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Investigating controller performance in hybrid SOFC systems in the presence of unknown nonlinearities

William Nowak Jr

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INVESTIGATING CONTROLLER PERFORMANCE IN HYBRID SOFC SYSTEMS WITH UNKNOWN NONLINEARITIES

by

William J. Nowak, Jr.

A Thesis Submitted in Partial Fulfillment of the Requirements for the Degree of Master of Science in Mechanical Engineering

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Investigating Controller Performance in Hybrid SOFC Systems in the Presence of Unknown Nonlinearities

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Abstract

Solid oxide fuel cells (SOFCs) are energy conversion devices that offer many benefits over various other fuel cell types as a result of high operating temperatures (800 $-\ 1000^\circ C$). Unfortunately, SOFCs tend to possess poor load following capabilities due to delays along the fuel path and complex system dynamics. Maintaining safe operating conditions during changes in power demand is addressed using a controller designed to regulate the fuel cell current based on fuel flow measurement. In order to compensate for the resulting mismatch between demanded and delivered power, the SOFC system is hybridized with an energy storage device, such as an ultra-capacitor. Prior research at the HySES laboratory at RIT has led to control designs that guarantee robustness to uncertainties in system parameters such as power electronics efficiencies. However, existing controllers for this system were developed under assumptions made about the unknown dynamics of the fuel supply system (FSS), such as exponential or bounded tracking. Retaining these controller designs, this thesis develops a general set of closed loop system equations in which the prior assumptions about the FSS are relaxed. The FSS behavior is treated as an unknown nonlinearity. Thereafter, concepts of absolute stability, Lyapunov stability and linear system approximation are used to evaluate the closed-loop system. The analysis leads to analytical conditions relating the controller gains and the local behavior of the FSS, predicting the onset of instability in the closed-loop system. The results are validated using simulations and using a hardware-in-the-loop test stand. Additionally, the problem of transient fuel utilization control of SOFCs is revisited and addressed by using a nonlinear observer design and an auxiliary hydrogen injection strategy. These approaches aim to compensate for fuel path delays and maintain desired operating conditions during transient loading conditions. Findings are validated using desktop simulations.
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Chapter 1

Introduction

1.1 Motivation

The need for alternative energy sources is a collectively global issue in today’s society. Growing energy concerns due to soaring fossil fuel prices, possible resource depletion, and fiercely debated environmental impacts all have brought energy security to the spotlight, sparking a public interest in finding new and improved energy sources. As a result, renewable energy in the form of wind, solar, and biomass systems are typically favored as potential solutions [5]. Of these, fuel cells have emerged for many as a viable supplement to increasing energy demands.

Long hailed as the technology of the future, fuel cells have been considered a “promising technology” since the 1960’s due to their clean, quiet and efficient operation [6]. Modern advancements have since revived this technology, making it desirable for a wide range of applications from distributed energy systems and stationary power production to mobile installations [7, 8, 9]. Fuel cells are considered advantageous because they are fueled by hydrogen, the most abundant element in the universe. Since hydrogen can be produced from any number of sources, fuel flexibility is a significant advantage [10].

Fuel cells operate by utilizing the electrochemical combination of hydrogen fuel and oxygen to produce water and electric current. This is achieved by feeding hydrogen and oxygen into the fuel cell’s anode and cathode, which is separated by a
permeable membrane or electrolyte. Ion’s pass through the electrolyte to react and form water while, simultaneously, electrons travel around an external circuit producing current [1]. Specifics of this process are unique to each fuel cell type and electrolyte material, resulting in a wide range of behaviors.

Of the various fuel cell designs, solid oxide fuel cells (SOFC) have recently gained popularity. Due to their high operating temperatures, SOFC’s tend to tolerant to fuel impurities and lend themselves well to be combined with bottoming cycles [1, 11, 8]. The following section will provide the basic operation principles of these solid oxide fuel cells.

1.2 Basic SOFC Operating Principles

Solid oxide fuel cell’s produce electric current from the electrochemical reaction between hydrogen and oxygen. This process occurs when hydrogen-rich gas is fed into the anode while oxygen enters the cathode. Oxygen ions pass through an oxide ion-conducting ceramic electrolyte to react with the hydrogen in the anode to produce water and release electrons. SOFC’s are considered relatively simple because they are completely solid-state devices and only require gas and solid phases to operate. In addition, since operating temperatures can reach as high as 1000°C, internal fuel reforming is possible. This characteristic lends SOFC’s to be tolerant of fuel impurities [1, 12].

Although SOFC’s are versatile, clean and efficient, they do have certain limitations. When exposed directly to large power fluctuations, hydrogen starvation can occur. This is often detrimental to cell operation and integrity as it can cause voltage drop and anode oxidation [13]. To monitor this phenomenon, system performance is typically quantified using a single performance variable known as $U$, or fuel utilization. Fuel utilization is defined as the ratio of hydrogen consumption to the net available hydrogen in the anode [14]. In order to balance fuel efficiency and safe
Figure 1.1: Basic schematic of an SOFC [1]

operation, target $U$ values typically range between 80% and 90% [14, 15, 16]. Since maintaining safe operating conditions during loading is imperative, previous research has focused on several techniques to compensate for inherent delays introduced fuel delivery system. Some of these existing techniques will be reviewed in the following section.

1.3 Literature Review

As mentioned in the previous section, maintaining fuel utilization at a safe target value is necessary to safe and efficient operation in SOFC’s [14, 4, 17]. Das, et al. [17] demonstrates that the formulation of steady state fuel utilization can be used effectively as an open loop control law for SOFC’s during transient operation. It is also observed that steady state fuel utilization is an invariant property, a function of fuel flow and current output, independent of internal reactions, temperatures and pressures. Furthermore, a feedback based current regulation scheme used for transient control of fuel utilization in terms of fuel flow and current demand is implemented and verified using simulations [17].

To further expand the capabilities of closed loop fuel utilization control, methods
have been investigated that build on previous work. Observer based transient control of $U$ is a proposed work in this research. In this regard, the use of observers in fuel cell systems has been attempted in previous work for sensor reduction in PEM and SOFC systems \cite{18, 19, 20, 21, 22, 23}. In \cite{21}, Tsourapas, et al. developed a feedback-based controller using a linear observer for a proton exchange membrane (PEM) fuel cell. By using only reactor temperatures as controller inputs, reportedly satisfactory closed-loop transient performance was recorded. Using a more theoretical approach, Gorgun, et al. \cite{22} provided observer designs for various fuel processing reactors. The novelty of this work lies in the fact that chemical composition estimates were obtained without knowledge of the reaction rate expressions. Although the authors stopped short of physical implementation, estimating species concentrations would prove valuable in improving the control of fuel cell systems.

In \cite{23}, Arcak et al. propose an adaptive nonlinear observer to estimate hydrogen partial pressures at the inlet and outlet of a fuel cell anode using available voltage, current and total pressure measurements. In this method, the fuel supply system dynamics are treated as a slowly varying parameter which is estimated using the voltage measurement. This is then used in the estimation of the anode pressures. Simulation results were favorable, however, there was an increased sensitivity to measurement disturbance at high hydrogen partial pressures. Improved performance could be realized by using the fuel supply system’s pressures and temperatures, instead of the voltage \cite{23}.

Focusing on SOFC systems, \cite{19} proposed an observer design that requires four species sensors; two at both the reformer and anode exits. From these measurements, the remaining eight species concentrations, fuel flow in and out of the anode and all reaction rates can be estimated. In \cite{18}, the previous design was improved upon by lowering the number of sensors. Ultimately, an observer design was developed using three sensors and the cell voltage measurement. It was concluded that parameter error bounds could be made arbitrarily small, verifying the proposed design. Simulations
were given. Several other observer designs have been developed that assume prior knowledge of reaction rates [24, 25]. When considering implementation, the merit of existing observer designs are diminished since they still require significant extra sensing and are heavily model-dependent.

Another area of focus is to improve transient fuel utilization through direct injection of hydrogen into the anode. This approach is equivalent to treating stored hydrogen as an energy buffer. Although little work has been done on energy buffering using hydrogen on SOFC’s, PEM fuel cells often take advantage of this concept [26, 27]. In [26], Capobianco, et al. explore a process called hydrogen production-separation and propose a reservoir volume used to store pure hydrogen. This buffer allowed for a simplified control strategy due to the fact that the feed back control of the reactor feeding could be avoided. By doing this, and only controlling reactor temperature and pressure, a simpler control strategy was developed. It was concluded that the reservoir was theoretically a valid strategy to improve transient qualities of the fuel cell. Additionally, the PEM fuel cell experimental setup in [27] included hydrogen storage bottles.

Similar setups used to mitigate transient effects during loading have also been proposed for SOFC’s. Mazumder, et al. proposed a pressurized-hydrogen tank buffer and investigates its effects on the performance and durability of a comprehensive SOFC model [28]. It was shown that the energy-buffering device provided additional energy requirements and degrading effects of load transients were significantly minimized. However, since the hydrogen tank effects were similar to that of a battery, specific data concerning the hydrogen storage was omitted. Rancrueil, et al. also simulates energy buffering hardware on a SOFC system. Here it was stated that fuel and energy buffering devices can significantly speed up system response to any load perturbation [29].

Similar in theory to hydrogen injection, another method of energy buffering has been proposed in previous work. This method uses an electrical storage device to
supply supplementary power during times of large load demand transients. In the Hybrid Sustainable Energy Systems Lab (HySES) at Rochester Institute of Technology, previous work has focused on using an ultra-capacitor to compensate for the slow transient response of an SOFC system [3, 30, 31]. The robust control techniques used in [30] and adaptive control techniques used in [31] successfully stabilized the system during load transients without knowing exactly DC/DC converter efficiencies or the fuel supply system dynamics. However, this incorporated the assumption the error in the FSS is bounded which simplifies the unknown dynamics behavior. The forthcoming work will relax this assumption and examine the effects of more general unknown FSS behaviors using existing control theory techniques.

A formulation for this hybrid system will be proposed such that it is represented as a cascaded connection between the fuel supply system and the error in the ultra-capacitor’s state of charge. This driver/driven system relationship is beneficial because it allows for the use of previous work to examine the effects of the unknown driver system on system stability. Since cascaded systems are a well researched area, many control-based textbooks contain extensive content pertaining to this class of systems [32, 33]. In Nonlinear Systems by Hassan Khalil, interconnected systems are addressed in topics such as Lure systems and absolute stability [32]. Similarly, [33] uses concepts of input-to-state stability to assess stability of cascaded systems [33].

One technique that has been used to prove stability for a range of unknown driver system dynamics is absolute stability [32, 34, 35, 36, 37]. By posing systems as a feedback interconnection of a linear and nonlinear system, also known as a Lur’e system, absolute stability concepts can be used to prove asymptotic stability for a class of unknown nonlinear systems [32, 34]. Although this is a novel application of these concepts on a hybrid fuel cell system, absolute stability is widely used on control problems relating to atomic force microscopes and other microelectromechanical systems [34, 35]. In [37], an actuator fault tolerant control (FTC) design is formulated
using concepts of absolute stability. It was stated that because of the generality incorporated into absolute stability, the results could be used on a wide range of FTC applications. In [36], a review of an application of absolute stability is presented for a half-plane axis flying vehicle model and the unknown flight characteristics associated with it.

When incorporating parameter estimation into the controller as in [31], the FSS behavior was also assumed bounded. Relaxing this condition will result in completely unknown dynamics of the fuel supply system. In order to examine stability during these uncertainties, a system linearization about an equilibrium point will be used. This technique is common in system dynamic and control text books and can be very valuable in simplifying complex nonlinearities [32, 38, 39, 40].

It is apparent through literature research that the transient control of SOFC’s is an important component of further fuel cell improvements. Methods such as hydrogen injection are showing promise to alleviate control concerns while still going relatively unexplored in depth. At the same time, observer based control methods have been investigated but still require significant improvement to warrant actual implementation. Likewise, the existing control techniques develop for the hybrid system are effective but based on simplifying assumptions about certain system components. In hopes to classify these controller for a broader range of unknown fuel supply systems, further analysis must be done by relaxing these assumptions and considering entire classes of dynamic behaviors.

1.4 Objectives

It is apparent from previous work relating to the control of SOFC systems that maintaining desired operating conditions during times of large transient load demands is vital. That being said, the objectives of this thesis are twofold. Firstly, maintaining a desired $U$ value during large load changes is important and one proposed method
of improving this includes using an observer to calculate actual fuel concentrations entering the anode. In doing so, delays relating to the reformer and FSS will be compensated for. The other method utilizes external hydrogen storage to compensate for delays associated with the FSS and allow for the fuel cell to follow changes in current more precisely. The implemented observer will be simulated using two comprehensive SOFC system models, representing two different stack geometries and reformers. Simulations are presented using MATLAB/Simulink® and conclusions are made based on the findings.

Secondly, given that previous controller designs for the SOFC/UC hybrid system made simplifying assumptions about the unknown FSS dynamics, further analysis is needed to relax these assumptions and broaden the results. The first controller, based on a robust feedback linearization technique, is revisited using a generalized cascaded form of the system equations. Absolute stability concepts are used to include entire classes of possible FSS behaviors, broadening the results beyond the original assumptions made about the fuel supply system. Also, a parameter estimation-based controller is considered in a generalized form in order to reexamine the assumptions made about the fuel supply system. Here, a system linearization approach is used to approximate a range of possible FSS behaviors. In doing so, conclusions are made based on analytical work about the limits of possible fuel supply system dynamics based on the controller gains and system parameters. Results are verified using MATLAB/Simulink® simulations and a hardware-in-the-loop test stand.
Chapter 2

SOFC Systems

The following sections will present both the steam reforming tubular SOFC model and the POX reforming planar SOFC model, both of which were developed in previous research. Further details of both models can be found in [3], [2] and [41].

2.1 System 1: Steam Reforming Tubular SOFC

Much like a physical fuel cell system, the System 1 model is comprised of three main components. These components include the steam reformer, fuel cell stack, and combustor. The schematic of this system is included in Figure 2.1 below [4]:

![Figure 2.1: Schematic diagram of steam reforming SOFC system model - System 1](image-url)
Methane fuel enters the reformer where steam reacts with the fuel in the presence of a catalyst to produce hydrogen-rich gas [42]. This reformed fuel, \( \dot{N}_m \), then travels to the anode of the fuel cell stack. This stack is modeled as a tubular geometry such that the anode and cathode are arranged in concentric tubes. As current is drawn, electrochemical reactions occur in the stack causing steam-rich gas to flow out of the anode. This exhaust \( \dot{N}_o \) enters the combustor while a known amount, \( k \), is recirculated back to the steam reformer. This recirculation sustains the endothermic reactions occurring in the reformer. While the combustion process burns excess fuel leftover from the electrochemical reactions in the stack, it also preheats \( \dot{N}_{air} \) amount of air before it enters the cathode. The exhaust from the combustor finally circulates back to the steam reformer, further contributing to the reforming reactions [4].

