{Z}\{1, .., t\}$ as a worker could not have performed a number of replications greater than the number of opportunities he had to perform the task. Substituting possible values of k for $\sum_{q=1}^{t} X_{ijq}$ in Constraint(10) yields the productivity of a worker i on a task j in a period t who has done the task k times (P_{ijtk}).

Figure 3 shows all the possible productivity levels for an example worker. With the addition of binary variables Z_{ijtk} we can reformulate the nonlinear Constraint (10) in the same way as the above equation $X = (g(Y))^3$. The necessary variables and constraints to perform the reformulation are discussed below.

Figure 3: Example of Worker P_{ijtk} Values with Three Example Assignment Paths



3.3.1 Reformulation Variables & Parameters

The following variables and parameters are required for the reformulation.

Z_{ijtk} - binary - indicates if worker i has been assigned to task j k times before and including period t where $k \leq t \in \mathcal{T}$

P_{ijtk} - parameter precalculated outside of model following equation (14) - Productivity of worker i who has performed k replications on task j before and including period t where $k \leq t \in \mathcal{T}$

$$P_{ijtk} = I_{ij} + K_{ij}\left[1 - exp\left(-\frac{1}{L_{ij}}k\right)\right] exp\left[\frac{1}{F_{ij}}(k-t)\right] \quad \forall i \in \mathcal{I},\ j \in \mathcal{J},\ t \in \mathcal{T},\ k < t \in \mathcal{T} \tag{14}$$

M - Arbitrary large number

3.3.2 Reformulation Constraints

The following constraints effectively replace constraint (10) using the reformulation variables and parameters.

$$\sum_{k=1}^{t}(k * Z_{ijtk}) \leq \sum_{c=1}^{t} X_{ijc} \quad \forall i \in \mathcal{I},\ j \in \mathcal{J},\ t \in \mathcal{T} \tag{15}$$

$$\sum_{k=1}^{t} Z_{ijtk} = X_{ijt} \quad \forall i \in \mathcal{I},\ j \in \mathcal{J},\ t \in \mathcal{T} \tag{16}$$

$$O_{ijt} \leq X_{ijt} * M \quad \forall i \in \mathcal{I},\ j \in \mathcal{J},\ t \in \mathcal{T} \tag{17}$$

$$O_{ijt} \leq S_j * \sum_{k=1}^{t}(Z_{ijtk} * P_{ijtk}) \quad \forall i \in \mathcal{I},\ j \in \mathcal{J},\ t \in \mathcal{T} \tag{18}$$

$$O_{ijt} \geq U * S_j * \sum_{k=1}^{t}(Z_{ijtk} * P_{ijtk}) \quad \forall i \in \mathcal{I},\ j \in \mathcal{J},\ t \in \mathcal{T} \tag{19}$$

$$\sum_{t=k}^{T} Z_{ijtk} \leq 1 \quad \forall i \in \mathcal{I},\ j \in \mathcal{J},\ k \in \mathcal{T} \tag{20}$$

The productivity of the worker is set as a parameter defined by the index k representing the number of replications of worker i has performed task j by period t. Instead of solving the exponential function as a part of the optimization, the feasible values are passed into the model as parameters calculated in constraint (14). Binary variables then decide which productivity values are to be used at which times. The k index in the variable Z_{ijtk} represents the number of times a worker has been assigned to a task as does the sum of X_{ijt} over the time. Constraint (15) ensures that values the chosen values of Z_{ijtk} correspond with the assignments up to the current period. Without Constraint (15) workers would not follow their learning and forgetting curves. Constraint (16) ensures that when worker i is assigned to a task j one of the variables Z_{ijtk} is set high and when worker i is not assigned Constraint (16) ensures that no Z_{ijtk} takes on a high value. Constraint (17) ensures that the X variables take values of 1 when corresponding worker output is positive. Constraint (18) sets the output of a worker to be less than the product of the task standard production rate, the Z decision variable, and the productivity. Constraint (19) sets the output of a worker to be greater than the product of the minimum worker utilization, the task standard production rate, the Z decision variable, and the probability. Constraint (20) essentially ensures that each worker can only perform a n^{th} replication of a task once. For example, a worker cannot be assigned to a task for the 5th time more than once.

3.4 Practicing Reduction

The original model formulation allowed for the possibility that a worker be assigned to a task but not produce anything. While workers do not produce any output, their assignment is still included in the productivity calculations. Hence, they learn but do not produce anything, so these events are refers to as practicing. Examine constraint (2) to see how a worker's output can be 0 while a worker is assigned to a task. Because the model uses experiential learning and forgetting curves where learning is based on actual completion of a task, the presence of practicing yields overestimate of a worker's productivity since the learning curve assumes that workers always produce as much as they can. Changing the inequality in Constraint (2) to an equal-to inequality would eliminate the practicing, but would greatly restrict the flexibility of worker assignments. The change would require workers to be 100% utilized during their assignments. In an unbalanced line, requiring workers to be fully utilized makes assignments difficult and often makes lines less productive. Because of the worker learning rate heterogeneity, perfectly balanced lines are seldom seen, so a different solution is needed.

Constraints (18) and (19) in combination, aim to control practicing while setting a minimum worker utilization. Constraint (18) sets the maximum productivity for a worker and Constraint (19) ensures that if the worker has been assigned to the task his utilization on that task is at least $100 * U\%$ of his maximum productivity at that time. Higher values of U reduce worker "practicing" which occurs when a worker is assigned to a task but performs the task less times than expected but is counted as having learned an amount which corresponds with his maximum productivity. A value of 0 for U could yield episodes of worker practicing, thus overestimating his productivity in future periods. Alternatively, in the presence of an unbalanced line a value of 1.0 for U would lead to poor product flow requiring assigned workers to be 100% utilized with zero waiting time allowed. This leads to many periods where most workers are not assigned to any task as they are waiting for buffers to fill before being able to produce. Implementors should pick values of U

according to their worker utilization requirements. Constraint (20) also controls another possibility of practicing. Without constraint (20) worker could repeat task replications. For example, a worker could work steadily on a task for 5 periods, gaining skill following his learning curve and then, in the next period, the worker could perform as though it was his first time being assigned on the task. Constraint (20) makes that infeasible.

4 Research Strategy

In this section, we discuss how individual experimental factors will be included in the analysis. The goal of this work is to demonstrate how the model can allocate workers to tasks under different system structures. The model optimally allocates workers by maximizing the production of demanded items while considering inventory constraints. How differing inventory constraint structures affect optimal worker allocations and ultimately the productivity of the system has been left unstudied due to problem size constraints observed when solving similar models with human learning and forgetting Sayin and Karabati [2007]. The strategy used in this work is to run multiple models with differing system configurations and compare the solutions. In this way, the effects of work structure on cross-training can be better understood. The experiments will show how the model could help managers of systems with various work structures better utilize their workforce.

4.1 Model Responses

To evaluate the performance of the model, two performance metrics are considered: the model solution time and the production of the products in the system. The solve time of an experiment is an important metric to track as anyone seeking to use the model needs to know how long the model will likely take. Longer solve times are a concern for industrial use as production changes can occur rapidly, requiring quick response. Solve times will be reported in seconds. The production of a system is the second performance metric of interest. After model termination, the objective value is reported which represents how many due dates were met and how much product the system produced under the solution at the time of model termination. There may arise differences in production due to due dates, worker set, or another design factor which need be explored.

4.2 Experimental Design Factors

The list of experimental factors follows:

1. ***The number of periods.*** This factor is set at 25 periods. While there is interest in using math programming models for long-term individual assignments, today's firms face the need to react quickly to things such as worker attrition, consumer demand changes, or new product introduction. The planning horizon of the model is approximately one month and 25 periods is aimed to approximately represent the staffing decisions for the planning horizon.

2. ***The number of tasks.*** This factor is set at 15 tasks. Hewitt et al. [2013] identifies this as a desirable increase from comparable studies. Larger increases in quantity of tasks yields numerous possible work structures as more tasks can be performed in parallel. The chosen level of tasks is closer to industrial examples and allows a multitude of structures and instances to be tested.

3. ***The number of workers.*** This factor is treated at one level with 7 workers corresponding to a staffing level of 50%, identified as common in DRC systems by Molleman and Slomp [1999], to gauge the effect of the worker-task ratio on performance metrics. The idea of using less than full staffing levels is that DRC systems need to have fewer workers than machines as the job shop type work done is these systems does not require work to be done on all the machines at the same time. An increase in staffing level may decrease the amount of worker multifunctionality seen in the solutions as workers will not need to rotate as often. Hewitt et al. [2013] show that, surprisingly, as the number of workers increases, the problem becomes easier to solve.

4. ***Worker Skill Mix.*** This is a group of factors which are generated together to create an heterogenous set of workers. Two randomly generated worker sets are tested for the experiments to show how worker mix affects performance of the model.

    (a) *Worker Forgetting Rate.* For each task, workers are assigned a random forgetting rate between 10 and 35 where lower values represent faster forgetting.

    (b) *Worker Learning Rate.* For each task, the workers's learning rates are treated at a random level between 2 and 10 where lower values represent faster learning.

    (c) *Worker Steady State Productivity.* For each task, the workers are assigned are assigned a random steady state productivity between 0.5 and 0.9.

    (d) *Worker Initial Productivity.* For each task, the workers are assigned are assigned a random initial productivity between 0.1 and 0.9.

5. ***Product Demand.*** The product demand quantity for each end product is assigned a value between 2 and 9. Larger demanded quantities will likely yield greater worker mulitfunctionality and lower task tenure while workers have to switch tasks more often to push product through the system.

6. ***Due Dates.*** Due dates and multiple products are being modeled following the logic presented in Corominas et al. [2010]. The differing product structures include those which have multiple products which must be accounted for. A due date is considered to be met if the demand of a product is fulfilled before the assigned due date which is between period 10 and period 25. Not all due date are required to be met, but meeting due dates is highly desirable and given high weight of 1000 in the objective function.

