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Evaluation of some multivariate CUSUM schemes

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by

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1.2 Abstract

This paper will review and compare five multivariate CUSUM techniques. Two of these are proposed by Crosier (1986), the multivariate CUSUM and the CUSUM of T (COT). It will also compare two proposed by Pignatiello (1986), the multivariate CUSUM #1 (MC1) and the multivariate CUSUM #2 (MC2). The fifth method which will be compared is the multivariate Shewhart method. A discussion of the method of computation and a comparison of results of all the above methods using the same data set will be included. Additionally, a short commentary on the cusum method by Woodall and Ncube is enclosed. Graphical interpretation is also provided to make differences more readily apparent.

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2. INTRODUCTION

The Cumulative Sum (CUSUM) chart is used to maintain control of a process. It is generally more advantageous to use than an ordinary Shewhart chart since it can be equally effective at less expense. It has the ability to pick up a sudden or persistent change more quickly than a comparable Shewhart chart and it can also be more precise as to when the change took place in the process.

When several variables are involved, with correlation existing between them, it is applicable to use multivariate cusum control charts. Recently, some schemes have been developed that take this correlation into account and which exhibit greater control than the past use of several univariate cusum charts. A measure of the power of a CUSUM procedure is Average Run Length (ARL), or the average number of sample points that will be plotted before a control scheme picks up a specific change in the process.

This thesis will involve the comparison of some multivariate CUSUM schemes by means of looking at some specific ARLS and associated plots of control ellipses to visually demonstrate the different schemes power in comparison to each other. This analysis will be based on two variables for simplicity at Type I error of .005 and will be done at several different levels of correlation to view differences in control.

Multivariate CUSUM methods to be compared are the following:

1. Multivariate CUSUM #1 (MC1) and Multivariate CUSUM #2 (MC2) by Joseph J. Pignatiello, Jr. (1986).

2. Multivariate CUSUM and the CUSUM of T with and without Fast Initial Response (FIR) feature by Ronald B. Crosier (1986).

3. A comment on two schemes, by William Woodall and Matoteng M. Ncube (1985), one basically involving use of univariate cusum and the other involving principal components.

3. Hotelling's T Square Statistic

Control charts, first developed by Walter Shewhart, are one of the most powerful and commonly used tools in statistical process control. The standard chart is usually used to detect significant shifts of the process level from a standard. Most charts are based on dealing with single variables. However, charts involving two or more characteristics measured on a process can also be used. One of these such charts that is known is the Hotelling's T square procedure developed by Harold Hotelling (1931).

The purpose or advantage in using multivariate control is that rather than having a chart for each variable under study there is one chart and one answer to the question of whether the process is in control or not. Furthermore, the Type I error is maintained, possible correlations that may exist between the variables under study are taken into account and lastly, should the process be out of control multivariate control charts provide some insight into the trouble or cause of the problem [see Jackson (1985)].

The following equation is Hotelling's T square:

$$T^2 = (x - \bar{x})' S^{*-1} (x - \bar{x})$$

where S^{-1} is the inverse of the covariance matrix and x is an observation vector. The T^2 distribution, derived by Hotelling, is a function of the number of variables and the number of observations used in estimating the covariance matrix. T^2 is related to the F distribution and can be approximated by the ChiSquare distribution with p degrees of freedom (for a reasonably sized base period). For this reason, the chi square average run lengths are used for the multivariate Shewhart method.

The multivariate Shewhart chart signals when $T^2 \geq$ SCL, the Shewhart control limit being:

$$T^2 = [p(n-1)/(n-p)]\, F_{p,\, n-p}$$

4. UNIVARIATE CUMULATIVE SUM CHART

The Cumulative Sum chart was first proposed by a British statistician, E.S. Page (1954). The CUSUM chart, unlike the Shewhart chart which is based on the just the last observation or subgroup, is based on all the data. Its primary use is for maintaining the current flow of a process. The Cumulative Sum chart, (CUSUM) advantages over the Shewhart chart are that it is usually just as effective as a Shewhart chart at less expense. It is less expensive because it detects sudden and persistent changes in the process average more rapidly. The CUSUM typically has smaller average run lengths than the standard control chart for detecting certain kinds of shifts in the process. Another advantage of the cusum chart is that it can pinpoint the time the change occurred more closely.

The method involves the use of a V mask which is used to decide if a significant change in the process has occurred at each sampling. The sum of the deviations from some reference value are plotted sequentially:

$$S_t = \sum_{i=1}^{t} (x - a)$$

S_t ,the sum of the deviations, is calculated by computing the summation of the differences from the aimed at target, a, and then plotted at time t on the control chart.

The point zero on the V mask is placed on the last point plotted . If any of the previous points plotted are outside the V mask, then the process is out of control. For instance, if the lower part of the mask covers a point, then the process has shifted upwards. If the upper part of the mask covers a point, the process has shifted downwards.

The V mask is based on two parameters, d and θ. These parameters determine the V mask's shape. Both, are determined by the type of operating characteristics desired for the control chart. The equations for these parameters follow:

$$d = (2/\delta^2) \ln((1-\beta)/\alpha)$$

$$\theta = \tan (D /2k)$$

$$\delta = D\sqrt{n}/\sigma$$

where α represents the Type I error, β the Type II error, k being a scale factor and D being the shift from the process mean.

5. MC2

5.1 The statistic.

This method, due to Pignatiello (1986) uses the square of the sample mean from the target value and then accumulates the values of Hotelling's statistic.

Using :

$$d_i = n\,[x_i - a]'\, V^{-1}\, [\, x_i - a\,]$$

where n is the size of the subgroup.

The statistic is as follows:

$$t = \sum_{i=1}^{t} d_i \qquad (\ 5.1\ )$$

The statistic is zeroed out when $\Delta t - t(\ p + \lambda^2/2) \leq 0$ where λ^2 is a specified distance from the target value. Thus, the number for which one would use to zero out is changing with each additional observation. This procedure will be illustrated with the data set provided in Table 3 for p=2 variables along with a covariance matrix of:

$$V = \begin{bmatrix} 1 & .5 \\ .5 & 1 \end{bmatrix}$$

The subgroup size will be n=1.

5.2 Example.

The first computation is as follows:

$$d_1 = 1\,[-1.19 \;\; .59\,]'\, V^{-1}\, [\,-1.19 \;\; .59\,] = 3.29$$

$$t_1 = \sum_{i=1}^{1} d_i = d_1 = 3.29$$ which is just Hotellings T^2 in this

first case. Then the check to see if the statistic should be reset to zero is,

$$d_1 - 1(\,2 + 1/2\,) \le 0$$

using $\lambda^2 = 1$ for good ARL's as suggested by Pignatiello in his paper. Then for the next observation,

$$d_2 = 1\,[\,-.8619 \;\; .4273\,]'\; V^{-1}\,[\,-.8619 \;\; .4273\,] = .96$$

$$t_2 = \sum_{i=1}^{2} d_i = d_1 + d_2 = 3.29 + .96 = 4.24$$

Checking to see if we need to zero out,

$$(\,d_1 + d_2\,) - 2(\,2 + \lambda^2/2\,) \le 0$$

$$[4.24 - 2(2.5)] < 0$$

which is in fact less than zero and the statistic, t_2, becomes zero and the t representing time is also zeroed out.

5.3 Advantages.

The advantages of the MC2 over the traditional multivariate Shewhart charts are :

1. It cumulates past information from previous data.

2. This method can be designed to detect a specific shift in the process mean.

6. CUSUM OF T (COT)

6.1 The statistic.

Crosier (1986) proposes the CUSUM of T (COT) which is similar to MC2 except for the reference value k. k is defined as d/2 where d is the distance from mean to the target value a. It also involves the use of Hotelling's T^2 statistic. The statistic is defined as follows:

$$COT_i = \max(0,\ COT_{i-1} + T_i - k) \qquad (6.1)$$

where COT ≥ 0 and k ≥ 0. The statistic signals when $COT_i \geq h$, the decision parameter. This amounts to taking the square root of Hotelling's T^2, subtracting some set constant k, and adding to previous COT_{i-1} letting this be COT_i provided it is greater than zero, otherwise COT_i becomes reset to zero and then procedure continues with accumulating the T.

6.2 Example.

Using the same data as in the previous section,

for observation n=1, $T^2 = [\ -1.19\ \ .59\ ]'\ V^{-1}[\ -1.19\ \ .59\ ] = 3.2884$

Hence, T = 1.8134 and using k = 1.41 $COT_1 = .4034$

Continuing on with the next observation group,

$$T^2 = [\ .12 \quad .90\ ]'\ V^{-1}\ [\ .12 \quad .90\ ] = .9562$$

so that,

$$T = .9772 \text{ and then,}$$

$$COT_2 = \max(0,\ COT_1 + T_1 - k)$$

and $$COT_2 = \max(0,\ .4034 + .9772 - 1.41) < 0$$ so that COT_2 becomes zero.

6.3 Advantages.

The advantages of the COT are the same as MC2 plus:

1. The ability to use the Fast Initial Response (FIR) feature by Lucas and Crossier(1982a) which is discussed later in this paper.

