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# **Using a 2-D Simulation Program to Support Interactive Learning of 3-D Vehicle Dynamics**

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## **Abstract**

The author has developed an undergraduate course to introduce students to the basic principles of land vehicle dynamics. Many students, though they are intrinsically interested in the subject, have difficulty grasping the physical principles needed to enable a sufficient understanding of the subject to support their aspirations of making informed vehicle design decisions. An intriguing potentially useful fact is that the computer gaming world has developed widely available very sophisticated vehicle computer models with realistic human driver interfaces. However, while very entertaining, these computer games are not suitable for helping undergraduate engineering students understand the basic principles needed for land vehicle design. Therefore, as part of this course, the students develop their own dynamic models of vehicles (using Working Model 2D) starting with simplified quarter-vehicle suspension systems and working up to a steerable four-wheel drive vehicle traveling over a rough road. The innovation from an instructional standpoint is that students can get a good understanding of some three-dimensional effects, and thus the design principles derivable from this understanding, through employing a fairly simple to program and use widely available 2-D dynamic simulator software. The paper outlines the methodology for incorporating 3-D effects for land vehicle problems, and this methodology is illustrated with some specific examples. In the paper presentation some models will be run in real time using parameter input from the audience.

## **Introduction**

The author developed a new senior elective mechanical engineering technology course in Land Vehicle Dynamics, which was first offered in Spring 2006. Students taking the course are highly interested in the subject matter due to their own personal experiences and in some cases a desire to ultimately work in the transportation industry. It is critical to capture that high interest level and channel it into learning of the fundamentals underlying the subject. This means keeping the students in touch with the application while they are absorbing the details of viscoelastic materials properties, vibration theory, spectral density transformations, rigid body mechanics and 3-D geometry. The author found that quickly being able to convert theory, once understood by the students, to visible results for a real simulated cars (a 2006 Pontiac G6 GTP and a 2006 Toyota RAV4 to be specific), kept the students especially motivated. This required using a simulation package that was easy to program and has easily understood graphical output display options.

## Vehicle Dynamics Fundamentals

While several aspects of vehicle dynamics are covered in the course, this paper focuses on the aspects that are considered as part of building a quasi 3-D model of a simplified Pontiac AWD G6GTP where the students can accelerate, brake and turn the car over a simulated rough terrain using working Model 2D. This is called the steerable (ST) accelerating (A) rough (R) terrain (T) (or START) model.

While the complex 3-D tire/wheel motions that occur with real vehicle suspensions are covered in the course, the START model only allows the individual wheel motions to occur in the direction of vehicle travel ( $x$ ) and vertical ( $z$ ) plane (and primarily in the  $z$  direction) relative to a vehicle-fixed coordinate system –which is an ideal suspension system approximated by most real suspension systems during normal use. (What this amounts to is a planar version of the McPherson strut suspension controlled in the  $x$ -direction by a 4-foot-long trailing arm.) Each tire/wheel/suspension system consists of a linear spring modeling the tire vertical stiffness,  $k_t$ , a linear spring,  $k_s$ , modeling the suspension spring and a viscous damper,  $c_s$ , with two directionally dependent damping coefficients modeling the shock absorber. Furthermore, for analysis simplicity, the front wheels are lumped together and the back wheels are lumped together so that a four wheel vehicle is effectively being modeled as a two-wheel vehicle. Figure 1 shows this half-car model.