7. ***Work Structure.*** This factor is treated at multiple levels. Different structures may have a different number of end tasks with differing demands. A graphical representation of the work structures can be seen in Figure 4 below. The number of units of output from task j required to complete task j' ($U_{j,j' \in \text{ Parents}j}$) is also random between pairs of j and j' for each work structure tested. Additionally, the standard production rate (S_j) which represents the difficulty of a tasks is set to a random level of 1 or 2 for each task. Figures 4 and 5 show how product flows through the chosen experimental work structures (labeled A-K). The structures do not encompass all the possible flows the model could be used to solve but are used to demonstrate the flexibility of the implemented inventory constraints to cover a larger set of systems than seen in the literature. Work structures A through F consist of simpler linear production flows starting with a single flow in structure A and increasing to eight parallel flows in structure F. These or very similar structures can be found in industry and vary the number of tasks that can be performed in parallel. Work structures G through K are less common orientations of the same 15 tasks and are included to demonstrate the robustness of the modeling method. Structures G through K include numerous instances of tasks which require multiple inputs from other tasks, a characteristic seen in industry, but seldom observed in the literature.

8. ***Task Standard Production Rate.*** To ensure that the production lines in the systems being tested are close to balanced the standard productivity rates for individual tasks are set to be within ± 1-10% of the balanced line productivity rate. Imbalanced lines are avoided because they yield results in which workers are often not allocated to tasks due to waiting for inventory to flow downstream.

Figure 4: Experimental Product Flows for Work Structures A-F

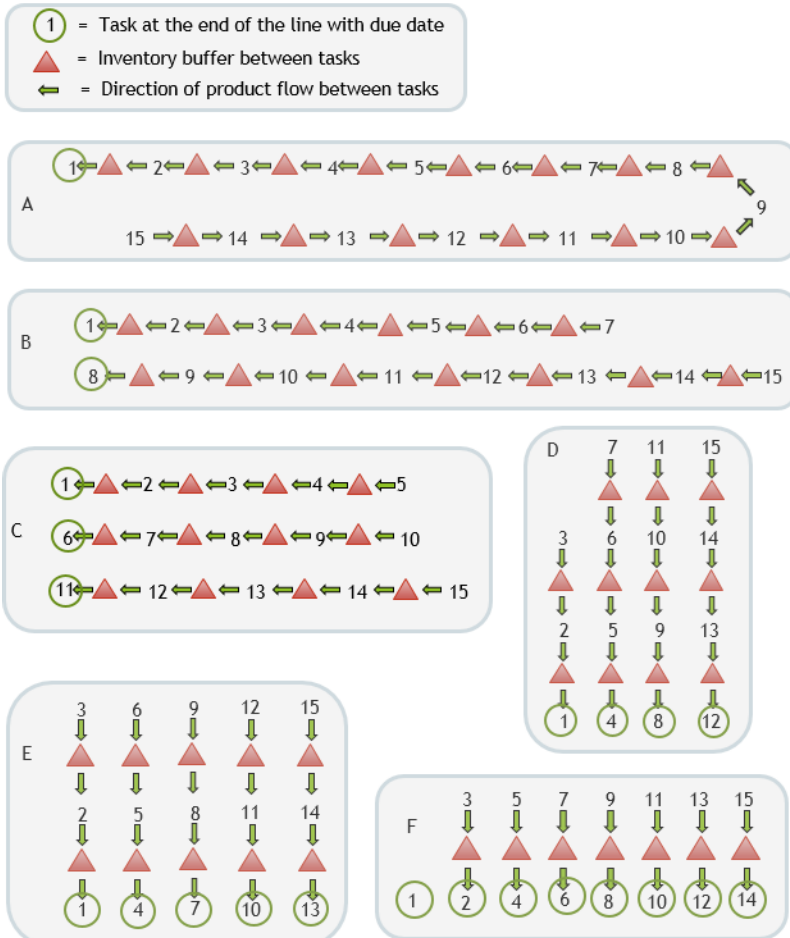
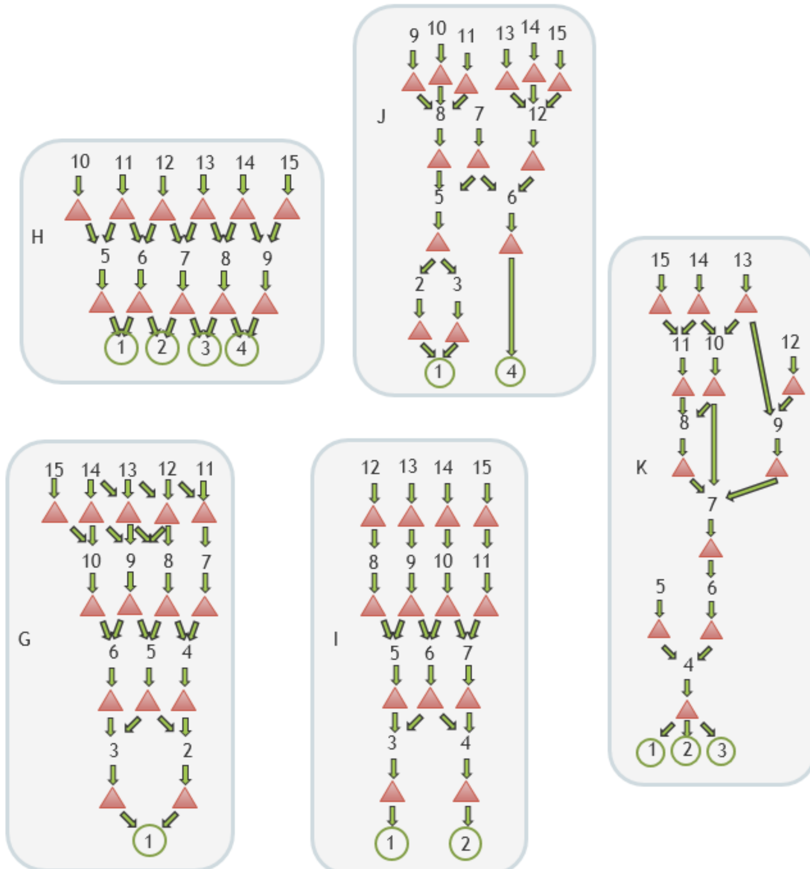


Figure 5: Experimental Product Flows for Work Structures G-K



4.3 Generation of Factor Random Values

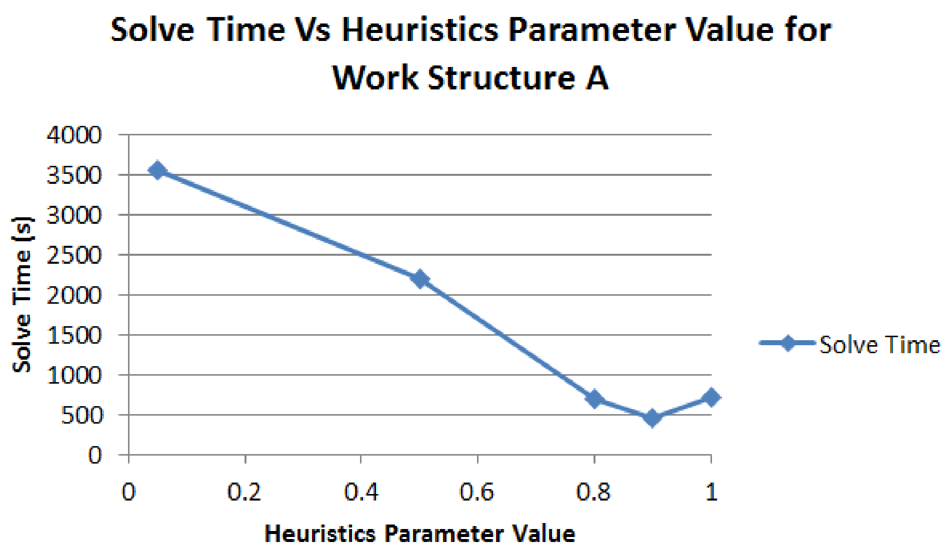
Random values for the experimental factors are generated using the Python function randint(). The randint() function produces pseudo-random integers contained in the range passed during call. The randint() function is part of the "random" library in Python and has roots in the random() function which produces pseudo-random floats in the range (0.0, 1.0]. Both random() and randint() use the Mersenne Twister, a widely used generator, at their core [van Rossum, 2011].

4.4 Algorithm Tuning

In implementing the model it may be necessary to adjust some solver settings to find better solutions faster. Gurobi offers algorithm tuning tools which repetitively solve models under different parameter configurations and reports back those configurations which perform better than others.

Because the solve times for the models in this work are so large, the standard Gurobi parameter tuning tools were not considered. Some initial testing revealed that when the solver used a heuristics solution, the optimality gap was decreased by finding a new incumbent solution. Experimentation with Gurobi's Heuristics parameter was found to have an interesting impact on the solve time of the models as shown in Figure 6. For Work Structure A, the best performing level tested for the Heuristics parameter was 0.9 and thus was used for the rest of the experimental testing.

Figure 6: Gurobi Heuristics parameter has an impact on solve time



Computational Environment

To study the computational effectiveness of the model, the instances of the experimentation were solved using Gurobi 5.1 to a relative optimality tolerance of 0.75% or an absolute optimality tolerance of 7. Prior testing pointed to these tolerances to balance model solve times and solution quality. Experiments were performed on a computer cluster where each node has 32-64 cores with AMD Opteron 2.2 GHz or AMD Interlagos 2.6 bulldozer processors and 128-256 GB of memory. For experimentation, the model instances were run on 6 cores with 24 GB of memory. When solving each instance of the model, Gurobi was given a time limit of 15 hours.