2. This method can be used in conjunction with the Shewhart method as a combined Shewhart-COT easily.

3. Additionally, this method uses the reference value k.

6.4 FAST INITIAL RESPONSE (FIR).

The purpose of the FIR feature is to provide quicker detection of an initial off-aim condition at start up. For the COT procedure, quicker detection of an initial off-aim condition is obtained by starting with COT_0 equal to h/2 rather than zero. If the process is off aim, the CUSUM of T will signal more quickly because of the headstart. If the process is not off-aim, the headstart will probably be removed by subtraction of k at each observation. The use of the FIR feature significantly reduces the ARLS as can be seen by referring to Table 3.

7. MC1

7.1 The statistic.

MC1 differs from MC2 in that instead of the statistic being based on accumulated squared distances, MC1 is based on, Γ_t the square of the distance of the accumulated sample averages from a. [see Pignatiello (1986)].

Let:

$$C_t = \sum_{i=1}^{n} [x - a]$$

Then:

$$\Gamma_t = n/t (C_t' V^{-1} C_t)$$ which is the test statistic and

represents the square of the distance of the accumulated average vector from the target value. n, again represents the subgroup size. The test statistic, Γ_t is zeroed out every time that

$$\Gamma_t - (p + \lambda^2/2) \leq 0$$

where λ^2 is a specified distance from the target value as in section 5.1.

Using p=2, n=1, and $\lambda^2 = 0$ since Pignatiello recommends it in his paper. For the data in Table 3, the statistic Γ_t would be zeroed out at:

$$\Gamma_t - (2 + 0/6) \leq 0 \text{ or } \Gamma_t \leq 0.$$

7.2 Example.

$$C_1 = [x_1 - a] = [-1.19 \quad .59]$$

$$\Gamma_1 = 1/1 [-1.19 \quad .59]' V^{-1} [-1.19 \quad .59] = 3.29$$

then for the next group of observations,

$$C_2 = \Sigma_{i=1} [x_i - a] = [-1.19 \quad .59] + [.12 \quad .90]$$

$$= [-1.07 \quad 1.49]$$

$$\Gamma_t = 1/2 [x - a]' V^{-1} [x \quad a] = 1/2 C_2 V^{-1} C_2 = 3.31$$

Thus far, it has not been necessary to zero out the statistic. The procedure would continue on in this fashion. Further computation can be reviewed in Table 3.

7.3 Advantages.

This method can have an added advantage over the previously discussed methods in that it has a directional nature providing some indication of where the mean has shifted. It tends to have better ARLs since the method allows observations in the opposite direction from the target value to cancel each other out. This cancelling occurs more frequently as the process mean is on target.

8. CUSUM

8.1 The statistic.

The calculation involves the following:

$$C_i = + ([S_{i-1} + x_i - a]' V^{-1} [S_{i-1} + x_i - a]) ** .5$$

where $S_i = 0$ if $C_i \le k$, $S_0 = 0$, $k \ge 0$ and

$$S_i = [S_{i-1} + x_i - a] (1 - k / C_i) \text{ if } C_i \ge k.$$

The test statistic is $S_i' V^{-1} S_i$ which signals when it is greater than some specified H, being the decision interval. C_i represents the length of

$(S_{i-1} + x_i - a)$, where a is the target

value, S_i is the standard deviation, and x_i represent the observation for two variables.

8.2 Example.

Again, using the data from Table 3, first calculate:

C_1 based on $S_{i-1} = S_0 = 0.$

$$C_1 = ([x_1 - a]' V^{-1} [x_1 - a]) ** .5$$

$$= ([-1.19 \quad .59]' V^{-1} [-1.19 \quad .59]) ** .5$$

= 1.813 which greater than k =.5, k being some reference value.

Next,

$$S_1 = [S_0 + x_1 - a] (1 - k / C_1)$$

$$= [-1.19 \quad .59] (1 - .5 / 1.813)$$

$= [-.8619 \quad .4273]$ and finally the test statistic being:

$[-.8619 \quad .4273]' V^{-1} [-.8619 \quad .4273] = 1.3134$ which is less than h= 5.50 so that process is in control so far.

Next, compute $C_2 = ([[.8619 \quad .4273] + [.12 \quad .90] - [0 \quad 0]]' V^{-1} [[.8619 \quad .4273] + [.12 \quad .90] - [0 \quad 0]] **.5$

= 2.096 which is greater than k=.5 and hence

$$S_2 = [same] (1 - .5 / 2.096)$$

$$= [.5650 \quad 1.0108]$$

and test statistic is $S_2' V^{-1} S_2 = 1.5966$ less than h = 5.50 and process is still in control.

8.3 Advantages.

This method as much the same advantages of MC1 over the previous methods discussed. In addition it has the added advantage of being able to design schemes to detect specific shifts in mean vectors. It can use the FIR feature which makes it better than MC1 in comparison of ARLs. It also provides insight into the origin of the shift from the target value.

8.4 Fast Initial Response For Crosier's CUSUM

For Crosier's CUSUM, using the FIR requires running the COT method simultaneously in order to obtain the value for H which is the parameter that is used to provide the early detection.

H is the specified decision interval which is determined by it giving an acceptable on target ARL. When the FIR feature is used in this case H = H/2 and thereafter

$$H_i = \min [H, H_{i-1} + \max (0 , k* - Ti)] \text{ for } i=1,2,3....$$

k* is the k value for the COT procedure designed to detect the same deviation as the multivariate CUSUM scheme. It can be seen be seen by looking at table 5 of Crosier's paper that the ARL's are significantly reduced using the FIR feature. It has been stated that if the process is indeed off target that the FIR is very effective in the early detection of it. However, should the process not be off-aim, the ARL's are increased slightly. This can be compensated for by using a slightly bigger H value. The effect of the FIR on CUSUM can be seen in Table 1.

9. Comment on Multivariate CUSUM procedure by Woodall and Ncube.

Another multivariate CUSUM procedure has been proposed by Woodall and Ncube (1985). This method essentially obtains univariate CUSUM procedures for each variable separately and uses Bonferroni bounds for the results. They also did this for principal components of the original variables with generally smaller ARL's. A comparison of these results with the others would require a series of simulations which is beyond the scope of this thesis. Crosier(1986) has done some work along these lines and has concluded for his examples that the ARL's of the multivariate CUSUM were less or equal to those of Woodall and Ncube.

10. Graphs

To compare the methods, it is necessary to be able to obtain the ARLs for any specific distance from the aim for any procedure. These were were obtained by using polynomial interpolation, a numerical analysis technique, and graphical interpolation to determine distances based on tables provided by Pignatiello (1986) and Crosier (1985) which they arrived at by simulation. These are summarized in Table 1. Then, using appropriate covariance matrices depending on which correlation was be investigated, an equation for an ellipse was derived.

$$[m_1 - m_2]' \, [V^{-1}] \, [m_1 - m_2] = d^2$$

m_1 and m_2 representing the values of the variables.

Multiplying the above out led to a quadratic equation which was solved by supplying values for one of the unknowns. After the roots were obtained these were plotted to form the graph of the ellipses. For example, using r=.5 meant that the covariance matrix was as follows:

$$V = \begin{pmatrix} 1 & .5 \\ .5 & 1 \end{pmatrix}$$

After getting the inverse of the covariance matrix and substituting in the above equation the following was obtained.

$$4/3\ m_1^2 - 4/3\ m_1 m_2 + 4/3\ m_2^2 = d^2$$

If we set $m_2 = k$ and solve for m_1 the equation looks like:

$$4/3\ m_1^2 - 4/3\ m_1 k + 4/3\ k - d^2 = 0$$

where a = 4/3, b = -4 /3 k and c = 4/3 k - d^2 for use in the quadratic formula.

Points were then plotted for each ellipse with r = 0, .5, and .9 and corresponding d values for the different ARLS of 5, 10, 20, and 50. Since it was clear from onset that Crosier's CUSUM and COT were considerably better with the use of the FIR feature, both were graphed with the FIR feature in comparing MC1 and MC2.

Figure 1 through 9 are graphs of CUSUM w/FIR, COT w/FIR, MC1, MC2, and Multivariate Shewhart using different correlations and ARLs. Figure 1 has r=0 and a ARL=10, figure 2 has r=0 and ARL=20 and figure 3 has r=0 and ARL=50.

Figure 4 has r=.5 and ARL=10, figure 5 has r=.5 and ARL=20 and figure 6 has r=.5 and ARL=50. Figure 7 has r=.9 and ARL=10, figure 8 has r=.9 and ARL=20 and figure 9 ha r=.9 and ARL=50. Figure 10 through 12 are all graphs of the CUSUM w/FIR for r=0, r=.5 and r=.9 respectively.

After reviewing the graphs, it can be observed that Crosier's CUSUM is consistently the best of all the methods followed by Pignatiello's MC1, Crosier's COT, Pignatiello's MC2 and then the multivariate Shewhart for these ARLS that were investigated.