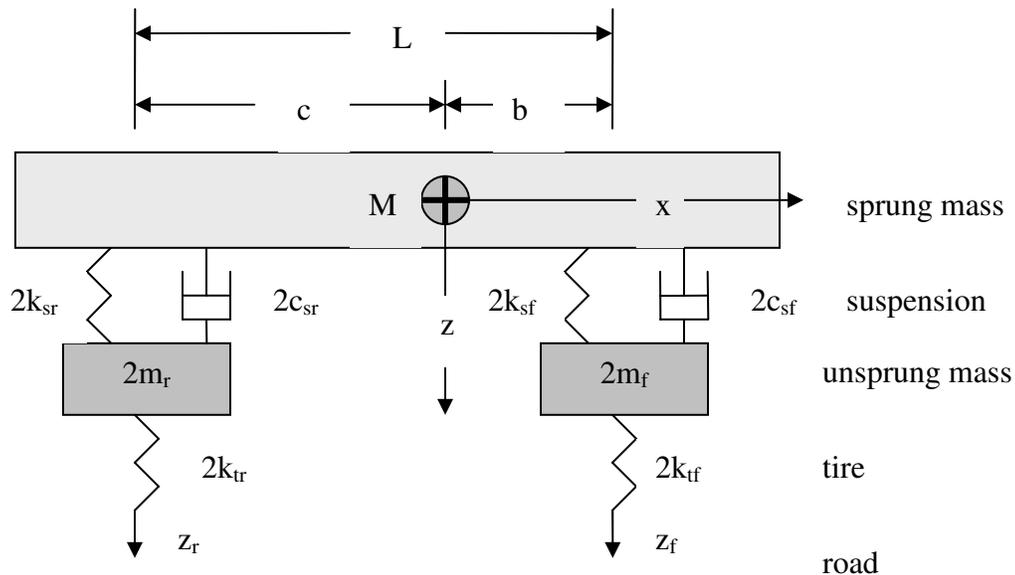


Figure 1 Half-car Model

In Figure 1 the subscript  $r$  refers to the rear parameters and  $f$  to the front parameters. SAE convention for vehicle analysis<sup>1</sup> has the  $x$  axis in the direction of vehicle motion and the  $z$  axis directed downwards towards the ground. The distances  $c$  and  $b$  locate the center of mass of the vehicle, which does not include the tires, wheels and suspension components which form the vehicle unsprung mass, which is divided between the front and rear in the START model.

(Also, once the students understand the effect of tire stiffness on the system by using the half-car model by itself with representative time-varying road profiles, for simplicity of graphical representation and focusing on other more important parameters, the tire stiffness is removed from the START model leaving only the front and rear suspension stiffness and damping.)

The road surface for START has been simplified to be a repetitive man-made all-terrain park with  $z_r$  (or  $z_f$ ) given by the equation

$$z_r = S[A_1\cos(0.02\pi x)\cos(0.02\pi y) + A_2\cos(0.2\pi x)\cos(0.2\pi y) + A_3\cos(2\pi x)\cos(2\pi y)] \quad (1)$$

where  $A_i$  are coefficients that define the amplitude of the roughness of the surface corresponding to the three wavenumbers (0.01, 0.1 and 1 cycles/ft) chosen and  $S$  is a scale factor that can be real-time user controlled to adjust the surface from perfectly smooth to very rough. (Note that so-called English ANSI units are commonly used to measure vehicle performance in the US and are thus used in the course and this paper.)  $x$  and  $y$  are the surface distances in feet from an arbitrary starting point in the “park”.

Directional control for the vehicle is provided by turning the vehicle’s front (steerable) wheels by an average angle  $\delta$ . Where the tire contacts the road, the tire angle differs from the wheel angle by an amount known as the tire slip angle,  $\alpha$ . The tire in effect twists about a vertical axis and generates a lateral force perpendicular to the wheel forward direction. This lateral force  $F_y$  is what steers the car and is related to the slip angle by the relationship<sup>2</sup>

$$F_y = CC_\alpha F_z \alpha \quad (2)$$

where  $CC_\alpha$  is the cornering coefficient for the tire (and is a function of the tire vertical load,  $F_z$ , the tire material characteristics, the tire shape and construction, and the road surface type and condition). (A typical value for  $CC_\alpha$  is 0.15 pounds of cornering force per pound vertical load per degree of tire slip angle.)

At slow speeds (less than 5mph on a dry road surface) a perpendicular to the average front wheel turned angle  $\delta$  will intersect a straight line drawn through the centers of the rear wheels at the center of the turn. In this case the front wheel angle is given by the Ackerman steering angle<sup>1</sup>,  $\delta = L/R$  (measured in radians), where  $L$  is the car wheelbase (distance between the front and rear axles) and  $R$  is the radius of the turn. As the car is driven faster in a turn both the front and rear wheels will develop slip angles in order to generate a lateral force sufficient to keep the car turning in the desired direction. Figure 2 shows the relations of these angles. As a function of the weight distribution of the car and the tire cornering coefficients (plus some other lesser factors like suspension geometry and which wheels are the driving wheels) a car will either turn more than predicted by pure Ackerman steering (known as the oversteer condition) or less than predicted (known as the understeer condition). Most vehicles are designed so that they exhibit understeer in normal to extreme driving conditions as this is considered to be most easily dealt with by an average driver. In the limit when a car is driven fast enough on a turn the lateral force required to keep the car turning at one or more wheels will exceed the normal force on the tire times the coefficient of static friction,  $\mu_s$ , between the road and the tire. In this case the tire will start to slide on the road surface and the lateral force will be limited to

