Image restoration by selective short space spectral subtraction

Rose M. Korte
IMAGE RESTORATION BY SELECTIVE SHORT
SPACE SPECTRAL SUBTRACTION

by

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A thesis submitted in partial fulfillment
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Submitted to the Photographic Science and Instrumentation Division in partial fulfillment of the requirements for the Bachelor of Science degree at the Rochester Institute of Technology

ABSTRACT

Image restoration by short space spectral subtraction has been applied recursively to a photographic system in an attempt to increase the signal to noise ratio proportionally to the amount of optical density present. The image is smoothed in the frequency domain a small space at a time based on the power spectrum at a given density level. The method is a variable filter that is a function of photographic density.
ACKNOWLEDGMENTS

Special thanks and recognition are offered to the following: Tom Lianza for his guidance and teaching, EIKONIX Corporation of Bedford, Mass. for the use of equipment, Barbara McNamara of the Rochester Division of TEKTRONIX for her cooperation in obtaining computer time, and the United States Central Intelligence Agency for a grant funding this research.
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INTRODUCTION

When a scene is recorded photographically and the film is scanned with a microdensitometer to obtain a density profile of the image, the representation is an image degraded by the additive noise of the film's grain structure and the blurring of the scanning aperture of the measuring instrument. The restoration of the image to contain only the information present in the scene (signal without noise) is desirable and has been attempted with various filtration techniques over the years.

Present techniques for the restoration of images degraded by blurring and additive noise include Weiner-Kolmogoroff filtering,\(^1,2\) power spectrum filtering,\(^3\) and least mean-square-error filtering.\(^4,5\) The common approach is for restoration to be based on a random stationary field image model and a linear space invariant filter whose frequency response is a function of the ideal image power spectral density, which is a white noise estimation made by averaging over different images that have the same suspected content as the image to be filtered. However, for most images, the scene changes sufficiently from one point in space to another to make the assumption of a random stationary field image an invalid one. Also, the described estimation of power spectral density
usually is not a good approximation for the image’s true power spectral density due to all the assumptions and averaging which go into the estimation.\textsuperscript{6}

Recent work by J.S. Lim\textsuperscript{7} develops a new image restoration technique by attempting to reduce the effects of the aforementioned difficulties. The new technique is called Image Restoration by Spectral Subtraction and is implemented on a short space basis to reduce the effect of the nonstationarity of real images. Lim also uses an estimation of the power spectral density but his is estimated from the degraded image function that is to be restored, not averages of many images. The image data is divided into many subgroups which are restored separately by having the estimated power spectral density subtracted from the degraded image. Recombining the subgroups yields a restored image.

An idea for carrying Lim’s method a step further in an application to photographic systems where the filtering would be applied recursively to a degraded image came from Gonsalves and Lianza of EIKONIX Corporation, Bedford, Mass. It is inherent in the nature of photographic systems -- its recording materials and the scanning process of measuring on them -- that the level of noise increases with a signal or density level increase (Selwyn’s Law). If the restoration filter could be sensitive to signal level and appropriately adjustable in its application, it follows that a more truly restored image should result from a restoration method which removes
noise most where its level is greatest, and less where least noise is present. This paper deals with one dimensional density profiles and its purpose is to build an algorithm which will restore those images selectively, according to density level at any point in the scene.
FOOTNOTES FOR CHAPTER I


6 Lim.

7 ibid.
THEORY

The following is a block diagram for the process of image degradation.

\[ s(x) \rightarrow g(x) \rightarrow h(x) \rightarrow i(x) = h(x) \ast [s(x) \star n(x)] \]

Figure 1. Image degradation process

where \( s(x) \) is a scene, \( h(x) \) is the impulse response of the mensuration system (convolution with microdensitometer aperture), \( g(x) \) is a noisy image, \( n(x) \) is additive noise due to the grain structure of the recording material, and \( i(x) \) is the resulting degraded image.