In comparison of these four methods, Polynomial Interpolation was used to get more exact ARLs for specific ARLs desiring to avoid simulation but allowing the ability to compare the power of the methods to each other. However, it was necessary to adjust some of these numbers by looking at graphs as the numerical analysis was not accurate enough for values on the ends of the interval. Specifically, the ARL = 5 was a problem and is clearly on the tail of the interval which is 1.2 to 200 approximately based on charts accompanying the papers. There probably could be some contention in a few cases about which method proved better than another but the only way to know would to be to use simulation or perhaps find a better numerical analysis method to do the interpolation. Regardless, there was no doubt which method gave best results and which the worst.

11. Discussion.

All four methods, the CUSUM of T, CUSUM, MC1, and MC2 use $\alpha = .005$ for the Type I error in these comparisons which procludes an ARL of 200 when the process is on aim. All methods incorporate a distance formula of the type:

$$[x - a]' V^{-1} [x - a]$$

In the case of MC2 and COT, it is the these distances which are accumulated and in the case of MC1 and the multivariate CUSUM, the observation vectors are summed prior to computing the distance. This is the preferred method. Within these pairs, Crosier's techniques are always slightly better in terms of ARL's than Pignatiello's. With the use of FIR, these differences are more pronounced. Although Crosier's multivariate CUSUM with FIR has the smallest ARL's, it is also has the most complex formula but the use of a computer will make these differences in complexity negligible.

All method's ARLS are dependent on the mean vector and the covariance matrix V only through the noncentrality parameter:

$$d \; + [x - a]' V^{-1} [x - a] ** .5$$

In addition to losing power because it works with each observation vector separately, the multivariate Shewhart has the disadvantages of it lacking robustness and it being sensitive to multivariate outliers. It does a fair job for detecting larger departures from the target value but the ARLS for smaller shifts are not very good, especially, when one compares the CUSUM/FIR to it. See Tables 1 and 2.

The final consideration should be the ease of the method and the amount of control or power being awarded for this ease or visa versa.

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Table 1.

Average Run Lengths for the Different Methods

d	chisquare	MC2	Cot	Cot/FIR	MC1	Cusum	Cusum/FIR
0.00	200.00	202.25	201.00	182.00	203.89	200.00	183.00
0.50	115.28	90.26	84.40	71.80	30.98	28.80	22.90
1.00	41.49	25.78	22.10	15.90	9.67	9.35	6.62
1.50	15.87	9.74	9.33	5.99	4.94	5.94	3.80
2.00	7.02	4.81	5.47	3.41	3.15	4.20	2.42
2.50	3.63	2.94	3.81	2.38	2.25	3.26	1.92
3.00	2.20	2.03	2.93	1.84	1.74	2.78	1.57
4.00	1.23	1.27	2.08	1.29	1.22	2.10	1.20

Table 2. Distances for specific ARLS.

ARL	COT	COT/FIR	CUSUM	CUSUM/FIR	MC1	MC2
5	2.145	1.75	1.75	1.30	1.489	1.972
10	1.452	1.30	.969	.8345	.9782	1.4824
20	1.047	.9123	.6163	.537	.6427	1.1073
50	.6831	.6146	.3676	.326	.3659	0.7409
100	.4375	.385	.2103	.175	.1848	0.4542

Table 3. Results of different methods using Crosier's data set.

n	x1	x2	T^2	T	MC2	COT	COT/FIR	MC1	CUSUM w/wo FIR	H for CUSUM w/ FIR
1	-1.19	0.59	3.29	1.81	3.29	0.40	2.42	3.29	1.31	2.75
2	0.12	0.90	0.96	0.98	0.00	0.00	1.99	3.31	1.60	3.20
3	-1.69	0.40	4.92	2.22	4.92	0.81	2.80	7.29	3.20	3.20
4	0.30	0.46	0.22	0.47	5.14	0.00	1.86	5.79	2.83	4.40
5	0.89	-0.75	2.70	1.64	7.84	0.23	2.09	2.01	0.69	4.40
6	0.82	0.98	1.11	1.05	0.00	0.00	1.73	2.03	0.89	4.70
7	-0.30	2.28	7.96	2.82	7.96	1.41	3.14	5.68	3.13	4.70
8	0.63	1.75	3.14	1.77	11.1	1.77	3.50	7.77	4.33	4.70
9	1.56	1.58	3.29	1.81	14.4	2.18	3.91	8.75	5.14**	4.70
10	1.46	3.05	9.31	3.05	23.7	3.82	5.55*	13.85*	7.68***	4.70
Criteria for signal:				SCL	Chisq df=tp	H	H	Chisq df=2	H**	
				3.2552	none out	4.04	4.04	10.6	5.5	

*** CUSUM w/o use of FIR
** CUSUM w/FIR
* COT/FIR

It should be noted that H, the criteria for determining if the CUSUM is out is one value for the CUSUM w/o FIR, 5.5. However, with the use of the FIR feature H changes with each new observation. Hence, the last column.

note: The data is based on unit variation and correlation of r=.5.

The population mean is (0,0) for the first five observations and then (1,2) for the last five observations (Crosier, 1986).

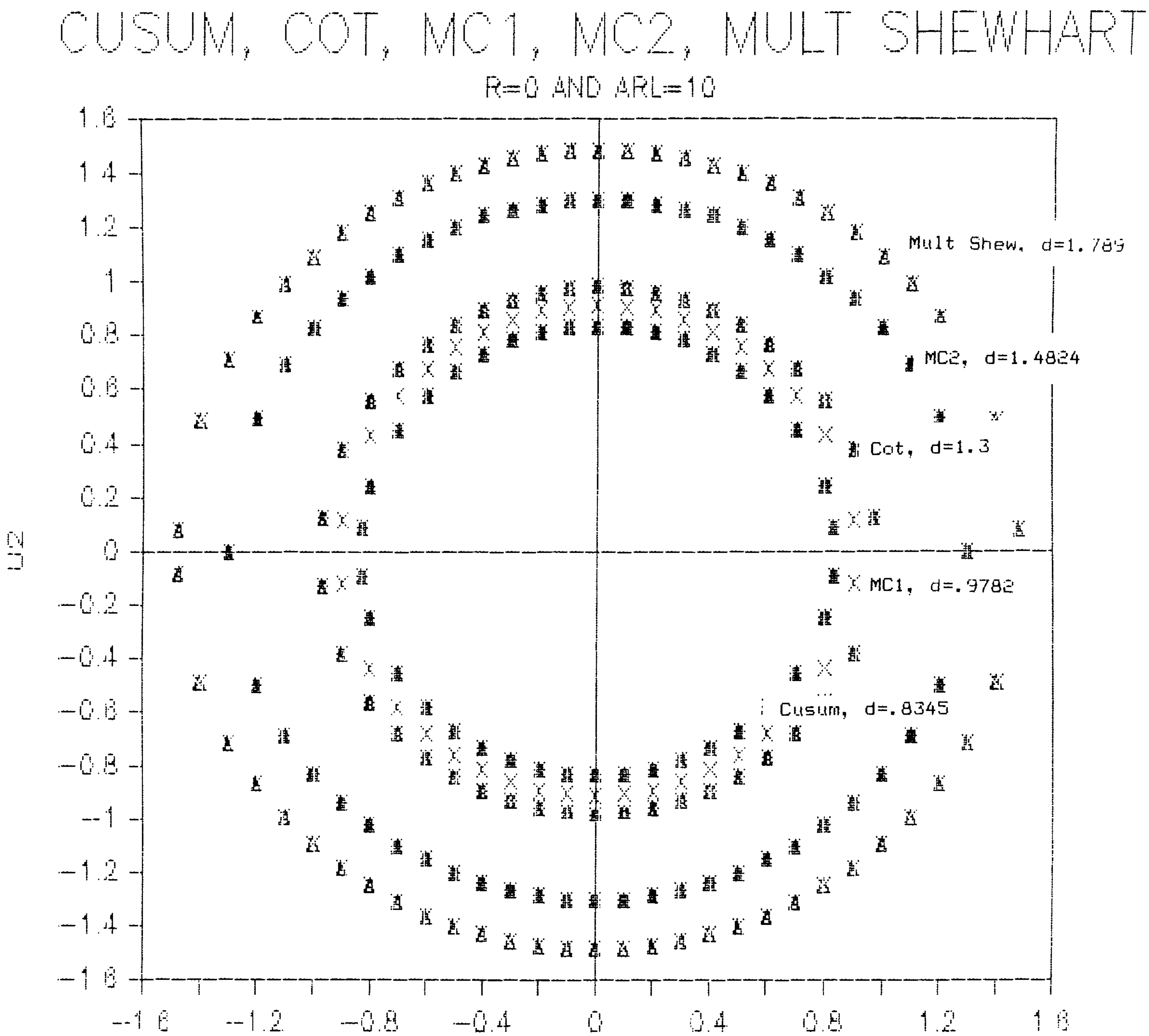
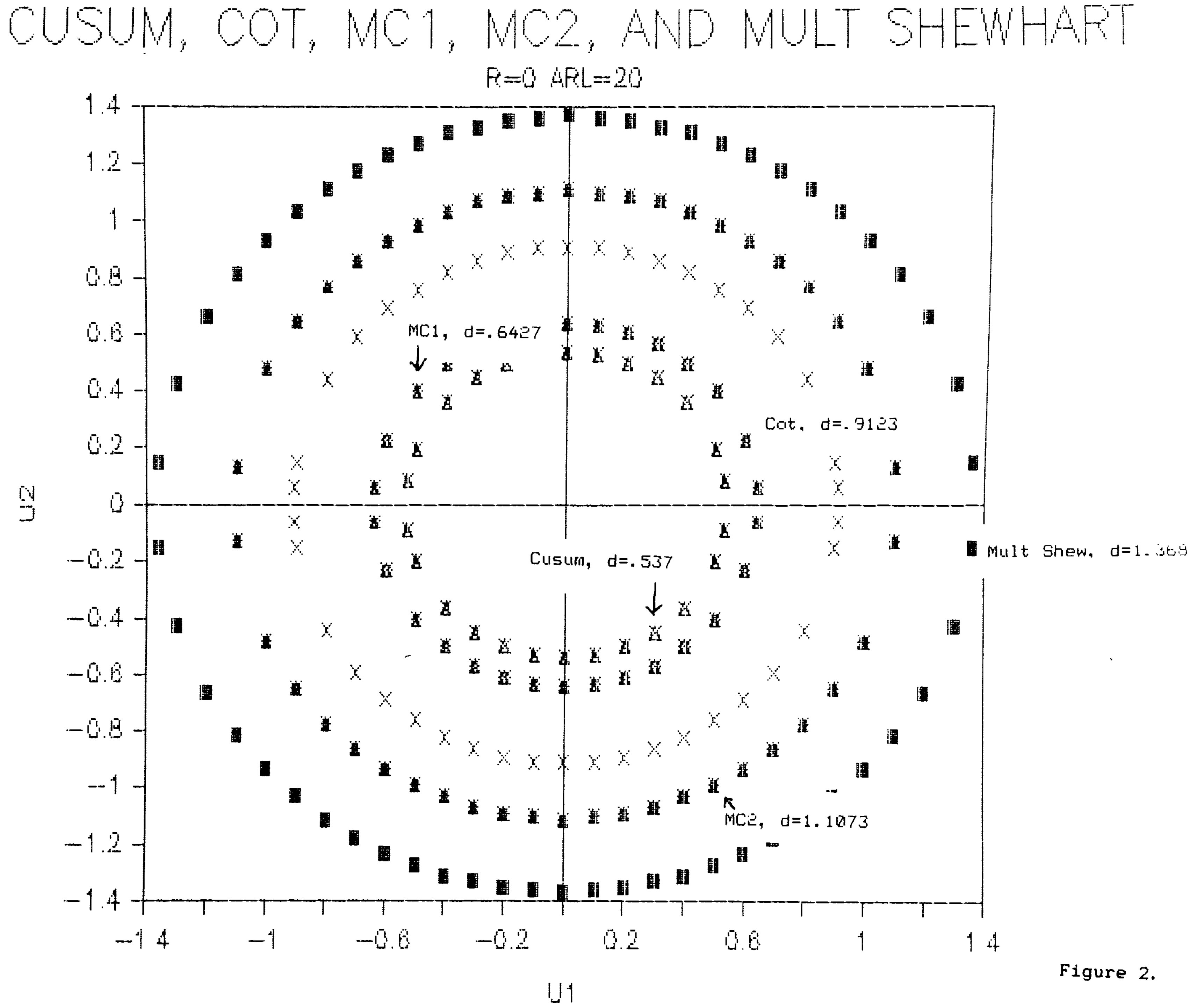