$$F_{y\text{lim}} = \mu_s F_z \quad (3)$$

After  $F_{y\text{lim}}$  is exceeded the tire force will drop to  $\mu_k F_z$  (where  $\mu_k$ , the kinetic coefficient of friction, is less than  $\mu_s$ ) until the lateral force on the tire required to maintain control is less than  $\mu_k F_z$ . Once control is achieved again, equation (3) describes the limiting lateral forces for the tires.

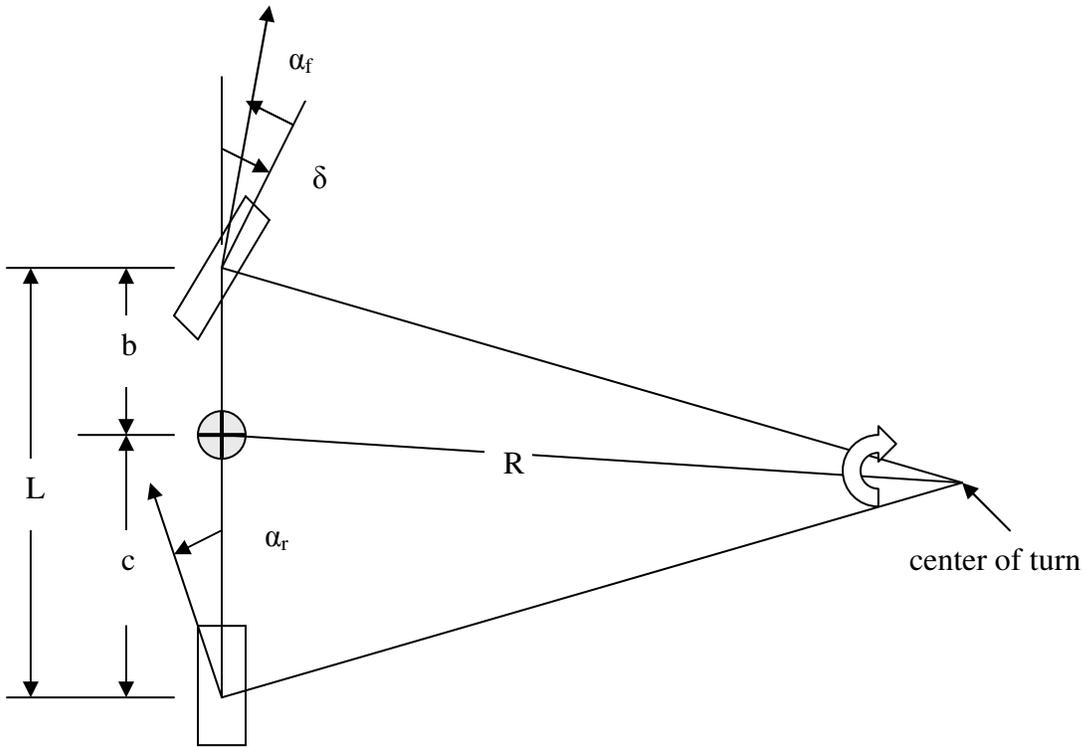


Figure 2 Tire Slip Angles When Turning

Braking and acceleration are provided by applying a positive (engine generated) or negative (braking) thrust at the front and rear axels. The thrust is nominally the same at both axels representing an open differential transfer case. However the thrust is limited in the model to the minimum of the input thrust or the limits of tire to road friction similar to equation (3). In the “park” the tires often have light or no contact with the terrain surface, so braking and acceleration (as well as steering) can be erratic (as they often are in reality).

#### START Model Using Working Model 2D

Working Model 2D<sup>3</sup> is a two dimensional simulator that takes user information described using graphical input and parameter modification (which can include BASIC-like formulas) and develops a set of internal ODEs which it solves through numerical integration and presents its results graphically or through exportable tables. For the START model, the plane in which the

vehicle makes its large movements is the x-y plane and Working Model graphically outputs the movement of the car in plan view as it traverses the all-terrain park with real-time adjusted user controls of the vehicle steering angle, accelerator/brake and the terrain roughness scale factor. The vehicle dimensions, weight characteristics, spring and shock characteristics and tire characteristics and the terrain park frequency component relative roughness amplitudes are adjusted before the simulation starts.