The reactions that take place in the are defined by I, II and III in Equation 2.1 while reactions I - IV also occur in the anode [43].

\[
\begin{align*}
(I) \quad & CH_4 + H_2O \leftrightarrow CO + 3H_2 \\
(II) \quad & CO + H_2O \leftrightarrow CO_2 + H_2 \\
(III) \quad & CH_4 + 2H_2O \leftrightarrow CO_2 + 4H_2 \\
(IV) \quad & H_2 + O^{2-} \rightarrow H_2O + 2e^- 
\end{align*}
\]

(2.1)

Where reformer reactions, \( r_1, r_2 \) and \( r_3 \) represent the reaction rates of equations I,II and III respectively. Therefore, the rates of consumption can be given in terms of these reaction rates such that \( (r_1 + r_3), (-r_1 + r_2), -(r_2 + r_3), -(3r_1 + r_2 + 4r_3) \) and \( (r_1 + r_2 + 3r_3) \) describe the rates at which \( CH_4, CO, CO_2, H_2 \) and \( H_2O \) are being consumed, respectively. Letting \( R_1, R_2, R_1, a \) and \( R_2, a \) represent rates of formation of \( CH_4 \) and \( CO \) in the reformer and anode, the rates of consumption of \( CH_4, CO, CO_2, H_2 \) and \( H_2O \) can then be represented in terms of the rates of formation of \( CH_4 \) and \( CO \). Using these representations, the mass balance equations of the reformer and
anode are as follows [3]:
\[
\begin{align*}
\frac{d}{dt}(N_rX_{1,r}) &= k\dot{N}_oX_{1,a} - \dot{N}_{in}X_{1,r} + R_{1,r} + \dot{N}_f \\
\frac{d}{dt}(N_rX_{2,r}) &= k\dot{N}_oX_{2,a} - \dot{N}_{in}X_{2,r} + R_{2,r} \\
\frac{d}{dt}(N_rX_{3,r}) &= k\dot{N}_oX_{3,a} - \dot{N}_{in}X_{3,r} - R_{1,r} - R_{2,r} \\
\frac{d}{dt}(N_rX_{4,r}) &= k\dot{N}_oX_{4,a} - \dot{N}_{in}X_{4,r} - 4R_{1,r} - R_{2,r} \\
\frac{d}{dt}(N_rX_{5,r}) &= k\dot{N}_oX_{5,a} - \dot{N}_{in}X_{5,r} + 2R_{1,r} + R_{2,r}
\end{align*}
\] (2.2)

\[
\begin{align*}
\frac{d}{dt}(N_aX_{1,a}) &= k\dot{N}_{in}X_{1,r} - \dot{N}_oX_{1,a} + R_{1,a} \\
\frac{d}{dt}(N_aX_{2,a}) &= k\dot{N}_{in}X_{2,r} - \dot{N}_oX_{2,a} + R_{2,a} \\
\frac{d}{dt}(N_aX_{3,a}) &= k\dot{N}_{in}X_{3,r} - \dot{N}_oX_{3,a} - R_{1,a} - R_{2,a} \\
\frac{d}{dt}(N_aX_{4,a}) &= k\dot{N}_{in}X_{4,r} - \dot{N}_oX_{4,a} - 4R_{1,a} - R_{2,a} - r_e \\
\frac{d}{dt}(N_aX_{5,a}) &= k\dot{N}_{in}X_{5,r} - \dot{N}_oX_{5,a} + 2R_{1,a} + R_{2,a} + r_e
\end{align*}
\] (2.3)

Where the rate of electrochemical reaction is given as

\[
r_e = i_{fc}N_{cell}/nF
\] (2.4)

In Equations 2.2 and 2.3, \(X_{i,a}\) and \(X_{i,r}\) are molar concentrations of species in the anode and reformer, with \(i=1,2,3,4,5\) representing \(CH_4, CO, CO_2, H_2\) and \(H_2O\) respectively. \(\dot{N}_{in}\) and \(\dot{N}_o\) are the flow rates in and out of the reformer, \(N_r\) and \(N_a\) represent the molar contents of the reformer and anode, \(\dot{N}_f\) is the fuel flow rate, and \(k\) is the amount of exhaust being recirculated. In Equation 2.4, \(i_{fc}\) is the fuel cell current, \(N_{cell}\) is the number of cells, \(n\) is the number of electrons participating in the electrochemical reaction and Faraday’s constant, \(F = 96485.34\) coulomb/mole.

### 2.2 System 2: POX Reforming Planar SOFC

System 2 operates in much the same way as first. One main difference relating to the stack is the geometrical layout, where the planar anode and cathode components are flat plates set face to face. Additionally, the POX reforming process differs significantly from the steam reforming process. The SOFC system schematic is shown in Figure 2.2.
In this system, a mixture of methane and air with a known molecular concentration ratio of oxygen to methane \((O2C)\) enters the reformer where hydrogen rich gas is generated by reacting the fuel with a sub-stoichiometric amount of oxygen \([42]\). The reformed fuel then enters the anode of the planar SOFC where electrochemical reactions occur as current is drawn from the stack. The used fuel then leaves the anode for the combustion chamber to be burned and preheat air entering the cathode \([3]\).

![Figure 2.2: Schematic diagram of POX reforming SOFC system model - System 2][2]

In the POX reformer, the following reactions occur in addition to Equation 2.1-I, II and III from the steam reformer \([44, 45]\):

\[
\begin{align*}
\text{(V)} \quad CH_4 + 0.5O_2 & \rightarrow CO + 2H_2 \\
\text{(VI)} \quad CH_4 + 2O_2 & \rightarrow CO_2 + 2H_2O \\
\text{(VII)} \quad CO + 0.5O_2 & \rightarrow CO_2 \\
\text{(VIII)} \quad H_2 + 0.5O_2 & \rightarrow H_2O
\end{align*}
\]

(2.5)

Considering \(\gamma\) as the fraction of oxidized \(CH_4\) being reacted in V, \(\beta\) as the fraction of \(CO\) generated by V that will be oxidized through VII, and \(\alpha\) as the fraction of \(H_2\) generated by V that will be oxidized through VIII, the net oxidation can be shown
as:

$$CH_4 + [0.5\gamma + 2(1 - \gamma) + \gamma\alpha + 0.5\gamma\beta]O_2 \rightarrow$$

$$(1 - \beta)\gamma CO + 2(1 - \alpha)\gamma H_2 + (1 - \gamma + \gamma\beta)CO_2 + 2(1 - \gamma + \gamma\alpha)H_2O \quad (2.6)$$

Letting $R_{ox}$ represent the net rate of oxidation of $CH_4$, the reformer mass balance equations can be shown as [46]:

$$\frac{d}{dt}(N_rX_{1,r}) = -\dot{N}_{in}X_{1,r} + R_{1,r} - R_{ox} + \dot{N}_fX_{1,f}$$

$$\frac{d}{dt}(N_rX_{2,r}) = -\dot{N}_{in}X_{2,r} + R_{2,r} + R_{ox}(1 - \beta)\gamma$$

$$\frac{d}{dt}(N_rX_{3,r}) = -\dot{N}_{in}X_{3,r} - R_{1,r} - R_{2,r} + R_{ox}(1 - \gamma + \gamma\beta)$$

$$\frac{d}{dt}(N_rX_{4,r}) = -\dot{N}_{in}X_{4,r} - 4R_{1,r} - R_{2,r} + 2R_{ox}(1 - \alpha) \quad (2.7)$$

$$\frac{d}{dt}(N_rX_{5,r}) = -\dot{N}_{in}X_{5,r} + 2R_{1,r} + 2R_{2,r} + 2R_{ox}(1 - \gamma - \gamma\alpha)$$

$$\frac{d}{dt}(N_rX_{6,r}) = -\dot{N}_{in}X_{6,r} + \dot{N}_{in}X_{6,f}$$

$$\frac{d}{dt}(N_rX_{7,r}) = -\dot{N}_{in}X_{7,r} - R_{ox}[0.5\gamma + 2(1 - \gamma) + \gamma\alpha + 0.5\gamma\beta] + \dot{N}_fX_{7,f}$$

Where subscripts 6 and 7 represent $N_2$ and $O_2$ respectively, and $X_{1,f}, X_{6,f},$ and $X_{7,f}$ represent the known molar fractions of $CH_4$, $N_2$, and $O_2$ in the fuel. Additionally, the anode mass balance equations are the same as in Equation 2.3 for System 1.

### 2.3 Fuel Utilization

Fuel cell performance is typically quantified using a single performance parameter known as fuel utilization. Fuel utilization, $U$, is defined as the ratio of hydrogen consumption to the net available hydrogen in the anode [14]. If hydrogen consumption is low relative to the amount of hydrogen available, fuel is being wasted. However, if nearly all the hydrogen in the anode is being consumed hydrogen starvation can occur. This can be detrimental to stack integrity as it may cause voltage drop and anode oxidation [13]. Because of these risks, ideal fuel utilization values typically range between 80% and 90% [14, 15, 16].
Mathematically, fuel utilization is defined as [4]:

\[ U = 1 - \frac{\dot{N}_o(4X_{1,a} + X_{2,a} + X_{4,a})}{\dot{N}_{in}(4X_{1,r} + X_{2,r} + X_{4,r})} \]  

(2.8)

Where \(X_{1,a}, X_{2,a}, X_{4,a}\) and \(X_{1,r}, X_{2,r}, X_{4,r}\) represent molar species concentrations of \(CH_4\), \(CO\) and \(H_2\) in the anode and reformer, respectively. \(\dot{N}_{in}\) and \(\dot{N}_o\) are flow rates in and out of the anode.

Figure 2.3: SOFC response to a step change in current demand [3]

Figure 2.3 demonstrates the response of System 1 to step changes in current. The simulation is run with \(N_{cell} = 50\) and \(\dot{N}_f = 7 \times 10^{-4}\), which yields a steady state \(U\) of \(U_{ss} \approx 85\%\). It is apparent from plot (b) that fuel utilization is drastically affected by a change in \(i_{fc}\). For a 0.5A step change in current, \(U\) increases to 95% while a 1A step change in current causes utilization to go to 100% and voltage to drop to 0V. Since these conditions can damage the fuel cell stack, it is clear that a method of controlling fuel flow is necessary to maintain a safe fuel utilization value.

### 2.4 Open Loop Control of \(U\)

It is apparent from Section 2.3 that a method of controlling fuel flow is necessary to maintain a specified \(U\) value. For System 1, this requires the use of the molar balance equations of species from Section 2.1. Setting the left hand side of Equations 2.2 and
2.3 to zero yield the steady state molar balance equations. Therefore, Equations 2.2, 2.3, 2.4 and 2.8 can be reduced to the following form [4, 3]:

\[ U_{ss} = \frac{1 - k}{(4nF\dot{N}_f/i_{fc}\dot{N}_{cell}) - k} \]  

System 1  \hspace{1cm} (2.9a)

Equation 2.9a is independent of reaction rates \( R_{1,r}, R_{2,r}, R_{1,a}, R_{2,a} \), and flow rates \( \dot{N}_{in} \) and \( \dot{N}_o \). Additionally, \( U_{ss} \) is invariant with respect to variations in operating pressures and temperatures, reforming catalyst mass, air flow rate and the Steam-to-Carbon ratio (STCR) [41]. Therefore, steady state fuel utilization \( U_{ss} \) is shown to have an invariant relationship with current draw \( i_{fc} \) and fuel flow \( \dot{N}_f \) [30].

Likewise, a method of fuel utilization control can be formulated for the POX reformer based system. By considering the molar balance equations for the reformer, and noting the oxygen is being completely consumed, Equation 2.7 leads to:

\[ R_{ox} = \dot{N}_f\alpha\dot{X}_1,f/[0.5\gamma + 2(1 - \gamma) + \gamma\alpha + 0.5\gamma\beta] \]  

\[ = \dot{N}_f02C\alpha\dot{X}_1,f/[0.5\gamma + 2(1 - \gamma) + \gamma\alpha + 0.5\gamma\beta] \]  

(2.9b)

From Equations 2.3, 2.4, 2.7, 2.8 and 2.9b, the expression for \( U_{ss} \) is given as [46]:

\[ U_{ss} = \frac{i_{fc}\dot{N}_{cell}}{2nF\dot{N}_f\alpha\dot{X}_1,f(2 - 02C)} \]  

System 2  \hspace{1cm} (2.9c)

As with the \( U_{ss} \) formulation for System 1, Equation 2.9c is also independent of reaction rates, flow rates, operating pressures and temperatures, reforming catalyst mass and air flow rate. Because Equations 2.9a and 2.9c are invariant properties of the fuel cell systems, they can easily be implemented to calculate fuel flow based on a current draw and desired steady state fuel utilization.

For System 1, Equation 2.9a can be rearranged such that a demanded fuel flow can be calculated given a demanded current \( i_{fc,d} \). Thus, as the current demand changes, fuel flow can be adjusted accordingly such that the targeted \( U \) is achieved. This formulation is given as [4, 3]:

\[ \dot{N}_{f,d} = \frac{i_{fc,d}\dot{N}_{cell}}{4nFU_{ss}} [1 - (1 - U_{ss})k] \]  

System 1  \hspace{1cm} (2.10a)
Where $\dot{N}_{f,d}$ is the amount of fuel needed to maintain the target $U$ value given $i_{fc,d}$. Similarly, Equation 2.9c for System 2 yields [46]:

$$\dot{N}_{f,d} = \frac{i_{fc,d}N_{cell}}{2nFX_{1,f}U_{ss}(2 - O2C)}$$  
System 2  
(2.10b)

Equations 2.10a and 2.10b can now be used to calculate the amount of fuel necessary to maintain a desired $U_{ss}$ given a current demand. These equations can be implemented as a form of open loop control for their respective systems. A schematic of this control layout is shown in Figure 2.4.

![Figure 2.4: Open loop control schematic](image)

Using this control architecture, Figure 2.5 gives simulation results for System 1 where targeted $U_{ss} = 85\%$, $i_{fc,d} = 10A$ for $t < 200s$ and 11, 14, 18, 22A for $t \geq 200s$.

![Figure 2.5: Open loop control - System 1](image)

The simulations given for System 1 show significant improvements over the simulation results in Figure 2.3. In contrast to the constant fuel flow approach, the open
loop control strategy is adequate in maintaining a steady state $U$ during step changes in current demand of up to 8A. Likewise, similar results are shown for System 2 in Figure 2.6.

![Figure 2.6: Open loop control - System 2](image)

Where $U_{ss} = 85\%$, $i_{fc,d} = 10A$ for $t < 100s$ and 20, 30A for $t \geq 100s$. It is clear the System 2 also exhibits favorable results when using Equation 2.10b to calculate necessary fuel flow. For a step change of 5A, fuel utilization fluctuates and returns to the target value following the load transient. However, as with System 1, fuel starvation occurs during the larger step change in current.

According to these simulations, this open loop strategy can fail when the systems are subjected to changes in demanded current. This response is due in part to the delays caused by the components of the fuel delivery path. The following section will examine these delays have an adverse effect on controlling $U$.

### 2.5 Delays in the Fuel Path

Revisiting the results in Figures 2.5 and 2.6, it was demonstrated that hydrogen starvation occurred in both systems as the change in current demand increased. Although the fuel flow rate $\dot{N}_{f,d}$ was calculated to meet the demands of the changing current, delays in the fuel path prevent the commanded fuel from reaching the anode when
it is needed. In the case of both the planar and tubular SOFC, delays in the fuel delivery path must be considered to avoid potentially harmful operating conditions during transient loading conditions.

The dominant delays in these SOFC systems are due to the fuel supply system dynamics and the reformer, labeled $D_1$ and $D_2$ in Figure 2.7 respectively. Delay $D_1$ is attributed to delays induced by the fuel supply system dynamics, as a result of valves and other hardware. Delay $D_2$ is caused by the dynamics of the fuel reformer in each system. Both the steam and POX-type reformers have specific dynamics associated with their operation, contributing to the overall slower response time of the fuel delivery process.

Figure 2.7: Delays D1 and D2 along fuel path

Figure 2.7 illustrates the impact of $D_1$ and $D_2$ on $\dot{N}_f$ for a given $\dot{N}_{f,d}$ command input. This flow lag proves detrimental when the fuel cell is commanded to produce an instantaneous change in power, and fuel demands calculated by an open loop control law are not immediately satisfied. The following section introduces a strategy to compensate for delay $D_1$ attributed to the fuel supply system and thereby improve the transient behavior of $U$. 
2.6 Transient Control of $U$

Section 2.4 demonstrates the utility of the invariant property $U$ in controlling demanded fuel flow $\dot{N}_{f,d}$. Extending this idea, a current regulation control can be developed to address delay $D1$ described in Section 2.5 for the two systems. Closing the control loop will reduce the occurrence of hydrogen starvation conditions during transient load demands.

