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NONLINEAR CUSP DIFFRACTION CATASTROPHE
AND VORTEX QUADRUPOLES FROM
A SMOOTH INITIAL BEAM

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Vortex quadrupoles and a nonlinear optical cusp diffraction catastrophe were observed
at the output face of a self-defocusing nonlinear medium. The initial beam had a smooth
cross-sectional intensity profile, but was elongated to an aspect ratio of 2 : 1. The power-
dependent evolution of the beam is described.

Keywords: Optical vortex; optical catastrophe; phase singularity; nonlinear refraction;
self-action effect; self-defocusing; thermal nonlinearity.

OCIS numbers: 190.4420; 190.5940; 080.1510

1. Introduction

Self-action effects such as self-defocusing\(^{40,30,41,65,3,20}\) and self-bending\(^{36,14}\) are
well-known nonlinear optical phenomenon dating back to the 1960s. Until recently the so-called blooming or lensing effect of a symmetric beam has been a
main topic of investigation. A smaller number of experiments using asymmetric beams were reported (see, for example Khoo\(^{37}\), Golub\(^{29}\), and Swartzlander\(^{58,59}\)).
Whereas many early experiments revealed complex patterns,\(^{65,20,49,28,2,21,15}\) an explanation of the observed phenomena predated modern mathematical tools and
paradigms for describing complex pattern formations, e.g. the theory of optical catastrophes.\(^{60,46,5,11,9,45}\) In recent years the formation of optical vortices has
guided much work in nonlinear refractive optics (see, for example, Refs. 18, 57,
As in linear optics, vortices have been found to accompany complex nonlinear diffraction patterns. Here we extend the work of several groups who have investigated the propagation of an elongated (or astigmatic) beam in a nonlinear material. In addition to verifying the formation of a single quadrupole of vortices, we observed for the first time as many as three sets of quadrupoles at relatively low powers (tens of mW), and a striking transition at higher powers (in the 100 mW range) leading to a new nonlinear phenomenon, namely a nonlinear cusp diffraction catastrophe.

An optical vortex is characterized by a helical phase front, causing the phase velocity of the beam to exhibit circulation and orbital angular momentum. The direction (or sign) and magnitude of this vorticity is called the topological charge. Vasnetsov et al. showed that a smooth beam having a circular cross-section becomes astigmatic and develops nonlinear phase distortion while propagating through an anisotropic photorefractive crystal (LiNbO$_3$). The output beam profile contained four vortices arranged as a quadrupole. Ackemann et al. introduced an initially astigmatic beam into sodium vapor and, under self-focusing conditions, also observed a vortex quadrupole. Treating the nonlinear material as a thin film, Kreminskaya et al. developed a theoretical basis for understanding the formation of the quadrupole in these cases.

The theory of linear vortices is the basis for understanding the physics of nonlinear optical vortices. Linear optical vortices appear, for example, in the diffraction fields of caustics (i.e. bright regions where many rays intersect), forming complex interference patterns when the light is coherent. A cusp field, first computed by Pearcey in 1946, is an example of a caustic. The cusp field was later studied in the context of so-called optical catastrophes and was found to contain an ordered array of vortex dipoles. A general scheme for deriving the diffraction integrals of caustics, based on catastrophe theory, soon followed.

One of the frequently observed caustics is the two-dimensional cusp, defined by the equation $y^2 = -x^3$, $x < 0$. This is the familiar pattern seen in a coffee cup when light reflects from the inside walls of the container. Cusps are also found in the vertices of astroids. Perhaps the grandest cusp was observed during the occultation of a star by Mars. The Martian atmosphere refracted starlight toward the slightly elliptical planetary shadow, forming a giant (100 km) astroid-shaped caustic across the Earth.