The type of image restoration process used in this work is shown in the following figure:

\[ |I(f)|^2 \xrightarrow{\neg} (\text{j}^2(f) \xrightarrow{\neg} 1/|H(f)|^2) \xrightarrow{\neg} s(f) \]

\[ [H(f) \cdot N(f)]^2 \]

Figure 2. Image restoration process

where \( I(f) \) is the discrete Fourier transform of the degraded image. \( N(f) \) is the frequency domain estimation of the noise, \( 1/|H(f)|^2 \) is the inverse transfer function of the system, and the inverse Fourier transform of \( s(f) \) is \( s(x) \), the image restored to the scene.
Consider the following noisy image function, \( i(x) \), that is \( M \) points long.

\[
\begin{array}{c}
i(x) \\
0 \quad L/2 \quad L \quad M
\end{array}
\]

Figure 3. Noisy image function

Processing will be done with subgroups \( L \) units wide at \( L/2 \) intervals. Windowing in this manner satisfies the condition that when the data is recombined, the effect of windowing is that of multiplying everything by one, i.e., the windowing procedure cannot corrupt the data. The window is a triangle function of height 1 and width \( L \) and will be used to isolate the subgroups of data for the smoothing operation.

\[
\begin{array}{c}
i(x) \\
0 \quad L/2 \quad L
\end{array} \times \begin{array}{c}
\text{tri}(x) \\
0 \quad L/2 \quad L
\end{array} = \begin{array}{c}
g(x) \\
0 \quad L/2 \quad L
\end{array}
\]

Figure 4. The windowing operation
The fourier transformation of the product function, $g(x)$, is written in polar form: $F\{g(x)\} = A(f)e^{i\phi}$. At this point, the amplitude of the function is modified so that

$$\hat{A}(f) = \begin{cases} 0 & \text{when } A^2(f) \leq P(f) \\ \left[|A(f)|^2 - P(f)\right]^{\frac{1}{2}} & \text{when } A^2(f) > P(f) \end{cases}$$

where $P(f)$ is the power spectrum of the image (resultant from photographic material + measuring instrument). Here, unlike Lim's method the power spectral weights are not an estimated flat noise but are calculated from the signal; the modulus squared of the fourier transform of the signal is its power spectrum. The measured power spectral density is not white noise and varies with density level. If the filter can be made to discriminate between density levels, it would be able to apply the appropriate spectral weights in the amplitude comparison and subtraction step described above, and therefore be a selective smoother.

Computing the inverse fourier transform of $\hat{A}(f)e^{i\phi}$ returns the function $\hat{g}(x) = \hat{f}(x) \times \text{tri}(x)$, where $\hat{f}(x)$ is $f(x)$ with the spectral power component due to $P(f)$ removed.

The algorithm is repeated for each subgroup of the data until the whole image has been operated on. That allows for the probable nonstationarity of the scene. Then all the modified subgroups of data are recombined to form the restored image. If the image is windowed like this:
Figure 5. Windowed image data

the windowing process introduces no artifacts to the restored data (except the first and last \( \frac{L}{2} \) points). That is, if the algorithm is applied and no amplitude modification takes place, the original data is return by the algorithm. That's important because if the data is noiseless when input, we don't want the "smoother" to corrupt it. If the spectral weights are such that the algorithm forces all the modified amplitudes to 0, the image is effectively averaged in blocks of 16, thus adding some signal distortion.
METHODOLOGY

All the images in this work are one-dimensional density profiles. Several uniform densities patches are required and some Ealing high resolution bar targets are used to produce variable density profiles.

Data was gathered from three uniform density patches made on Kodak Professional Copy film 4125 (0.44, 1.55, 2.21, ±0.2 density units as measured on a transmission macro densitometer) so that the power spectrum of the film at various density levels could be calculated. That was necessary to get an estimate of the shape and amplitude of the power spectra change as a function of density. The data gathered from the variable density profiles was used to test the algorithm.