Equation 2.9a derived for System 1 can be rearranged to calculate an allowable current draw based on the measured fuel flow $\dot{N}_f$. By using this measured value, the current demand will be limited according to the rate of fuel actually being supplied to the reformer. This current regulation equation is represented as the following:

$$i_{fc} = \frac{4nFU_{ss}\dot{N}_f}{N_{cell}} \frac{1}{[1 - (1 - U_{ss})k]} \quad \text{System 1} \quad (2.11a)$$

Where $i_{fc}$ is the regulated current based on the measured fuel flow $\dot{N}_f$ and a target $U_{ss}$ value. In the same manner, using Equation 2.10b for System 2 yields:

$$i_{fc} = \frac{2nFU_{ss}\dot{N}_fX_{1,f}(2 - O2C)}{N_{cell}} \quad \text{System 2} \quad (2.11b)$$

Assuming the measurement of $\dot{N}_f$ is available, Equations 2.10a and 2.11a can be used in a closed loop control configuration for System 1 to target a desired $U_{ss}$. Equations 2.10b and 2.11b are used in a similar fashion for System 2. This strategy is depicted in Figure 2.8.

Figure 2.8: Open and closed loop control schematic [4]
In Figure 2.8, when the switch is in the OL or open loop position, the open loop strategy from Section 2.4 is being implemented. Likewise, the closed loop position CL illustrates the closed loop transient control method.

Under closed loop operation, the raw current demanded signal \( i_{fc,d} \) is used to calculate demanded fuel flow rate \( \dot{N}_{f,d} \). This rate is then fed into the fuel cell system where delays begin influencing the flow. Downstream of the fuel supply system, a measurement \( \dot{N}_f \) is taken and fed back to calculate a regulated amount of current depending on the actual amount of fuel entering the reformer.

![Figure 2.9: Transient closed loop control - System 1](image)

Figure 2.9 demonstrates the effectiveness of this closed loop strategy on the steam reforming tubular SOFC system, where \( U_{ss} = 85\% \), \( i_{fc,d} = 10A \) for \( t < 150s \) and 18, 30, and 50A for \( t \geq 150s \). Plot (b) shows the actual fuel flow \( \dot{N}_f \) that is being fed back to regulate the current drawn. Plot (a) demonstrates the current regulation as compared to the demanded current \( i_{fc,d} \). Plot (c) shows the fuel utilization which does not fluctuate more than \( \pm 5A \) given a change in demanded current of up to 40A. It is apparent that this feedback control results in an attenuated fluctuation of \( U \) during transient loading conditions.

The simulations in Figure 2.10 were run with targeted \( U_{ss} = 85\% \), \( i_{fc,d} = 10A \) for \( t < 100s \) and 20, 30A for \( t \geq 100s \). As with System 1, \( i_{fc} \) is shown to be properly regulated according to fuel flow \( \dot{N}_f \). This regulation is shown to be effective with step
changes in current draw of up to 15A. During these transient conditions, $U$ does not fluctuate beyond ±2%. 

Figure 2.10: Transient closed loop control - System 2
Chapter 3

Transient Control Strategies

3.1 Nonlinear Observer

Introduced in Section 2.5, the delays in the fuel supply path significantly affect the flow of commanded fuel and can potentially cause hydrogen starvation during transient loading. Because of this, Section 2.6 made use of a current regulation method to compensate for delay $D_1$. While the current regulation method reduced fluctuations in $U$, further improvements can be made by considering delay $D_2$.

As previously shown in Figure 2.7, $D_2$ is a byproduct of the fuel cell system’s reformer. Accounting for this delay by measuring fuel flow after the reformer would theoretically improve the current regulation method’s effectiveness. However, the reformer produces varying and unknown concentrations of chemical species which are not easily measurable. This issue motivates the use of a nonlinear observer to estimate species concentrations exiting the reformer and reduce the effects of delays attributed to the reforming process.

3.1.1 Observer Design for System 1

Designing an observer for System 1 requires revisiting the mathematical definition of $U$ given in Equation 2.8. By assigning $\zeta_r$ and $\zeta_a$ as the combined species concentrations that yield hydrogen in the reformer and anode respectively, $U$ can be expressed
as,

\[ U = 1 - \frac{\dot{N}_o \zeta_r}{\dot{N}_{in} \zeta_a} \]  

(3.1)

where

\[ \zeta_r = 4X_{1,r} + X_{2,r} + X_{4,r} \]

\[ \zeta_a = 4X_{1,a} + X_{2,a} + X_{4,a} \]  

(3.2)

Considering Equations 2.2, 2.3, 2.4 and 3.2, the following form of the mass balance equations for the reformer and anode can be shown:

\[ \frac{d}{dt}(N_r \zeta_r) = k\dot{N}_o \zeta_a - \dot{N}_{in} \zeta_r + 4\dot{N}_f \]

\[ \frac{d}{dt}(N_a \zeta_a) = -k\dot{N}_o \zeta_a + \dot{N}_{in} \zeta_r - i_{fc}N_{cell}/nF \]  

(3.3)

Using Equations 3.1 and 3.3, a steady state fuel utilization relationship can be derived as a function of state \( \zeta_r \) such that

\[ U_{ss} = \frac{i_{fc}N_{cell}}{nF\dot{N}_{in} \zeta_r} \]  

(3.4)

Rearranging to calculate \( i_{fc} \), Equation 3.4 takes the form:

\[ i_{fc} = \frac{U_{ss} nF \dot{N}_{in} \hat{\zeta}_r}{N_{cell}} \]  

(3.5)

Assuming the estimate \( \hat{\zeta}_r \) is available and \( \dot{N}_{in} \) is measured, Equation 3.5 can be used as a current regulation method to calculated \( i_{fc} \). This configuration is illustrated in Figure 3.1, where the \( i_{fc,d} \) signal enters Equation 2.10a and \( \dot{N}_{f,d} \) is calculated. For the previous current regulation method, \( \dot{N}_f \) is fed back to calculate \( i_{fc} \) and compensate for delay \( D_1 \). Alternatively, in the observer-based approach flow rate \( \dot{N}_{in} \) is fed back to the observer equations and those estimates are used in Equation 3.5 to calculate \( i_{fc} \).

Based on the molar balance equations given in Equation 3.3 and considering the simplifying assumption that \( N_r \) and \( N_a \) are constants, the proposed observer equations are given as the following:

\[ N_r \dot{\hat{\zeta}}_r = k\dot{N}_o \hat{\zeta}_a - \dot{N}_{in} \hat{\zeta}_r + 4\dot{N}_f \]

\[ N_a \dot{\hat{\zeta}}_a = -\dot{N}_o \hat{\zeta}_a + \dot{N}_{in} \hat{\zeta}_r - i_{fc}N_{cell}/nF \]  

(3.6)
Where \( \hat{\zeta}_r \) and \( \hat{\zeta}_a \) are estimates of the species concentrations \( \zeta_r \) and \( \zeta_a \), respectively. It is apparent that the equations in 3.6 are independent of species concentrations, negating the need for costly species concentration sensors. That being said, the proposed observer equations do require the knowledge of \( N_r \) and \( N_a \). This design also operates under the assumption that there are existing sensors that can measure the temperature and pressure at the reformer and anode. This, paired with the known volumes and universal gas constant, will allow for \( N_r \) and \( N_a \) to be calculated. In addition, measurements of \( \dot{N}_{in} \) and \( \dot{N}_o \) are needed, increasing the amount of sensing needed over the default current regulation method.

### 3.1.2 Observer Design for System 2

From Equations 2.3, 2.4, 2.7, 2.9b and 3.2, System 2’s mass balance equations are presented as:

\[
\frac{d}{dt}(N_r \zeta_r) = -\dot{N}_{in} \zeta_r + 2\dot{N}_f (2 - O2C) \\
\frac{d}{dt}(N_a \zeta_a) = -\dot{N}_o \zeta_a + \dot{N}_{in} \zeta_r + i_{fc} N_{cell}/nF
\]  

(3.7)

Considering Equation 3.7, and again making the simplifying assumption that \( N_r \) is constant, the proposed observer equation for System 2 is presented as the following:

\[
N_r \dot{\hat{\zeta}}_r = -\dot{N}_{in} \hat{\zeta}_r + 2\dot{N}_f (2 - O2C)
\]  

(3.8)
Using Equation 3.5 from System 1 to calculate $i_{fc}$, the observer equation will estimate species concentrations exiting the reformer based on flow rate $\dot{N}_{in}$. This schematic is shown in Figure 3.2.

![Figure 3.2: Block diagram of observer based estimation - System 2](image)

As with the observer formulation for System 1, the estimate of $\zeta_r$ is dependent on the knowledge of $N_r$. This value is assumed to be available using existing temperature and pressure sensors.

### 3.2 Hydrogen Injection

Revisiting Figure 2.7, inherent delays in the fuel delivery system drive the need for a feedback current regulation technique. While effective, this method limits the fuel cell system’s load following capabilities. This load following issue then motivates the idea of an energy buffering device that can compensate for power lost during current regulation. In this section, the use of a mechanical energy buffer in the form of an auxiliary hydrogen storage device is proposed. The goal is to compensate for the slow dynamics of the system with a direct hydrogen injection system capable of significantly faster response times to transient loading.

This hydrogen injection technique will use knowledge of $\dot{N}_{f,d}$ and $\dot{N}_{f}$ to improve load following capabilities relative to the current regulation method. The auxiliary hydrogen will be used to provide the difference in fuel flow between $\dot{N}_{f,d}$ and $\dot{N}_{f}$.
caused by delay $D1$. The proposed hydrogen injection schematic is shown in Figure 3.3.

![Figure 3.3: Block diagram of hydrogen storage and fuel path delays](image)

For this preliminary simulation-based study, the hydrogen injection system is assumed to act as a stable first order system. This assumption proves to be valid by considering the injection system operating under its own control system that guarantees stability and zero steady state error. Additionally, this assumption neglects the need to consider tank capacity, pressure variations and hydrogen depletion during operation. For simplicity, temperature is considered to be constant throughout the injection process.

Considering only System 1, $\dot{N}_{f,d}$ is calculated using Equation 2.10a. The measurement of $\dot{N}_f$ is then used to calculate the amount of additional hydrogen based on the difference between the measured $\dot{N}_f$ and the required $\dot{N}_{f,d}$. Considering the reactions in the steam reformer given in Equation 2.1 by noting that one $CH_4$ molecule can yield at most four $H_2$ molecules, the amount of additional hydrogen needed is designed as:

$$\dot{N}_{H_2} = 4(\dot{N}_{f,d} - \dot{N}_f)$$

The allowable current is then recalculated using a form of Equation 2.11a given as:

$$i_{fc} = \frac{4nF U_{ss} \dot{N}_{total}}{\dot{N}_{cell}} \frac{1}{[1 - (1 - U_{ss})k]}$$

(3.10)

where

$$\dot{N}_{total} = \dot{N}_{H_2} \frac{1}{4} + \dot{N}_f$$

(3.11)
Such that $\dot{N}_{\text{total}}$ is the total amount of fuel entering the anode after hydrogen injection takes place. With this extension of the current regulation control, $i_{fc}$ will now account for the hydrogen that is directly injected into the anode and allow for more aggressive changes in current.

### 3.3 Simulations and Discussion

#### 3.3.1 Observer Based Current Regulation

For Figure 3.4, target $U_{ss} = 85\%$, $i_{fc,d} = 10\text{A}$ for $t < 200\text{s}$ and $20, 50\text{A}$ for $t \geq 200\text{s}$. The system response to the step changes in current were recorded using both the existing current regulation method and the observer based estimation.

![Figure 3.4: Response to step input - System 1](image)

Figure 3.4 shows simulation results for System 1 in response to step changes in current. Plots (a) and (d) demonstrate the regulated current due to a $10\text{A}$ step change and a $50\text{A}$ step change respectively. It is apparent that the observer for System 1 does allow for a slightly more aggressive current signal to reach the fuel cell in both instances. Examining plots (b) and (e), it is clear that the maximum fluctuation
in fuel utilization is reduced by nearly 1% for a 10A step change and nearly 2% for the 40A step change. Additionally, the estimations of $\hat{\zeta}_r$ are shown to converge on the actual value about 10 seconds after the current step. It can be concluded that there are benefits to compensating for delay $D2$, including slight improvements in load following and reducing potentially harmful fluctuations of $U$.

Likewise, Figure 3.5 demonstrates the response of System 2 for step changes in current draw. Plots (a) and (d) illustrate the regulated current in response to a 5A and 15A step in current respectively. Also, fuel utilization and $\zeta_r$ estimation plots are given. It is apparent that the use of this observer does not significantly improve fuel cell operation over the current regulation method for the POX reformer. This is most likely due to the fact that relative to the steam reformer, the POX reformer in System 2 is more compact and responds faster to transient loading conditions. Because of this, delay $D2$ is likely to have less of an impact on the fuel path.
3.3.2 Assumptions Relating to the Observer Formulation

While formulating the observer equations for Systems 1 and 2, it was assumed that \( N_r \) and \( N_a \) were constant. This assumption not only simplified the process of computing \( N_a \) and \( N_r \), but also eliminated the need for knowledge of \( \dot{N}_r \) and \( \dot{N}_a \). The following aims to validate these assumptions by revisiting the observer equations for both systems.

Considering Equation 3.3 for System 1, relaxing the assumption that \( N_r \) and \( N_a \) are constants yields:

\[
\dot{N}_r \zeta_r + N_r \dot{\zeta}_r = k \dot{N}_o \zeta_a - \dot{N}_{in} \zeta_r + 4N_f \\
\dot{N}_a \zeta_r + N_a \dot{\zeta}_a = -k \dot{N}_o \zeta_a + \dot{N}_{in} \zeta_r - i_{fc} N_{cell}/nF
\]  

(3.12)

Resulting in observer equations:

\[
\dot{N}_r \hat{\zeta}_r = k \dot{N}_o \hat{\zeta}_a - (\dot{N}_{in} + \dot{N}_r) \hat{\zeta}_r + 4N_f \\
\dot{N}_a \hat{\zeta}_a = -(\dot{N}_o + \dot{N}_a) \hat{\zeta}_a + \dot{N}_{in} \hat{\zeta}_r - i_{fc} N_{cell}/nF
\]  

(3.13)

Implementing this form of the observer equations on System 1 will demonstrate the accuracy of the initial assumptions. Figure 3.6 compares the original observer design with the form found in Equation 3.13.

![Figure 3.6: Observer results with \( \dot{N}_r \) and \( \dot{N}_a \) - System 1](image-url)
Plots (c) and (d) in Figure 3.6 illustrate $\dot{N}_r$ and $\dot{N}_a$ compared to $\dot{N}_{in}$ and $\dot{N}_o$ during a 15 A step change in current. Examining Equation 3.13, it is clear that $\dot{N}_r$ and $\dot{N}_a$ have little effect in the presence of $\dot{N}_{in}$ and $\dot{N}_o$. Additionally, plots (a) and (b) demonstrate almost no improvement in load following capabilities and minimal improvement in reducing fluctuations of $U$. It is apparent that the effects of $\dot{N}_r$ and $\dot{N}_a$ on the estimation of $\zeta_r$ and $\zeta_a$ are minimal. These results support the assumption of considering $N_r$ and $N_a$ as constants in the original observer formulation.

Revisiting Equation 3.7 for System 2 and considering the effects of $\dot{N}_r$ and $\dot{N}_a$ yields the following:

$$\dot{N}_r \zeta_r + N_r \dot{\zeta}_r = -\dot{N}_{in} \zeta_r + 2\dot{N}_f (2 - O2C)$$

$$\dot{N}_a \zeta_a + N_a \dot{\zeta}_a = -\dot{N}_o \zeta_a + \dot{N}_{in} \zeta_r + i_{fc} N_{cell} / nF$$

Equation (3.14)

Which leads to the following observer equation:

$$N_r \dot{\zeta}_r = -(\dot{N}_{in} + \dot{N}_r) \dot{\zeta}_r + 2\dot{N}_f (2 - O2C)$$

Equation (3.15)

As with System 1, the implementation of Equation 3.15 on System 2 shows little improvement over the use of Equation 3.8. Plot (c) reinforces the notion that in the presence of $\dot{N}_{in}$, $\dot{N}_r$ has little effect of the estimation of $\zeta_r$. Additionally, plots (a) and (b) demonstrate few differences between considering $\dot{N}_r$ as in Equation 3.15 and the original formulation in Equation 3.8.