Model Investigations

Two experiments are setup utilizing combinations of the Experimental Design Factors. The first experiment is an exploratory experiment which is meant to demonstrate the flexibility of the model. The second experiment is 2^4 factorial experiment which is meant to explore how system structure, product demand, product due dates, and worker sets affect the model responses.

Table 3: Experimental Design

Factors	***Factor Levels***
Experiment 1	
Worker Skill Mix	Worker Set 1
Product Demand	Random $\in\{2..9\}$ for each product
Due Dates	Random Period $\in\{10..25\}$
Work Structure	All 11 Work Structures $\in\{A..K\}$
Experiment 2	
Worker Skill Mix	Worker Sets $\{1, 2\}$
Product Demand	Demand $\in\{3, 8\}$ for each product
Due Dates	Periods $\{12, 22\}$ for each product
Work Structure	Work Structures $\{A, J\}$

5 Results

Table 4 shows a sample allocation solution for Structure A taken from the algorithm tuning trials. The model was run to a relative optimality gap of 0.5%.

Table 4: Sample Solution for Structure A

	Worker						
	0	1	2	3	4	5	6
Period	Tasks						
1	7	13	14	8	9	6	15
2	7	13	2	8	10	11	15
3	7	5	3	12	9	11	15
4	4	5	14	8	10	9	15
5	7	13	14	12	3	9	1
6	7	13	14	8	10	2	1
7	4	13	2	12	3	11	1
8	4	13	2	12	10	6	15
9	4	5	14	12	3	9	1
10	7	13	14	8	3	9	11
11	4	13	14	8	10	9	11
12	7	5	11	12	10	6	15
13	7	13	2	12	15	6	11
14	12	13	14	11	3	6	15
15	4	5	14	8	3	9	11
16	4	5	14	12	10	9	11
17	4	5	2	11	3	6	1
18	7	5	2	8	10	6	1
19	4	13	2	12	10	9	1
20	4	5	2	8	3	6	9
21	7	5	2	12	10	6	11
22	4	1	2	12	3	9	15
23	7	13	2	11	3	6	1
24	4	5	14	8	3	15	1

Workers perform and focus on between 2 and 3 tasks throughout the planning horizon. With a staffing level of 50%, that level of worker multifunctionality is to be expected.

To contrast Table 4, Table 5 shows a sample allocation solution for Structure J observed during Experiment 2 that shows the differences between the allocations for two work structures. The model was run to an absolute optimality gap of 7 units. The chosen gap is used for the experiments because it was found to allow the models to solve in a reasonable amount of time while producing well performing solutions.

Table 5: Sample Solution for Structure J

	Worker						
	0	1	2	3	4	5	6
Period	Tasks						
1	12	14	15	5	6	11	13
2	12	14	15	9	6	7	13
3	8	14	15	4	6	7	13
4	12	14	15	4	6	10	13
5	8	14	15	4	6	7	13
6	12	14	15	4	6	3	13
7	12	14	15	4	6	7	13
8	12	14	15	4	6	2	13
9	12	14	2	4	6	1	13
10	12	14	15	4	6	10	13
11	12	14	15	4	6	3	13
12	12	14	15	4	6	9	13
13	1	14	15	9	6	7	11
14	12	14	15	4	6	7	13
15	12	14	15	5	6	7	13
16	12	14	15	4	6	1	13
17	12	14	15	4	6	1	13
18	12	2	15	4	3	1	13
19	12	14	15	4	6	13	11
20	12	14	15	5	6	4	13
21	NONE	8	2	4	11	7	13
22	NONE	3	15	NONE	6	10	11
23	NONE	8	15	9	6	7	2
24	5	NONE	NONE	4	12	10	8

Compared to the solution seen in Table 4, some workers perform and focus on fewer tasks throughout the planning horizon. Additionally, worker multifunctionality is not evenly distributed across the workers. Many workers focus on a single task while Worker 5 performs many tasks. This result may be because the production route to task 4 is simpler than that to task 1, causing more work to be done on tasks 13,14,15,12,and 6. Use Figure 5 for reference of structure J. There also seems to be significant end of period effects in this sample result for structure J. In particular, worker allocations follow a steady pattern up until the last few periods. The end of period effects are an artifact of the model formulation and effort could be taken in future work to eliminate them.

5.1 Experiment 1

Figure 7 plots the solution times after relative optimality gap was reduced to under 0.75% reported by Gurobi.

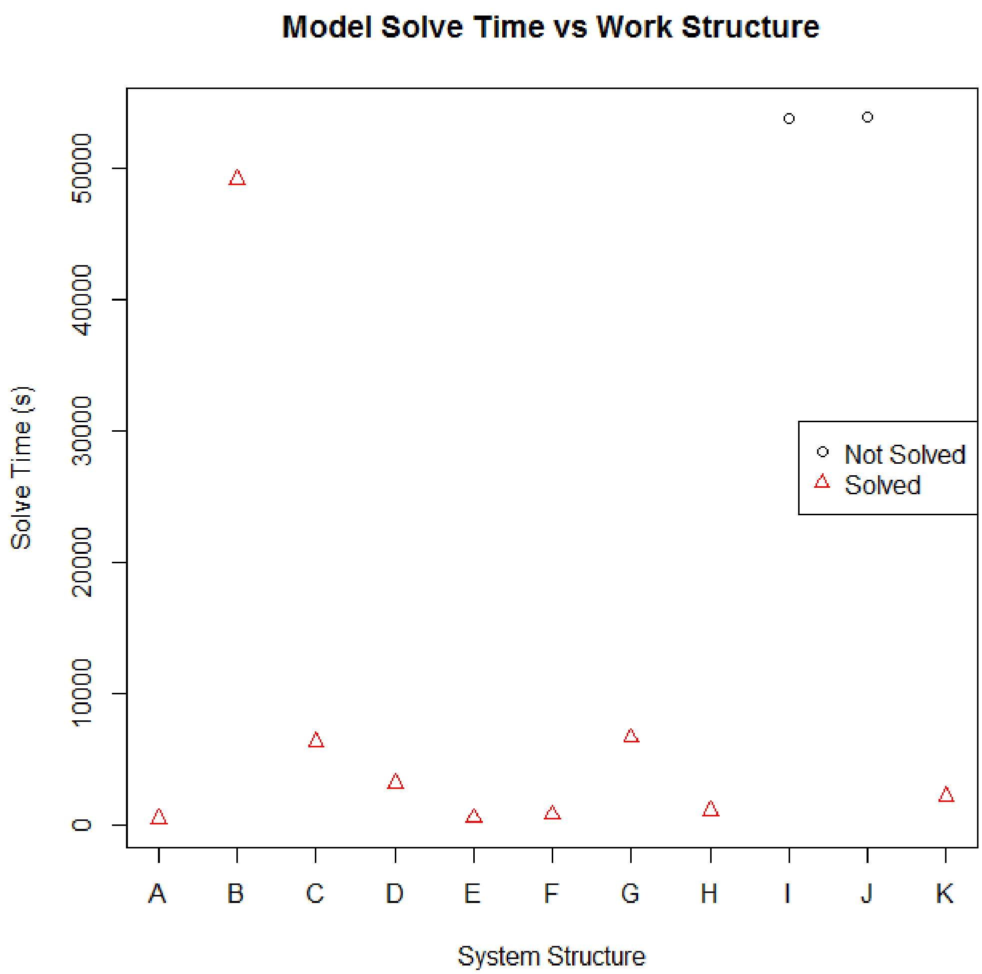


Figure 7: Solve time results for Experiment 1

We observe that Gurobi, given most of the tested work structures, is able to produce quality solutions in reasonable time frames. However, the cases with the most complex structures with many parallel tasks, I and J, take the longest to solve. Interestingly, the case with work structure K, which is similar to I and J, was solved in a similar amount of time as many of the other cases. The case with work structure B took longer than anticipated. While work structure B has two parallel lines, not too different from work structure A, C, or D, the case was required much more time to find an optimal solution than other, similar cases. Interestingly, all three structures that took longer to solve than other structures had 2 products. It may be the case that the long production lines paired with the multiple due dates for the products are driving up the solution times. While structure K does have long production lines and three products, the products are only differentiated by the last tasks which all pull inventory from the same task. The effect of due dates on structure J is explored further in Experiment 2.

Figure 8 reports the average units produced per product line for the cases tested in Experiment 1.