The data files used in the algorithm were made by sampling the density of film image samples as they were scanned by a microdensitometer. Three replicates of each uniform density patch were made to test the variability of the data gathering. A 10μm circular aperture on a Mann A.D.M MKII microdensitometer was used to take a density reading every 0.12 seconds as the stage moved at a rate of 5mm/minute. 512 samples were taken for each array. 512 is an appropriate power of 2 to be compatible with the FFT software used in
the smoother. A data collection program (see Appendix A) for a TEKTRONIX 4052 Graphic System Computer System, HP 3437A System Voltmeter and Mann Microdensitometer was used to gather the data arrays. The 4052 unit employs a particular form of BASIC and uses subroutines on the TEKTRONIX 4051R07 and 4052R08 Signal Processing ROM Packs to assist its graphic capabilities and to perform such operations as FFT, IFT and convolution. The FFT algorithm of the ROM Pack employs the Sande-Tukey decimation-in-frequency FFT. The ROM Pack's POLAR function, which is used to give the amplitude portion of the spectra with which the smoothing algorithm works, has an output of N/2 + 1 points for an input of N (power of 2) points. When 16 points are fast Fourier transformed, and the magnitude and phase components separated with the POLAR function, a nine point magnitude array results, in which the first point is a representation of the DC term of the spectrum.

All the data is in units of machine density, and remains so throughout the restoration process. The whole issue is dealt with from the perspective of removing noise from a signal; whether the signal is in terms of density, transmittance, exposure, voltage, or any other unit is of no significant importance.

The triangular windowing function is built and implemented by the restoration algorithm in the following manner:
This windowing procedure is done to isolate a small part of the scene during the restoration process so it will be operated on according to its actual parameters, not basing these on averages or estimates of what the short space signal is based on viewing the whole scene at once. Lim's idea of short space restoration avoids the assumption of stationarity of the image.

A triangular window was used because it gently forces the data to zero, thereby avoiding artifacts in the spectra due to a false edge discontinuity (such as those which would result if a rectangular window were used). The total effect of the triangular windowing operation on the data at any point is that of multiplying it by one, or leaving it essentially unchanged. This is desirable because the only purpose of the windowing operation is to subdivide the image and it must not affect the data in any other way. Of course, the first and last eight points (the window is a 16 point wide function) of the whole data array are corrupted because they only get multiplied by half a triangle once. Ideally, the overlap
ought to occur such that the last zero of one triangle is the first zero of the next and that those occur at the same point in space. Because computer indexing is integer, begins at one, and only one value can be assigned to each point, and because an even number of points is required for the FFT, to perform the windowing operation in a program, the function must be generated and implemented as shown in Figure 6— with the value of the first point of a triangle being 1/8 and starting at the 9th point of the previous triangle. In that way, the "summing to one" condition is maintained. The only anomaly introduced is a small phase shift since each window is not a centered triangle function. That however, does not effect this restoration process since the smoothing algorithm operates only on amplitudes.

After a particular subgroup of data has been windowed by multiplication of data array with triangle array, that product function is fourier transformed by the ROM Pack FFT subroutine. Magnitude and phase components are separated by the POLAR subroutine (see Appendix B) and the result is a nine point spectrum of image signal, M. Next there is an amplitude comparison between the square of the modulus of the last eight array M's components and another nine point array, D, of spectral weights. Array D may be the power spectrum of the original noisy image, or it may be filled with arbitrary values. (Note that the first term is never changed to maintain DC integrity.) The amplitude comparison subtracts the
value of D from that of M at a particular frequency if \( M > D \).

If \( M \leq D \), that comparison is assigned the value zero. That's done to avoid having negative densities in the restored image. If the contents of array D is the image's noise spectrum, after the subtraction, what remains is a noiseless image spectrum. If array D contains arbitrary values (large ones) such that the comparison forces all the eight magnitudes to zero, the only remaining value for that subimage is its DC. If the arbitrary values are zero, the magnitudes of the image array emerge from the amplitude comparison unaltered. In that case, the algorithm has just computed the power spectrum of the image data it's operating on (power spectrum = square of the modulus of the Fourier transform of the image function).

The next step to obtaining a restored image is to take the square root of the modulus squared, recombine the phase and amplitude components of the modified spectrum of this subimage and perform an inverse fourier transformation on it. After this whole procedure is repeated for enough subimages (64) to operate on the whole image array, all the modified subimages are just added back together (see program in Appendix B) to obtain a complete restored image.