Here we describe the power-dependent evolution of an astigmatic beam which develops peripheral vortex quadrupoles at low powers (10 mW range) and a nonlinear cusp diffraction catastrophe (CDC) at high powers (100 mW range). In contrast to linear catastrophes, where the beam is perturbed by a refracting object, the nonlinear catastrophe described in this report is produced by an intensity-dependent refractive index change induced in a homogeneous self-defocusing medium. The field of an optical vortex and a quadrupole of optical vortices is introduced in Sec. 2. The experimental apparatus is described in Sec. 3. In Sec. 4 we report the observation of peripheral vortex quadrupoles, each appearing after
increasing the power by an equal amount. The transformation of a high-power elliptical Gaussian beam into the nonlinear CDC pattern is discussed in Sec. 5.

2. Vortices and Vortex Quadrupoles

The scalar field of a single optical vortex with topological charge, $m_j$, and with the core at the point $(x_j, y_j)$ may be expressed as:

$$E(x, y) = A(r_j) \exp(im_j \theta_j) \exp(-ikz)$$

where $r_j = [(x - x_j)^2 + (y - y_j)^2]^{1/2}$, and $\theta_j = \arctan[(y - y_j)/(x - x_j)]$, are polar coordinates measured about the vortex core in the transverse plane, $A(r_j)$ is the core amplitude function (assumed to be identical for all $j$), $m_j$ is a signed nonzero integer, $z$ is the direction of propagation of the background field, and $k$ is the wave number. The intensity and phase of an optical vortex are shown in Fig. 1. In experiments, vortices are detected with the aid of interferometry. The interference of a vortex field with an equally intense off-axis plane wave generates a set of nearly parallel fringes with a bifurcation near the vortex center as shown in Fig. 1(c).

A single vortex quadrupole may be written as the product of four vortices having alternating charges at the corners of a rectangle, $(x_j, y_j) = (\pm a, \pm b)$, with $m_1 = m_3 = -m_2 = -m_4$ (where the subscript denotes the quadrant number). If the quadrupole exists in a beam having an elliptical Gaussian intensity distribution (with an otherwise flat phase front), then the field may be written

$$E(x, y, z = 0) = E_0 \exp(-x^2/w_x^2 - y^2/w_y^2) \prod_{j=1}^{N} A(r_j) \exp(im_j \theta_j) ,$$

where $N = 4$, $w_x$ and $w_y$ are the elliptical beam sizes, and $E_0$ is the characteristic amplitude of the field.

3. Experiment

A schematic diagram of the experimental apparatus is shown in Fig. 2. An initially collimated TEM$_{00}$ mode of a continuous-wave Argon ion laser, Ar, ($\lambda = 514$ nm) is elongated by a prism, Pr, to obtain an elliptical Gaussian intensity profile

$$I(x, y) = I_0 \exp(-2x^2/w_{x,0}^2 - 2y^2/w_{y,0}^2),$$

where $w_{x,0}$ and $w_{y,0}$ are the elliptical beam sizes.
where \( w_{x,0} = 2.8 \text{ mm} \) and \( w_{y,0} = 1.4 \text{ mm} \) are the characteristic beam sizes of the major and minor axes, respectively and the aspect ratio is \( q_0 = w_{x,0} : w_{y,0} = 2 : 1 \). The initial intensity, \( I_0 \), is related to the power of the beam, \( I_0 = 2P/\pi w_{x,0}w_{y,0} \). The beam is focused with a lens, \( (f_1 = 200 \text{ mm}) \), into the middle of a vertically aligned cell of maximum length \( L = 250 \text{ mm} \). At the output face a movable glass window, \( W \), was used to vary the nonlinear propagation length and prevent distortions from the meniscus. The cell contained methanol (with a linear refractive index of \( n_0 = 1.33 \)) and nigrosin dye (to achieve an absorption coefficient of \( \alpha = 2.8 \text{ m}^{-1} \)).