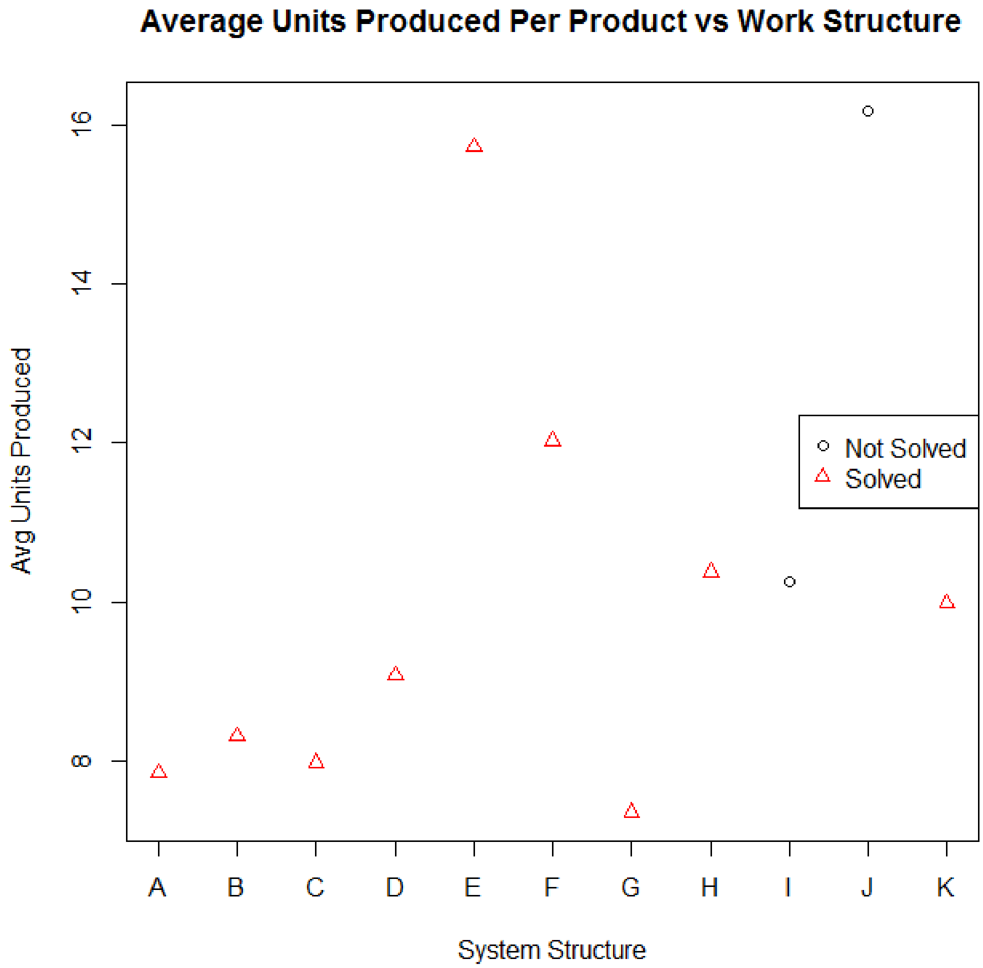


Figure 8: Production results for Experiment 1

Work structures A through D have very similar average values, while cases with work structure E through K have different average values of production. A replication of the experiment may show that the variation in production may be due to the random standard production rates.

Table 6 - 14 represent the solutions generated in Experiment 1. Tables for Structures I and J are missing because they did not reach the targeted optimality gap in the allotted time.

Table 6: Experiment 1 Solution for Structure A

	Worker						
	0	1	2	3	4	5	6
Period	Tasks						
1	14	10	2	8	4	3	15
2	14	5	13	7	6	9	4
3	15	12	13	7	5	11	1
4	4	12	13	8	3	9	2
5	14	5	NONE	12	6	11	15
6	4	11	13	8	15	7	1
7	4	12	13	10	14	5	15
8	5	7	13	10	1	6	9
9	10	NONE	2	14	6	5	3
10	4	9	11	8	NONE	5	12
11	7	10	NONE	14	4	5	12
12	2	4	NONE	11	6	9	3
13	4	5	2	8	11	9	3
14	12	1	2	14	NONE	13	3
15	2	15	NONE	8	10	7	11
16	NONE	7	3	15	6	10	NONE
17	2	9	11	8	6	13	1
18	2	13	9	10	3	NONE	1
19	2	4	11	3	10	13	1
20	15	7	3	14	9	NONE	12
21	15	12	NONE	14	6	7	11
22	1	11	3	15	NONE	5	12
23	4	5	2	14	11	13	10
24	1	12	5	8	9	13	3

Table 6 represents the assignment solution for work structure A in Experiment 1. Every worker except worker 3 is not assigned to a task in at least one period and perform many of the same tasks.

Table 7: Experiment 1 Solution for Structure B

	Worker						
	0	1	2	3	4	5	6
Period	Tasks						
1	4	15	2	8	3	13	11
2	4	5	2	14	13	9	15
3	4	5	13	12	14	6	11
4	1	10	13	12	15	9	4
5	7	15	10	5	3	6	14
6	12	7	2	5	3	6	11
7	4	11	13	14	9	7	1
8	12	7	13	14	3	6	11
9	10	7	14	5	3	6	9
10	15	10	2	5	3	7	4
11	14	7	15	5	3	6	1
12	7	10	11	12	15	9	1
13	4	9	13	5	3	12	1
14	4	5	2	14	3	6	13
15	7	5	2	8	14	6	1
16	4	10	2	5	15	6	11
17	4	7	13	8	3	6	14
18	12	10	13	8	7	9	1
19	7	11	2	8	15	6	12
20	4	5	7	8	3	6	1
21	14	5	13	10	3	9	4
22	15	5	10	12	7	6	11
23	7	5	13	8	15	9	1
24	14	11	2	8	15	9	12

Table 7 represents the assignment solution for work structure B in Experiment 1. Unlike the solution for work structure A, in the solution for structure B workers do not have periods of inactivity. Workers also exhibit longer average task tenure and perform fewer tasks than in the structure A solution.

Table 8: Experiment 1 Solution for Structure C

	Worker						
	0	1	2	3	4	5	6
Period	Tasks						
1	14	5	13	11	10	8	15
2	14	7	9	12	10	8	15
3	14	13	2	12	10	3	15
4	15	14	13	4	10	8	9
5	7	10	2	12	8	3	1
6	13	7	5	NONE	9	8	11
7	5	11	8	9	15	4	1
8	4	13	5	14	3	11	1
9	2	13	14	12	8	9	1
10	NONE	5	13	12	9	4	8
11	13	7	10	11	15	4	14
12	2	3	13	12	15	6	11
13	1	5	8	2	6	3	15
14	NONE	NONE	8	11	15	3	1
15	4	5	8	12	6	NONE	3
16	6	5	11	2	9	7	3
17	4	1	11	15	3	13	12
18	NONE	3	8	NONE	4	11	2
19	5	3	14	10	6	4	8
20	NONE	7	8	11	14	6	9
21	7	13	8	5	9	4	1
22	14	7	13	12	6	8	10
23	8	NONE	NONE	11	1	4	2
24	15	5	14	11	3	9	2

Table 8 represents the assignment solution for work structure C in Experiment 1. Many workers in this solution have periods of inactivity similar to the solution in table 6.

Table 9: Experiment 1 Solution for Structure D

	Worker						
	0	1	2	3	4	5	6
Period	Tasks						
1	7	13	2	9	3	11	1
2	14	13	2	8	3	11	1
3	9	10	2	5	3	8	1
4	6	10	2	NONE	3	11	1
5	12	7	2	11	3	9	1
6	6	5	2	10	3	11	1
7	5	1	2	NONE	3	9	11
8	5	10	2	8	3	14	15
9	7	10	14	8	3	9	1
10	7	10	2	NONE	3	8	15
11	12	NONE	13	8	3	9	4
12	6	1	13	8	3	9	15
13	4	NONE	2	5	3	6	1
14	15	12	NONE	5	8	NONE	4
15	4	NONE	NONE	10	1	11	2
16	4	12	NONE	5	1	7	2
17	4	NONE	NONE	5	6	NONE	1
18	14	1	2	NONE	13	11	3
19	1	9	2	NONE	6	8	NONE
20	5	9	2	NONE	NONE	8	1
21	NONE	NONE	2	NONE	9	7	1
22	14	5	2	NONE	11	8	1
23	NONE	2	5	10	4	6	3
24	NONE	3	2	15	1	7	4

Table 9 shows the results for structure D which, out of all of the solutions, shows the most periods when workers are not assigned to any task. Unlike in the previous solutions workers perform their initial tasks for many periods before switching to another task.

Table 10: Experiment 1 Solution for Structure E

	Worker						
	0	1	2	3	4	5	6
Period	Tasks						
1	9	5	2	8	3	4	1
2	9	NONE	2	8	3	4	1
3	9	NONE	2	8	3	13	1
4	9	NONE	2	8	3	6	1
5	9	NONE	2	7	3	5	1
6	9	7	2	8	3	5	1
7	NONE	14	2	8	3	10	1
8	15	12	2	7	3	10	1
9	13	6	2	5	3	NONE	1
10	15	11	2	5	3	6	1
11	12	7	14	15	3	13	1
12	14	9	2	NONE	3	13	1
13	12	7	2	8	3	4	1
14	15	13	2	7	3	11	1
15	14	10	2	NONE	3	NONE	1
16	15	NONE	2	7	3	13	1
17	15	11	2	NONE	3	13	1
18	14	NONE	2	11	3	7	1
19	NONE	NONE	2	11	3	13	1
20	14	9	2	8	3	NONE	1
21	13	1	2	10	3	9	4
22	7	9	2	10	3	NONE	1
23	12	1	2	15	3	5	4
24	14	7	2	15	8	6	1

Table 10 represents the assignment solution for work structure E in Experiment 1. Workers 2, 3, and 6 perform primarily only one task and because of that, the solution for structure E has the lowest average values for task redundancy and worker multifunctionality and highest value for average task tenure.

Table 11: Experiment 1 Solution for Structure F

	Worker						
	0	1	2	3	4	5	6
Period	Tasks						
1	9	13	2	8	15	11	1
2	9	13	5	8	10	4	1
3	9	12	11	8	15	7	1
4	9	13	14	8	3	11	1
5	9	12	11	8	3	4	1
6	14	5	11	7	10	4	1
7	9	13	2	7	3	11	1
8	9	12	2	8	10	5	1
9	14	13	12	8	4	NONE	1
10	14	13	6	10	15	11	1
11	7	5	14	2	15	NONE	1
12	6	12	14	10	15	11	1
13	4	13	NONE	10	15	3	1
14	NONE	12	2	11	6	3	1
15	14	12	3	10	15	13	1
16	4	5	3	11	15	6	1
17	14	12	2	10	NONE	4	1
18	2	9	5	10	12	NONE	1
19	4	11	3	8	6	5	1
20	10	5	3	NONE	6	11	1
21	2	15	14	11	6	7	1
22	11	2	3	9	10	NONE	1
23	NONE	13	2	11	10	NONE	1
24	9	11	15	10	3	7	1

Table 11 represents the assignment solution for work structure F in Experiment 1. The solution shows workers assigned to their initial tasks for multiple periods and worker 6 only assigned to task 1.

