3-28-2007

Using Descriptive Statistics in Statistical Process Control

Donald Holmes
Stochos, Inc.

A. Erhan Mergen
Rochester Institute of Technology

Northeast Decision Sciences Insitute

2007 Conference Proceedings

Renaissance HarborPlace Hotel
March 28-30, 2007
Baltimore, Maryland

Program Chair: Rhonda A. Hyde
University of Delaware

Proceedings Editor: Doug White, Roger Williams University

[IT1] GENERATION X, TELEVISION AND GAMING

8:00 AM to 9:30 AM *Suite 10059*

Chair

Matthew H. Roy, University of Massachusetts - Dartmouth

Analyzing Employee Commitment: The Case of Generation X

Matthew H. Roy, University of Massachusetts - Dartmouth

Father Knows Best? The Influence of Television Versus Personal Advice on Career Decisions

Emily T. Porschitz & Sinead G. Ruane, University of Massachusetts - Amherst

The Societal and Academic Implications of Electronic Games Use

Neset Hikmet, Deborah Lifland Decker & V. Kayhan
University of South Florida, Sarasota - Manatee

[IS3] DECISION SYSTEMS AND AI

8:00 AM to 9:30 AM *Pride of Baltimore*

Chair

Janet J. Prichard, Bryant University

A New Sigmoid Function-Based Consensus Ranking Method

Madjid Tavana, Frank A. LoPinto & James W. Smither, LaSalle University

Implementing Guided-Inference for Bayesian Intelligent Systems

Jinchang Wang, Richard Stockton College of New Jersey

Multi Agent Strategies for Decision Support Applications

David West & Scott A. Dellana, East Carolina University

[QU2] QUALITY POTPOURRI

8:00 AM to 9:30 AM *Suite 9059*

Chair

S. Bruce Han, Merrimack College

Capability Maturity Model: Investigating Success Factors

Adenekan Dedeke, Suffolk University

Do Type of Ownership, Size and Quality Orientation of an Organization Matter?

S. Bruce Han, Merrimack College
Shaw K. Chen, University of Rhode Island

Using Quality Function Deployment in Strategic Capital Budgeting Decisions

Fariborz Y. Partovi, Anatoly K. Kotlarsky, Susan F. Davis & Andrew K. Janas
Drexel University

Using Descriptive Statistics in Statistical Process Control

Donald S. Holmes, Stochos, Inc.
A. Erhan Mergen, Rochester Institute of Technology

USING DESCRIPTIVE STATISTICS IN STATISTICAL PROCESS CONTROL

Donald S. Holmes, Stochos Inc. P.O. Box 247, Duanesburg, N.Y. 12056.
(518) 895-2896, dsholmes@stochos.com
A. Erhan Mergen, Rochester Institute of Technology, Saunders College of Business
Decision Sciences, 107 Lomb Memorial Drive, Rochester, N.Y. 14623-5608.
(585) 475-6143, emergen@cob.rit.edu

ABSTRACT

The purpose of this paper is to demonstrate the various uses of descriptive statistical analysis in statistical process control (SPC). Some fairly well-known descriptive techniques, along with the lesser known methods, will be discussed for their potential use in SPC. A numerical example will be provided.

Keywords: Statistical process control, descriptive statistics, process improvement.

DISCUSSION

SPC is a key component of every quality philosophy, e.g., Total Quality Management, Six Sigma, etc. It is process-oriented, preventive, and helps identify types of variation in a process. SPC has several components, such as developing guidelines to run the process (i.e., standard operating procedures (SOP); describing the current process performance (e.g., using some descriptive statistics); monitoring the process over time (e.g., using proper control charts); assessing the capability of the process (e.g., using capability estimates such as capability indices); and providing review and feedback. For a more detailed description of SPC see, for example, Montgomery [11] and Duncan [4].

In this paper, we will look at the descriptive statistics component of SPC. Most SPC applications either totally ignore or do not take sufficient advantage of the use of descriptive statistics. Descriptive statistic analysis would provide valuable information about the current performance of the process, as well as providing significant input to the proper selection and use of process control and process capability methods. Descriptive statistics in a sense provide a picture of the process at a given time, i.e., what the process has done up to that point. In this regard descriptive statistics can be considered static measures, rather than dynamic measures like control charts.

Examples of the Use of Descriptive Statistical Analysis

i. The Histogram when compared to customer specification limits gives preliminary information about the process performance up to that point in graphical format, e.g., does the process seem to be meeting the customer expectations; is the process average near the customer target (if there is a customer target); how wide the width of the process compared to customer tolerance; etc.? These are all visual analyses but it could be very helpful in understanding the process performance.

ii. Calculating mean, standard deviation, skewness, kurtosis, etc., would provide additional information about the process during the period in which the sample data is gathered. For example, the estimated mean value can be compared to the customer target directly; and the standard deviation can be used to determine the width of the process, which in return can be checked against the specification limits. Skewness and kurtosis can be used to get a rough estimate of the shape of the process distribution. Skewness deals with the question of the symmetry of the curve relative to its center as measured by the average; and kurtosis describes the tendency of the curve to have long tails and a high center. A "Normal" curve is symmetric about the mean value; the skewness is zero for such a curve. The kurtosis measure for a normal curve is 3. Skewness and kurtosis measures can be computed as follows:

$$\text{Skewness} = \frac{\frac{\sum (X - \overline{X})^3}{n}}{SD_X^3} \tag{1}$$

$$\text{Kurtosis} = \frac{\frac{\sum (X - \overline{X})^4}{n}}{SD_X^4} \tag{2}$$

where X's are the individual observations, n is the number of observation, $\overline{X}$ and SD_X are the mean and standard deviation of the X's , respectively.