Figure 3.7: Observer results with $\dot{N}_r$ - System 2
3.3.3 Observer Robustness Considerations

While the observer approach did not yield significant results under typical operating conditions, it is possible that it’s utility may become apparent in other scenarios. The following simulations consider varying reformer sizes and flow constants. These alterations to the system simulate an increase in the effects of delay D2 on the fuel delivery path.

Figure 3.8a: 2x Reformer CV - System 1

Figure 3.8b: 4x Reformer CV - System 1
Figure 3.8a demonstrates the effects of increasing the reformer volume, therefore creating more severe delays in the fuel path. It is apparent from (b) and (e) that during a step change in current, the observer shows significant improvements in controlling $U$ over the current regulation method. Additionally, plots (a) and (d) depict improved load following capabilities during a step change in current.

As in the previous simulation, Figure 3.8b shows that the implementation of the observer yields drastic improvements over the current regulation method. In plots (b) and (e), the observer manages to limit fluctuations in $U$ to about 3% whereas the current regulation method results in a fuel utilization as much as 95%. Along with reducing fluctuations in $U$, again the observer manages to allow for better load following during transient conditions.

Another method of adding delays to the system is through artificially restricting flow through the reformer. This was simulated by increasing the reformer flow constant from the initial value of $1e-1$ kg/s/Pa in Figures 3.9a, 3.9b and 3.9c.

![Figure 3.9a: Reformer flow constant (kg/s/Pa) = 1e-1/10 - System 1](image1)

![Figure 3.9b: Reformer flow constant (kg/s/Pa) = 1e-1/20 - System 1](image2)
As a result of the increased reformer flow constant while using the current regulation method, destabilization is clear in Figures 3.9b and 3.9c. However, the use of the observer has proved useful in stabilizing the system during transient input conditions. Also, Figure 3.9a demonstrates an improvement in load following capabilities over the current regulation control. These simulations suggest that there are benefits to implementing observer-based estimation on systems with significant delays affecting the fuel path.

### 3.3.4 Hydrogen Injection

For System 1, the hydrogen injection method was simulated as depicted in Figure 3.1 with target $U_{ss} = 85\%$, $i_{f,d} = 10A$ for $t < 200s$ and 20, 50A for $t \geq 200s$. These results are then compared with the existing current regulation method given in Section 2.6.

![Figure 3.9c: Reformers flow constant (kg/s/Pa) = 1e-1/30 - System 1](image)

Figures 3.10a and 3.10b demonstrate the effectiveness of the direct hydrogen injection setup for improving transient load following. Plots (a) and (d) clearly illustrate
the near perfect tracking of $i_{fc,d}$ as compared to the current regulation during 10A and 40A step changes. The auxiliary hydrogen flow rate that allows for this improvement is shown in plots (c) and (f), as well as the existing $\dot{N}_f$. However, while the improvements in load following are significant, fuel utilization is fluctuating considerably in plots (b) and (e) from the hydrogen injection. This is likely due to the delays and dynamics of the reformer, fuel cell stack and the hydrogen injection system itself. Ultimately, this method is effective at drastically improving performance at the expense of efficiency.

### 3.3.5 Observer Based Current Regulation with $H_2$ Injection

As mentioned in the previous section, large fluctuations in $U$ during hydrogen injection are caused by delays due to the reformer, stack and the hydrogen injection system. By implementing the observer from Section 3.1.1, these fluctuations may be attenuated during transient loading conditions while the hydrogen injection system is in use.

Figure 3.11 shows the schematic of the observer with hydrogen injection system. Similar to the system described in Section 3.2, $H_2$ is contained in a storage tank and is then injected directly into the anode when a difference in demanded fuel flow to actual fuel flow is measured. Whereas measurements previously could only be taken upstream of the reformer, the observer can be used to estimate the species concentrations of the fuel entering the anode. In doing so, the hydrogen injection can compensate for both delays $D_1$ and $D_2$. 

![Figure 3.10b: Hydrogen Injection](image-url)
By using information about $\dot{N}_{in}$ to estimate the species concentrations in the fuel downstream of the reformer, the injection system will provide hydrogen based on the following:

$$\dot{N}_{H_2} = 4(\dot{N}_{f,d} - \dot{N}_{in}\hat{\zeta}_r)$$  \hspace{1cm} (3.16)

Where $\dot{N}_{f,d}$ is the calculated demand fuel flow rate, $\dot{N}_{in}$ is the flow rate downstream of the reformer and $\hat{\zeta}_r$ is the estimate of hydrogen entering the anode. As compared to Equation 3.9, the use of the observer will be able to compensate for delays due to the fuel supply system and the reformer. This strategy should therefore reduce the fluctuations in $U$ during the injection process.

In Figure 3.12, target $U_{ss} = 85\%$, $i_{f,c,d} = 10\text{A}$ for $t < 200\text{s}$ and $50\text{A}$ for $t \geq 200\text{s}$. The results from this hydrogen injection using an observer are then compared with the original hydrogen injection results.

It is apparent from plot (b) that the observer improved the transient control of fuel utilization, limiting fluctuations to about $\pm 10\%$. Additionally, plot (a) demonstrates that the load following of the system became less responsive with the implementation of the observer. This is due in part to the fact that $\dot{N}_{in}\hat{\zeta}_r$ is used to calculate $\dot{N}_{H_2}$ while using the observer, which introduces some delay from to the estimation of $\zeta_r$. While the use of the observer with hydrogen injection reduces load following relative
Figure 3.12: Hydrogen Injection

to the original hydrogen injection scheme, improvements are apparent over the current regulation method.
Chapter 4

Hybrid System

In Section 2.6, a method to regulate the fuel cell current was introduced. This regulation not only ensures $U$ remains close to a safe operating point, but also protects the fuel cell from potentially harmful power fluctuations. While this method is effective in maintaining safe fuel cell operating conditions, it reduces the load following capabilities of the system. To compensate for power that has been limited by the current regulation strategy, a faster responding energy storage device has been added to the system. The following chapter will introduce this hybrid system setup and propose a general formulation that will allow for the limitations of existing controllers to be evaluated.

4.1 SOFC/Ultra-capacitor System

For the hybrid system, an ultra-capacitor was used to compensate for the regulated fuel cell current. While ultra-capacitors have low energy storage density, they offer a high power density and fast response [47, 48]. By pairing this device with the higher energy density characteristics of a fuel cell, this hybridization will simultaneously deliver the advantages of high energy density and high power density.

The hybrid system schematic is shown in Figure 4.1. Both the fuel cell system and ultra-capacitor are connected to the electrical bus through DC/DC converters $C_1$ and $C_2$. $C_1$, connected to the fuel cell, is a unidirectional DC/DC converter which
Figure 4.1: Hybrid Fuel Cell System [4]
holds bus voltage $V_L$ to 24V. $C_2$ is a bidirectional DC/DC converter which controls the ultra-capacitor current $i_{uc}$. The efficiencies of the converters, represented as $\eta_1$ and $\eta_2$, vary with operating conditions and are treated as unknown but bounded. The ultra-capacitor current $i_{uc}$ and fuel demand $\dot{N}_{f,d}$ are treated as control inputs for the system. Ultra-capacitor current $i_{uc}$ is controlled using the DC/DC converter $C_2$ and the fuel demand $\dot{N}_{f,d}$ is calculated using Equation 2.10a. This equation shows that we can equivalently consider $i_{fc,d}$ or $\dot{N}_{f,d}$ to be the control input. It is also assumed that measurements of the fuel cell voltage $V_{fc}$, ultra-capacitor voltage $V_{uc}$, load current $i_L$, fuel cell current $i_{fc}$ and actual fuel flow $\dot{N}_{f}$ are available.

From the system schematic, the instantaneous power balance is expressed as [4]:

$$V_L i_L = \eta_1 V_{fc} i_{fc} + \eta_2 V_{uc} i_{uc} \quad (4.1)$$

In this hybrid configuration, the current regulation method limits the amount of power drawn from the fuel cell based on the measured value of $\dot{N}_{f}$. Simultaneously, the ultra-capacitor discharges to compensate for the current being limited by the fuel cell. Once the ultra-capacitor begins discharging, the fuel cell adjusts its power level to regain the ultra-capacitor’s desired SOC. In doing so, over-charging or over-discharging can be avoided. The following section will develop the equations for the hybrid system and propose a cascaded system arrangement for this setup.

### 4.2 SOFC/UC as a Cascaded System

As covered in Section 4.1, it is necessary to maintain the ultra-capacitor’s SOC at a target value to avoid over-charging and over-discharging. Therefore, the development of the system state equations begins by considering error between the ultra-capacitor’s SOC and the target SOC. This can be represented as

$$E_s = S - S_t, \quad S = \frac{V_{uc}}{V_{max}} \quad (4.2)$$
where \( S \) is the ultra-capacitor SOC, \( S_t \) is the target SOC, \( V_{uc} \) is the ultra-capacitor voltage and \( V_{max} \) is the maximum ultra-capacitor voltage. Taking the derivative of Equation 4.2 and using the fundamental equation of a capacitor yields

\[
\dot{V}_{uc} = \frac{-i_{uc}}{C} \quad \Rightarrow \quad \dot{E}_s = -\frac{i_{uc}}{CV_{max}} \tag{4.3}
\]

From this relationship of \( \dot{E}_s \), the hybrid system power balance expression in Equation 4.1 can be used to represent the error in the ultra-capacitor’s SOC as the following:

\[
\dot{E}_s = -\left( \frac{1}{CV_{max}} \right) \left[ \left( \frac{V_L i_L}{\eta_2 V_{uc}} \right) - \left( \frac{\eta_1 V_{fc}}{\eta_2 V_{uc}} \right) i_{fc} \right] \tag{4.4}
\]

However, \( i_{fc,d} \) rather than \( i_{fc} \), is the control input. Additionally, in contrast to the basic current regulation method illustrated in Figure 2.8, hardware limitations of our specific set-up prevents drawing \( i_{fc} \) directly from the fuel cell. Instead, \( i_{fc} \) is indirectly achieved by commanding \( i_{uc} \), which is a control input. This indirect approach of commanding the fuel cell current introduces an intermediate variable \( i_{fc,t} \). This variable can be considered as the target fuel cell current draw as required for current regulation. This leads to the following two error variables:

- The error between the demanded current \( i_{fc,d} \) and the current \( i_{fc,t} \) which is given as

  \[
  E_{fc,t} = i_{fc,t} - i_{fc,d} \tag{4.5}
  \]

- The current \( i_{fc} \) is converged to \( i_{fc,t} \) by making small adjustments to \( i_{uc} \), indirectly prompting the fuel cell to alter its current draw. The error between the targeted current \( i_{fc,t} \) and actual fuel cell current \( i_{fc} \) is the second error variable given as

  \[
  E_{fc} = i_{fc} - i_{fc,t} \tag{4.6}
  \]

Since \( i_{fc,d} \) is currently being calculated as a control input to the system, the following expression for \( i_{fc} \) is used considering Equations 4.5 and 4.6:

\[
i_{fc} = E_{fc} + i_{fc,t} = E_{fc} + E_{fc,t} + i_{fc,d} \tag{4.7}
\]
Equations 4.4 and 4.7 then result in the following expression for $\dot{E}_s$:

$$
\dot{E}_s = - \left( \frac{1}{CV_{max}} \right) \left[ \left( \frac{V_{L,iL}}{\eta_2 V_{uc}} \right) - \left( \frac{\eta_1 V_{fc}}{\eta_2 V_{uc}} \right) (E_{fc} + i_{fc,t}) \right]
$$

(4.8)

Because input $E_{fc,t}$ is affected by the unknown dynamics of the fuel supply system, this system can be interpreted as the cascaded connection between two systems. In the case of this hybrid system, the cascaded connection is between the fuel supply system dynamics and the dynamics $E_s$. This connection is shown in Figure 4.2.

Since the dynamics of the fuel supply system are unknown, certain assumptions about its behavior must be made for further analysis. These assumptions must simplify the analysis process while at the same time allow for general results that remain valid for a variety of potential FSS behaviors. The following section will discuss these assumptions in further detail.

### 4.3 Prior Assumptions about FSS Dynamics

Introduced in Section 2.5, the fuel supply system can be comprised of various pumps, valves and fuel flow controllers. Because of these system components and their associated dynamic behavior, previous controller designs for the hybrid system were done without a mathematical expression for the FSS. Instead, the FSS is only assumed to possess general characteristics. While the ensuing work in this thesis will assume the fuel supply system to be an unknown nonlinearity, this section presents the assumptions made about the FSS in existing controller designs.

In [31], the authors made the following assumptions about the FSS:
The FSS is a closed loop system comprised of pumps, valves and a controller. It delivers $\hat{N}_f$ in response to $\hat{N}_{f,d}$.

The system equations for the FSS are considered to be unknown, where $\dot{N}_f$ is a state.

Fuel flow $\dot{N}_f$ tracks $\dot{N}_{f,d}$ such that

\[
|\dot{N}_f(t)| \leq \beta(|\dot{N}_f(t_0)|, t - t_0) + \gamma \left( \sup_{t_0 \leq \tau \leq t} |\dot{N}_{f,d}(\tau)| \right)
\]

(4.9)

where $\beta$ is a class $\mathcal{KL}$ function and $\gamma$ is a class $\mathcal{K}$ function. Comparison functions $\beta$ and $\gamma$ are defined in [32]. This assumption guarantees ultimate boundedness of $\dot{N}_f$ by the function $\gamma(\sup_{t_0 \leq \tau \leq t} |\dot{N}_{f,d}(\tau)|)$, which is dependent on the magnitude of $\dot{N}_{f,d}(\tau)$ [32].

From this, the following theorem is stated:

**Theorem 1.** If $\dot{N}_f$ satisfies Equation 4.9, then $E_{fl}$, given as

\[
E_{fl} = \dot{N}_f - \dot{N}_{f,d}
\]

satisfies

\[
|E_{fl}(t)| \leq \beta_e(|E_{fl}(t_0)|, t - t_0) + \gamma_e \left( \sup_{t_0 \leq \tau \leq t} |\dot{N}_{f,d}(\tau)| \right)
\]

(4.11)

Proof. Given in the Appendix □

From Equation 2.10a, $E_{fl}$ and $E_{fc,t}$ are related by a known constant $\sigma$ given as

\[
E_{fl} = \sigma E_{fc,t}, \quad \sigma = \frac{N_{cell}}{4nFU_{ss}}[1 - (1 - U_{ss})k]
\]

(4.12)

Resulting in the following assumed FSS behavior

\[
\sigma|E_{fc,t}(t)| \leq \beta_e(\sigma|E_{fc,t}(t_0)|, t - t_0) + \gamma_e \left( \sup_{t_0 \leq \tau \leq t} \sigma|i_{fc,d}(\tau)| \right)
\]

(4.13)

In [3], the author assumes the following behavior where $E_{fl}$ is given in Equation 4.10:

\[
|E_{fl}(t)| \leq \gamma|E_{fl}(t_0)|e^{-\alpha t}, \quad \gamma, \alpha > 0, \quad \forall t > 0
\]

(4.14)
This assumed behavior can similarly be related to $E_{fc,t}$ using the relationship given in Equation 4.12 resulting in

$$|E_{fc,t}(t)| \leq \bar{\gamma}|E_{fc,t}(t_0)|e^{-\alpha t}, \quad \bar{\gamma}, \alpha > 0, \quad \forall t > 0 \quad (4.15)$$

This assumption implies exponentially convergence to zero of the FSS. These assumptions were made to describe the unknown dynamic behavior of the FSS in order to cover a range of possible FSS dynamic behaviors such as first order, ramped and rate limited responses. This thesis, however, will not make these assumptions and only assume an unknown nonlinearity. In the following section, the cascaded system formulated in Section 4.2 will be generalized in order to investigate the effects of the unknown fuel supply system.