Half the beam power was absorbed over \( 250 \text{ mm} \). In the linear case, the calculated sizes of the beam in the focal plane of the lens are

\[
\begin{align*}
\text{linear size:} & \quad w_{x,f} = \frac{\lambda f_1}{\pi w_{x,0}} = 11.5 \mu m \\
\text{linear size:} & \quad w_{y,f} = \frac{\lambda f_1}{\pi w_{y,0}} = 23 \mu m \\
\text{aspect ratio:} & \quad q_f = w_{x,f} : w_{y,f} = 1 : 2 
\end{align*}
\]

The sizes are expected to vary with propagation distance:

\[
w_i(z) = w_{i,f} \left[ 1 + \left( \frac{z - L/2}{Z_i^2} \right)^2 \right]^{1/2} (i = x, y) \tag{4}
\]

where \( Z_i = \pi w_{i,f}^2 n_0/\lambda \) are the characteristic diffraction lengths of the focused beam, and \( (z - L/2) \) is the distance from the focal plane of the lens (we ignore the small shifts of the waist positions due to the astigmatism). At the input and output faces of the cell \( (z - L/2 = \pm 125 \text{ mm}) \) the calculated linear beam sizes are \( w_{x,L} = 1.34 \text{ mm} \) and \( w_{y,L} = 0.68 \text{ mm} \) and the aspect ratio is \( q_L = w_{x,L} : w_{y,L} = 2 : 1 \). Owing to different rates of diffraction the beam becomes circular \( (w_x = w_y = 25 \mu m) \) at \( z - L/2 = \pm 2 \text{ mm} \), in the so-called “circle of confusion” region, and the major and minor axes of the ellipse are interchanged for \( |z - L/2| < 2 \text{ mm} \).

A wedged glass plate, BS, and attenuator, \( \text{At}_1 \), are used to direct light from the output face to a CCD camera. The output beam profile is imaged onto the CCD array with a lens \( (f_2 = 100 \text{ mm}) \). An interferometer which includes a beam splitter, BS, lens \( (f_3 = 50 \text{ mm}) \), and attenuator, \( \text{At}_2 \), is used to detect vortices and their
The nonlinear index change in the thermal medium is proportional to the temperature: $\Delta n = (\partial n/\partial T)\Delta T$, where $\partial n/\partial T = 3.9 \times 10^{-4} \text{ K}^{-1}$ in methanol. The thermally induced index change in the center of an axially symmetric beam weakly depends on the beam size in the steady state, $n = (\partial n/\partial T)AP = 4\alpha P/4\pi\kappa$ (5)

where $\kappa = 0.202 \text{ W/mK}$ is the heat conductivity of the medium, and $\alpha$ is a dimensionless parameter which depends on time and the boundary conditions. In an unbounded convectionless medium, $A = \ln(1+t/t_c)$, where $t_c = w^2/8D$, $w$ is the radial beam size (see Eq. (4)), and $D = \kappa/c_p\rho = 0.1 \text{ mm}^2/\text{s}$ is the temperature conduction coefficient of the thermal medium, and $c_p = 2.5 \text{ J/g/K}$ and $\rho = 0.8 \text{ g/cm}^3$ are heat capacity and density of methanol, respectively. We estimate that at the input, $t_{c0} \approx w_{x,L}^2/8D = 2s$, and in the focal plane, $t_{cf} = w_{x,f}^2/8D \approx 2 \times 10^{-4} \text{ s}$. Thus we calculate that $A \approx 2$ at the input face and $A \approx 13$ in the focal plane. Unlike a Kerr medium where the refractive index would be roughly three orders of magnitude larger in the focal region, here we find a difference of less than an order of magnitude.

4. Weak Nonlinearity

In the low power regime, $P < 42 \text{ mW}$, we observed self-defocusing of the beam primarily along the $y$-axis and the generation of phase singularities in the periphery of the beam. The beam at the output face of the cell under linear propagation conditions ($P < 4 \text{ mW}$) is elliptical, as shown in Fig. 3(a), having an aspect ratio of $2:1$. Nonlinear refraction becomes evident in the intensity profile at $P = 19 \text{ mW}$, where we see in Fig. 3(b) two dark segments oriented parallel to the major axis at the perimeter of the beam. By examining the interference fringes from the opposite sides of each segment (not shown) we determined that a $\pi$ phase dislocation causes the appearance of the dark stripes. At slightly higher powers, we found that each edge dislocation transformed into a vortex dipole. Interferometry showed the dipoles formed a vortex quadrupole with the topological charge in the first quadrant $m_1 = 1$. The location and charge of each vortex is demarked in Fig. 3 with “+” and “−” signs. Initially the vortices appeared near the intersections of the beam perimeter and the $y$-axis. However, as the power was increased the vortices migrated along the perimeter and toward the $x$-axis. At $P = 26 \text{ mW}$ this migration essentially stopped, at the position shown in Fig. 3(c), and a second quadrupole started developing at the intersection of the beam perimeter and the $y$-axis. After migrating to the positions shown in Figs. 3(d) and 3(e), at $P = 34 \text{ mW}$ and $42 \text{ mW}$ respectively, a third and fourth quadrupole started to develop.