At the same time Working Model is running a 2-D system that corresponds to the x-y motion of the plan view of the vehicle, it can be set up to run and display a 2-D system in the x-z vehicle-fixed coordinate system that correspond to the vertical motion of the car, where the idealized front and rear suspension system connects the car to the road surface corresponding to the global x-y coordinates of the front and rear of the vehicle. Through the formula equations describing the Working Model system constraints, the two systems are coupled so that each system affects the other in real time as the ODEs are integrated by the software.

Figure 3 is a screen shot of the simulation window before the simulation starts running. Figure 4 is a screen shot of the simulation in progress. Note that the time varying tire forces show up on the model as arrows applied at the front and rear axels in the plan view. The car weight and body-fixed x-acceleration forces also show up as arrows on the profile view.

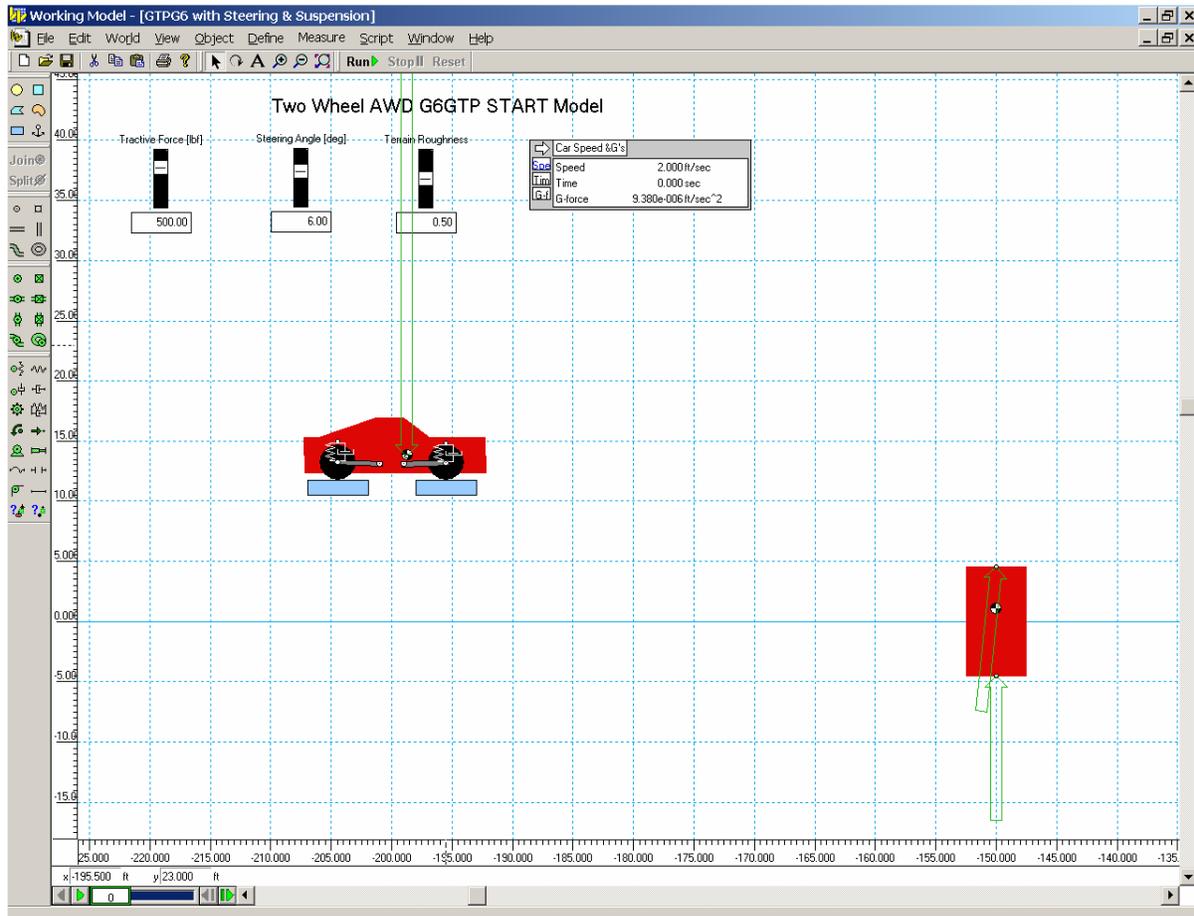


Figure 3 Screenshot of START Model Before Execution

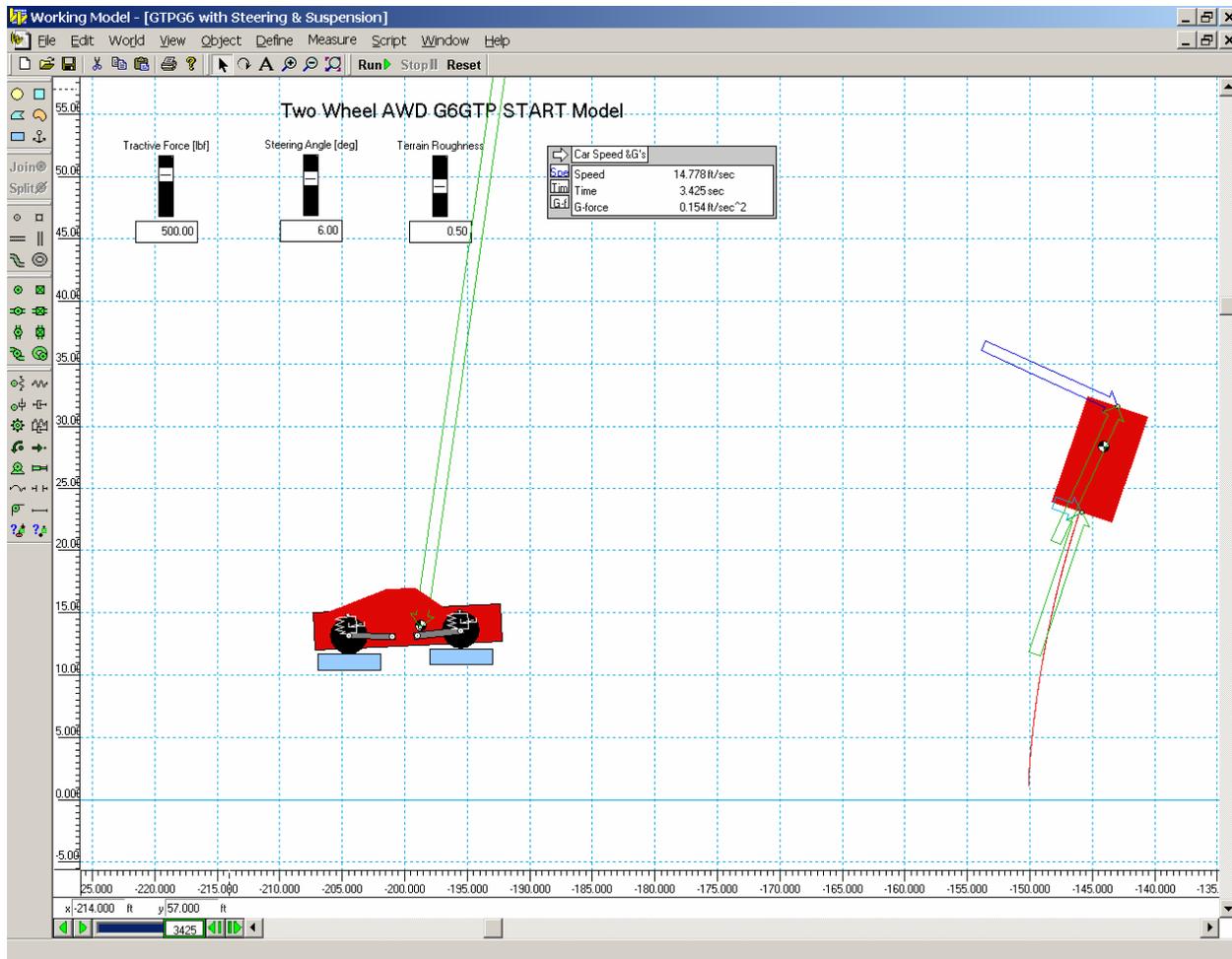


Figure 4 Screenshot of START Model While Running

Figure 5 is a screen shot of the formula description of the lateral force acting on the front wheels. “Normalforce(24,49).y” is the WM expression for the vertical contact force between the front tire and the terrain surface from the profile view. The “if” expression is used to start applying the tire lateral constraint force after 0.1 seconds of simulation time to avoid creating initial instability in the model. The “min” expression limits the tire lateral force to the maximum allowable by static friction, with  $\mu_s=0.9$ . 0.15 is the value of the cornering coefficient. “Output[15].y2” is the WM name for an expression that is evaluated using the system outputs and in this case is an expression for the slip angle for the front tires. Note the boxes checked that establishes the constraint as being in polar coordinates fixed to the body. This option is quite desirable for vehicle dynamics problems. “Input[13]” is the front wheel steering angle that the user inputs. In START this is done with a slider input so the steering angle can be changed in real time thus allowing the car direction to be changed by the user as the car moves through the terrain park. A similar slider input is used for the accelerator/brake control. These slider inputs can be seen in Figures 3 and 4 near the top of the figures.

Figure 6 is a screen shot of the formula description of the front wheel terrain position. Equation (1) has been entered using the WM formula language. In this screenshot  $A_1=2.5$ ,  $A_2=0.5$  and  $A_3=0.1$ . Output[59].y4, which is labeled “total”, includes everything in equation (1) except the

scale factor  $S$ .  $S$  is input by the user using a slider control and is near the other sliders in Figures 3 and 4. Changing  $S$  as the simulation runs allows the user to drive the car into smoother or rougher areas without restarting the simulation.