### 4.4 Generalized Cascaded System

The cascaded system formulated in Section 4.2 consists of the unknown FSS dynamics acting as a driver system and Equation 4.8 acting as the driven system. Considering this arrangement shown in Figure 4.2 and from Equation 4.8, the proposed generalized cascaded system is illustrated in Figure 4.3, where

![Diagram of cascaded system](image)

Figure 4.3: Cascaded system in the general form

$$i_{ic,t} \equiv y, \quad i_{fc,d} \equiv r, \quad x \equiv E_s, \quad \dot{x} = h_1 + h_2 y \quad (4.16)$$

and

$$h_1 = -\frac{1}{\eta_2} \left( \frac{V_L i_L}{C V_{max} V_{uc}} \right), \quad h_2 = \frac{\eta_1}{\eta_2} \left( \frac{V_{fc}}{C V_{max} V_{uc}} \right) \quad (4.17)$$
Note that $f(y, r)$ is the unknown nonlinearity representing FSS. The overall state-space model of the system is therefore given by

$$\dot{y} = f(y, r), \quad \dot{x} = h_1 + h_2y$$

(4.18)

Revisiting Equation 4.8, note that the term $E_{fc}$ is absent in Equation 4.18. $E_{fc} = i_{fc} - i_{fc,t}$ is the error between the actual and targeted fuel cell current. The error occurs due to the hardware limitation of the specific experimental setup that requires commanding the ultra-capacitor current instead of the fuel cell current. Since this is a setup specific error and experiments show that it can be corrected at a significantly smaller time-scale compared to the rest of the control, its effect on the dynamics of $x \equiv E_s$ is neglected. With this, it will be assumed in the following analysis that $E_{fc} \approx 0$. 

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Chapter 5

Stability Analysis without Uncertainties

In the previous chapter, the SOFC/UC hybrid system was introduced and a general form of the equations were proposed as a cascaded system arrangement. This representation will be used in this chapter to explore the stability characteristics of the hybrid system. In this chapter, the quantities $h_1$ and $h_2$, given in Equations 4.17 will be considered known and constant. Using this assumption, the concepts of absolute stability and Lyapunov stability theory will be applied to the general form as a means to evaluate stability limitations. In the forthcoming chapters, the aforementioned assumption that $h_1$ and $h_2$ are known will be relaxed.

5.1 Lur’e System Formulation

Lur’e systems are defined as the feedback interconnection of a linear time invariant system and a nonlinear counterpart. This representation is beneficial because it allows the use of absolute stability concepts toward investigating asymptotic stability in the presence of all nonlinearities in a given sector [32]. This representation is shown in Figure 5.1. Further, it is show that the system dynamics given by Equation 4.18, together with a feedback control law, can be expressed as a Lur’e system with the FSS dynamics being treated as the unknown nonlinearity.
Considering Figure 4.3, the fuel cell’s unknown fuel supply system dynamics are represented as $\dot{y} = f(y, r)$ from Equation 4.17. The dynamics of the state $E_s$, represented by $x$, is given as $\dot{x} = h_1 + h_2 y$ in Equation 4.17. Here, $h_1$ and $h_2$ are treated as known and constant and driver system input $r$ and driven system input $y$ from Figure 4.3 are related by the error equation

$$y = e + r \quad (5.1)$$

Driver system input $r$ is designed using a feedback linearization approach where

$$r = \frac{h_1}{h_2} - kx \quad (5.2)$$

From Equations 4.17, 5.1 and 5.2, the closed loop system equations is represented as

$$\dot{x} = -h_2 k x + h_2 e \quad (5.3)$$

Considering Equation 5.1 and the general FSS equation from Equation 4.17 in Section 4.4,

$$\dot{y} = f(y, r) \rightarrow \dot{e} + \dot{r} = f(e + r, r) \quad (5.4)$$

From Equations 5.3 and 5.4, and denoting $E = \frac{\dot{r}}{\dot{x}} \left( \frac{h_1}{h_2} \right)$, the error state equation can be represented as

$$\dot{e} = f(e + r, r) - \dot{r}$$
$$= f(e + r, r) - E + k \dot{x}$$
$$= f(e + r, r) + k h_2 e - E - h_2 k^2 x \quad (5.5)$$
Since $h_1$ and $h_2$ are considered known and constant, $\mathcal{E} = 0$. This results in the closed loop system state equations below:

$$
\dot{x} = -h_2 k x + h_2 e
$$

$$
\dot{e} = f(e + r, r) + k h_2 e - h_2 k^2 x
$$

where $f(e + r, r)$ represents the unknown nonlinearity, consider the following coordinate transform:

$$
z = e - k x
$$

then,

$$
\dot{z} = f(e + r, r)
$$

$$
\dot{e} = f(e + r, r) + k h_2 z
$$

Given this closed loop state-space model in Equation 5.8, the remaining work will consider the case where unknown nonlinearity $f(e + r, r) \equiv f(e)$. While this is a simplifying assumption, it is not considerably restrictive due to the nature of the general formulation of the FSS dynamics proposed in Equation 4.17. Physically, the only implication of this assumption is that the general FSS dynamics $\dot{e} = f(e + r, r)$ induces no steady state error. Considering this the case where $\dot{e} = e = 0$, the resulting expression $0 = f(r)$ is independent of nonlinearity $f(e + r, r)$ at steady state. Letting $u = -\psi$ where the nonlinearity $\psi = -f(e)$ from Figure 5.1, the state-space representation of the linear system in Equation 5.8 can be given as

$$
\dot{X} = AX + Bu, \quad X = \begin{bmatrix} z \\ e \end{bmatrix}, \quad A = \begin{bmatrix} 0 & 0 \\ kh_2 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}
$$

with output $y = CX + D$ where $C = [0 \quad 1]$ and $D = 0$. Shown in Figure 5.1 as $G(s)$, the linear system can be constructed from the state-space model in Equation 5.9. Considering the formula for the transfer function of a linear system $G(s) = C(sI - A)^{-1}B + D$, the resulting transfer function is given as

$$
G(s) = \frac{s + kh_2}{s^2}
$$
From the Lur’e system schematic given in Figure 5.1, the generalized system proposed in the state space model from Equation 5.9 can be arranged in this format. Using transfer function $G(s)$ from Equation 5.10 and the previously defined $\psi = -f(e)$, the generalized closed loop system is illustrated in Figure 5.2 in a Lur’e system configuration.

![Figure 5.2: Generalized hybrid system as a Lur’e system](image)

### 5.1.1 Absolute Stability Analysis

Using the formulation given in Figure 5.2, concepts of absolute stability can be used to determine what restrictions must apply to the fuel supply system dynamics to ensure stability. This is done by considering not a given nonlinearity, but an entire class of nonlinearities. Such a class is defined as a group of nonlinearities that meet a given sector condition. If the system can be shown to be uniformly asymptotically stable for all nonlinearities in a given sector, the system is said to be absolutely stable [32]. The ensuing analysis will apply the Circle Criterion [32] to guarantee absolute stability of the closed loop system for nonlinearities in a sector, $[K_1, \infty]$.

First testing the case where $K_1 = 0$ such that nonlinearity $\psi \in [0, \infty]$, the Circle Criterion states that if transfer function $G(s)$ is strictly positive real for the given sector condition $\psi \in [0, \infty]$ then the system is absolutely stable. From Definition 6.4, given in [32], transfer function $G(s)$ can be shown to be strictly positive real if
$G(s - \epsilon)$ is positive real for some $\epsilon > 0$. Using the generalized transfer function $G(s)$ given in Equation 5.10, Definition 6.4 is used to test if $G(s - \epsilon)$ is positive real.

$$G(s - \epsilon) = \frac{s - \epsilon + kh_2}{(s - \epsilon)^2}$$

(5.11)

Resulting in poles

$$s_{1,2} = \epsilon > 0$$

(5.12)

Since the poles of $G(s)$ are $Re[s] > 0$ for some $\epsilon > 0$, $G(s)$ is not strictly positive real. This means that the system cannot be shown to be absolutely stable for nonlinearities in sector $[0, \infty]$. Therefore, the Circle Criterion must be used to perform a loop transformation that will impose a more restrictive sector condition on the nonlinearity, limiting the possible dynamic behavior of the nonlinearity $\psi$. This sector condition is given as $\psi \in [K_1, \infty]$, where $K_1 > 0$ and transfer function $\hat{G}(s) = G(s)[I + K_1G(s)]^{-1}$.

Applying this loop transformation to $G(s)$ results in the following transformed transfer function $\hat{G}(s)$:

$$\hat{G}(s) = G(s)[I + K_1G(s)]^{-1} = \frac{s + kh_2}{s^2 + K_1s + K_1Kh_2}$$

(5.13)

With this transformed transfer function, the Circle Criterion is used to prove absolute stability for all nonlinearities in the sector $\psi \in [K_1, \infty]$ where $K_1 > 0$. Again using Definition 6.4, $\hat{G}(s)$ is strictly positive real provided that

- **Poles of all elements of $\hat{G}(s - \epsilon)$ are $Re[s] \leq 0$:**

Let $\bar{s} = s - \epsilon$ such that

$$\hat{G}(\bar{s}) = \frac{\bar{s} + kh_2}{\bar{s}^2 + K_1\bar{s} + K_1kh_2}$$

(5.14)

From Equation 5.14, the characteristic equation $\bar{s}^2 + K_1\bar{s} + K_1kh_2 = 0$ leads to roots $s_1$ and $s_2$ where

$$s_{1,2} = \frac{-K_1 \pm \sqrt{K_1^2 - 4K_1kh_2}}{2} + \epsilon$$

(5.15)

Implying that roots $s_1$ and $s_2$ have negative real parts for infinitesimally small $\epsilon > 0$ and for $kh_2 > 0$. 49
\[ \hat{G}(j\omega - \epsilon) + \hat{G}(-j\omega - \epsilon) \text{ is positive semidefinite} \]

\[ \hat{G}(j\omega - \epsilon) + \hat{G}(-j\omega - \epsilon) = \frac{j\omega - \epsilon + kh_2}{(j\omega - \epsilon)^2 + K_1(j\omega - \epsilon) + kK_1 h_2} + \frac{-j\omega - \epsilon + kh_2}{(-j\omega - \epsilon)^2 + K_1(-j\omega - \epsilon) + kK_1 h_2} = (2K_1 - 2\epsilon - kh_2)\omega^2 + (2K_1 + kh_2 - 2\epsilon)\epsilon^2 + 2(k^2 K_1 h_2^2 - \epsilon k K_1 h_2) \]

Assuming \( 1 >> \epsilon > 0 \), \( \hat{G}(s) \) is strictly positive real when

\[ (2K_1 - kh_2)\omega^2 + 2(k^2 K_1 h_2^2) > 0 \quad \forall \quad K_1 > \frac{kh_2}{2} \quad (5.16) \]

The result of the above analysis imposes two conditions on the closed loop system which must be met in order to guarantee absolute stability. The first condition given in Equation 5.15, states that in order for roots \( s_1 \) and \( s_2 \) of transfer function \( \hat{G}(s) \) to have negative real parts \( K_1 > 0 \) and \( kh_2 > 0 \). Since \( \epsilon \) can be an infinitely small positive number, this condition is not restrictive as long as \( (-K_1 \pm \sqrt{K_1^2 - 4K_1 kh_2})/2 > \epsilon > 0 \).

The second condition given in Equation 5.16, is more restrictive as compared to Equation 5.15. This gives a relationship between the sector condition \( K_1 \) placed on the unknown nonlinearity and feedback gain \( k \) and \( h_2 \) defined in Equation 4.17. Given this result, feedback gain \( k \) can be designed based on the sector condition \( [K_1, \infty] \) according to the type of nonlinearity of the FSS such that \( K_1 > kh_2/2 > 0 \). The following simulations will be used to test sample general FSS behaviors \( f(\epsilon) \) that meet the sector condition \( [K_1, \infty] \) and also behaviors that violate this condition.

### 5.1.2 Simulation Results

In the previous section, the Circle Criterion was used to analytically concluded that the system is asymptotically stable if nonlinearity \( \psi = -f(\epsilon) \) satisfied a sector condition such that \( \psi \in [K_1, \infty] \). Using Equation 5.16, a condition can be calculated based feedback gain \( k \) and parameter \( h_2 \). For these simulations, \( h_2 \) is considered a known and constant parameter whose functional form is given in Equation 4.17. Using the
resulting relationship from Equation 5.16 the hybrid system should remain stable for all nonlinearities in the given sector when \( K_1 > \frac{kh_2}{2} > 0 \). The following will present simulation results to verify these findings.

![Phase Portrait](figure5.3.png)

**Figure 5.3:** Absolute stability simulation case 1: \(-f(e) \in [K_1, \infty]\)

For the first case, the sector condition criteria was verified by simulating a simple function \( \psi = -f(e) = ae \) for the general fuel supply system dynamic equation. Using the closed loop system from Equation 5.8, the phase portrait is shown in Figure 5.3 using \( kh_2 = 1 \) and \( f(e + r, r) \triangleq f(e) \) where \( f(e) = -ae \). The closed loop system for this simulation takes the form

\[
\begin{align*}
\dot{z} &= -ae \\
\dot{e} &= -ae + z
\end{align*}
\]

Using Equation 5.16, the sector condition is required to meet the condition \( K_1 > 1/2 \) due to the chosen value of \( kh_2 = 1 \). Therefore, \( a = 2 \) was chosen such that \( \psi = -f(e) \) met the given sector condition. The results of this simulation verified the findings from the implementation of the *Circle Criterion* because the system remains stable when function \( \psi \) meets the sector condition.

Shown in Figure 5.4 is the phase portrait of the same system when the sector condition is not met. As in the previous simulation, the sector condition requirement is \( K_1 > 1/2 \) from Equation 5.16 and setting \( kh_2 = 1 \). In order to purposefully violate this condition, function \( f(e) = -ae \) was chosen such that \( a = 0.2 \). This results in a
function $\psi$ that does not lie within the sector condition $\psi = -f(e) \in [K_1, \infty]$. From the simulation, it is clear that although the sector condition is not met the system remains stable for an array of initial conditions. As compared to Figure 5.3, the states appear to be considerably more oscillatory but still converge to the origin.

![Phase Portrait](image1)

Figure 5.4: Absolute stability case 2: $-f(e) \notin [K_1, \infty]$

From these simulations, it can be concluded that absolute stability can be used to ensure stability of this generalized system for a class of nonlinearities. However, these results prove to be conservative due to the fact that it is possible for the system to remain stable when the sector condition is not met as in Figure 5.4. The following section will explore a Lyapunov based stability approach for the general cascaded system.

### 5.2 Lyapunov Stability Analysis

In the previous section, the generalized closed loop system from Equation 5.8 was posed as a Lur'e system in which absolute stability concepts could be used to guarantee stability for a class of unknown nonlinearities. As a result, a requirement for sector condition $K_1$ was derived in Equation 5.16 that could ensure system stability. However, based on simulation results given in Figure 5.4, it was observed that when these conditions were violated the system could potentially remain stable. This led
to the conclusion that the absolute stability result was conservative. In an effort to improve the estimation of the ranges of nonlinearities that will guarantee stability, the following section will apply a traditional Lyapunov stability analysis.