The smoother also calculates a mean nine point spectrum from the 64 subimage spectral and calculates the standard deviation of that mean of each frequency. So the algorithm itself can be used to calculate the power spectra of various density levels.
A question of some importance is what nine numerical values ought to go into the power spectral array $D$. A reasonable approach, since the image data was multiplied by a triangle in space, would be to convolve the power spectrum of the image data with a $\text{sinc}^2$ function in the frequency domain then choose nine points of that function for spectral weighting input. While this method is simple in theory, it can lead to some problems in practice. The record lengths and sampling must be chosen to adequately describe the spectrum. Many "cuts" must be made to characterize the spectrum as a function of input signal level. To adequately record structure, the DC term must be removed from the signal before calculation and this invariably leads to further complications in the restoration process. Because the record lengths used to compute power spectrum are much longer than those used in the smoother algorithm, frequency increments are much smaller. One must decide whether to use single estimates at the frequencies required by the smoother or use local averages for frequencies close to the nominal required.

In light of the above problems, it is very convenient and quite sensible to let the smoothing algorithm, itself, calculate the power spectra estimates of the various different uniform density profiles; the result of each is a nine point array (appropriate for comparison to the nine point image array, $M$, for the spectral subtraction step) that is an average (over all the subimages) power spectrum representation
for a particular density level. It must be decided if and how those spectra change with respect to density. If the smoother can also identify what general density level it is operating on, it can apply the spectral weighting array that corresponds to that density level. That discrimination is accomplished by computing the mean of each sixteen point subimage. As stated earlier, the mean is returned as the first point from the FFT algorithm employed. Now the algorithm has become a selective short space spectral subtraction filter.

However, before the algorithm can be put to work on continuously variable density images, the relationship between changes in amplitude and shape of the power spectra and changes in density must be characterized. If only the amplitude, not shape, changes with density, the spectra calculated for each density level could be normalized by its DC term and in filtering, be multiplied by a scaling factor appropriate for the current mean density level. If, however, the various spectra amplitudes are not linear with respect to density changes, the filter will be a bit more difficult to implement.

The following figures (7, 8 and 9) led me to the conclusion that the amplitude of the spectra is not strictly linear with respect to density over all frequencies. Figures 7 and 8 are partial plots of the power spectra calculated for three density levels. They show that a linear approximation for amplitude vs. density no longer holds at some density less than 1.5., i.e., the power spectrum shape changes with density
Figure 7. Power spectra for three density levels as calculated by the algorithm.
Figure 9. Mean DC value of power spectrum of various density levels
over the density range 0.0 to 1.5. Above a density of 1.5 it seems to remain linear, only scaled by amplitude. The calculation of power spectra at more density levels, especially between 0.4 and 1.5 would better characterize how the spectra changes with density. Because time and equipment constraints allowed for the calculation of only three densities, the approximations shown in Figure 9 were made. Figure 9 is a plot of the amplitude of the DC term of the mean frequency values as calculated by the algorithm as a function of three density levels. The solid line represents the relationship as approximated by a linear regression. The broken line represents the expected function which is of the form \( A = KD^2 \). The dotted lines are two graphically linear approximations which succeed in differentiating high from low density regions. These two approximations will yield the same spectral amplitude at a density of 1.0, and will treat densities on either side of 1.0 differently from each other. That is what Figure 8 indicates to be necessary. So in operation, the filter identifies the general density level of a particular subimage and if it is a "low" density (<1.0), the filter applies a different set of spectral weights than if it is a "high" density.