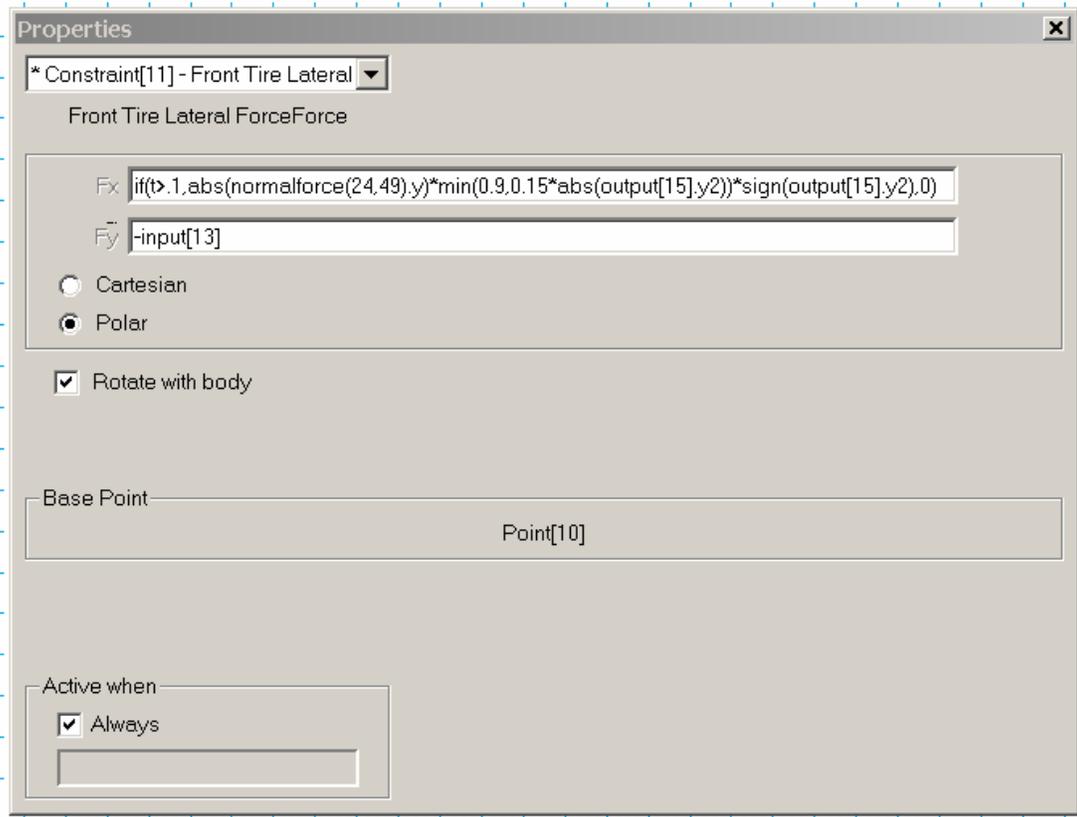


Figure 5 WM Front Tire Lateral Force Description

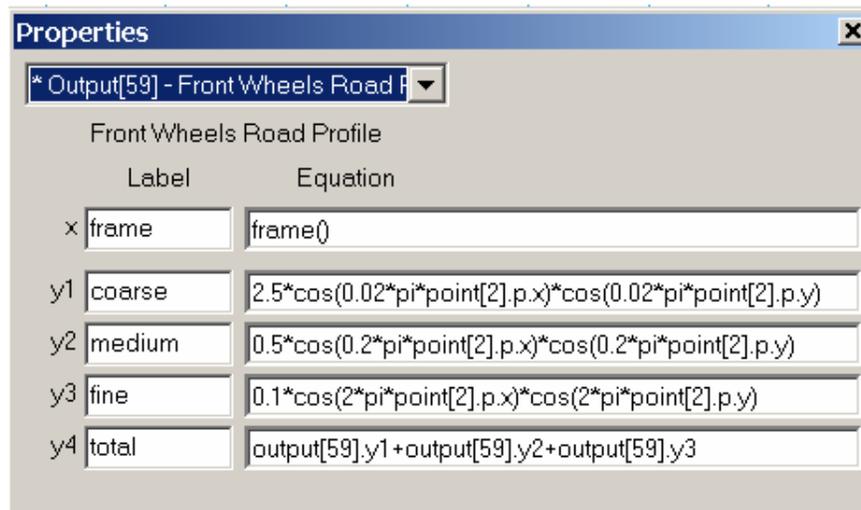


Figure 6 WM Front Tire Contact Point Position Description

## Results

When running the model in the smooth terrain mode ( $S=0$ ), students are able to see how the CG placement and coefficient of friction affect the maximum g-force the car can generate in a standard skid pad simulation. Interestingly the model comes very close (0.81g in FWD mode) to the experimental results, 0.82g, published in Road and Track<sup>4</sup> for the 2006 Pontiac FWD G6 GTP. The students also develop a full four wheel model (but with no suspension systems), which enables them to see the effect on maximum lateral g-forces of cornering coefficients,  $CC_{\alpha}$ , that are a function of the vertical tire load (rather than set at an average constant value). The effect is to slightly lower the maximum lateral g-force with all other factors remaining constant.

As expected, the rougher the terrain, the greater the steering angle has to be in order to effect a turn at a given radius for a given speed. Similarly, the engine thrust necessary to maintain a given speed in a turn is greater the rougher the terrain. For a given terrain roughness, the suspension spring rate and damping coefficients can be changed to maximize performance for the vehicle mass characteristics. Once they get their model working properly, students start to play with their model and develop criteria for a “good” suspension system. Considerations that generate diverse “optimums” are isolating the driver from g-forces that humans do not tolerate well and on the other hand providing the best control of the vehicle. A vehicle that is not in contact with the ground is not well-controlled, but the ride quality while flying through the air could be considered excellent! Students working independently from each other using the START model do not come up with the same answer for what is the best spring and damper constants for a given car mass distribution. This is also reflected in practice in vehicle design.