CUSUM, COT, MC1, MC2, MULT SHEWHART
R=0 AND ARL=10
Mult Shew, d=1.789
MC2, d=1.4824
Cot, d=1.3
MC1, d=.9782
Cusum, d=.8345
U2
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
-1.6
-1.6
-1.2
-0.8
-0.4
0
0.4
0.8
1.2
1.6

Figure 1.



CUSUM, COT, MC1, MC2, AND MULT SHEWHART
R=0 ARL=20
MC1, d=.6427
Cot, d=.9123
Cusum, d=.537
MC2, d=1.1073
Mult Shew, d=1.369
U2
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
U1
-1.4
-1
-0.6
-0.2
0.2
0.6
1
1.4

Figure 2.

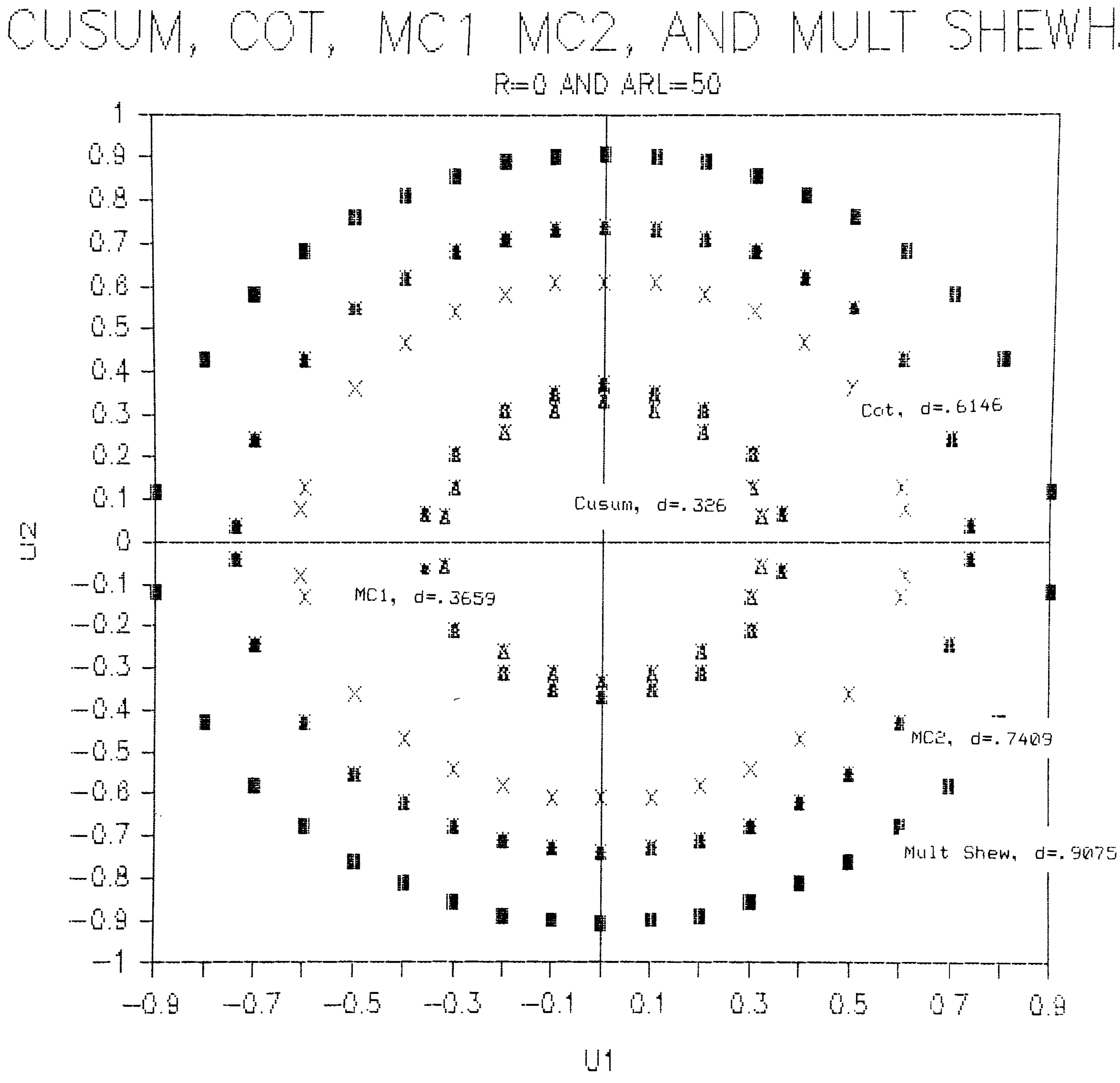

CUSUM, COT, MC1 MC2, AND MULT SHEWHART
R=0 AND ARL=50
Cot, d=.6146
Cusum, d=.326
MC1, d=.3659
MC2, d=.7409
Mult Shew, d=.9075
U2
U1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
−0.1
−0.2
−0.3
−0.4
−0.5
−0.6
−0.7
−0.8
−0.9
−1
−0.9
−0.7
−0.5
−0.3
−0.1
0.1
0.3
0.5
0.7
0.9

Figure 3.