Summarizing our main findings from this set of measurements, we observed that (1) the beam developed mostly in the $y$-direction, (2) the charges of the vortices
were identical within a given quadrant, and (3) the vortices within a given quadrant migrated along the perimeter and toward the $x$-axis. The process by which the vortices are created can be understood as the breaking of an edge dislocation. Curiously, however, the migrating vortices do not annihilate as they approach those of opposite charge near the $x$-axis. Let us now provide an explanation for these observations.

To understand why the beam self-defocuses mostly along the $y$-axis, we examine the refractive index (or temperature) gradients throughout the nonlinear cell. Since the radius of the cell, $R = 12.7$ mm is much greater than the input size of the beam, we neglect the effect of the circular boundary on the temperature distribution. In an unbounded medium, the characteristic size of the temperature distribution is proportional to the size of the heat source. Therefore, a higher temperature gradient is expected along the minor axis of an elliptical beam. Along 98% of the cell length the minor axis lies along the $y$-axis, and thus, self-refraction of light may be expected mainly in the $y$-direction.

When relatively small amounts of power are deflected away from the $x$-axis, we may estimate the affect on the beam profile by superimposing the original elliptical beam, $E_{\text{lin}} = \exp(-x^2/4 - y^2)$, with a weak self-deflected planar beam, $E_{\text{ref}} = \xi \exp[i \pi \eta y^2]$, where $\xi$ represents the perturbation amplitude and $\eta$ represents the magnitude of nonlinear refraction. The intensity of the superimposed fields, $E_{\text{lin}} + E_{\text{ref}}$, is shown in Fig. 4. Edge dislocations appear near the $y$-axis when $\eta = 1.5$ and
Fig. 4. Interference of a linear field, $E_{\text{lin}} = \exp(-x^2/4 - y^2)$, and a weak refracted field, $E_{\text{ref}} = \xi \exp(\pi \eta y^2)$. Edge dislocation (a) appears at $\eta = 1.5$, $\xi = 0.075$. Vortex quadrupole (b) is formed at $\eta = 3.0$, $\xi = 0.15$.

Fig. 5. Calculation of additional phase shift in the center of a beam due to a quadrupole of optical vortices in the periphery. The center of each vortex makes an angle $\pm \beta$ with y-axis. The angular position, $\theta_j$, of the origin with respect to the local coordinate system is shown with grey arcs. The amount of phase shift in the center contributed by a vortex of charge $m_j$ in $j$th quadrant is $m_j \theta_j$. Black arrows show the direction of motion of vortices with increasing power.

$\xi = 0.075$. Doubling the wavefront curvature and the amplitude by setting $\eta = 3$ and $\xi = 0.15$ we find that the edge dislocation breaks, leaving four black points as shown in Fig. 4(b). The phase profile (not shown) confirms that these points are vortices having the same quadrupole arrangement of topological charge as seen in Fig. 3(c).

In our experiment the creation of quadrupoles occurred at a regular power interval of 8 mW. In a defocusing medium, the phase in the center of a beam, where the intensity is higher, is delayed relative to the periphery. Therefore, the power increment may be compared to the phase delay in the beam center when new vortices are spawned. The phase at the center of the beam, $\Phi_0$, may be found with the aid of Fig. 5, showing a quadrupole arranged along the perimeter of an ellipse. The net phase at the origin is found by summing over the phase contributions from each vortex: $\Phi_0 = \sum_{j=1}^{4} m_j \theta_j$, where $\theta_j$ is the angular position of the origin. Expressing $\theta_j$ in terms of the angle, $\pm \beta$, subtending each vortex and the y-axis (see Fig. 5) we find $\Phi_0 = -4 \beta$. We measured the positions of vortices at the threshold powers corresponding to the creation of a new quadrupole. For
example, the threshold between the first and second set of quadrupoles occurred at $P = 26$ mW (see Fig. 3(c)), and the average value of the angle was $\beta = 66^\circ$. Thus, the value of the central phase $\Phi_{0,1} = -1.48\pi$.