## Summary

For the student there are three stages to working with the START model. First they have to understand the basic vehicle force and motion relationships. This is no different than what a student would need to do in order to do a standard homework problem in a vehicle dynamics course. Secondly the student must relate the vehicle forces to the elements available in Working Model to represent dynamic systems. Fortunately the Working Model elements are simple and intuitive so mostly this step involves the students accurately entering formulas for the constraint forces and entering parameter values of the correct magnitude with consistent units. Thirdly, and most importantly, the student must decide how to set up a set of “experiments” where the model is run with different parameter values and the results are observed. These observations then need to be examined and analyzed to develop some insight into how the system behaves and how the system parameters might be changed to optimize some measure of performance. As a corollary to this, and as something that students enjoy most in the third step, the system parameters are also investigated to see what values make the system becomes unstable in some way. (Watching the vehicle awkwardly spin out of control or bounce off of the screen is a hoot!) As a result of going through the three stages, students (and the instructor) get insights into vehicle design that would not be possible with a more conventional analytical approach to teaching the course.

## References

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## Author's Biography

DR. GEORGE H. SUTHERLAND is professor and chair of the Manufacturing & Mechanical Engineering Technology and Packaging Science Department at Rochester Institute of Technology. Dr. Sutherland is interested in the dynamics of high speed machinery and vehicle dynamics. He was formally an associate professor in ME at Ohio State, a manager at General Electric, a VP at CAMP Inc and President of Washington Manufacturing Services.