Considering the generalized system given in Equation 5.8, the following analysis will consider the unknown FSS behavior proposed in Section 4.4 to be of the form $\dot{y} = -f(y, r)$. Considering $y = e + r$,

$$\dot{y} = -f(y, r) \Rightarrow \dot{e} + \dot{r} = -f(e + r, r) \quad (5.17)$$

The previous assumption made about the FSS was that $f(e + r, r) = f(e)$ and similarly it will be assumed that $-f(e + r, r) = -f(e)$ in the forthcoming analysis. With this, the previous general closed loop equation given in Equation 5.8 will become

$$\dot{z} = -f(e)$$
$$\dot{e} = -f(e) + kh_2 z \quad (5.18)$$

Using Equation 5.18, a single second order expression is derived such that,

$$\ddot{e} + \frac{df}{de} \dot{e} + kh_2 f(e) = 0 \quad (5.19)$$

where $\dot{e}$ and $e$ are considered the states. Since this equation resembles the form of a generalized nonlinear mass-spring-damper system, the Lyapunov function candidate is chosen as the energy storage function. This is shown in the following:

$$V = \frac{1}{2} \dot{e}^2 + \int_0^e kh_2 f(\kappa) d\kappa \quad (5.20)$$

Taking the derivative of Equation 5.20 along the state trajectories and using 5.19 results in the following:

$$\dot{V} = \dot{\dot{e}}(-\frac{df}{de} \dot{e} - kh_2 f(e)) + kh_2 f(e) \dot{e}$$
$$= -\frac{df}{de} \dot{e}^2 \leq 0 \quad (5.21)$$

Since this expression for $\dot{V}$ can only be concluded to be negative semidefinite, the invariance principle is used [32]. Considering Equation 5.21,

$$\dot{V} = 0 \Rightarrow \dot{e} = 0 \Rightarrow \ddot{e} = 0 \quad (5.22)$$
From Equation 5.19, \( f(e) = 0 \Rightarrow e = 0 \). Therefore, in can be concluded that Equation 5.21 is negative definite and hence the system is asymptotically stable provided that
\[
\frac{df}{de} > 0 \forall e
\]
This result states the system will be stable for all nonlinearities \( f(e) \) where \( \frac{df}{de} > 0 \). As compared to the result from Equation 5.16, this condition is independent of the system characteristics \( k \) and \( h_2 \). The following section will verify these findings using desktop simulations.

5.2.1 Simulation Results

In order to verify the result from the previous section stating that the closed loop system is stable for \( \frac{df}{de} > 0 \), the generalized closed loop system from Equation 5.8 was simulated. The simulation conditions were set such that \( kh_2 = 1 \), and initial conditions \( e(0) = -20 : 5 : 20 \) and \( z(0) = -20 : 5 : 20 \). With this array of initial conditions, multiple state trajectories can be examined for a variety of FSS functions \( f(e) \).

![Phase Portrait](image)

Figure 5.5: Phase portrait with a positive \( df/de \)

Figure 5.5 provides results when \( df/de > 0 \forall e \), hence meeting the asymptotic stability criteria from Equation 5.23. In this case, all state trajectories converge to the
origin which agrees with the analytical result. In contrast to the previous plot, Figure 5.6 demonstrates a FSS behavior that is $df/de < 0 \forall e$. Because of this violation, all state trajectories diverge from the origin which results in instability. This also agrees with the analytical result in Equation 5.23 from the previous section.

Since fuel supply system dynamics may exhibit nonlinear behavior and their slopes may vary significantly with respect to $e$, behaviors that increase and decrease or saturate are examined. In Figure 5.7, the FSS behavior acts in such a way that $df/de > 0$ if $-13 < e < 13$, else $df/de < 0$. From the results, it is clear that the system converges to the origin for all $-5 \leq z(0) \leq 5$ even though the slope of $f(e)$ is
not positive for any $e$. In a similar manner, Figure 5.8 demonstrates a FSS behavior where $df/de > 0$ if $-15 < e < 15$ and $df/de = 0$ if $-15 > e$ or $e < 15$. This behavior results in the state trajectories converging to the origin when $-5 \leq z(0) \leq 5$ and $-10 > z(0) > 10$.

From these simulations, it can be concluded that when the FSS behavior meets the condition $df/de > 0 \ \forall \ e$ the system will converge to the origin. These results support the analytical work and verify the closed loop stable can be guaranteed to remain stable when the FSS behavior is met. However it is apparent that the simulations given in figures 5.7 and 5.8 do in fact remain stable under certain initial conditions. Since the FSS behavior does not meet the condition derived, it can be concluded that this result is conservative. The following work will continue to investigate the effects of the FSS behavior during uncertainties in the system.
Chapter 6

Stability Analysis with Unknown System Parameters

In the previous analysis, $h_1$ and $h_2$ given in Equation 4.17 have been treated as known and constant. Because of this, a feedback linearization approach could be adopted. From there, absolute stability and Lyapunov stability concepts were utilized to determine restrictions on the fuel supply system’s dynamic behavior. However, since $h_1$ and $h_2$ are unknown and variable in the physical system, robustness must be incorporated in the control design.

Previous work on this hybrid system focused specifically on two robust controllers, [30] and [31]. The first of these used a Lyapunov redesign approach which made use of a robustness switching term based on the known bounds of the converter efficiencies. The use of this switching term, from an analytical perspective, would be problematic in taking the derivative of the control reference signal. Because of this, the second controller was used in the following analytical work. This controller uses an adaptive control law to estimate the unknown parameters which, as will be shown, results in a form of integral control for the closed loop system. This method results in smoother estimations and lends itself well to further analysis. The following sections will formulate the generalized adaptive controller in order to determine limitations of the existing controller. Desktop and HIL simulations will be given to verify these findings. Further information regarding the these existing controller can be found in

57
and [31].

6.1 Generalized Adaptive Control Strategy

This analysis will begin by considering the generalized hybrid system equations presented in Section 5.1, Equations 4.18, 5.1 and 5.17 yield the following system equations:

\[
\begin{align*}
\dot{x} &= h_1 + h_2(e + r) \\
\dot{e} &= -f(e + r, r) - \dot{r}
\end{align*}
\] (6.1)

As stated in Section 5.2, the fuel supply system behavior will be assumed to follow the general behavior \(-f(e + r, r) = -f(e)\) and \(h_1\) and \(h_2\) given in Equation 4.17 are either constant or slowly varying, unknown and bounded quantities. Control input \(r\) shown in the following equation is designed to cancel out the effects of \(h_1\) in Equation 6.1 and stabilize error in the ultra-capacitor SOC \(E_s = x\).

\[
r = -\frac{\dot{h}_1}{h_2} - kx
\] (6.2)

Since \(h_1\) and \(h_2\) are unknown, estimates \(\hat{h}_1\) and \(\hat{h}_2\) of \(h_1\) and \(h_2\) respectively are used in the control law. Implementing the control design from Equation 6.2 on the generalized system equation from 6.1, the closed loop system equations are presented as in the following where parameter error \(e_{12} = \frac{h_1}{h_2} - \frac{\dot{h}_1}{h_2}\) and \(\mathcal{E} = \frac{d}{dt} \left( h_1 \right) / h_2\),

\[
\begin{align*}
\dot{x} &= h_2 e_{12} + h_2(e - kx) \\
\dot{e} &= -f(e) + k h_2 e_{12} + k h_2(e - kx) + \mathcal{E}
\end{align*}
\] (6.3)

Using the previous coordinate transform \(z = e - kx\) from Equation 5.7, Equation 6.3 becomes

\[
\begin{align*}
\dot{z} &= -f(e) + \mathcal{E} \\
\dot{e} &= -f(e) + \mathcal{E} + k h_2 e_{12} + k h_2 z
\end{align*}
\] (6.4)

Since \(h_1\) and \(h_2\) are considered slowly varying parameters, it is assumed that the effects of their rates of change are negligible. This leads to the following derivative of
parameter error $e_{12}$ where
\[ \dot{e}_{12} = -\frac{d}{dt} \left( \frac{\dot{h}_1}{h_2} \right) = -\mathcal{E} \quad (6.5) \]

In order to estimate the parameter error, the proposed adaptation law is
\[ \dot{e}_{12} = -\gamma x \quad (6.6) \]

Considering Equation 6.4, the closed loop system equation is represented as a single second order equation such that
\[ \ddot{e} = -\frac{df}{de} \dot{e} + \dot{\mathcal{E}} + k h_2 \dot{e}_{12} + k h_2 \dot{z} \quad (6.7) \]

Noting that from coordinate transform $z = e - k x$,
\[ \dot{z} = \dot{e} - k \dot{x} \quad (6.8) \]

Equations 6.4, 6.5, and 6.6 lead to the following
\[
\begin{align*}
\ddot{e} &= -\frac{df}{de} \dot{e} + \dot{\mathcal{E}} + k h_2 \dot{e}_{12} + k h_2 \dot{z} \\
&= -\frac{df}{de} \dot{e} + \dot{\mathcal{E}} + k h_2 \dot{e}_{12} + k h_2 \gamma x \\
&= -\left( \frac{df}{de} - \gamma k \right) \dot{e} - (k h_2 - \frac{\gamma}{k}) f(e) - \frac{\gamma^2}{k} x \\
&= -\left( \frac{df}{de} - \frac{\gamma}{k} \right) \dot{e} - (k h_2 - \frac{\gamma}{k}) f(e) - \frac{\gamma^2}{k} x \\
\end{align*}
\]

Considering Equation 6.8, $\dot{x}$ can be represented as
\[
\begin{align*}
\dot{x} &= \frac{1}{k} [\dot{e} - \dot{z}] \\
&= \frac{1}{k} [\dot{e} - f(e) - \gamma x] \\
&= -\frac{\gamma}{k} x + \frac{1}{k} (f(e) - \dot{e}) \\
\end{align*}
\]

Therefore the generalized closed loop system with adaptive control can be represented as the following system equations:
\[
\begin{align*}
\ddot{e} + \left( \frac{df}{de} - \frac{\gamma}{k} \right) \dot{e} + \left( k h_2 - \frac{\gamma}{k} \right) f(e) &= -\frac{\gamma^2}{k} x \\
\dot{x} &= -\frac{\gamma}{k} x + \frac{1}{k} (f(e) - \dot{e}) \\
\end{align*}
\]

Next, the following transform is proposed:
\[ v = \gamma x - \dot{e} - f(e) \quad (6.12) \]
Applying this transformation to Equation 6.11 yields
\[
\ddot{e} + \frac{df}{de} \dot{e} + kh_2 f(e) = -\frac{\gamma}{k}(\gamma x - \dot{e} - f(e)) \\
\dot{x} = -\frac{1}{k}(\gamma x - \dot{e} - f(e))
\] (6.13)
such that
\[
\ddot{e} + \frac{df}{dx} \dot{e} + kh_2 f(e) = -\frac{\gamma}{k}v \\
\dot{x} = -\frac{1}{k}v
\] (6.14)
Solving for \(\dot{v}\) from Equation 6.12 results in
\[
\dot{v} = \gamma \dot{x} - \ddot{e} - \frac{df}{dx} \dot{e} \\
= -\frac{\gamma^2}{k} x + \frac{\gamma}{k}(\dot{e} + f(e)) + \frac{\gamma}{k}v + \frac{df}{dx} \dot{e} - \frac{df}{dx} \dot{e} + kh_2 f(e) \\
= -\frac{\gamma}{k}(\gamma x - \dot{e} - f(e)) + \frac{\gamma}{k}v + kh_2 f(e) \\
= kh_2 f(e)
\] (6.15)
From Equation 6.14 and 6.15, the system equations as a results of the coordinate transform from Equation 6.12 result in
\[
\ddot{e} + \frac{df}{dx} \dot{e} + kh_2 f(e) = -\frac{\gamma}{k}v \\
\dot{v} = kh_2 f(e)
\] (6.16)
It is important to note that the input of the second order equation above is the integral of a function of the error in the fuel supply system, \(kh_2 f(e)\). As stated before, the use of this adaptation law results in a form of integral control on the closed loop system.
The following section will provide results regarding the limitations of this approach.

6.2 Stability Analysis of Adaptive Control

Considering the generalized closed loop system with adaptive control from Equation 6.16, the state space representation is formulated where \(x_1 = e\), \(x_2 = \dot{e}\) and \(x_3 = v\). The state space form is given as,
\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3
\end{bmatrix} =
\begin{bmatrix}
x_2 \\
-kh_2 f(x_1) - \frac{df}{dx_1} x_2 - \frac{\gamma}{k} x_3 \\
kh_2 f(x_1)
\end{bmatrix}
\] (6.17)
In order to analytically examine limitations of the adaptive controller with unknown fuel supply system behavior, this system is linearized about the origin. This approximation will characterize the dynamics of the FSS in a manner that is conducive to further analysis. The linear approximation of the FSS behavior will be as follows:

\[ f(x_1) = ax_1 \]  

(6.18)

Where \( f(x_1) \) will be assumed to be a linear function with slope \( a \). Forming linear system matrix \( A_{lin} \), the Jacobian matrix of the closed loop system at \( x_1 = x_2 = x_3 = 0 \) is formulated. Using Equations 6.17 and 6.18 leads to:

\[
A_{lin} = \begin{bmatrix}
0 & 1 & 0 \\
-kh_2a & -a & -\frac{\gamma}{k} \\
kh_2a & 0 & 0
\end{bmatrix}
\]  

(6.19)

From Equation 6.19 the characteristic equation is derived using the property \( 0 = \text{det}(sI - A_{lin}) \), resulting in

\[ 0 = s^3 + as^2 + kh_2as + h_2a\gamma \]  

(6.20)

Using this third order characteristic equation, *Routh’s Stability Criterion* from [49] can be applied. It can be shown that if \( a_1a_2 > a_0a_3 \) and \( a_n > 0 \) when the characteristic equation is of the form \( a_3s^3 + a_2s^2 + a_1s + a_0s^0 = 0 \) all roots will have negative real parts. Since *Routh’s Stability Criterion* offers a necessary and sufficient condition that all roots will lie in the left half plane, if it is not met the system will potentially face instability. From this criterion and Equation 6.20, the following relationship is established:

\[ ka > \gamma \]  

(6.21)

Using this stability result, the feedback gain \( k \) and adaptive parameter gain \( \gamma \) can be designed to ensure stability given a slope \( a \) representing the approximation of the fuel supply system about the origin.
6.3 Simulation Results

In order to verify the results from the previous section, basic simulations were run of the closed loop system with adaptive control given in Equation 6.16. For all simulations, initial conditions were set as $\dot{e}(0) = 10$, $e(0) = 10$, $v(0) = 5$, $h_2 = 1$ and $ka = 1$.

![Simulation Results](image)

Figure 6.1: Generalized adaptive controller simulation results

Plots (a), (b) and (c) are the results of $\dot{e}$, $e$ and $v$, respectively. The system was simulated with $\dot{e}(0) = 10$, $e(0) = 10$ and $v(0) = 5$ when $k = 1$, $a = 1$ and $\gamma = 0.9$. With these conditions, the stability criterion is met and simulations confirm that the system remains stable. Likewise, plots (d), (e) and (f) depict the simulation results when $\gamma = 1$ such that the condition is not met. It is apparent that the system is
marginally stable, due to the fact that the roots of the characteristic equation cannot be guaranteed to have positive or negative real parts. Finally, plots (g), (h) and (i) show simulation results when $\gamma = 1.1$ thus violating the Routh stability result. The simulations confirm that the system states become unstable due to the fact that the roots of the system lie in the right half plane.

It is clear from these results that the use of the Routh’s Stability Criterion is useful in determining the limits of the adaptive controller gain given a feedback gain and fuel supply system approximation. The following chapter will introduce the HIL system and the adaptive controller used on the system. This general stability result from section 6.2 will then be mapped to the existing controller in such a way that the results will be applicable to the hybrid system. Desktop and HIL simulation results will be given.
Chapter 7

Hybrid System Results

Section 6.1 proposed a stability criterion for the closed loop system with adaptive control, involving the feedback gain $k$, FSS dynamics characterized by slope $a$ around $e = 0$, and adaptation gain $\gamma$. This relationship will be tested on the existing adaptive controller developed in [31]. Results for both desktop and HIL simulations will be provided.