The failure of the vortices to migrate to the $x$-axis and annihilate may be explained by examining the change in phase as vortices move from $\beta = \pi/2 - \delta$ to the annihilation position $\beta = \pi/2$. According to Eq. (6), $\Phi_0 \rightarrow -2\pi$ in the limit $\delta \rightarrow 0$, whereas an absence of vortices requires that $\Phi_0 = 0$. If, as the beam power increased, vortices migrated to the annihilation point, the central phase would step from $\Phi_0 \rightarrow -2\pi$ to $\Phi_0 = 0$. Such a sudden nonlinear phase shift is unphysical, and thus it is not surprising that vortices seem to avoid meeting at the $x$-axis. Rather than jumping to zero, the central phase continues to decrease with increasing power, thereby driving the creation of additional vortices.

An expression for $\Phi_0$ is easily generalized for the case of $n$ sets of quadrupoles:

$$\Phi_{0,n} = \sum_{j=1}^{4n} m_j \theta_j = -4 \sum_{k=1}^{n} \beta_k$$

where $\beta_k$ is the angular vortex position in $k$th set. We measured the angles and found $\beta_1 = 73^\circ$, $\beta_2 = 52^\circ$ for $P_2 = 34$ mW, and $\beta_1 = 76.3^\circ$, $\beta_2 = 60.7^\circ$, $\beta_3 = 43.6^\circ$ for $P_3 = 42$ mW. The last set of $\beta_j$ is approximate since the fourth quadrupole did not appear. The central phase is found for each transition according to Eq. (6): $\Phi_{0,2} = -2.8\pi$, and $\Phi_{0,3} \simeq -4.0\pi$. Analyzing the values of $\Phi_{0,i}$ for $i = 1, 2, 3$, we conclude that the central phase changes by roughly $-1.5\pi$ per each 8 mW of power, i.e. $\Delta \Phi_0/\Delta P = -0.6$ rad/mW. Let us now equate the central phase with the phase $\Phi_0 = \Phi_{\text{lin}} + \Phi_{\text{NL}}$, where $\Phi_{\text{lin}} = \text{const}$ is attributed to the wavefront curvature of the diffracted beam, and $\Phi_{\text{NL}} = 2\pi L \Delta n/\lambda$, where $\Delta n$ is given by Eq. (5). The effective value of the parameter in Eq. (5) may be estimated: $A_{\text{eff}} = (\Delta \Phi_0/\Delta P)(\partial \eta/\partial T)^{-1}2\kappa\lambda/(\alpha L) = 0.45$. Thus, we compute that the refractive index changes by $\Delta n = 1.5 \times 10^{-6}$ between the subsequent generation of quadrupoles. We note that $A_{\text{eff}}$ is less than that expected for an unbounded thermal medium. We attribute this discrepancy to thermally-induced convection within the cell.

5. Large Nonlinearity

Further cascading of quadrupoles was not observed beyond 42 mW; rather, the beam exhibited strong diffraction along the minor axis and acquired complex interference-dominated structure in the center. The quadrupoles vanished into the darkness beyond the beam, and a new pattern exhibiting the characteristics of a cusp diffraction catastrophe emerged at $P > 100$ mW. To develop an understanding of the complex structure in the beam center, we varied the length of the nonlinear cell while holding the power constant at $P = 156$ mW. The most striking profile, shown in Fig. 6(a), occurred at a length $L = 215$ mm we observed a single quadrupole having vortices near cusp points. The arrangement of topological charge was the same as we observed in the low power regime ($m_1 = +1$), as indicated in
Fig. 6. Trace of a cusp diffraction catastrophe. (a) Intensity profile at distance $L = 215$ mm from the input face of the nonlinear cell. Incident power $P = 156$ mW. A grey astroid having an aspect ratio of $4 : 3$ outlines the complex structure of the beam near caustic. The oval intensity minima near the vertical cusps of the astroid are pairs of oppositely charged vortices as shown by the interferograms in (b) and (c). The topological charge of each vortex is indicated in (b) and (c) by “+” and “−” signs.