Table 12: Experiment 1 Solution for Structure G

	Worker						
	0	1	2	3	4	5	6
Period	Tasks						
1	15	10	2	9	6	13	4
2	5	7	NONE	8	15	3	11
3	7	11	13	12	4	3	15
4	14	9	2	8	10	NONE	15
5	4	5	11	12	6	7	15
6	7	5	10	11	1	13	15
7	14	7	13	NONE	8	NONE	12
8	4	NONE	NONE	8	9	3	12
9	14	10	13	NONE	1	3	4
10	14	12	5	11	4	NONE	NONE
11	13	7	NONE	8	1	5	4
12	14	9	6	11	13	NONE	12
13	6	11	2	8	NONE	9	1
14	13	5	2	NONE	6	7	9
15	NONE	5	2	NONE	10	3	NONE
16	15	13	2	9	10	3	12
17	14	12	2	9	NONE	5	1
18	6	1	NONE	NONE	NONE	NONE	10
19	12	5	13	11	NONE	3	1
20	NONE	9	NONE	8	6	NONE	NONE
21	13	9	2	15	NONE	7	3
22	4	10	13	NONE	14	3	6
23	NONE	13	2	15	6	3	1
24	4	5	11	8	14	7	9

Table 12 represents the assignment solution for work structure G in Experiment 1. The solution for work structure G shows many workers with multiple periods of inactivity.

Table 13: Experiment 1 Solution for Structure H

	Worker						
	0	1	2	3	4	5	6
Period	Tasks						
1	14	7	2	12	9	6	11
2	14	13	2	12	8	6	11
3	5	7	2	12	8	6	11
4	7	13	2	12	8	6	11
5	14	13	2	12	8	6	11
6	NONE	13	2	12	15	6	3
7	5	10	2	12	9	6	11
8	15	7	2	12	NONE	6	3
9	15	7	2	12	11	6	3
10	5	10	2	12	NONE	6	11
11	10	5	2	12	1	8	11
12	5	NONE	2	12	1	6	15
13	4	10	2	12	NONE	7	11
14	5	10	2	8	1	7	14
15	4	9	2	5	1	NONE	8
16	13	1	2	5	7	3	8
17	4	13	2	11	8	7	10
18	9	7	5	11	1	3	2
19	15	NONE	5	11	1	13	NONE
20	5	1	7	2	13	3	6
21	4	5	13	2	3	1	8
22	1	10	8	5	9	7	NONE
23	12	11	2	1	10	4	6
24	8	5	13	12	6	7	2

Table 13 represents the assignment solution for work structure H in Experiment 1. The solution shows some end of period effects as the pattern of assignments change in the last four periods.

	Worker						
	0	1	2	3	4	5	6
Period	Tasks						
1	14	13	6	8	3	9	15
2	14	10	11	7	3	6	4
3	4	5	2	1	3	8	15
4	7	10	2	4	3	13	15
5	6	5	13	4	3	8	12
6	9	4	2	12	3	13	11
7	12	7	5	4	6	9	2
8	14	10	2	8	1	7	9
9	14	1	11	10	9	8	12
10	14	13	5	11	9	12	2
11	1	5	2	12	6	9	4
12	4	12	2	5	1	8	9
13	15	13	2	7	6	10	8
14	NONE	12	2	11	10	13	NONE
15	4	13	3	7	6	8	2
16	15	5	14	7	11	10	2
17	15	13	5	NONE	8	NONE	2
18	9	10	14	11	4	7	2
19	1	NONE	2	15	12	5	10
20	15	12	2	11	9	8	7
21	15	12	14	11	8	6	2
22	4	5	2	10	6	8	13
23	15	9	13	4	11	8	2
24	14	7	2	9	8	4	10

Table 14: Experiment 1 Solution for Structure K

Table 14 represents the assignment solution for work structure K in Experiment 1. In the solution, the majority of the tasks are not assigned to the same worker for more than one period consecutively. Instead, tasks are often performed by different workers in sequential periods.

Many of the solutions show periods where workers are not assigned to a task, due to inventory constraints. The solutions from the various work structures show differences in the optimal allocations. The following figures and tables summarizes how work structures affect task redundancy, task tenure, and worker multifunctionality.

Figure 9: Task Redundancy results for Experiment 1

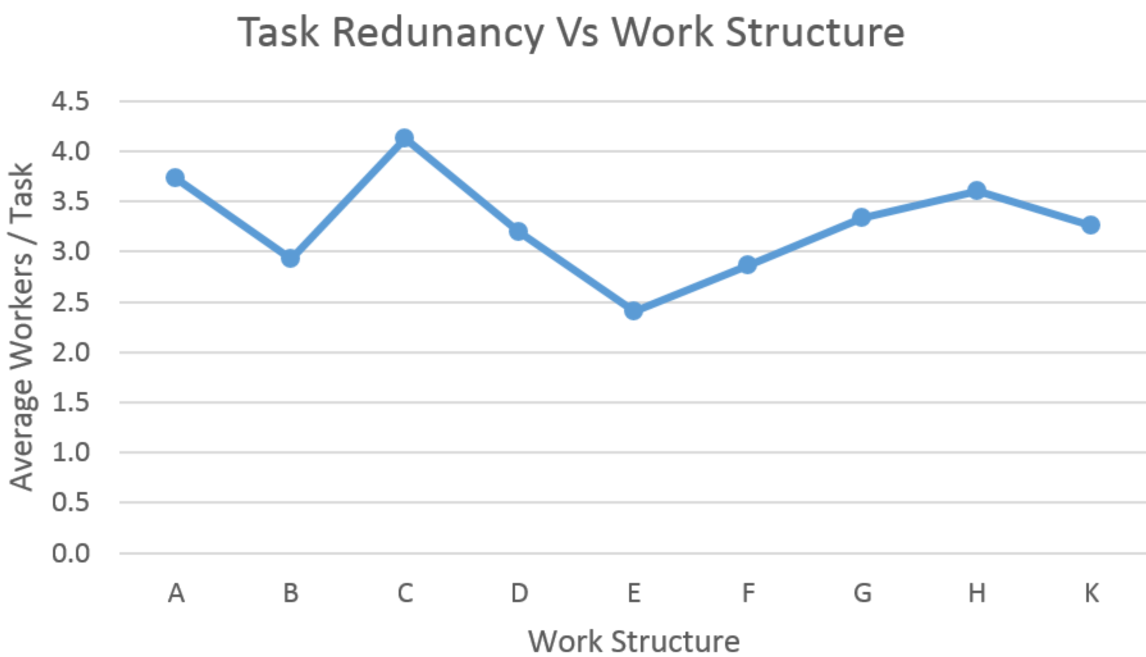


Figure 9 shows the differences in task redundancy between solutions with various work structures. The solutions do not differ greatly in average worker task redundancy as most tasks are performed by an average of 3.3 different workers.

Figure 10: Task Tenure results for Experiment 1

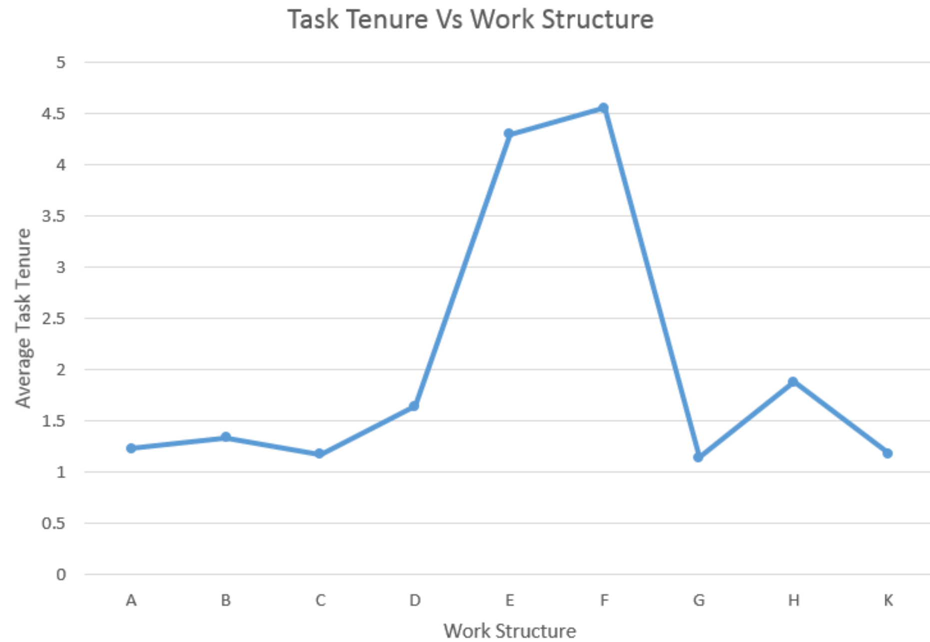


Figure 10 shows the differences in average worker task tenure between solutions with various work structures. While many of the solutions do not differ greatly in task tenure, structures E and F yielded solutions in which workers on average stay on tasks longer.