7.1 Existing Adaptive Controller

In the SOFC/UC hybrid system, the fuel cell is the primary power source with the ultra-capacitor compensating for power lost due to the current regulation method during transient loading conditions. However, as covered in Section 4.1, hardware limitations prevent the fuel cell from being directly controlled. In order to indirectly control the fuel cell, fuel cell current demanded $i_{fc,d}$ is designed in such a way that prompts changes in fuel flow $\dot{N}_{fc,d}$ to the fuel cell based on external power demands and error in the ultra-capacitors SOC. Using the power balance equation from Equation 4.1, $i_{fc,d}$ is designed to provide all the load demand power. Since realistically transient loads will lead to a mismatch in power production and power demanded, a term to correct error in the SOC $E_s$ of the ultra-capacitor is used to indirectly command the
fuel cell to alter its power output. This is shown in the following design of $i_{fc,d}$,

$$i_{fc,d} = \frac{V_L i_L}{\eta_1 V_{fc}} - k_s E_s, \quad E_s \triangleq S - S_t, \quad S = \frac{V_{uc}}{V_{max}}, \quad k_s > 0 \quad (7.1)$$

where $S$ is the ultra-capacitor’s state of charge, $S_t$ is the desired SOC, $V_{uc}$ is the ultra-capacitor voltage and $V_{max}$ is the ultra-capacitor’s maximum voltage. Additionally, $i_{fc,d}$ is an algebraic function of the control input $\dot{N}_{f,d}$. This relationship is given as Equation 2.10a and is designed to satisfy the target $U_{ss}$. The ultra-capacitor current is designed to provide power when the fuel cell does not meet the power demand $i_L V_L$.

This is done by using the power balance equation given in Equation 4.1 such that,

$$i_{uc} = i_{uc,c} = \frac{(V_L i_L - \eta_1 V_{fc} i_{fc,t})}{(\eta_2 V_{uc})} + h(E_{fc}),$$

$$E_{fc} \triangleq i_{fc} - i_{fc,t},$$

$$h(E_{fc}) = k_p E_{fc} + k_d \dot{E}_{fc}, \quad k_p, k_d > 0 \quad (7.2)$$

where $h(E_{fc})$ is designed to ensure $E_{fc}$ goes to zero in the presence of uncertainties by adjusting the commanded $i_{uc}$. Also, $i_{fc,t}$ is calculated by Equation 2.11a which is based on the desired $U_{ss}$ and measured fuel flow $\dot{N}_f$. Since both $i_{uc}$ and $\dot{N}_{f,d}$ depend on the estimates of the DC/DC converter efficiencies, the following parametric forms of these efficiencies are defined by

$$\beta_1 = \frac{1}{\eta_1}, \quad \beta_2 = \frac{1}{\eta_2}, \quad \beta_{12} = \eta_1 \eta_2 \quad (7.3)$$

with the estimates being represented as

$$\bar{\beta}_1 = \frac{1}{\bar{\eta}_1}, \quad \bar{\beta}_2 = \frac{1}{\bar{\eta}_2}, \quad \bar{\beta}_{12} = \bar{\eta}_1 \bar{\eta}_2 \quad (7.4)$$

Using Equations 7.3 and 7.4, the estimation errors are

$$e_1 \triangleq \beta_1 - \bar{\beta}_1, \quad e_2 \triangleq \beta_2 - \bar{\beta}_2, \quad e_{12} \triangleq \beta_{12} - \bar{\beta}_{12} \quad (7.5)$$

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With this, the parameter adaptation laws are designed as follows:

\[
\dot{\beta}_1 = -\frac{V_L i_L E_s}{CV_{max} V_{uc}} \gamma_1 + g_1 \\
\dot{\beta}_2 = \frac{V_L i_L E_{fc}}{V_{uc}} \gamma_2 + g_2 \\
\dot{\beta}_{12} = -\frac{V_{fc} i_{fc} E_{fc}}{V_{uc}} \gamma_{12} + g_{12}
\]  

(7.6)

\[
\gamma_1, \gamma_2, \gamma_{12} > 0
\]

Where \(\gamma_1, \gamma_2, \text{ and } \gamma_{12}\) are constant parameter adaptation gains. The adaptation laws are designed to maintain boundedness by avoiding adaptation when the estimates are outside the known bounds and are not converging toward the true value. This is achieved by designing \(g_1, g_2, \text{ and } g_{12}\)

\[
g_1 = \begin{cases} 
-d_1 & \text{if } \bar{\beta}_1 \geq \beta_{1,\text{max}} \text{ and } d_1 > 0 \\
& \text{or } \bar{\beta}_1 \leq \beta_{1,\text{min}} \text{ and } d_1 < 0 , \quad d_1 = -\gamma_1 V_L i_L E_s/(CV_{max} V_{uc}) \quad (7.7) \\
0 & \text{otherwise}
\end{cases}
\]

\[
g_2 = \begin{cases} 
-d_2 & \text{if } \bar{\beta}_2 \geq \beta_{2,\text{max}} \text{ and } d_2 > 0 \\
& \text{or } \bar{\beta}_2 \leq \beta_{2,\text{min}} \text{ and } d_2 < 0 , \quad d_2 = \gamma_2 V_L i_L E_{fc}/V_{uc} \quad (7.8) \\
0 & \text{otherwise}
\end{cases}
\]

\[
g_{12} = \begin{cases} 
-d_{12} & \text{if } \bar{\beta}_{12} \geq \beta_{12,\text{max}} \text{ and } d_{12} > 0 \\
& \text{or } \bar{\beta}_{12} \leq \beta_{12,\text{min}} \text{ and } d_{12} < 0 , \quad d_{12} = -\gamma_{12} V_{fc} i_{fc} E_{fc}/V_{uc} \quad (7.9) \\
0 & \text{otherwise}
\end{cases}
\]

where \(\beta_{1,\text{max}}, \beta_{2,\text{max}}, \beta_{12,\text{max}}, \beta_{1,\text{min}}, \beta_{2,\text{min}}, \beta_{12,\text{min}}\) represent the upper and lower bounds of the parameters \(\beta_1, \beta_2, \beta_{12}\) respectively. These bounds are defined as

\[
\beta_{1,\text{min}} = 1/\eta_{1,\text{max}}, \quad \beta_{2,\text{min}} = 1/\eta_{2,\text{max}}, \quad \beta_{12,\text{min}} = \eta_{1,\text{min}}/\eta_{2,\text{max}} \\
\beta_{1,\text{max}} = 1/\eta_{1,\text{min}}, \quad \beta_{2,\text{max}} = 1/\eta_{2,\text{min}}, \quad \beta_{12,\text{max}} = \eta_{1,\text{max}}/\eta_{2,\text{min}}
\]

(7.10)

Using the prior formulation, the adaptive controller schematic is shown in Figure 7.1.
7.2 Mapping the General Results

In order for the general result given in Equation 6.21 to be applicable to the existing controller, the general terms \( k, a \) and \( \gamma \) must be mapped to the actual system parameters. Beginning first with feedback gain \( k, i_{fc,d} \) from Equation 7.1 was ultimately generalized to the variable \( r \) in Equation 6.2 such that

\[
i_{fc,d} = \frac{V_L i_L}{\hat{h}_1 V_{fc}} - k_s E_s \quad \Rightarrow \quad r = -\frac{\hat{h}_1}{\hat{h}_2} - k x
\]  

(7.11)

Since \( \hat{h}_1 = -1/\bar{\eta}_2 (V_L i_L/C V_{max} V_{uc}) \), \( \hat{h}_2 = \tilde{\eta}_1/\bar{\eta}_2 (V_{fc}/C V_{max} V_{uc}) \) and \( x \equiv E_s \), it can be concluded that \( k \equiv k_s \). Likewise, the fuel supply system approximation \( f(e) = -a e \) given in Equation 6.18 is directly related to the assumed fuel supply system dynamics in the SOFC model. In this model, the FSS is assumed to take the form:

\[
\frac{Y(s)}{R(s)} = \frac{1}{\tau s + 1}
\]

(7.12)

where \( \hat{N}_{f,d} \) is the transfer function input and \( \hat{N}_f \) is the output. Noting that from Equations 2.10a and 2.11a \( \hat{N}_{f,d} \) and \( \hat{N}_f \) are algebraically related to \( i_{fc,d} \) and \( i_{fc} \), performing an inverse Laplace transform, Equation 7.12 results in

\[
\dot{y} = -\frac{1}{\tau} (y - r) = -\frac{1}{\tau} e
\]

(7.13)
This form of the FSS dynamics is similar to the assumed linear approximation presented in Equation 6.18. Therefore, there is a direct relationship between the two such that \( a \equiv 1/\tau \). Lastly, in order to determine the actual value of \( \gamma \), the parameter adaptations laws must be revisited. From Equation 7.6

\[
\dot{e}_1 = -\dot{\beta}_1 = \frac{V_L i_L E_s}{CV_{\text{max}} V_{\text{ac}}} \gamma_1 + g_1
\]  (7.14)

where

\[
\dot{e}_1 = \frac{d}{dt} \left( \frac{1}{\eta_1} - \frac{1}{\bar{\eta}_1} \right)
\]  (7.15)

From the general adaptation law in Equation 6.6, the estimated parameter was defined as \( \dot{e}_{12} = -\gamma x \) where

\[
\dot{e}_{12} = \frac{d}{dt} \left( \frac{h_1}{h_2} - \frac{\hat{h}_1}{\hat{h}_2} \right)
\]  (7.16)

Noting that \( \hat{h}_1 = -1/\bar{\eta}_2 (V_L i_L/CV_{\text{max}} V_{\text{ac}}) \) and \( \hat{h}_2 = \bar{\eta}_1/\bar{\eta}_2 (V_{fc}/CV_{\text{max}} V_{\text{ac}}) \), from Equation 7.16

\[
\dot{e}_{12} = - \left( \frac{V_L i_L}{V_{fc}} \right) \frac{d}{dt} \left( \frac{1}{\eta_1} - \frac{1}{\bar{\eta}_1} \right)
\]  (7.17)

In order to find the relationship between generalized \( \gamma \) and actual \( \gamma_1 \), Equation 7.14 must be multiplied by a factor of \(-V_L i_L/V_{fc}\). From equations 6.6, 7.14, and 7.17:

\[
-\frac{(V_L i_L)^2 E_s}{CV_{\text{max}} V_{\text{ac}} V_{fc}} \gamma_1 = -\gamma x
\]  (7.18)

Leaving

\[
\frac{(V_L i_L)^2}{CV_{\text{max}} V_{\text{ac}} V_{fc}} \gamma_1 = \gamma
\]  (7.19)

Therefore, the general stability result being applied to this system would take the form:

\[
ka > \frac{(V_L i_L)^2}{CV_{\text{max}} V_{\text{ac}} V_{fc}} \gamma_1
\]  (7.20)

### 7.3 Hybrid System Desktop Simulations

Using Equation 7.20, desktop simulations can be run to verify these findings. Initially, the MATLAB/Simulink® hybrid system model will be run with \( E_{fc} = 0 \). As
discussed in Section 4.4, this simplifying assumption was made due to its negligible effects on $E_s$. While this assumption was made to simplify the general analysis, it will be reintroduced in the simulations during the ensuing discussion.

The following simulations were run using the tubular SOFC model discussed in Section 2.1 as the main power source. For these simulations, the system was set to $V_L = 24V$, $U_{ss} = 80\%$, target SOC $S_t = 0.8$, ultra-capacitor $C = 250F$ and $V_{max} = 16.2V$. Since $E_{fc} = 0$ the only parameter to be estimated is $\eta_1$ and its initially estimation value is $\bar{\eta}_1 = 0.92$. The FSS is modeled as $\dot{N}_f(s)/\dot{N}_{f,d}(s) = 1/(2s + 1)$ but is treated as unknown. Parameter estimation is switch on when $t \geq 250$ seconds.

![Figure 7.2: Hybrid system simulation with adaptive controller when $iL = 10A$ and $\gamma_1 = 1535$](image)

In Figure 7.2, results are given for a adaptation gain of $\gamma_1 = 1535$, feedback
gain $k = 70$ and $a = 1/2$. From Equation 7.20, this chosen value of $\gamma_1$ results in $\gamma \approx 35$ which should drive the system near marginal stability. It is clear from these simulations that the system is highly oscillatory, but it tends to diverge from the equilibrium values. This phenomena is due in part to the uncertainty in the calculation of $\gamma$. Since Equation 7.20 is a function of $V_{uc}$ and $V_{fc}$, the fluctuation in these values will alter the actual value of $\gamma$. Also, since parameters $h_1$ and $h_2$ The utility of this technique, however, can be shown in the following simulation.

Figure 7.3: Hybrid system simulation with adaptive controller when $iL = 10A$ and $\gamma_1 = 1000$

Figure 7.3 gives simulation results when adaptation gain of $\gamma_1 = 1000$, feedback gain $k = 70$ and $a = 1/2$, resulting in $\gamma \approx 23$. From this scenario it can be shown that while the system is largely oscillatory, it system tends to remain stable for
the given adaptation gain. These results agree with the analysis from the previous section, providing an estimation of the allowable $\gamma_1$ gain given the necessary system parameters.

Figure 7.4: Hybrid system simulation with adaptive controller when $i_L = 20A$ and $\gamma_1 = 344$

Figure 7.4 gives simulation results for a 20A load current. Since $\gamma$ is also dependent on $i_L$ and $V_L$, a $\gamma_1$ value of 344 now results in $\gamma \approx 35$. For $k = 70$ and $a = 1/2$, this theoretically should drive the system to be marginally stable. As in Figure 7.2, the system does appear to go unstable. This behavior can be expected based on the previous simulations. This result does however verify the previous findings and supports the idea that the derived stability result holds true for various load demands. The simulation results from Figures 7.2, 7.3 and 7.4 all demonstrated that Equation
7.20 is useful in approximating the limit of adaptation gain $\gamma$ given the necessary parameter of the system.

Since it was initially assumed that $E_{fc}$ has negligible effects on the dynamics of $E_s$, the following simulations relax this condition and include these effects. The simulations are run with $V_L = 24\text{V}$, $U_{ss} = 80\%$, target SOC $S_t = 0.8$, ultra-capacitor $C = 250F$ and $V_{max} = 16.2\text{V}$. The parameter estimates $\hat{\eta}_1$ and $\hat{\eta}_2$ are initialized at 0.92. The FSS is modeled as $\dot{N}_f(s)/\dot{N}_{fd}(s) = 1/(2s + 1)$ but is treated as unknown to the controller.

Figure 7.5 shows the system response to a constant input of $i_L = 10\text{A}$. This simulation was run with $\gamma_1 = 2000$ when $t < 450$ seconds and $\gamma_1 = 1000$ when $t \geq 450$ seconds. Using Equation 7.20, these values were chosen to test the calculated limit of $\gamma_1 = 1535$. Similarly, Figure 7.6 shows the system response to an input of $i_L = 20\text{A}$. The given simulation was run with $\gamma_1 = 400$ when $t < 450$ seconds and $\gamma_1 = 200$ when $t \geq 450$ seconds. Again, these $\gamma_1$ values were chosen to test the calculated limit of $\gamma_1 = 344$. The controller adaptation is switched on $t = 250s$ in both figures.

In both simulations, it is clear that the system tends to go unstable when $\gamma_1$ is set above the calculated stability threshold. Expectedly, the system begins to stabilize when $t = 450$ seconds and $\gamma_1$ is switched to a gain value below the threshold. Although this method is fairly accurate at predicting stability conditions, the following section will discuss an aspect of this method that induces error.

### 7.3.1 Variability in Calculating $\gamma$

The stability result given in Equation 6.21 gave conditions at which the general closed loop system would be stable or unstable given feedback gain $k$, FSS slope approximation $a$ and adaptation gain $\gamma$. In Section 7.2, a mapping was proposed based on the actual existing adaptive controller and hybrid system which was verified with simulations in Section 7.3. The results were favorable however, error was present predicting
Figure 7.5: Hybrid system simulation with adaptive controller when $i_L = 10A$
Figure 7.6: Hybrid system simulation with adaptive controller when $i_L = 20A$
the stability of the system based on the Equation 7.20. This section will examine how
the changing state of the system components effects the calculation of \( \gamma \).