It would seem reasonable to use the nine point mean power spectrum output by the smoothing algorithm as input to the spectral weighting array, D. The standard deviation, however, is on the order of those mean values for most of the higher
frequencies, filtering with that spectral array makes for very gentle smoothing. Maximum smoothing will occur when the spectral weights are chosen such that they are slightly less than the threshold values which force the amplitudes to zero. Using the mean + three sigma spectral values causes this kind of gross smoothing some places in the image. Experimentation with various series of spectral weights including mean power spectrum + one, two or three sigma values, and arbitrary values such as $1/X^n$ where $X$ is the frequency slots, 1 to 9, and $n$ varies from 1 to 4, led to using a spectral weighting function in array $D$ that is a normalized mean + two sigma power spectrum value in the final selective smoothing algorithm. Those two normalized arrays (for "high" and "low" densities) are scaled proportionally to the DC of the identified density level in the frequency domain before the amplitude comparison between signal and noise is made. See program in Appendix B for specific details.

The analysis here is aimed at discovering whether or not this algorithm will act as a variable filter and smooth signals proportionally to their magnitudes. Showing that using this algorithm on a noisy signal reduces the variance in the signal demonstrates that it is working as a smoother. Showing that the amount of smoothing that takes place in high amplitude regions of the signal is different from (greater than) the amount of noise removed from low amplitude regions demonstrates that the algorithm is a selective smoother, or self-adjusting variable filter.
RESULTS

It has been found that one dimensional images degraded by additive noise can be restored by a filter which varies as a function of photographic density on a short space basis. The subimage approach allows different parts of the image to be restored to different degrees as they need to be. The algorithm identifies what density level it is operating on and applies an appropriate spectral weighting to the frequency domain subtraction. Signal to noise ratios have been increased by a factor of roughly three for all the density levels tested (range of 0.44 to 3.2). It has also been discovered that this algorithm is not suitable for edge trace analysis.

An early test of whether or not the algorithm was working was to calculate the mean and variance of both original and restored data and perform a statistical t-test of the mean and F-test of the variance. It was expected that there be no difference between the means because the DC term of the transform was not operated. No real difference was found (see table). Restoration has implied in its definition the reduction of signal variance. The degree of variance reduction is an indication of the degree of restoration.
<table>
<thead>
<tr>
<th>$\bar{D}$</th>
<th>Spectral Weighting</th>
<th>$\sigma$</th>
<th>$\bar{D}/S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>.44</td>
<td>none</td>
<td>.081</td>
<td>5.51</td>
</tr>
<tr>
<td></td>
<td>mean power spectrum</td>
<td>.068</td>
<td>6.58</td>
</tr>
<tr>
<td></td>
<td>mean + 1 std. dev.</td>
<td>.042</td>
<td>10.63</td>
</tr>
<tr>
<td></td>
<td>mean + 2 std. dev.</td>
<td>.026</td>
<td>17.31</td>
</tr>
<tr>
<td>1.55</td>
<td>none</td>
<td>.097</td>
<td>16.08</td>
</tr>
<tr>
<td></td>
<td>mean power spectrum</td>
<td>.065</td>
<td>24.01</td>
</tr>
<tr>
<td></td>
<td>mean + 1 std. dev.</td>
<td>.060</td>
<td>25.81</td>
</tr>
<tr>
<td></td>
<td>mean + 2 std. dev.</td>
<td>.029</td>
<td>53.20</td>
</tr>
<tr>
<td>2.02</td>
<td>none</td>
<td>.117</td>
<td>18.88</td>
</tr>
<tr>
<td></td>
<td>mean</td>
<td>.132</td>
<td>16.70</td>
</tr>
<tr>
<td></td>
<td>mean + 1 std. dev.</td>
<td>.007</td>
<td>28.67</td>
</tr>
<tr>
<td></td>
<td>mean + 2 std. dev.</td>
<td>.042</td>
<td>52.42</td>
</tr>
</tbody>
</table>

Table 1
Signal to noise improvement for various degrees of restoration

The "summing to one" property of the windowing procedure discussed earlier was tested by building a ramp data file and running it through the algorithm, making no amplitude alterations in the frequency domain. The ramp came out of the filter unaltered (except, of course, for the first and last eight points).