Figure 11: Worker Multifunctionality results for Experiment 1

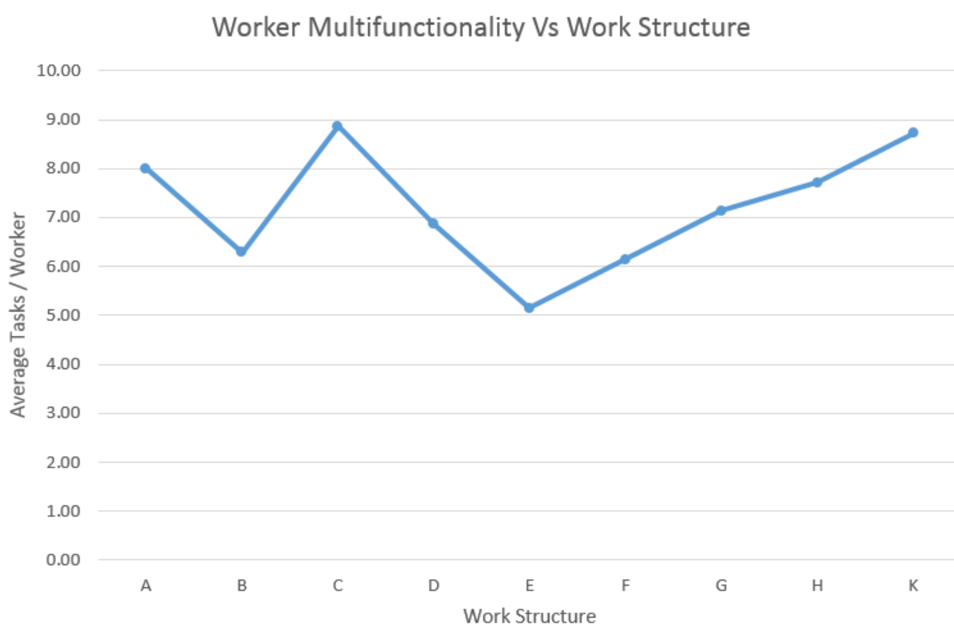


Figure 11 shows the differences in worker multifunctionality between solutions with various work structures. While the pattern of average worker multifunctionality across the work structures is similar to the task redundancy, the solutions show greater variance in multifunctionality than in task redundancy. In the 24 periods, workers perform an average of 7.2 different tasks.

5.2 Experiment 2

Table 15 displays the data collected in Experiment 2. The experimental factors along with the solution times, optimality gap, and objective values at the end of the trial are reported.

Table 15: Results of Experiment 2

Structure	WorkerSet	Demand	Date	Solved	Solve Time(s)	Solved Gap(%)	Objective Value
A	1	3	12	Yes	7407	0.7	1008
A	1	8	12	No	53866	Not Reported	9
A	1	8	22	Yes	12532	0.7	1008
A	1	3	22	Yes	10443	0.7	1008
A	2	3	22	Yes	11351	0.7	1008
A	2	8	22	Yes	20771	0.7	1008
A	2	8	12	No	53978	9932.0	10
A	2	3	12	Yes	16261	0.7	1008
J	1	3	12	Yes	18382	0.3	2027
J	1	3	22	Yes	14106	0.3	2027
J	1	8	12	No	52649	96.6	1032
J	1	8	22	No	52747	96.8	1032
J	2	8	22	No	53907	0.5	2022
J	2	8	12	No	53919	97.2	1029
J	2	3	22	Yes	4479	0.3	2026
J	2	3	12	Yes	25038	0.3	2026

Many of the trials did not solve before the 15 hour cut-off time; whether trials solved is reported in the "Solved" column. The objective values of solved trials differ by the Structure as Structure A has one product while Structure J has two products. It can be seen in the table that different combinations of due dates and demands have noticeable impacts on the solve times. Specifically, larger demands earlier due dates increase solve times. Also notice that the trials with structure A with a demand of 8 and due date of period 12 did not solve for either worker set despite having larger final production values than other trials for structure A. It may have been the case that the due date was unable to be reached in an optimal solution but the bound did not reflect the fact.

Table 16 reports the Experiment 2 ANOVA results for the Solve Time model response.

Table 16: Experiment 2: ANOVA Table Factors vs Solve Time

Source	Degrees of Freedom	Sum Sq Error	Mean Sq Error	F-Value	Pr(>F)
WorkerSet	1	1.93E+07	1.93E+07	1.087	0.331757
Demand	1	3.81E+09	3.81E+09	214.694	1.65E-06
Due Date	1	6.40E+08	6.40E+08	36.043	0.00054
Structure	1	4.91E+08	4.91E+08	27.657	0.001175
Demand * Due Date	1	1.43E+08	1.43E+08	8.028	0.025282
Demand * Structure	1	1.93E+08	1.93E+08	10.86	0.013203
Due Date * Structure	1	1.67E+08	1.67E+08	9.401	0.018166
Demand * Due Date * Structure	1	5.95E+08	5.95E+08	33.539	0.000669
Residuals	7	1.24E+08	1.78E+07		

Most of the factors including interactions between factors are shown to have a statistically significant impact on the model solution time at an α value of 0.05. Work structure J is more complex than work structure A in that it has more parameters to describe the inventory flow and larger summations in the inventory constraints. This may explain the differences in solve times observed between the two structures. Also observed in the table, the Worker Set Factor did not have a statistically significant impact on the solution time. This is important because as the model is applied, the workers are going change. The model solve time does not appear to be affected by the group of workers being allocated.

Table 17 reports the Experiment 2 ANOVA results for the Productivity model response.

Table 17: Experiment 2: ANOVA Table Factors vs Average Output Per Product

Source	Degrees of Freedom	Sum Sq Error	Mean Sq Error	F-Value	Pr(>F)
WorkerSet	1	2.92	2.92	1.789	0.223
Demand	1	3.93	3.93	2.407	0.165
Due Date	1	2.52	2.52	1.547	0.254
Structure	1	118.84	118.84	72.888	6.00E-05
Demand * Due Date	1	2.48	2.48	1.522	0.257
Demand * Structure	1	0.17	0.17	0.102	0.759
Due Date * Structure	1	0.01	0.01	0.005	0.947
Demand * Due Date * Structure	1	0.03	0.03	0.018	0.898
Residuals	7	11.41	1.63		

The System Structure is the only factor shown to have a statistically significant impact on the average output per product at an α value of 0.05. It is interesting that most of the trials for structure J did not solve while the production in the solutions for structure J were often higher. It may have been the case that the one product was difficult to meet the due date while the other was simple. This case would lead to larger average output per product while non terminating because due dates were not met for one of the products.

The experimental results also show that the solve times are affected by multiple model parameters such as product due dates, demand, and structure. Additionally, it appears from the results of Experiment 2 that how solve time is affected by the due date and product demand parameters changes according to work structure observed in Table 16 from the statistically significant interaction effects between the due date factor, product demand factor, and system structure factor.

6 Discussion

Experiment 1 demonstrates the capability to model multiple work structures for worker allocation models with human learning and forgetting. The results show that work structure can affect solve times of the allocation models but also seem to suggest that production managers would be able to represent their lines and find a solution in a reasonable amount of time. It would seem that work structures with two products take longer to solve than the others. This may be attributable to the due date fulfilment objective weighting.

Some of the work structures allow tasks to be performed in parallel. It was expected that those structures would require workers to perform smaller diversity of tasks than the other structures, but this result was not observed across all the structures. However, the results for structures E and F, structures with many tasks being performed in parallel, do show decreased task redundancy and worker multifunctionality, aligning with the expectation. The heightened task tenure results for structures E and F are understandably related to the task redundancy and worker multifunctionality results.

Experiment 2 sought to investigate effects of due dates, demand, and worker sets on allocations for work structures A and J. Except for the worker set factor, all of the other factors had statistically significant effects on the model solve times when all solve time results were considered including the trials which did not solve. The following ANOVA tables summarizes the results from Experiment 2 without unsolved trials.

Table 18: Experiment 2: ANOVA Table Factors vs Solve Time Excluding Trials without Solutions

Source	Degrees of Freedom	Sum Sq Error	Mean Sq Error	F-Value	Pr(>F)
WorkerSet	1	2.26E+07	2.26E+07	0.714	0.460
Demand	1	1.66E+07	1.66E+07	0.524	0.521
Due Date	1	8.92E+07	8.92E+07	2.820	0.192
Structure	1	3.42E+07	3.42E+07	1.082	0.375
Demand * Worker Set	1	1.71E+07	1.71E+07	0.541	0.515
Due Date * Worker Set	1	7.34E+07	7.34E+07	2.321	0.225
Residuals	3	9.49E+07	9.49E+07		

After removing the trials that did not solve, the factor effects were not statistically significant. There seems to be a relationship between the factors and whether the models solved. Particularly, when demand was high and due dates sooner, the model often failed to find an optimal solution. Perhaps, changes to the objective variable weighting factors or Gurobi modeling parameters could be used to further tune the models to find better feasible solutions faster.

Overall, Experiments 1 and 2 show that practitioners could use the proposed modeling approach to pursue worker allocation solutions for their production environments. The results suggest that the work structure and other factors can have an effect on how workers should be allocated, and that in many cases a solution can be found in a reasonable amount of time.

7 Conclusions and Future Work

This thesis sought to expand workforce allocation models with learning and forgetting to include variable work structure and has identified a method to make that possible. Production managers in industry face difficult staffing decisions, but previous works that demonstrated the importance of including leaning and forgetting into workforce allocation decisions have not been adaptable to fit the production lines in which production managers need to make decisions. In production systems inventory flows from one end of the line to another, but like rivers, there are forks and junctions where inventory can flow in parallel. To capture the different flows of various production systems, flexible inventory constraints were added to a workforce allocation model so that production managers could receive staffing decisions that were tailored to their production system and guided by human learning and forgetting.

The output of the optimization model presented in this thesis can help managers make important worker allocation decisions. Managers can run a model and retrieve staffing decisions for a group of workers in an area of production. Managers can then use the same model with a different data set to address a new group of workers and a different structure of tasks. The ability to use the same model to address different areas of production may be increase the use of these models as practitioners become more comfortable with one model.