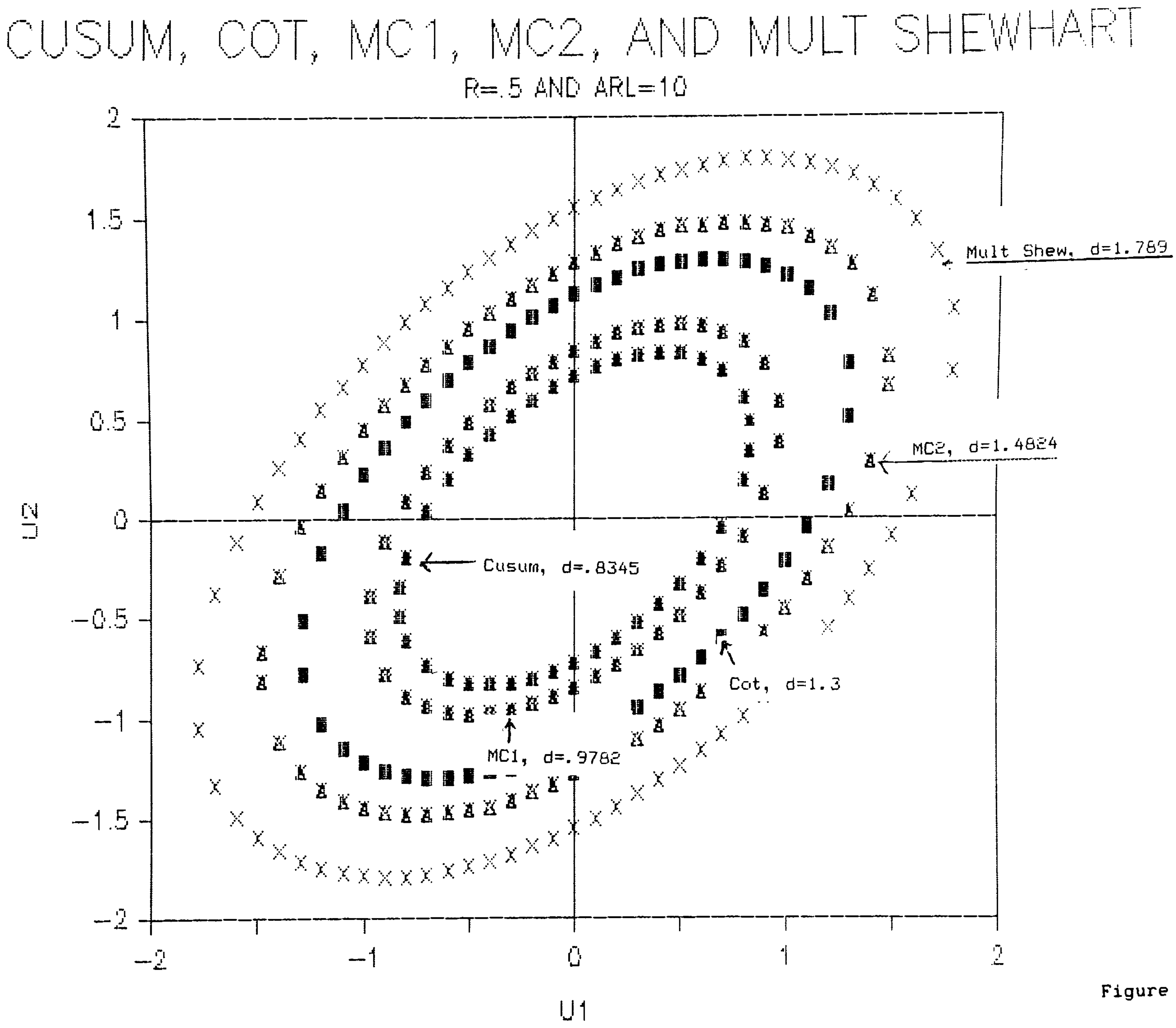
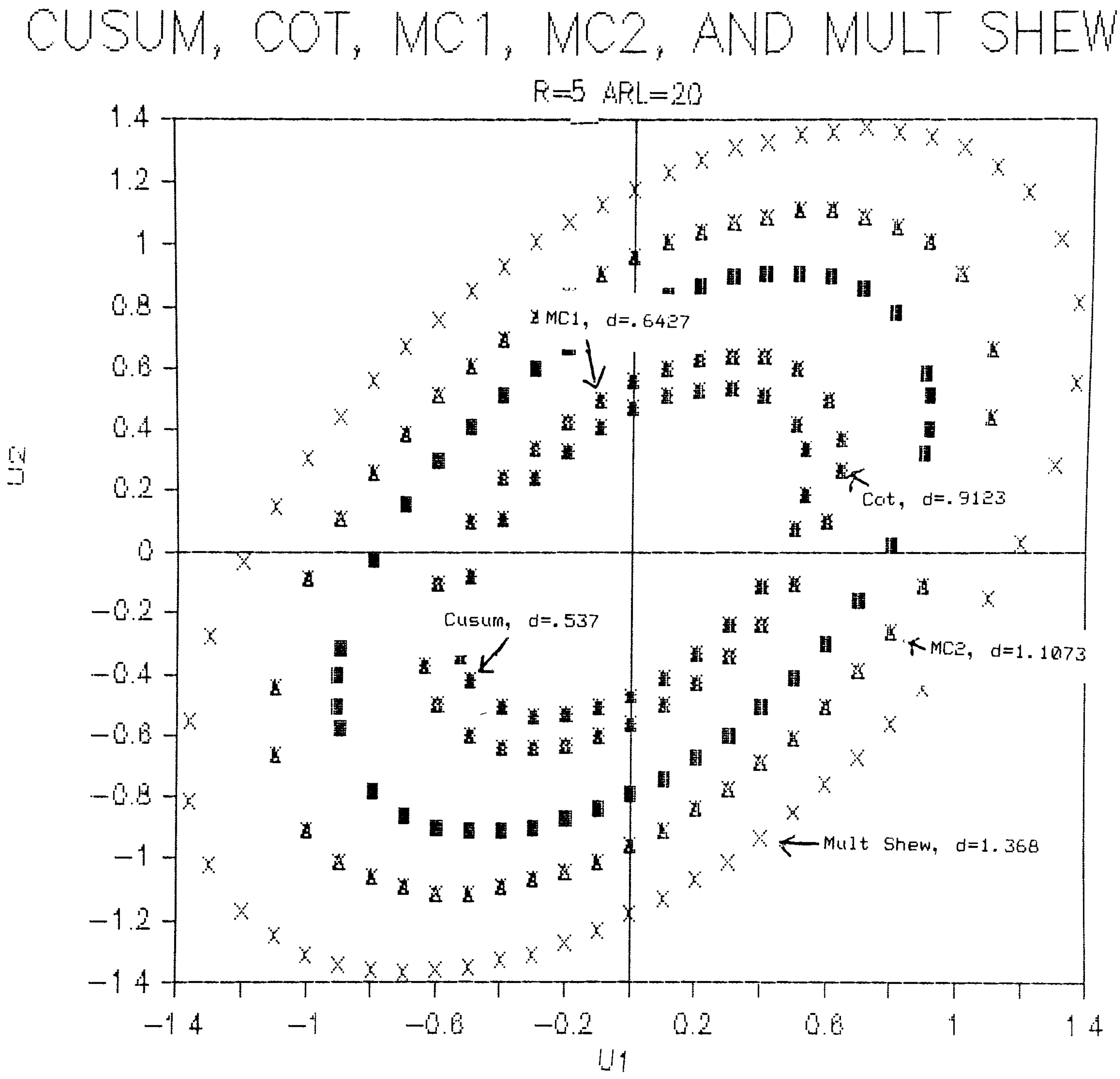

CUSUM, COT, MC1, MC2, AND MULT SHEWHART
R=.5 AND ARL=10
Mult Shew, d=1.789
MC2, d=1.4824
Cusum, d=.8345
Cot, d=1.3
MC1, d=.9782
U2
U1
2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
−2
−1
0
1
2

Figure 4.


CUSUM, COT, MC1, MC2, AND MULT SHEWHART
R=5 ARL=20
MC1, d=.6427
Cot, d=.9123
Cusum, d=.537
MC2, d=1.1073
Mult Shew, d=1.368
U1
U2