The algorithm was made to be selective by calculating the average density of each subimage and multiplying all the spectral weights in the frequency domain by the DC term of the subimage. The two spectral weighting arrays finally settled upon are listed in Appendix C. As explained before, when the spectral weights are greater than the signal, the
amplitude magnification that occurs is that it is set to 0. Since the DC term isn't operated on, it is the only value left in a subimage that is not zero so that whole "chunk" is plotted at its DC value. An example of this can be found in Figure 11. This is equivalent to averaging the signal in "chunks" of 16 points. The original image was a degraded bar target, shown in Figure 10.

Application of the algorithm to bar target image data results in a ringing phenomenon on all the edges. See Figure 12. A series of different spectral weighting inputs were applied in an attempt to reduce the effect. It did not go away. Taking the transform of such a discontinuous function as the edges within a short space array adds an artifact to the data in the frequency domain.

Edge detection and avoidance methods were investigated and tried. Finally it was found that adding together (1) data which has been transformed, amplitude altered (or not), and inverse transformed, and (2) data which has not gone through the forward and reverse transformations will generate artifacts. The triangle window in space is a sinc\(^2\) function in the frequency domain and to do the inverse tranformation it must be truncated at some point since it is an infinite function. The inverse transformed data is no longer purely triangularly influenced. The problems associated with alternate forms of edge smoothing are beyond the scope of this work, the problem with the short space smoother occurs only
Figure 10. Degraded bar target image data
Figure 11. Gross smoothing of degraded bar target image data
Figure 12. Artifacts resultant from application of algorithm to perfect bar target
when a very sharp discontinuity occurs within a short space array. The properties of the algorithm have been demonstrated using the uniform density patches.

Figures 13 through 16 show a series of successive degrees of harshness of filtering ending with the result of using a mean + 2 standard deviation power spectrum in the frequency domain subtraction. The signal to noise improvement by the end of the series is a factor of 3.3.

Using the filter in the non-selective mode requires that the signal be known and the proper absolute spectral weighting array be provided. By making the two linear approximations described in the Methodology section, and inputing two corresponding normalized spectral weighting arrays, the filter can operate in a selective mode on any signal in the density range of 0.0 to 3.0 with the same spectral input. Plots of original data and restored data with the filter in selective mode for each of the three density levels are shown in Figures 17 through 22.

A visual comparison of the amount of noise removed at high and low density levels can be made from Figures 23 and 24. This demonstrates that the filter does indeed eliminate more noise at high densities where most is present, and less noise at low densities where less is generated. Figure 25 is the difference between medium density data restored in the non-selective and then the selective modes. The small differences indicate that the linear approximation scaling process used yields very reasonable results.
Figure 13. Degraded medium (1.55) density data
\[ S/N = \frac{\bar{x}}{s} = 16.08. \]
Figure 14. Degraded medium density data filtered by subtraction of a $[x+1s]$ power spectrum. $S/N = 24.01.$
Figure 15. Degraded image smoothed by spectral subtraction of a $[x+ls]$ power spectrum. $S/N = 25.81$. 
Figure 16. Degraded image restored by spectral subtraction of a \([x+2s]\) power spectrum. Final S/N = 53.20.
Figure 17. Original degraded low (0.44) density data
Figure 18. Degraded low density data restored by spectral subtraction of a \([x+2s]\) power spectrum.
Figure 19. Original degraded medium (1.55) density data
Figure 20. Degraded medium density data restored by spectral subtraction of a $[\bar{x}+2s]$ power spectrum.
Figure 21. Original degraded high (2.21) density data
Figure 22. Degraded high density data restored by spectral subtraction of a $[x+2s]$ power spectrum.
Figure 23. Difference between original and restored low density data
Figure 24. Difference between original and restored high density data
Figure 25. Difference between medium density data restored by algorithm's selective and non-selective modes
CONCLUSIONS

The idea to extend Lim's restoration method to a recursive filter for photographic systems was a reasonable and useful one because it works on low contrast one dimensional images, fitting itself to the image's real noise spectrum from one point in space to another. The next step would be to apply the filter to two-dimensional images. It is important to explore the problem that occurs in smoothing edges; what density gradient constitutes an edge for this algorithm? If this idea is to be very useful for a variety of images, the ringing phenomenon that occurs at edges must be overcome.