The modeling of various production systems with dissimilar structures has been left unstudied largely because of the solving difficulties caused by the inclusion of mathematically complex human learning and forgetting models. This thesis addressed the issue by incorporating a reformulation technique that has shown promising results combatting the increased solution times. The results of the experimentation demonstrate that despite the added complexity of variable inventory constraints, multiple products and due dates, the reformulation makes it possible to solve numerous the worker allocation problems in a variety of production systems with different work structures within reasonable amounts of time.

While the solution time results are important for production managers and other implementors to understand, the success of this thesis opens new areas in research for theoretical implications. The models in this thesis could be altered to address new research questions. Some of the possible areas of future work are discussed below.

Generation of Initial Feasible Solutions

While many of the model runs ended by meeting the optimality gap, there were trials where the solve time limit was reached and an optimal solution was not found. Initial feasible solutions provide a branch-and-bound algorithm a feasible starting point are known to reduce solve times. Future research into generating initial feasible solutions for various work structure could decrease solve times such that optimal solutions can be found faster. Although starting the solver with an initial feasible solution is likely to reduce solve times, it is not without challenge. The inventory constraints along with variable work structure and human learning make generating a feasible solution difficult.

Upper Bound on the Objective Value

Initial feasible solutions are not the only way to combat large solve times. Another way to decrease solve times is to generate valid inequalities which further restrict the search volume. Generating an upper bound on the objective function could allow the models to be solved faster. Generating an upper bound by relaxation of the inventory constraints may be a good starting point for research but other methods which generate tighter bounds could be explored. More research in this area would likely be beneficial.

Task Certification and Similarity

While the author is not aware of works that consider it, some industrial processes require workers to be certified on machinery in order to perform their jobs. In a cross-training environment where forgetting can be a concern, ensuring that workers routinely cycle back to their certified operations is important so as not to lose their certifications. Adding a constraint and additional parameter to ensure certifications are kept may allow for certifications to be modeled. The constraint may take the following form:

$$\sum_{k=1}^{t}(P_{ijtk} * Z_{ijtk}) \leq CL_{jt} * X_{ijt} \quad \forall i \in \mathcal{I},\ j \in \mathcal{J},\ t \in \mathcal{T} \tag{21}$$

The parameter CL_{jt} would represent the minimum productivity level needed to get recertified on a task. The constraint would ensure that anyone who performs task j has a certain level of training. Different levels of required training could be modeled to explore the effect of difficult certifications on the systems.

Similar changes could be made to include task similarity. Olivella [2007] present an alteration to the Hyperbolic Recency Model which accounts for Task Similarity. It may be possible to apply a similar to the learning function used in this work and in Nembhard and Norman [2007].

Additional Learning Curves

Because a main contribution of this work will be a model that emphasizes flexibility and because of the debate on the subject, another learning curve could be included in another formulation of the model. Jaber and Sikström [2004] would suggest using the LFCM model as it is arguably more accurate than other learning models. Jaber and Sikström [2004] also present some sets of parameters which minimize the difference in learning and forgetting for individual workers modeled with different learning curves.

The learning curve currently in the current model has a fixed period length over which workers produce output at a certain productivity level. The possible productivities are in a finite and fixed domain and are a fraction of standard output possible during the fixed period. The LFCM on the other hand models learning as a continuous decreasing time to complete one unit of output. It should be possible to integrate the LFCM over the fixed period length to figure out the productivity of a worker in a given period.

Worker Attrition and Absenteeism

The loss of workers is an important concern of any manager. In a similar fashion as presented in Malhotra et al. [1993], workers who leave will be immediately replaced with an untrained worker. This could be achieved by constraining the X_{ijt} variable of the lost worker to be 0 after attrition, and constrain the X_{ijt} variable of the new worker to be 0 before attrition. It would then appear that a new worker has become available when a worker leaves. The attrition could be modeled in two ways: without warning, and with warning. In the without warning method, the model would be run until and optimal solution is found. The model would then be run again with the variables fixed to the optimal solution until a the period in which the worker leaves. The new worker would be introduced, and a new solution could be realized that accounts for having a new employee. Some mangers may want to know how to shift their employees around once one of the employees has given a notice of resignation. We could allow a new worker to become available at a certain point *before* the lost worker leaves to see how to deal with those situations. Absenteeism could be modeled in a very similar manor with a change in that the lost worker returns.

Two-Stage Optimization Approach

Sayin and Karabati [2007] presents a two-stage approach that first optimizes productivity and then the staff's learning, but note that due to model complexity, only single period models could be run. With the reformulation technique to simplify the model, it would be a promising area to explore. The change to the objection function for the two-stage optimization would be something like the following:

$$max \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \sum_{k=0}^{T} Z_{ijT\mathbf{k}} * PijT\mathbf{k} \tag{22}$$

Direct comparison of cross-training methods between workers in systems with different work structure has been left largely unstudied. The work presented in this thesis, while appealing directly to industry practitioners, could allow for researchers to evaluate the solutions to different structures to identify trends in optimal worker allocation solutions which could be turned into powerful heuristics. The expansion of workforce allocation models with learning and forgetting to include variable work structure has benefits in both practical and theoretical domains.

8 Bibliography

References

S Agnihothri, A Mishra, and D Simmons. Workforce cross-training decisions in field service systems with two job types. *Journal of the Operational Research Society*, 54(4):410–418, 2003. ISSN 0160-5682. doi: 10.1057/palgrave.jors.2601535.

D Biskup and D Simons. Common due date scheduling with autonomous and induced learning. *European Journal of Operational Research*, 159(3):606–616, 2004. ISSN 03772217. doi: 10.1016/j.ejor.2003.06.001.

J Bokhorst and G Gaalman. Cross-training workers in Dual Resource Constrained systems with heterogeneous processing times. *International Journal of Production Research*, 47(22):6333–6356, 2009. ISSN 0020-7543. doi: 10.1080/00207540802350724.

M Brusco. An exact algorithm for a workforce allocation problem with application to an analysis of cross-training policies. *IIE Transactions*, 40(5):495–508, 2008. ISSN 0740-817X. doi: 10.1080/07408170701598124.

G Campbell. A two-stage stochastic program for scheduling and allocating cross-trained workers. *Journal of the Operational Research Society*, pages 1–10, 2010. ISSN 0160-5682. doi: 10.1057/jors.2010.16.

A Corominas, J Olivella, and R Pastor. A model for the assignment of a set of tasks when work performance depends on experience of all tasks involved. *International Journal of Production Economics*, 126(2):335–340, 2010. ISSN 09255273. doi: 10.1016/j.ijpe.2010.04.012.

M Hewitt, A Chacosky, S Grasman, and B Thomas. A Reformulation Technique for Solving Non-linearWorkforce Planning Models with Learning. note, 2013.

W Hopp and M Oyen. Agile workforce evaluation: a framework for cross-training and coordination. *IIE Transactions*, 36(10):919–940, 2004. ISSN 0740-817X. doi: 10.1080/07408170490487759.

Faisal Hoque. The speed of business today @ONLINE. http://www.baselinemag.com/c/a/Innovation/The-Speed-of-Business-Today-720022/, 2009.

M Hottenstein and S Bowman. Cross-training and worker flexibility: A review of DRC system research. *The Journal of High Technology Management Research*, 9(2):157–174, 1998. ISSN 1047-8310. doi: http://dx.doi.org/10.1016/S1047-8310(98)90002-5.

S Iravani, B Kolfal, and M Van Oyen. Call-Center Labor Cross-Training: It's a Small World After All. *Management Science*, 53(7):1102–1112, 2007. ISSN 0025-1909. doi: 10.1287/mnsc.1060.0621.

M Jaber. Learning and Forgetting Models and Their Applications. In Adedeji B. Badiru, editor, *Handbook of Industrial and Systems Engineering*, Industrial Innovation Series, chapter 30, pages 30–1 —- 30–27. CRC Press, Boca Raton, FL, USA, 2006.

M Jaber and M Bonney. Production breaks and the learning curve: The forgetting phenomenon. *Applied Mathematical Modelling*, 20(2):162–169, 1996. ISSN 0307-904X. doi: http://dx.doi.org/10.1016/0307-904X(95)00157-F.

M Jaber and S Sikström. A numerical comparison of three potential learning and forgetting models. *International Journal of Production Economics*, 92(3):281–294, 2004. ISSN 09255273. doi: 10.1016/j.ijpe.2003.10.019.

M Jaber, H Kher, and D Davis. Countering forgetting through training and deployment. *International Journal of Production Economics*, 85(1):33–46, 2003. ISSN 09255273. doi: 10.1016/S0925-5273(03)00084-7.

M Malhotra, T Fry, H Kher, and J Donohue. The impact of learning and labor attrition on worker flexibility in dual resource constrained job shops. *Decision Sciences*, 24(3):641, 1993. ISSN 00117315.

J Mazzola, A Neebe, and C Rump. Multiproduct production planning in the presence of work-force learning. *European Journal of Operational Research*, 106(23):336–356, 1998. ISSN 0377-2217. doi: http://dx.doi.org/10.1016/S0377-2217(97)00278-6.

T McDonald. *Analysis of worker assignment policies on production line performance utilizing a multi-skilled workforce.* PhD thesis, Virginia Polytechnic Institute and State University, Ann Arbor, 2004.

E Molleman and J Slomp. Functional flexibility and team performance. *International Journal of Production Research*, 37(8):1837, 1999. ISSN 00207543.

D Nembhard. Heuristic approach for assigning workers to tasks based on individual learning rates. *International Journal of Production Research*, 39(9):1955–1968, 2001. ISSN 0020-7543. doi: 10.1080/00207540110036696.

D Nembhard and B Norman. Cross Training in Production Systems with Human Learning and Forgetting. In David A Nembhard, editor, *Workforce Cross Training*, chapter 4, pages 111–129. CRC Press, Boca Raton, FL, USA, 2007.