Figure

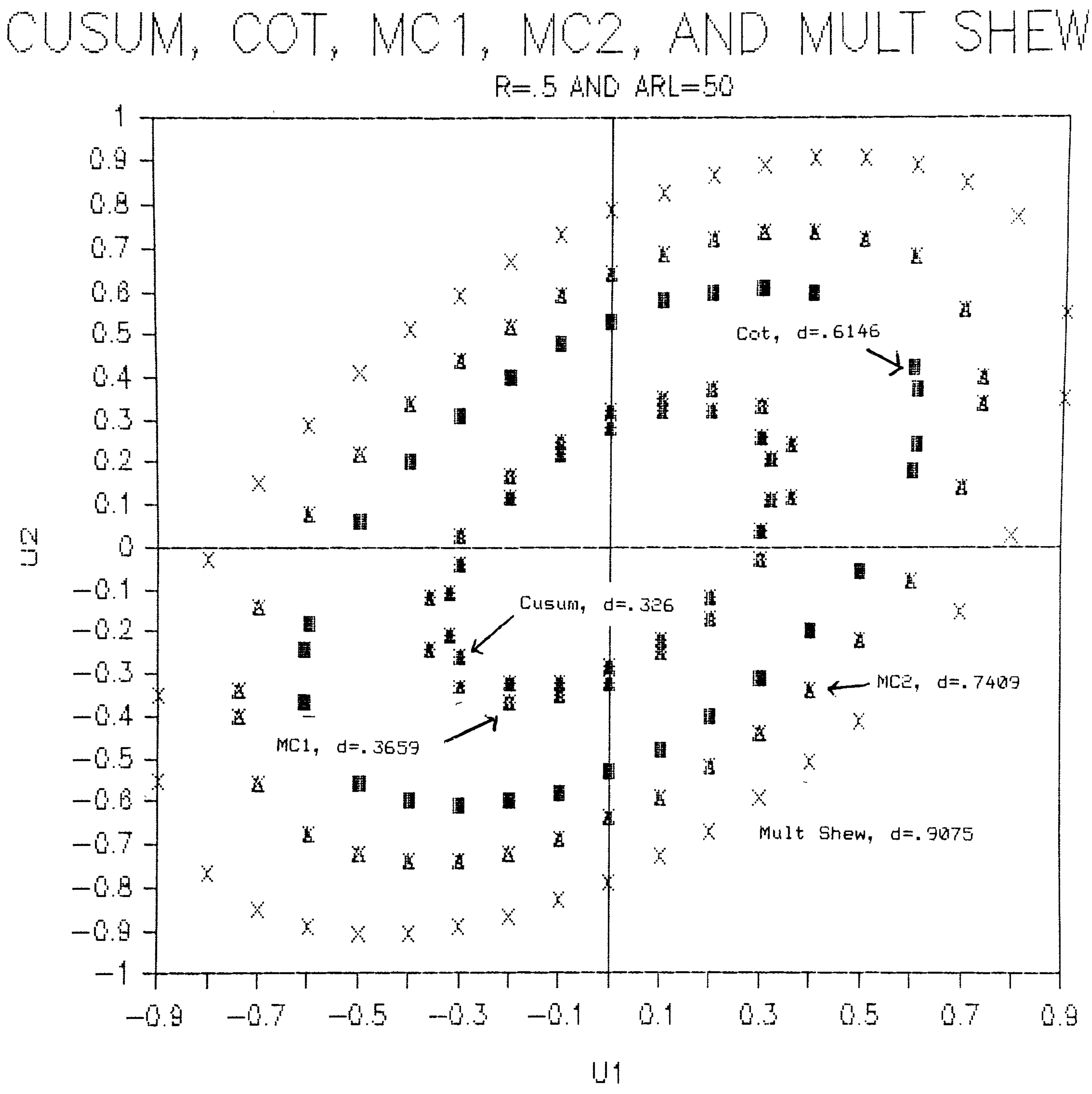

CUSUM, COT, MC1, MC2, AND MULT SHEWHART
R=.5 AND ARL=50
U2
U1
Cot, d=.6146
Cusum, d=.326
MC2, d=.7409
MC1, d=.3659
Mult Shew, d=.9075
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
−0.1
−0.2
−0.3
−0.4
−0.5
−0.6
−0.7
−0.8
−0.9
−1
−0.9
−0.7
−0.5
−0.3
−0.1
0.1
0.3
0.5
0.7
0.9

Figure 6.

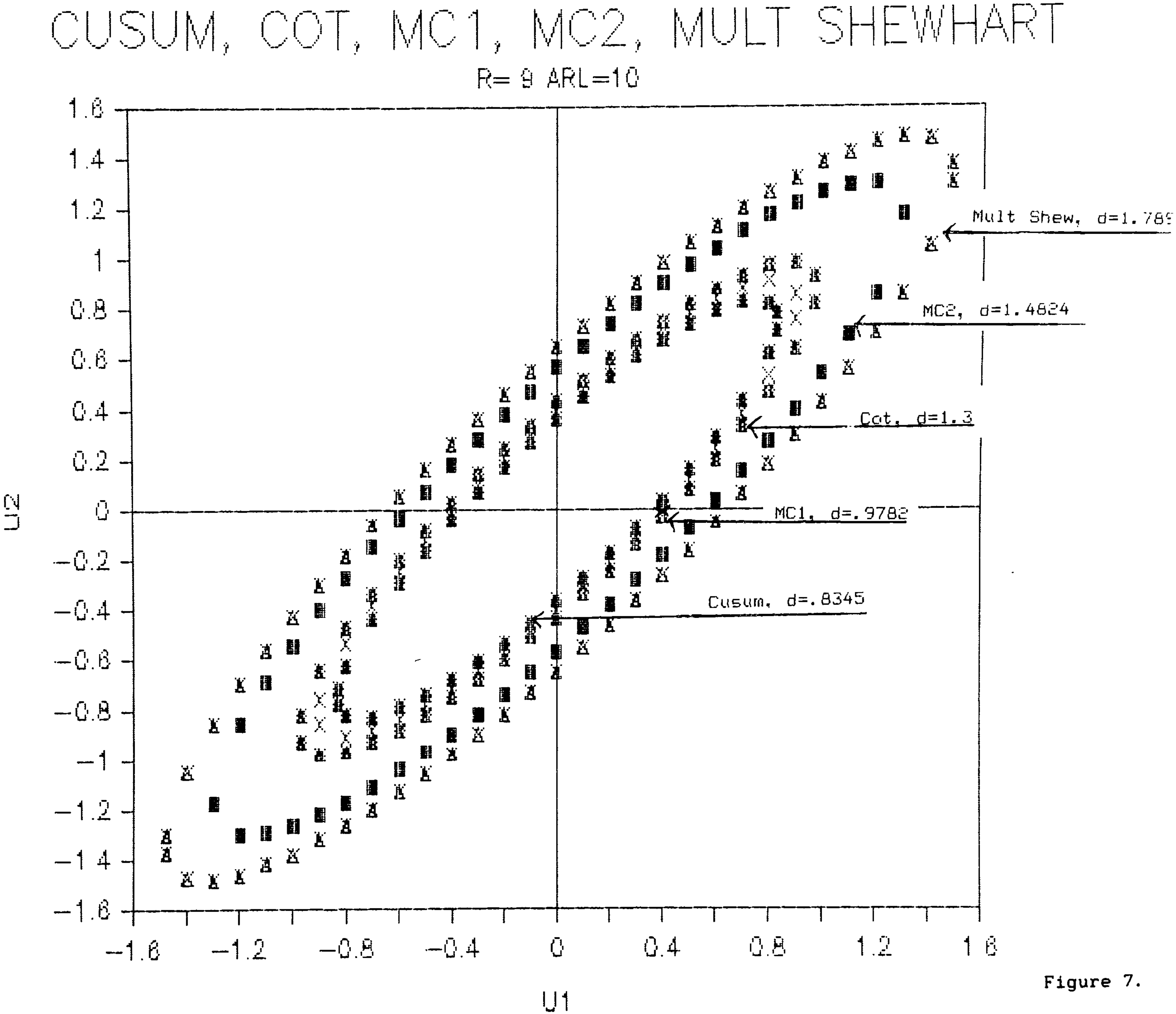

CUSUM, COT, MC1, MC2, MULT SHEWHART
R= 9 ARL=10
Mult Shew, d=1.789
MC2, d=1.4824
Cot, d=1.3
MC1, d=.9782
Cusum, d=.8345
U1
U2

Figure 7.

