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AN ALTERNATIVE METHOD TO TEST THE RESIDUALS IN A REGRESSION MODEL

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ABSTRACT

In this paper we propose an alternative method to the Durbin-Watson (DW) test for the fitness of a regression model (see, for example, Makridakis [5, pp. 267-268; 303-304; 630-631] and Wilson & Keating [8, pp. 182-184; 234-236] for DW statistic). The proposed method tests whether the residual terms (i.e., actual_t – model_t) display any sign of non-randomness by comparing two variance estimators of the residuals. This test is a transformation of the DW statistic into a standardized normal statistic, N(0,1), which is readily interpreted; and unlike the DW test does not require a special table. A numerical example is provided.

Keywords: Regression models, residuals, Durbin-Watson statistics, test for significance.

DISCUSSION

The Durbin-Watson (DW) statistic is a test statistic used to check for the presence of autocorrelation in the error terms in regression models. To test the significance of the DW statistic one must resort to DW tables. In this paper we are proposing an alternative test of significance to the DW test which is a transformation of DW statistic into a N(0,1).

The proposed approach uses the significance test of two variance estimators of the residuals: one is the regular variance estimate, the other is the one computed using mean square successive differences (MSSD) – see , for example, Neumann, et al. [6], Hald [2, pp.357-360], Holmes and Mergen [3], [4]. The MSSD is defined as

$$\text{MSSD} = \frac{1}{(n-1)}\sum_{i=1}^{n-1}(X_{i+1} - X_i)^2 \qquad (1)$$

where n is the number of observations.

Using these differences, an unbiased estimate for the process variance is given by Hald [2] as

$$q^2 = \frac{1}{2(n-1)}\sum_{i=1}^{n-1}(X_{i+1} - X_i)^2 \qquad (2)$$

Then the MSSD standard deviation is determined by taking the square root of q^2. Roes, et al. [7] suggested a minor correction factor in estimating the standard deviation when the MSSD approach is used. This factor disappears as the sample size gets bigger. The significance of the difference between the conventional and MSSD variance estimates can be tested using the test given in Dixon and Massey [1, pp. 353-354].

$$z = \frac{1 - \frac{q^2}{s^2}}{\sqrt{\frac{n-2}{(n-1)(n+1)}}} \tag{3}$$

where the usual variance estimator (s^2) is

$$s^2 = \frac{\sum_{i=1}^{n}(X_i - \overline{X})^2}{n-1} \tag{4}$$

Values of z between ±3 indicate that the difference between the two estimates is not significant, i.e., the data set seems to be random. Z values bigger than +3 and less than -3 indicate that the two estimates are significantly different and thus the data is not random. Z values bigger than +3 imply trend and long-term cycles in the data and values less than -3 indicate short term cycles. Thus we propose the use of this z statistic to check the randomness of the error terms in regression models. Within the context of the regression, X_i values are replaced by error terms (e_t), i.e., residuals. To calculate the proposed z statistic, two variance estimators will be calculated for the error terms, one using the MSSD approach and the other one using the regular approach as given in equation 4. Thus z values within ±3 imply that the error terms do not show any sign of non-random patterns. Since z is N(0,1) when n is bigger than 20, the use of z values between ± 3 gives about a 99.7% critical region for the test.

One can show the relation between the proposed test and the DW test as follows. DW statistics is given as

$$DW = \frac{\sum_{t=2}^{T}(e_t - e_{t-1})^2}{\sum_{t=1}^{T} e_t^2} \tag{5}$$

where e_t is the residual associated with the observation at time t. The DW statistic lies between 0 and 4, and its value around 2 implies no autocorrelation among the error terms. DW values near 0 indicate positive correlation, whereas values near 4 imply negative correlation. The DW statistic is very similar to the term $\frac{q^2}{s^2}$ given in equation 3 except the division by 2 in q^2. We can re-write equation 3 as follows:

$$z = \frac{1 - \frac{DW}{2}}{\sqrt{\frac{n-2}{(n-1)(n+1)}}} \tag{6}$$

Multiplying both sides by the term in the numerator in equation 6 will yield

$$\sqrt{\frac{n-2}{(n-1)(n+1)}}\,(z) = 1 - \frac{DW}{2} \tag{7}$$

And multiplying both sides by 2 will result in equation 8 as follows:

$$(2)\sqrt{\frac{n-2}{(n-1)(n+1)}}\,(z) = 2 - DW \tag{8}$$

Thus we can express DW as a function of z which is given below in equation 9.

$$DW = 2 - 2\sqrt{\frac{n-2}{(n-1)(n+1)}}\,(z) \tag{9}$$

Equation 9 clearly shows the relation between the DW statistics and the proposed z statistic. As can be seen in the equation, when z is at its desired value of 0 for randomness of the error terms, DW will be equal to its desired value of 2. This approach does not intend to replace the DW statistic test nor does it claim to be a better approach. It is proposed just as an alternative test to check the randomness of the error terms in regression and autoregressive models. Its advantage, as it is stated in the beginning, is that it does not require a special table to check the significance of the z values. We believe this will be a practical method to quickly check the randomness of the error terms in regression models. However, the drawback of the proposed approach is that it does not indicate the sign of the correlation between the error terms, if there is significant correlation.

EXAMPLE

The example data set (see the Appendix) is from a machine shop producing break drums. The data, which is disguised to maintain confidentiality, is on the diameter of sequentially machined hubs. The test for stability indicated that the time series data given in the Appendix is not stable. This was tied to the fact that there is an automatic control mechanism in the shop which adjusts the process automatically. This, in return, creates an autocorrelation in the process. The autocorrelation function given in Figure 1 supports this.