D Nembhard and N Osothsilp. An empirical comparison of forgetting models. *IEEE Transactions on Engineering Management*, 48(3):283–291, 2001. ISSN 00189391. doi: 10.1109/17.946527.

D Nembhard and N Osothsilp. Learning and forgetting-based worker selection for tasks of varying complexity. *Journal of the Operational Research Society*, 56(5):576–587, 2004. ISSN 0160-5682. doi: 10.1057/palgrave.jors.2601842.

D Nembhard and S Shafer. The effects of workforce heterogeneity on productivity in an experiential learning environment. *International Journal of Production Research*, 46(14):3909–3929, 2008. ISSN 0020-7543. doi: 10.1080/00207540600596981.

D Nembhard and M Uzumeri. Experiential learning and forgetting for manual and cognitive tasks. *International Journal of Industrial Ergonomics*, 25(4):315–326, 2000a. ISSN 01698141. doi: 10.1016/S0169-8141(99)00021-9.

D Nembhard and M Uzumeri. An individual-based description of learning within an organization. {*IEEE*} *Transactions on Engineering Management*, 47(3):370–378, 2000b. ISSN 00189391. doi: 10.1109/17.865905.

D Nembhard, H Nembhard, and R Qin. A Real Options Model for Workforce Cross-Training. *The Engineering Economist*, 50(2):95–116, 2005. ISSN 0013791X.

J Olivella. An Experiment on Task Performance Forecasting Based on the Experience of Different Tasks. In *Proceedings of I7th International Conference on Knowledge Management*, 2007.

S Sayin and S Karabati. Assigning cross-trained workers to departments: A two-stage optimization model to maximize utility and skill improvement. *European Journal of Operational Research*, 176(3):1643–1658, 2007. ISSN 03772217. doi: 10.1016/j.ejor.2005.10.045.

A. Smith. *An Inquiry Into the Nature and Causes of the Wealth of Nations*. Cosimo, Incorporated, 1776. ISBN 9781602069015.

G van Rossum. Python v3.2 documentation, 2011. URL `https://docs.python.org/release/3.2/library/random.html`.

KK Yang. A comparison of cross-training policies in different job shops. *International Journal of Production Research*, 45(6):1279–1295, 2007. ISSN 0020-7543. doi: 10.1080/00207540600658039.

9 Appendix

```
from gurobipy import *
from math import *
from random import randint

Starting = "F"
WritingLearning = "F"
WritingTasks = "T"
InitialBuffer = 0
MaxWorkers = 7
MaxPeriods = 25
MaxTasks = 15
Struc = "C"
Workers = list()

#Workers available to allocate
for i in range(MaxWorkers):
        Workers.append (i)
#-----------------------------------------------------------------------------------------------


#Periods available to plan for
Periods = list()
for t in range(1,MaxPeriods):
        Periods.append (t)
#-----------------------------------------------------------------------------------------------


Taskfile = Struc+str(MaxTasks)
ImportedTaskData = __import__(str(Taskfile), globals(), locals(), ['Tasks', '
    Parent', 'C', 'A', 'DTasks', 'BTasks', 'DD', 'DA'], -1)
Tasks = ImportedTaskData.Tasks
Parent = ImportedTaskData.Parent
C = ImportedTaskData.C
A = ImportedTaskData.A
```

```
DTasks = ImportedTaskData.DTasks
BTasks = ImportedTaskData.BTasks
DD = ImportedTaskData.DD
DA = ImportedTaskData.DA

#-------------------------------------------------------------------------------



m = Model("SpringRES")
m.modelSense = GRB.MAXIMIZE
#-------------------------------------------------------------------------------

if WritingLearning == "T":
        I = {}
        K = {}
        L = {}
        F = {}
        for i in Workers:
                for j in Tasks:
                        I[i,j] = float(randint(1,9))/10
                        K[i,j] = float(randint(5,9))/10
                        L[i,j] = randint(2,10)
                        F[i,j] = randint(10,35)
        S = open('Skills.py', 'w')
        S.write("I = " + str(I) + "\nK = " + str(K) + "\nL = " + str(L) + "\nF
            = " + str(F))
        S.close()
if WritingTasks == "T":
        B = open(Taskfile+'.py', 'a')
        B.write("\nC = " + str(C) + "\nA = " + str(A) + "\nDD = " + str(DD) +
            "\nDA = " + str(DA))
from Skills import *
#-------------------------------------------------------------------------------

m.update()
X = {}
```

```
O = {}
for j in Tasks:
        for i in Workers:
                for t in Periods:
                        X[i,j,t] = m.addVar(vtype=GRB.BINARY,
                                        name='X_'+ str(i)+'_'+str(j)+'_'+str(t
                                            ))

                        if j in DTasks:
                                O[i,j,t] = m.addVar(vtype=GRB.CONTINUOUS, obj
                                    =1.0,
                                                name='O_'+ str(i)+'_'+str(j)+'
                                                    _'+str(t))
                        else:
                                O[i,j,t] = m.addVar(vtype=GRB.CONTINUOUS, obj
                                    =0.0,
                                                name='O_'+ str(i)+'_'+str(j)+'
                                                    _'+str(t))

P = {}
Z = {}
setE = {}
for i in Workers:
        for j in Tasks:
                for t in Periods:
                        setE = range(1,t+1)
                        for k in setE:
                                P[i,j,t,k] = I[i,j]+(K[i,j]*(1-exp((float(-1)/
                                    L[i,j])*k))*(exp((float(1)/F[i,j])*(k - t)
                                    )))
                                Z[i,j,t,k] = m.addVar(vtype=GRB.BINARY,name='
                                    Z_'+str(i)+'_'+str(j)+'_'+str(t)+'_'+str(k
                                    ))
R = {}
for j in DTasks:
        R[j] = m.addVar(vtype=GRB.BINARY, obj= 1000.0, name='R_'+str(j))
```

```
BI = {}
B = {}
for j in Tasks:
        BI[j] = InitialBuffer
        BI[j] = A[j]
        for t in Periods:
                B[j,t] = m.addVar(vtype=GRB.CONTINUOUS,name='B_'+str(j)+'_'+
                    str(t),lb = 0)


#------------------------------------------------------------------------------------------------


m.update()
for index, j in enumerate(DTasks):
        print "Task ", j
        setDD = {}
        setDD = range(1,DD[j])
        print "DD: ",DD[j]
        print "DA: ",DA[j],"\n\n ----------------"
        m.addConstr(quicksum(O[i,j,a] for i in Workers for a in setDD),GRB.
            GREATER_EQUAL, DA[j]*R[j],"Due Dates")

m.update()
setD = {}
setE = {}
for i in Workers:
        for j in Tasks:
                for t in Periods:
                        setD = range(1,t+1)
                        setE = range(t,MaxPeriods-1)
                        m.addConstr(O[i,j,t],GRB.LESS_EQUAL, X[i,j,t
                            ]*50000000000,"If Output, Must do task")
                        m.addConstr(O[i,j,t],GRB.GREATER_EQUAL,0.8*A[j]*
                            quicksum(P[i,j,t,k]*Z[i,j,t,k] for k in setD),"
```

```
                    Assign Min Productivity")
                m.addConstr(O[i,j,t],GRB.LESS_EQUAL,A[j]*quicksum(P[i,
                    j,t,k]*Z[i,j,t,k] for k in setD),"Assign Max
                    Productivity")
                m.addConstr(quicksum(k*Z[i,j,t,k] for k in setD),GRB.
                    LESS_EQUAL,quicksum(X[i,j,r] for r in setD),"Set Z
                    ")
                #m.addConstr(quicksum(Z[i,j,t,k] for k in setD),GRB.
                    EQUAL,1,"One Z")
                m.addConstr(quicksum(Z[i,j,t,k] for k in setD),GRB.
                    EQUAL,X[i,j,t],"No Z if no X")
                m.addConstr(quicksum(Z[i,j,b,t] for b in setE),GRB.
                    LESS_EQUAL,1)
m.update()
for i in Workers:
    for t in Periods:
        setD = range(1,t+1)
        m.addConstr(quicksum(X[i,j,t] for j in Tasks),GRB.LESS_EQUAL
            ,1, "One Task per Worker")

for j in Tasks:
    for t in Periods:
        setD = range(1,t+1)
        m.addConstr(quicksum(X[i,j,t] for i in Workers),GRB.LESS_EQUAL
            ,1, "One Worker per Task")

for j in BTasks:
    for t in Periods[1:]:

        m.addConstr(quicksum(O[i,j,t] for i in Workers) + B[j,t-1] -
            quicksum(C[c,j]*O[i,c,t] for i in Workers for c in Parent[
            j]), GRB.EQUAL, B[j,t], "Inventory Constraint")

for j in BTasks:
    m.addConstr(quicksum(O[i,j,1] for i in Workers) + BI[j] - quicksum(C[c
        ,j]*O[i,c,1] for i in Workers for c in Parent[j]), GRB.EQUAL, B[j
```

```
            ,1],"Initial Buffer")
        #m.addConstr(B[j,(MaxPeriods-1)],GRB.GREATER_EQUAL,BI[j],"ConWIP")
#-----------------------------------------------------------------------------

if Starting == "T":
        WorkersAvail = list()
        for i in Workers:
                WorkersAvail.append (i)
        for j in Tasks:
                if len(WorkersAvail) > 0:
                        print len(WorkersAvail), WorkersAvail
                        maxI = 0
                        maxIindex = 0
                        for i in WorkersAvail:
                                if I[i,j] >= maxI:
                                        maxI = I[i,j]
                                        maxIindex = i
                        for t in Periods[:1]:
                                X[maxIindex,j,t].start = 1.0
                        print maxIindex
                        WorkersAvail.remove (maxIindex)

m.Params.MIPGap = 0.005
m.Params.Threads = 6
#m.Params.NodefileStart = 4
m.update()
# Optimize
m.optimize()
status = m.status
```