Figs. 6(b) and 6(c). The vortices, however, were unstable and vanished at slightly higher or lower power. We also noticed a striking feature in the beam profile: the perimeter of the interference region appeared to form an outline of an astroid.

It is known in linear optics that light is distributed along an astroid if the initial beam has the shape of a thin annular ellipse. Furthermore, linear diffraction catastrophe theory predicts that vortices will form within cusps such as those of the vertices of an astroid. In our experiment, however, the initial beam had an elliptical Gaussian profile. We wondered whether nonlinear refraction transformed the beam into an annular ellipse within the nonlinear cell, thereby causing the subsequent astroid.

Images were recorded at various nonlinear propagation distances to monitor the beam evolution. At roughly one-third of the cell length ($L = 76$ mm) we observed a beam with a rather flat intensity profile, shown in Fig. 7(a). Shortly before the focal plane of the lens, ($L = 103$ mm), we indeed observed an annular elliptical profile as shown in Fig. 7(b). The measured aspect ratio of the annular ellipse was found to be $a_e = a_x : a_y = 0.7 : 0.92$ or almost $3 : 4$. Beyond this plane the beam exhibited complex interference, as demonstrated in Figs. 7(c) and 7(d). The interference seen in Fig. 7(c) is reminiscent of the ringing observed in earlier experiments and is attributed to the spherical aberration of the induced nonlinear lens. The effects of astigmatism are particularly evident in Fig. 7(d), where the development of a complex diffraction region bounded by an astroid is seen.
Fig. 7. Development of the CDC in the nonlinear medium is shown as a set of intensity profiles at the output face of a cell having variable length, \( L \). The beam power is 156 mW. (a) An elliptical beam having almost uniform distribution of intensity at \( L = 76 \) mm from the input face of a nonlinear cell. (b) The beam area is minimal at \( L = 103 \) mm, and an annular ellipse of aspect ratio \( q_e = a_x : a_y = 0.70 \) mm : 0.92 mm \( \approx 3 : 4 \) is formed. The annulus starts radiating inward waves. (c) Interference between inwardly radiated wavefronts in the center at \( L = 122 \) mm. (d) All but three diffraction ovals have transformed into a spatially complex pattern at \( L = 139 \) mm. Three-wave interference regions appear near the cusps.

To describe this developing astroid, let us consider the linear propagation of light from an annular elliptical source\(^6\) using ray tracing techniques. The transverse components of the light are represented by rays in the \( xy \)-plane. The intensity profile of this source is restricted to the curve:

\[
X^2/a_x^2 + Y^2/a_y^2 = 1.
\]  

(7)

Rays drawn normal to this curve (shown in Fig. 8) are concentrated along an astroid-shaped caustic. In fact this astroid is the evolute of Eq. (7), and is defined by the curve:\(^{12,26}\)

\[
(Xa_x)^{2/3} + (Ya_y)^{2/3} = f_e^{4/3}
\]

(8)

where \( f_e = (a_x^2 - a_y^2)^{1/2} \). The astroid has four cusps at \( X = \pm f_x^2/a_x \) and \( Y = \pm f_y^2/a_y \). The aspect ratio of the astroid is \( (f_x^2/a_x) : (f_y^2/a_y) \) or \( a_y/a_x \), i.e. it is the reciprocal of the aspect ratio of the ellipse. Note that the rays inside the astroid cross in triplets whereas outside they only cross in pairs. In the latter case one may anticipate simple two-wave interference patterns, such as the rings observed in Fig. 7. In the former case, however, complex three-wave interference patterns are expected, with the possible appearance of optical vortices.\(^{10}\)
Fig. 8. Graphical construction of the caustic of ellipse. Rays are drawn from equidistant points on the ellipse, perpendicular to its surface. The intersection of rays forms a caustic curve in a shape of astroid. The aspect ratio of the astroid, \((f_x^2/a_x) : (f_y^2/a_y) = a_y : a_x\), is the reciprocal of the aspect ratio of the ellipse \(a_x : a_y = 3 : 4\).