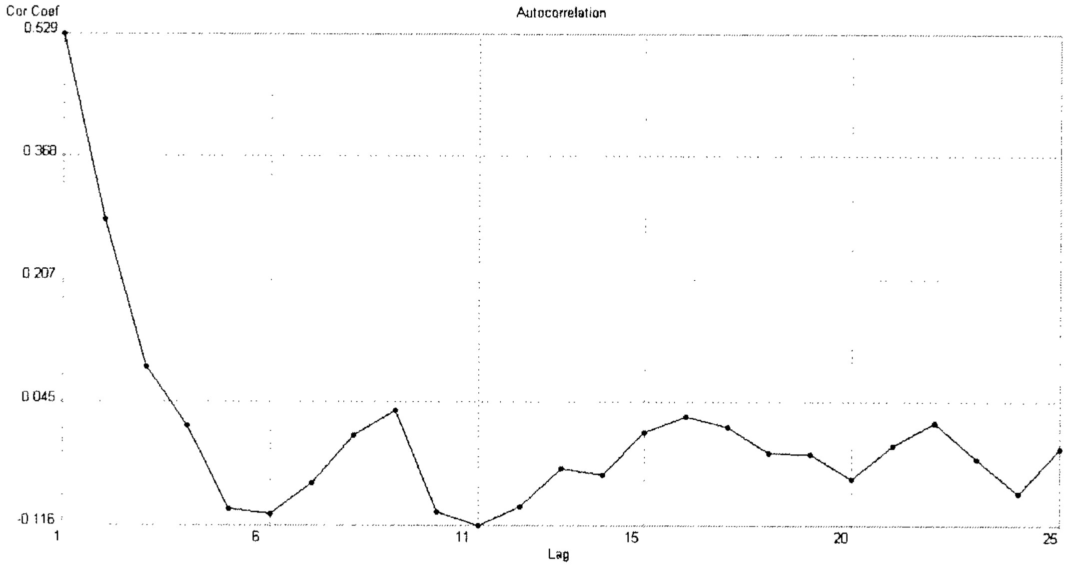

Figure 1. Autocorrelation chart on hubs diameter.

Thus we fit a first order autoregressive (AR) model to model the autocorrelation in the data (equation 10) and calculated the error terms, e_t, (equation 11):

$$\hat{X}_t = 1.395 + 0.53X_{t-1} \tag{10}$$

$$e_t = X_t - \hat{X}_t \tag{11}$$

where X_t is the actual observation and $\hat{X}_t$ is the estimate at time t, respectively.

To test the significance of this AR model we used both the DW test and the proposed method on the error terms (i.e., the actual – the model given in equation 10 for each time period). The DW statistic and the z value from the proposed method using equation 6 are given below:

DW = 2.003 and the z = -0.077.

Both the DW statistic and the z value from the proposed method indicate that the error terms are random, which implies that the first order AR model is a good fit for this data.

CONCLUSION

In this paper we proposed an alternative method to the Durbin-Watson (DW) test for the fitness of a regression model. The proposed method is a practical method to quickly check the randomness of the error terms in regression models, since it does not require a special table to check the significance of the z values computed by the equation 6.

APPENDIX

Data on hubs diameter

2.966	2.966	2.974	2.970	2.960	2.958
2.966	2.960	2.988	2.959	2.950	2.977
2.964	2.966	2.981	2.953	2.964	2.982
2.953	2.967	2.979	2.940	2.968	2.977
2.953	2.963	2.981	2.959	2.971	2.981
2.958	2.983	2.980	2.980	2.983	2.976
2.965	2.986	2.972	2.981	2.968	2.986
2.973	2.960	2.944	2.976	2.986	2.972
2.978	2.982	2.958	2.969	2.981	2.981
2.987	2.979	2.958	2.961	2.998	
2.977	2.978	2.952	2.953	2.974	
2.964	2.976	2.975	2.951	2.982	
2.966	2.976	2.976	2.948	2.987	
2.976	2.952	2.970	2.971	2.981	
2.967	2.973	2.968	2.960	2.976	
2.981	2.967	2.955	2.979	2.958	
2.983	2.980	2.944	2.972	2.950	
2.989	2.973	2.944	2.979	2.960	
2.979	2.970	2.973	2.976	2.964	
2.966	2.980	2.963	2.971	2.968	
2.965	2.983	2.960	2.976	2.978	
2.961	2.968	2.956	2.965	2.980	
2.968	2.955	2.965	2.967	2.984	
2.976	2.942	2.961	2.960	2.978	
2.973	2.955	2.968	2.962	2.966	
2.967	2.960	2.972	2.963	2.972	
2.964	2.962	2.973	2.961	2.975	
2.976	2.974	2.964	2.953	2.974	
2.965	2.968	2.966	2.964	2.977	
2.968	2.963	2.957	2.968	2.979	
2.968	2.967	2.968	2.959	2.978	
2.987	2.966	2.985	2.960	2.973	
2.976	2.963	2.976	2.972	2.971	
2.963	2.967	2.988	2.968	2.962	
2.964	2.964	2.984	2.990	2.956	
2.990	2.966	2.981	2.979	2.973	
2.995	2.964	2.967	2.964	2.968	
2.975	2.966	2.969	2.967	2.972	
2.980	2.987	2.978	2.978	2.969	
2.980	2.991	2.975	2.952	2.986	
2.966	2.975	2.964	2.962	2.977	
2.965	2.983	2.958	2.953	2.965	

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