CUSUM, COT, MC1, MC2, AND MULT SHEWHART

R= 9 AND ARL=20

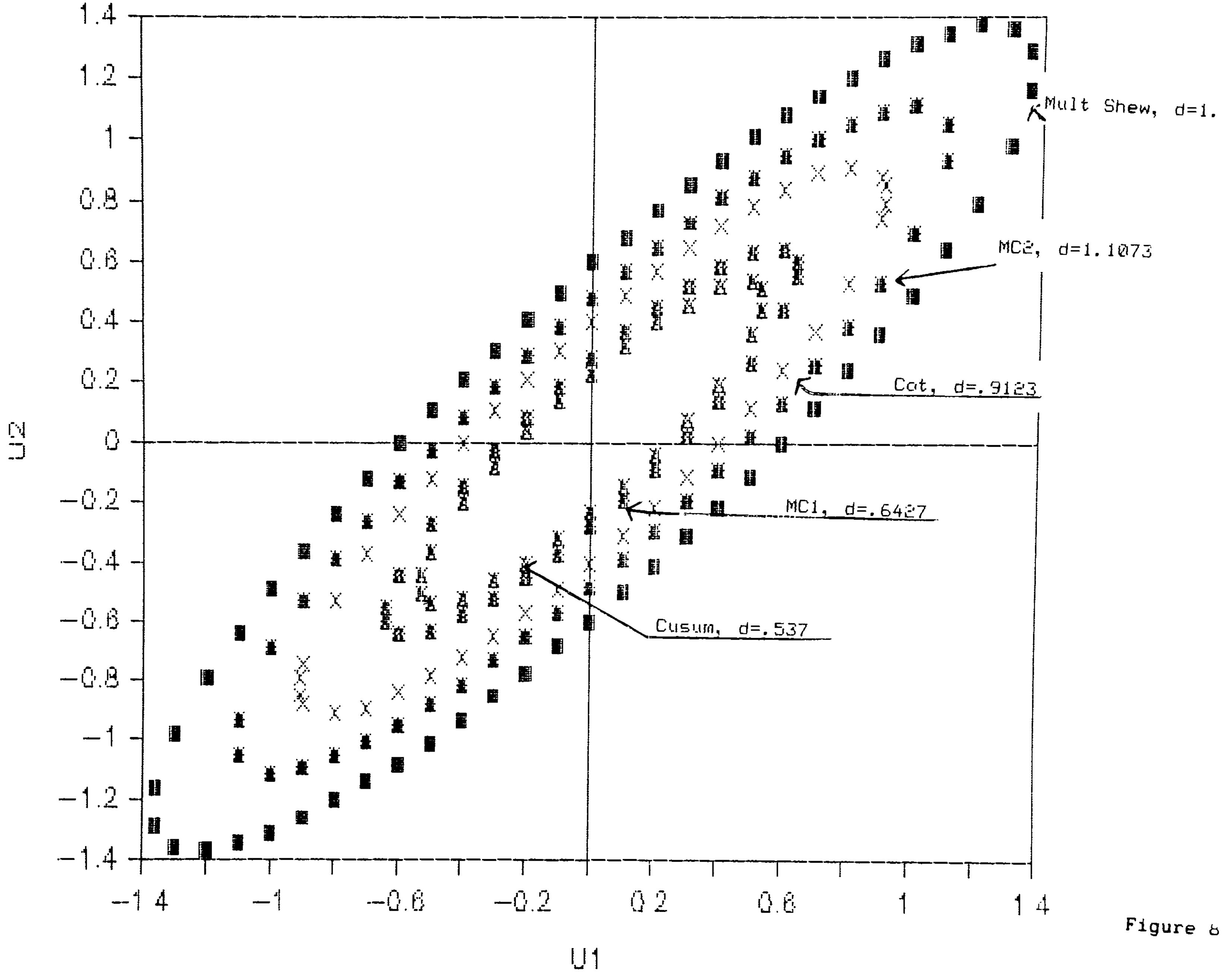


Figure 6.

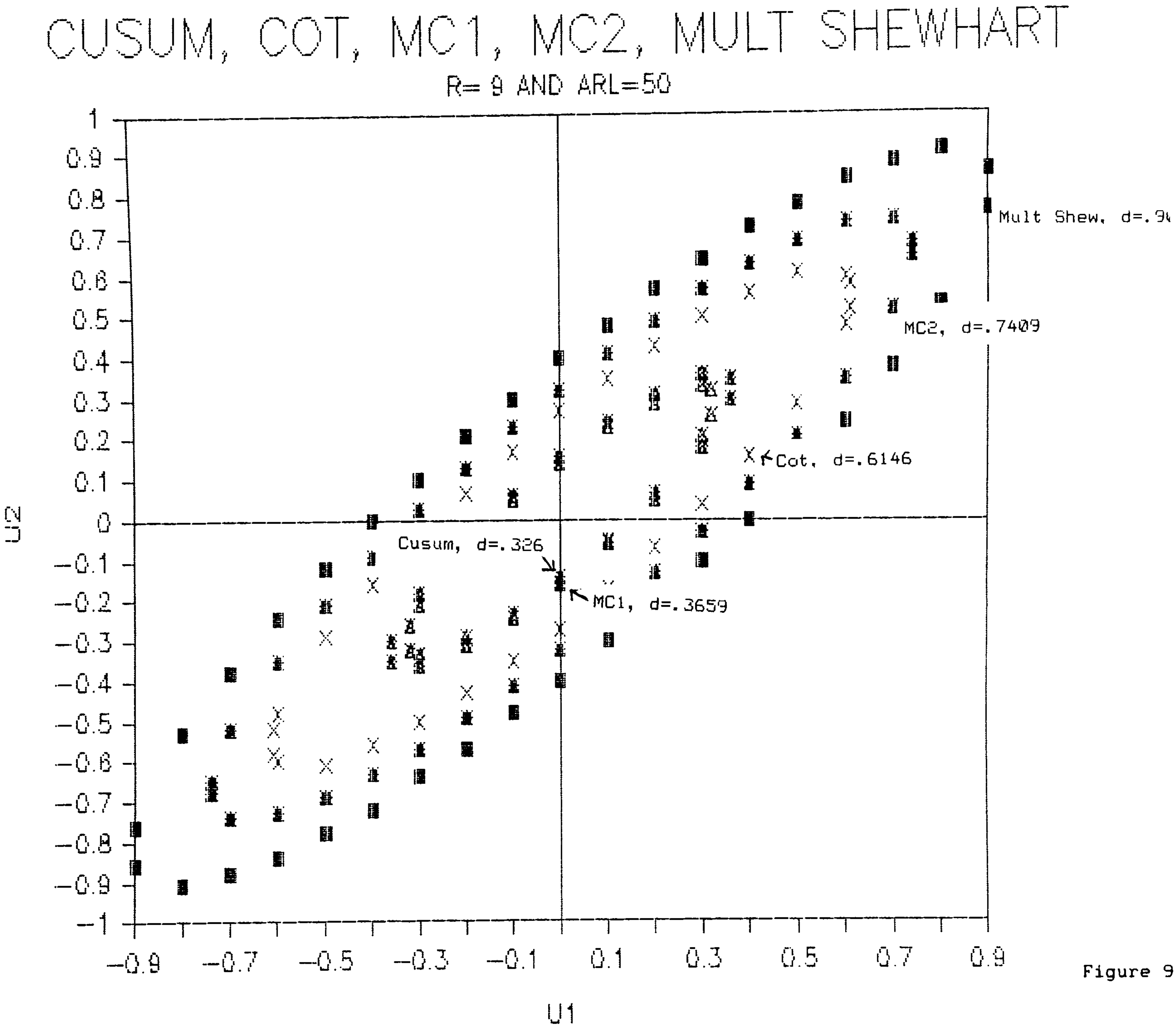

CUSUM, COT, MC1, MC2, MULT SHEWHART
R= 9 AND ARL=50
Mult Shew, d=.9
MC2, d=.7409
Cot, d=.6146
Cusum, d=.326
MC1, d=.3659
U2
U1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
-0.8
-0.9
-1
-0.9
-0.7
-0.5
-0.3
-0.1
0.1
0.3
0.5
0.7
0.9

Figure 9.

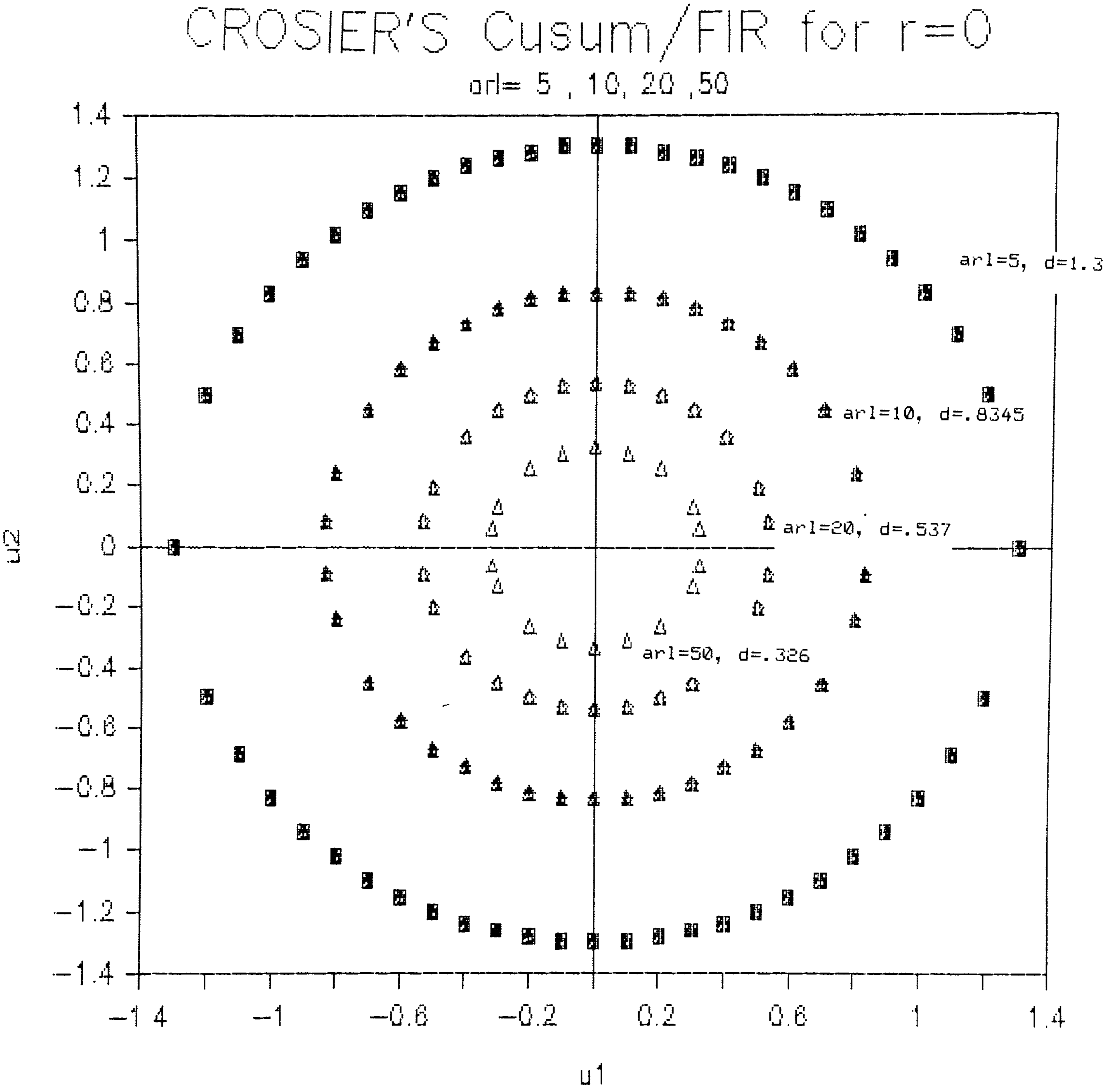

CROSIER'S Cusum/FIR for r=0
arl= 5 , 10, 20 ,50
arl=5, d=1.3
arl=10, d=.8345
arl=20, d=.537
arl=50, d=.326
u2
u1
1.4
1.2
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1
−1.2
−1.4
−1.4
−1
−0.6
−0.2
0.2
0.6
1
1.4

Figure 10.