An astroid having an aspect ratio, \(q_a = 4 : 3\), is plotted over the intensity distribution in Fig. 6(a). As anticipated, the plot agrees with the outline of the diffraction envelope. Further analysis of the structure inside the astroid is based on the fact that some caustics have a well-known diffraction fields associated with them. The caustic at the vertex of the astroid is a cusp whose field was first computed by Pearcey in 1946.\(^{48}\) The intensity and phase of the so-called Pearcey function are shown in Figs. 9(a) and 9(b) respectively. Berry and Upstill later recognized the cusp as a diffraction catastrophe.\(^{11}\) White (black) lines in Fig. 9(b) represent the path where the real (imaginary) component of the field vanishes; crossings of these lines coincide with the center of an optical vortex. These crossings occur in closely spaced pairs, forming a triangular array of vortex dipoles.\(^{11}\) The intensity at the dipoles appears as dark line segments in Fig. 9(a).

By varying the nonlinear propagation distance in our experiment we were able to find similar triangular arrays of dark notches over the range \((120 < L < 200)\) mm. Surprisingly, interferometric measurements showed no vortices in this range, i.e. the interference fringes did not split as shown in Fig. 1(c). In particular, we found no vortex dipoles at \(L = 185\) mm despite the similarity of the triangular arrays of intensity minima found in theory (Fig. 9(a)) and in the experiment (Fig. 9(c)). The nonlinearity of the system suggests that nonlinear refraction may account for the discrepancy between the small scale structure (e.g. the lack of vortices) in Figs. 9(a) and 9(c). Another possible source of distortion is convection owing to laser-heating in the vertical cell. Furthermore, we note that an ideal annular ellipse is not achieved in the experiment; the ellipse has a non-vanishing thickness and the interior region has a non-vanishing intensity. The apparent instability of the vortices in a CDC
may not be too surprising, given that the zero crossing lines between the vortex dipole pairs in Fig. 9(b) are nearly touching. We believe that the perturbations in our system drive these lines toward each other, causing the vortices to annihilate. Accordingly, the most robust dipole exists for vortex pairs having the greatest separation between the zero-lines in the region between the pair. From Fig. 9(b) we therefore expect the dipole at the vertex of a cusp to exhibit the greatest stability. Indeed, we only observed vortices at this location (see Fig. 6).

In conclusion, we experimentally studied the propagation dynamics of an elliptical Gaussian beam that was focused in a thick defocusing medium. In a low power regime, we observed up to three vortex quadrupoles in the periphery of the beam. Each quadrupole appeared after increasing the power by an equal amount, which is associated with \( \Delta 1.5\pi \) increase of the nonlinear phase in the beam center due to each quadrupole. At high powers, we found an annular ellipse near the focal plane and a complex diffraction pattern bounded by an astroid beyond the focal plane. We believe linear diffraction dominated the beam evolution after the annulus was formed. Although the intensity patterns near the cusps of the astroid were similar to those of CDC, the vortex dipoles associated with the nonlinear CDC were unstable.

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Fig. 9. Cusp diffraction catastrophe. (a) Intensity map of CDC. Each zero valued minimum of the intensity inside the cusp is produced by a vortex dipole. (b) Phase map of CDC. The vortex cores are located at the intersections of zero lines where the real (white) or imaginary (black) component of the field equals zero. (c) An experimental CDC intensity pattern is found at \( L = 185 \text{ mm}, P = 156 \text{ mW} \) (only top quarter of the beam is shown). Compare the triangular array of six intensity minima to (a). The elongated intensity minimum near the vertex (c) develops into a pair of optical vortices at \( L = 215 \text{ mm} \) as shown in Fig. 6.
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