There are several other things which still ought to be done with the algorithm the way it is. This algorithm in its selective mode ought to be tested on a continuous variable density image against some average spectral weighting subtraction in the frequency domain as a comparison to the other filtration techniques mentioned in the Introduction which are based on scene and power spectrum averages and estimations. A sine wave frequency series ought to be run through the filter to see where and in what ways the algorithm breaks down.
In image processing and reconnaissance applications, the edge artifacts introduced by the algorithm may actually be beneficial to the observer. It should be remembered that while the algorithm "rings" at the edges, it still smooths to either side of the edge. This reduces the noise in the surround and enhances the edge definition.
LIST OF REFERENCES
LIST OF REFERENCES


Lianza, T.A., personal communication.


APPENDICES
APPENDIX A

Table 2

Program For Data Collection

```
100 Print "Enter the number of readings/trigger"
110 Input N$
120 N=VAL(N$)
130 Print "Enter the delay between readings (.999999-0)"
140 Input A$
145 A=VAL(A$)
146 F=1/A
147 IF F<3000 then 150
148 PRI "Burst rate too fast; must use next program"
149 Go to 600
150 Print "Trigger type"
155 PRI "1=Internal"
160 PRI "2=External"
165 PRI "3=Manual"
170 Input T$
175 Print "Voltage range"
180 PRI "1=.1"
185 PRI "2=1.00V"
190 PRI "3=10.00V"
195 PRI "4=Auto range, remember trigger restrictions!"
200 Input R$
205 REM: Set up program for system voltmeter
210 PRI @ wr: "N", N$, "SD", A$, "ST", T$
215 R=VAL(R$)
220 IF R=4 then 360
230 PRI @ 24: "R", R$
235 DIM V(N)
240 Input @24: V
245 Print "Data collection complete"
250 Go to 800
255 REM: PAUTOZSCALE, routine
260 DIM V(N), V1(N), V2(N), V3(N)
265 REM
270 Print @ 24: "R1"
275 Input @ 24: V1
280 REM
285 PRI @ 24: "R2"
290 REM
295 Input @ 24: V2
```
APPENDIX A (Cont'd.)

450 REM
460 PRI @ 24: "R3"
470 REM
480 Input @ 24: V3
490 For I=1 to N
500 If V1(I)>0.9 then 530
510 V(I) = V1(I)
520 Next I
530 If V2(I)>9 then 560
540 V(I)=V2(I)
550 Next I
560 If V3(I)>90 then 600
570 V(I)=V3(I)
580 Next I
590 Go to 610
600 Print "Warning: Data is out of range"
610 END
800 Viewport 10,90,10,90
810 Window 0,N,0,10
815 Page
820 Axis 8,0,5
830 Move 0,0
835 V=V/-2
840 For I=1 to N
850 Draw I, V(I)
860 Next I
870 Home
880 Print "Do you wish to save the data?"
890 Input Z
900 If Z<1 then 100
910 Print "What file number on tape?"
920 Input Y
930 Find Y
940 For Y=1 to N
950 Print @ 33: V(Y)
960 Next Y
965 Delete V
970 Go to 100
980 END
APPENDIX B

Table 3

Selective Short Space Spectral Smoothing Algorithm

100 REM: Short space spectral smoother (normalized area)
110 DIM A(512), B(512), D(9), T(16), M(9), P(9), C(16), K(9), Y(9), G(512)
120 DIM U(9,63), W(9), W1(9), M5(9), Q(63)
130 REM=Ø triangle width=N=Power of two
140 N=16
150 S=N/2
160 For I=1 to S
170 T(I)=I/S
180 Next I
190 For I=S+1 to N
200 T(I)=1-(I-S)/S
210 Next I
220 REM Find data file to operate on
230 Print "What data file is to be smoothed?"
240 Input J
250 Find J
260 Input @ 33: A
270 G=A
280 Print "Input 9 spectral weights for low densities"
285 Print D1
290 Print "Input 9 spectral weights for higher densities"
295 Input D2
300 W1=Ø
310 W=Ø
320 B=Ø
330 J=Ø
340 Page
350 Viewport 45,210,25,100
360 Call "Max", A, M2, Z
370 Window 1,512,Ø,3
380 Axis 8,3/15
390 Move 1,Ø
400 For I=1 to 63
410 REM: Enter short space data file, C, from A
420 For K=1 to 16
430 C(K)=A(J+K)
440 Next K
441 C1=SUM(C)/16
APPENDIX B (Cont'd.)