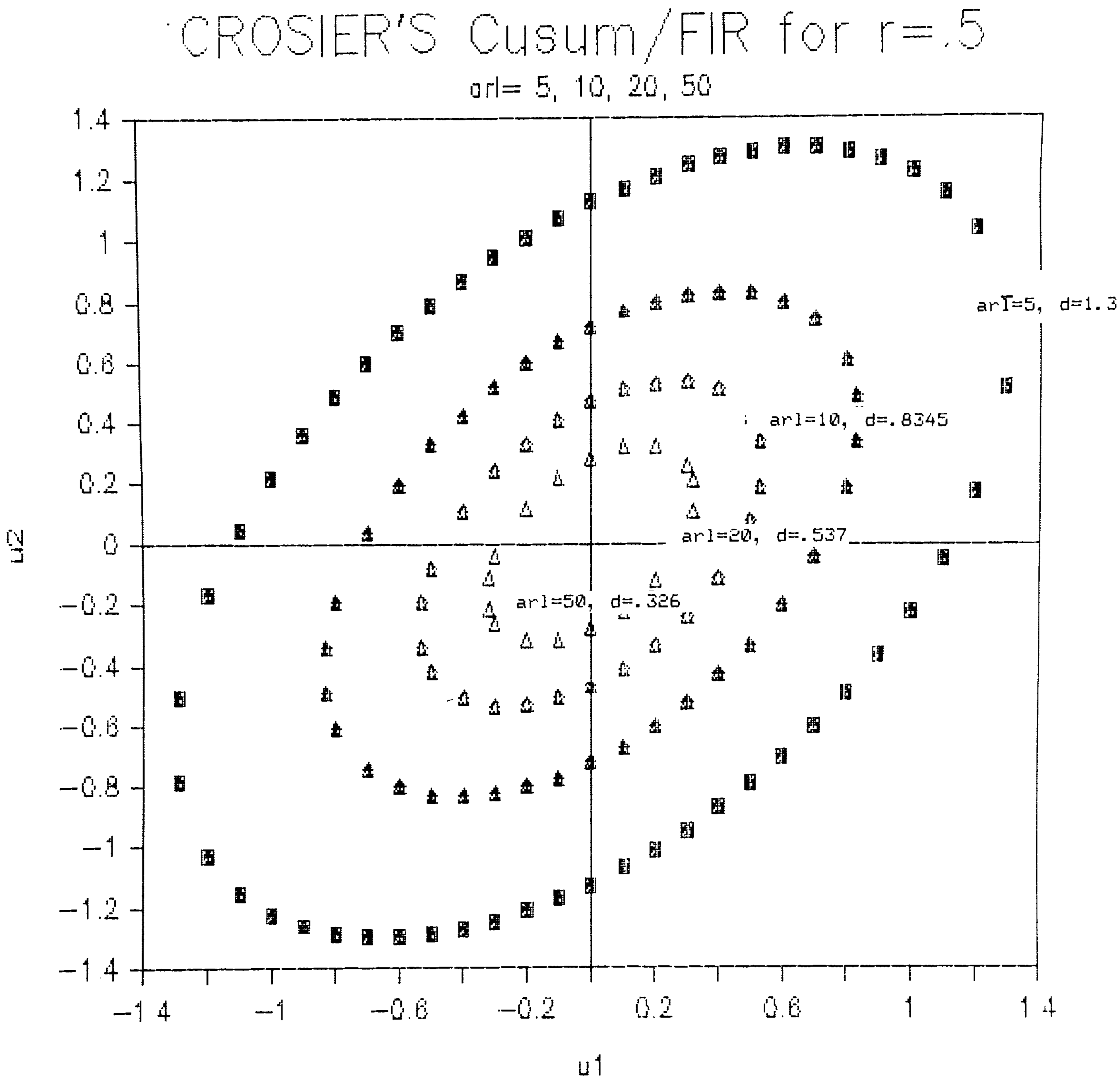

CROSIER'S Cusum/FIR for r=.5
arl= 5, 10, 20, 50
arl=5, d=1.3
arl=10, d=.8345
arl=20, d=.537
arl=50, d=.326
u1
u2
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
-1.4
-1
-0.6
-0.2
0.2
0.6
1
1.4

Figure 11.

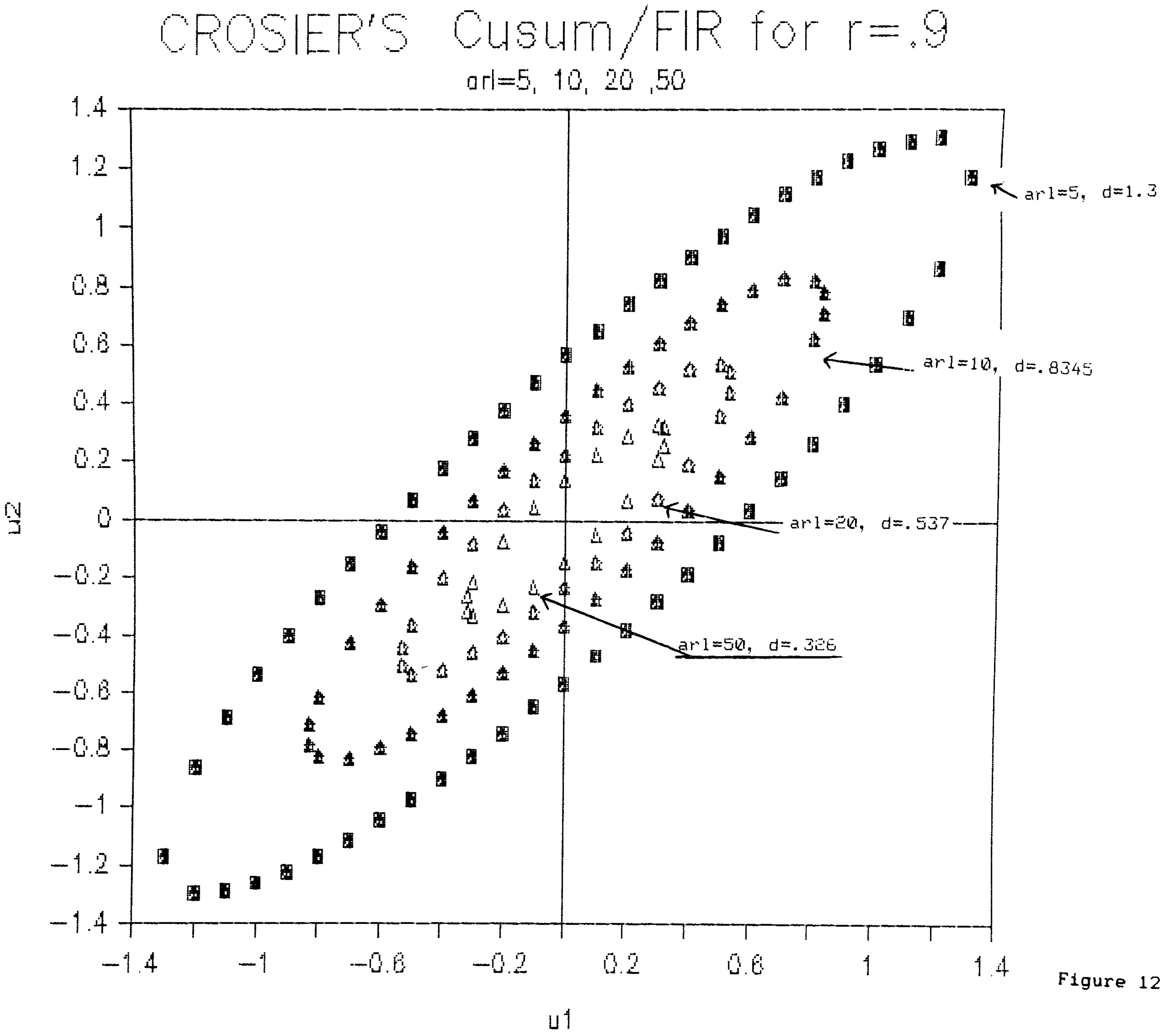

CROSIER'S Cusum/FIR for r=.9
arl=5, 10, 20 ,50
arl=5, d=1.3
arl=10, d=.8345
arl=20, d=.537
arl=50, d=.326
u1
u2
1.4
1.2
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1
−1.2
−1.4

Figure 12.