Considering first the mapping between the general adaptation gain \( \gamma \) and the
existing adaptive controller gain \( \gamma_1 \) in Equation 7.19, it is clear that this relationship is
depended on \( V_L, i_L, C, V_{\text{max}}, V_{\text{uc}} \) and \( V_{\text{fc}} \). While \( C \) and \( V_{\text{max}} \) are known characteristics
of the ultra-capacitor and \( V_L \) is held constant, \( i_L, V_{\text{uc}} \) and \( V_{\text{fc}} \) change with power
demand. The clear consequence of this is apparent when comparing simulation results
in Figure 7.5 and 7.6. In these cases the system is driven unstable then, with a change
in \( \gamma_1 \), is driven stable. The first case with a current demand of 10 A requires \( \gamma_1 = 2000 \)
for the system to become unstable and \( \gamma_1 = 1000 \) to be driven stable. In contrast,
the 20 A case requires only \( \gamma_1 = 400 \) for instability and \( \gamma_1 = 200 \) for stability. This is
because of the mapping’s dependency on the variables \( i_L, V_{\text{uc}} \) and \( V_{\text{fc}} \).

Furthermore, error is also induced when \( i_L \) is held constant. As the system be-
comes unstable with a constant power demand, the voltages of the system components
also begin to fluctuate. This, for example, can be seen in plot (c) of both figures 7.5
and 7.6. As the system is driven unstable, \( V_{\text{fc}} \) begins to fluctuate. This phenom-
ena theoretically changes the calculated value for \( \gamma_1 \), resulting in a changing stability
limit based on the mapping given in Equation 7.20. While this occurrence inher-
ently restricts the accuracy of this method of calculating allowable limits of gains
and FSS characteristics for the closed loop system, it is a beneficial guideline when
implementing the existing adaptive controller from Section 7.1 on a physical system.
The following section will discuss an existing hardware-in-the-loop test stand which
will be used to verify these findings in an actual hardware-integrated setup.

### 7.4 Experimental Test Stand

The experimental setup used is a hardware-in-the-loop system that allows for real-
time simulations of the SOFC/UC system without the need for a physical fuel cell
setup. This system is shown in Figure 7.7.

The SOFC system is emulated in the system using a detailed mathematical model on a dSPACE DS1103PPC controller board. The model used is of the tubular solid oxide fuel cell discussed in Section 2.1, and was developed in Matlab/SIMULINK environment. A SGA Series DC programmable power supply from Elgar Sorensen was used to provide the calculated fuel cell power output. The power supply is capable of providing 100V and 50A in real time to the hardware system. This simulated fuel cell is connected to the system using a SD-1000L-24 unidirectional DC/DC converter from Mean Wells Inc. This power converter maintains the load voltage $i_L$ at 24V and has a maximum output of around 40A. In parallel to this lies a 16.2V series BMOD0250-E016 ultra-capacitor from Maxwell Technologies which has a 250F capacitance and
around 4.1mΩ resistance. This is connected to the system using a DC5050F-SU bidirectional DC/DC converter from Zahn Electronics Inc. This power converter commands the ultra-capacitor current $i_{uc}$ and allows for the ultra-capacitor to both charge and discharge. To consume power from both the fuel cell and ultra-capacitor, a 1.8kW SLH series DC electronic load is used. The current consumed is controlled by the computer through the dSPACE board. A host PC is used for data capture and real time monitoring using the included dSPACE software Control-Desk. Further details pertaining to this system can be found in [3].

7.5 HIL Simulation Results

The following simulations are run with $V_L = 24V$, $U_{ss} = 80\%$, target SOC $S_t = 0.8$, ultra-capacitor $C = 250F$ and $V_{max} = 16.2V$. The parameter estimates $\hat{\eta}_1$ and $\hat{\eta}_2$ are both initialized at 0.92. As in the desktop simulations, the FSS is modeled as $\dot{N}_f(s)/\dot{N}_{f,d}(s) = 0.85/(2s + 1)$ but is treated as unknown. Also, the startup process is not shown in the following simulations and $t = 0$ seconds is the start of parameter estimation and data capture.

Figure 7.8 demonstrates the results of a constant 10A load demand. This simulation was run with $\gamma_1 = 3000$ when $t < 200$ seconds and $\gamma_1 = 1000$ when $t \geq 200$ seconds. Given the mapping from Equation 7.20, this results in actual $\gamma$ values of $\gamma = 68.4$ and $\gamma = 22.8$ for $\gamma_1 = 3000$ and $\gamma_1 = 1000$, respectively. These results return an approximately marginally stable system before $t = 200$ seconds which then becomes stable after the gain $\gamma_1$ is reduced. Similar results can be shown for $i_L = 20A$ in the following.

Figure 7.9 shows the simulation results for $\gamma_1 = 500$ when $t < 200$ seconds and $\gamma_1 = 200$ when $t \geq 200$ seconds, resulting in actual $\gamma$ values of $\gamma = 50.9$ and $\gamma = 20.4$ for $\gamma_1 = 500$ and $\gamma_1 = 200$, respectively. This simulation clearly shows the system gradually becoming unstable when $t < 200$ seconds. When $\gamma_1$ is lowered to a value
Figure 7.8: HIL simulation when $i_L = 10A$
Figure 7.9: HIL simulation when $i_L = 20\, \text{A}$
below the predicted threshold of $\gamma_1 = 344$, the system begins to stabilize.

These simulations support the idea that the generalized adaptive closed loop system equations can be used to determine when the system can potentially be driven unstable. Although there is error induced in various stages of the process, the general result provides a relatively good approximation. Additionally, it has been shown that the stability result is applicable for various load demands. It was determined that an increased load demand results in a lower allowable adaptation gain. This result would prove useful in avoiding unstable and potentially harmful conditions at higher power demands where the allowable gains are significantly lower.
Chapter 8

Discussion and Conclusion

8.1 Assumptions

This section will revisit assumptions made during the analysis in the previous chapters. These assumptions include the form of the nonlinearity assumed for the FSS and neglecting the effects of the variation in parameters $h_1$ and $h_2$. Analytical result will be used to verify these arguments.

8.1.1 Considering the FSS dynamics with input $r$

In Section 5.1, it was assumed that the general FSS equation $f(e + r, r)$ could be represented as $f(e)$ without significant loss of generality. This was due to the fact that if there is no steady state error, the value of $r$ does not affect the output of the FSS equation. This was shown with the simple example that if the system is at steady state without error, then $f(0 + r, r) = 0$. This section will seek to reintroduce the initial form of the FSS equation by relaxing this assumption and considering the effects of input $r$. This will be done analytically using a Lyapunov stability approach as in Section 5.2. To begin, the general nonlinearity form of the FSS dynamics will be assumed to take the following form:

$$
\dot{y} = -f(e, r) = -f_1(e) - f_2(r)
$$

(8.1)
From the proposed closed loop state equations presented in Section 5.1, Equation 5.8 and 8.1 can be represented as a single second order system given by
\[
\ddot{e} = -\frac{df_1}{de} \dot{e} - \frac{df_2}{dr} \dot{r} + kh_2 \dot{z} \quad (8.2)
\]
Noting again that the error between the input and output of the FSS is represented as \( e = y - r \),
\[
\dot{r} = -f_1(e) - f_2(r) - \dot{e} \quad (8.3)
\]
Equations 8.1, 8.2, and 8.3 result in
\[
\ddot{e} + \left( \frac{df_1}{de} - \frac{df_2}{dr} \right) \dot{e} + \left( kh_2 - \frac{df_2}{dr} \right) f_1(e) = \left( \frac{df_2}{dr} - kh_2 \right) f_2(r) \quad (8.4)
\]
This closed loop equation demonstrates that by considering the FSS to be a function of both \( e \) and \( r \), additional conditions must be considered to guarantee stability.

As mentioned above, it was initially considered that steady state error due to the FSS system was zero. This allowed for the fuel supply system behavior to be considered strictly a function of \( e \). This allowed for a simplified analysis without any loss of generality. However, relaxing the assumption that there is no steady state error allows for the implementation of the input \( r \). To see how this effects the closed loop system given in Equation 5.8, the general FSS behavior equation that is a function of both \( e \) and \( r \) will be considered at steady state. From Equation 8.1,
\[
0 = -f_1(e) - f_2(r), \quad f_1(e) = -f_2(r) \quad (8.5)
\]
To see how this would affect the closed loop system, the FSS equation is considered the sum of two linear functions. This same assumption was used in Section 6.2 for the general adaptive analysis. Given in Equation 6.18, it was assumed that \( f(x_1) \approx ae \). Likewise, the FSS expression in Equation 8.1 will be linearized using the relationship
\[
-f_1(e) - f_2(r) \approx -ae - br. \quad \text{Using this assumption, Equation 8.5 becomes } \quad ae = -br
\]
yielding the steady state error of
\[
\hat{e}_{ss} = -\frac{b}{a} r \quad (8.6)
\]
This result can be verified by simulating the closed loop system given in Equation 5.8, where \( f(e + r, r) = ae + br \). The simulation was run with \( k = 2, h_2 = 1, a = 2, b = 1 \) and input \( r = 5 \). The states \( e \) and \( z \) were initialized at \( e(0) = z(0) = 0 \). The results are given in Figure 8.1.

![Figure 8.1: Effects of input \( r \) on closed loop system](image)

Although a simple example, these results agree with the steady state error expression given in Equation 8.6. Considering \( a = 2, b = 1 \) and \( r = 5 \), it can be calculated from Equation 8.6 that \( e_{ss} = -2.5 \). Examining state \( e \) given in plot (c) in Figure 8.1, it is clear that it converges to a value of \( e = -2.5 \). This result supports the assumption that considering the FSS dynamic equation to be a function of both \( e \) and \( r \) results in a calculable steady state error.

### 8.1.2 Considering \( h_2 \) and \( h_1 \)

In Section 6.1, the general adaptive controller was formulated for the generalized \( E_s \) equation given in Equation 4.18. As compared to the derivation in Section 5.1 where \( h_1 \) and \( h_2 \) given in Equation 4.17 were considered known and constant, Chapter 6 focused on removing these assumptions and considering \( h_1 \) and \( h_2 \) unknown and slowly varying. This was done using an adaptive controller approach which was ultimately a generalization of an existing controller given in Section 7.1. Since, in the previous approach, \( h_1 \) and \( h_2 \) were considered slowly varying, their derivatives were assumed
to have negligible effects on the system. This section will reevaluate the system in the presence of these derivatives. Beginning with the state space representation from Equation 6.4

\[
\dot{z} = -f(e) + \mathcal{E} \\
\dot{e} = -f(e) + \mathcal{E} + kh_2 e_{12} + kh_2 z
\]

where

\[
e_{12} = \frac{h_1}{h_2} - \frac{\dot{h}_1}{h_2} \\
\dot{e}_{12} = \frac{d}{dt} \left( \frac{h_1}{h_2} \right) - \frac{d}{dt} \left( \frac{\dot{h}_1}{h_2} \right)
\]

Designing adaptation law to be \(\dot{e}_{12} = -\gamma x\) results in

\[
\mathcal{E} = \frac{d}{dt} \left( \frac{h_1}{h_2} \right) = \gamma x + \frac{d}{dt} \left( \frac{h_1}{h_2} \right)
\]

and

\[
\dot{\mathcal{E}} = \gamma \dot{x} + \frac{d^2}{dt^2} \left( \frac{h_1}{h_2} \right)
\]

From Equation 6.4, 8.10, and 8.8

\[
\ddot{e} = -\frac{df}{de} \dot{e} + kh_2 \dot{e}_{12} + kh_2 \dot{z} + \frac{d^2}{dt^2} \left( \frac{h_1}{h_2} \right) + k \frac{dh_2}{dt} (e_{12} + z)
\]

where

\[
\Psi = kh_2 \frac{d}{dt} \left( \frac{h_1}{h_2} \right) + \frac{d^2}{dt^2} \left( \frac{h_1}{h_2} \right) + kh_2 \frac{dh_2}{dt} (e_{12} + z)
\]

Using coordinate transform from Equation 5.7, \(\dot{x} = (\dot{e} - \dot{z}) / k\). From this relationship, Equations 6.4 and 8.11 result in

\[
\ddot{e} = -\frac{df}{de} \dot{e} + \frac{\gamma}{k} (\dot{e} - \dot{z}) - kh_2 f(e) + \Psi
\]

where

\[
\Omega = -\frac{\gamma}{k} \frac{d}{dt} \left( \frac{h_1}{h_2} \right) + \Psi
\]
Forming the $\dot{x}$ state equation, from Equation 6.4 and 8.9

$$\dot{x} = \frac{1}{k} (\dot{e} - \dot{z})$$

$$= \frac{1}{k} (\dot{e} + f(e) - \gamma x - \frac{d}{dt} \left( \frac{h_1}{h_2} \right))$$

$$= -\frac{1}{k} (\gamma x - \dot{e} - f(e)) - \frac{1}{k} \frac{d}{dt} \left( \frac{h_1}{h_2} \right)$$

(8.15)

Considering the system state equations for $\dot{e}$ and $\dot{x}$, the following coordinate transform is proposed:

$$v = \gamma - \dot{e} - f(e)$$

(8.16)

From Equations 8.11, 8.16, and 8.15

$$\dot{v} = \gamma \dot{x} - \frac{d}{dt} \dot{e}$$

$$= kh_2 f(e) - \frac{\gamma}{k} \frac{d}{dt} \left( \frac{h_1}{h_2} \right) - \Omega$$

(8.17)

Equations 8.11, 8.12, 8.14 and 8.17 thus lead to the following state equation formulation of the closed loop system:

$$\dot{v} = kh_2 f(e) - k \frac{dh_2}{dt} (e_{12} + z) - kh_2 \frac{d}{dt} \left( \frac{h_1}{h_2} \right) - \frac{d^2}{dt^2} \left( \frac{h_1}{h_2} \right)$$

$$\dot{e} + \frac{df}{de} \dot{e} + kh_2 f(e) = -\frac{\gamma}{k} v + k \frac{dh_2}{dt} (e_{12} + z) + (kh_2 - \frac{\gamma}{k}) \frac{d}{dt} \left( \frac{h_1}{h_2} \right) + \frac{d^2}{dt^2} \left( \frac{h_1}{h_2} \right)$$

(8.18)

Comparing the resulting closed loop system from Equation 8.18 with the previous form given in Equation 6.16, Equation 8.18 contains the same terms plus a number of perturbation terms consisting of the derivatives of $h_1$ and $h_2$. It is apparent that since $h_1$ and $h_2$ are in fact slowly varying, their derivatives will be very small. This result supports the idea that neglecting the derivatives of $h_1$ and $h_2$ is a viable assumption as it has little impact on the closed loop system.

8.2 Conclusion

This thesis work focused on techniques related to controlling SOFC systems during transient loading conditions and hybridization with an energy storage device. The SOFC system control is based on a current regulation technique which uses feedback
to compensate for delays attributed to the fuel supply system. The methods proposed in this work sought to minimize the effects due to delays in the reformer by implementing a state observer to calculate fuel species entering the anode. This resulted in noticeable improvements, however it was determined that this technique is most effective during times of increased flow restrictions or improperly sized reformer control volumes. The second transient control technique made use of a supplementary hydrogen storage device used to provide additional fuel during times of large changes in load. The implementation of a simplified hydrogen injection system resulted in little improvement in load following and fluctuations in fuel utilization. This was due to the complex dynamics of the reformer and fuel cell stack. Future work would include a more sophisticated controller to manage the injection amount, as well as a more comprehensive model of the hydrogen storage canister and injection mechanism.

The final focus of this work related to the hybridization of the SOFC system with an ultra-capacitor. Based on existing controllers, work was done to relax the initial assumptions placed on the FSS dynamics in these methods. For instance, exponential tracking or bounded tracking of the FSS were incorporated in a more generalized framework as nonlinear functions. Using a generalized cascaded system formation and absolute stability concepts, classes of nonlinear FSS behaviors were examined and their effects on system stability documented. While feedback linearization was sufficient when the system was assumed known, it was enhanced to an adaptive controller when the DC/DC converter efficiencies were considered unknown and varying. This existing parameter estimation technique was reexamined in a general form. Using a linearization, the onset of instability could be predicted through an inequality condition involving the FSS dynamics at the origin, feedback gain and parameter estimation gain. This result was verified using both desktop simulations and a HIL test stand. The simulations supported the initial relationship, holding true at varying load demand levels. This relationship would be useful in future work for sizing gains, preventing system instability and possible component damage.
References