450 REM: Now multiply by the triangle envelop
460 C=C*T
470 REM: Transform the short sequence
480 Call "FFT, C"
490 Call "POLAR", C,M,P
500 For L=1 to 9
510 V(L,I)=M(L)^2
520 Next L
530 REM: M is the magnitude vector and C is the phase
531 If Cl=> then 539
532 For L=2 to 9
533 If M(L)^2= D1(L)*M(1)^2 then 536
534 M(L)=0
535 Go to 537
536 M(L)=SQR(M(L)^2-D1(L)*M(1)^2)
537 Next L
538 Go to 600
539 For L=2 to 9
540 If M(L)^2=>D2(L)*M(1)^2 then 543
541 M(L)=0
542 Go to 544
543 M(L)=SQR(M(L)^2-D2(L)*M(1)^2)
544 Next L
600 For L=1 to 9
610 X(L)=M(L)*COS(P(L))
620 Y(L)=M(L)*SIN(P(L))
630 Next L
640 REM: Data is now ready to be packed
650 REM: Pack the data to be inverse transformed
660 Call "INLEAU", X,Y,C
670 Call "IFT,C"
680 REM: Add into B
690 For K=1 to 16
700 B(J+K)=B(J+K)+C(K)
710 Next K
720 J=I*8
730 Next I
740 For I=1 to 512
750 Draw I,B(I)
760 Next I
770 Home
780 For I=1 to 9
790 For J=1 to 63
800 W(I)=W(I)+V(I,J)
810 Next J
820 Next I
830 W=W/63
840 For I=1 to 9
APPENDIX B (Cont'd.)

850 Print using 860:I,W(I)
860 Image "W("1D,")="",4D.4D
870 Next I
880 REM: Examine standard deviations of the spectra
890 For I=1 to 9
900 W1(I)=0
910 For J=1 to 63
920 W1(I)=W1(I)+(V(I,J)-W(I))†2
930 Next J
940 W1(I)=SQR(W1(I)/62)
950 Next I
960 For I=1 to 9
970 Print using 980:I,W1(I)
980 Image "Sigma("1D,")="",6E
990 Next I
1000 For J=1 to 9
1010 For I=1 to 63
1020 Q(I)=V(J,I)
1030 Next I
1040 Call "MAX",Q,M1,L
1050 M5(J)=M1
1060 Next J
1070 For I=1 to 9
1080 Print using 1090:I,M5(I)
1090 Image "MAX("1D,")="",4D.4D
1100 Next I
1110 A+B
1120 Al=0
1130 S1=0
1140 For I=9 to 504
1150 Al=Al+A(I)/496
1160 Next I
1170 For I=9 to 504
1180 S1=S1+(A(I)-Al)†2
1190 Next I
1200 S1=SQR(S1/495)
1210 W=0
1220 W1=0
1230 Print "JJJ"
1240 Print "Mean=",Al
1250 Print "Std.Dev.=",S1
1260 Go to 280
1270 END
APPENDIX C

Selective Smoother's Normalized Spectral Weights

<table>
<thead>
<tr>
<th>Approximate frequency (mm(^{-1}))</th>
<th>Table 4 (\bar{x} + 2s) power spectrum for densities &lt;1.0</th>
<th>Table 5 (\bar{x} + 2s) power spectrum for densities &gt;1.0</th>
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<tbody>
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<td>1.0</td>
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<td>.006099</td>
<td>.006516</td>
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<td>.001673</td>
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<td>.001728</td>
</tr>
<tr>
<td>1.6</td>
<td>.006616</td>
<td>.001108</td>
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