The measures of skewness and kurtosis are primarily used to make a quick assessment of the normality of your curve. Skewness values outside of $0 \pm \frac{7.35}{\sqrt{n}}$ or kurtosis values outside of $3 \pm \frac{14.70}{\sqrt{n}}$ are indications that the curve is not a normal curve (Holmes [6]). If the test results reveal that the process curve is not normally distributed, SPC analysis can be modified accordingly. For example, capability analysis should be carried out using the indices developed for non-normal process distribution (see, for example, Holmes and Mergen [7]), since conventional indices assume a Normal process distribution. This way the over/under estimate of the capability would be avoided.

iii. The standard deviation, which is used to measure the process width in SPC, can be estimated in a variety of different ways, depending upon what question we are trying to answer. It is crucial to use the proper estimate of the standard deviation to have the right answer for the question being asked; otherwise the analysis may lead to erroneous results. The conventional estimate of the standard deviation (equation 3) gives the total estimate of the variation that currently exists in the process.

$$\text{Standard deviation} = \sqrt{s^2} \ \text{where}\ s^2 = \frac{\sum_{i=1}^{n} (X_i - \overline{X})^2}{n-1} \tag{3}$$

This estimate is good to check the current performance of the process with respect to meeting the customer specifications. However, the capability standard deviation of the process, i.e., the standard deviation when the process operates under only common causes of variation, could be different than this one if the process currently is not statistically stable (i.e., in control). It is this capability standard deviation that we need to use to estimate the real capability of the process. Thus, several standard deviations should be estimated as part of descriptive statistical analysis. These estimates would be very close to each other if the process is stable; otherwise they would differ. Thus, these different estimates would then be used, for example, to generate early signals to see if the process was in control during the period under review by checking the significance of the difference of the estimators. The result of such analysis could be a valuable input for managerial planning in terms of setting proper targets for the capability of the process to reduce the variation.

One such estimate for the capability standard deviation is the one computed using mean square successive differences (MSSD) – see , for example, Neumann, et al. [13], Hald [5], Holmes and Mergen ([8] [9]). The MSSD is defined as

$$MSSD = \frac{1}{(n-1)}\sum_{i=1}^{n-1}(X_{i+1} - X_i)^2 \qquad (4)$$

Using these differences an unbiased estimate for the process variance is given by Hald [5] as

$$q^2 = \frac{1}{2(n-1)}\sum_{i=1}^{n-1}(X_{i+1} - X_i)^2 \qquad (5)$$

Then MSSD standard deviation is determined by taking the square root of the q^2. Roes, et al. [14] suggested a minor correction factor in estimating the standard deviation when the MSSD approach is used. This factor disappears as the sample size gets bigger. The significance of the difference between the conventional and MSSD variance estimates can be tested using the test given in Dixon and Massey [3]:

$$z = \frac{1 - \frac{q^2}{s^2}}{\sqrt{\frac{n-2}{(n-1)(n+1)}}} \qquad (6)$$

Values of z between ±3 indicate that the difference between the two estimates is not significant, i.e., the process seems to be stable, in other words, operating under common causes only. Z values bigger than +3 and less than -3 indicate that the two estimates are significantly different and thus the process is not stable (values bigger than +3 imply trend and long-term cycles in the process and values less than -3 imply short term cycles in the process). Since z is N(0,1), then the use of z values between ± 3 gives about 99.7% critical region for the test.

Through analysis like this process managers not only would have preliminary information about the stability of the process, but at the same time, using the smallest standard deviation estimate, they could come up with the potential capability of the process.

iv. Another test which should be part of descriptive statistical analysis is the test for autocorrelation. Use of computer controlled machines and automatic process control mechanisms seem to increase the chance for autocorrelation in the process. When the data is autocorrelated, it violates the basic assumption of the Shewhart control charts, which assume independence of the data points. Thus the check for existence of autocorrelation should be a routine part of the descriptive statistical analysis. If the process displays the sign of autocorrelation, process control techniques, such as control charts, should be modified to take into account the variation due to autocorrelation. Failure to do so would result in an erroneous conclusion about the process. See, for example, the following studies on process control with autocorrelated data ([1] [2] [10] [12]).

v. In the case of multiple process variables, the variance/covariance matrix should be analyzed as part of descriptive statistics to see if all or some of those variables are correlated. Correlated variables should be analyzed using multivariate SPC techniques to minimize the type I and/or type II error.

EXAMPLES

The first data set that is used is on viscosity. The summary descriptive statistics are given below (all data sets are available from the authors upon request).

			Descriptive Statistics			
Mean	=	9.1		Median	=	9.3
Std Dev .	=	0.6		SE Mean	=	0.0
Range	=	3.0		# Observ	=	310
Minimum	=	7.4		Maximum	=	10.4
Skewness	=	-0.5		Kurtosis	=	2.4
Cap SD	=	0.2		Cap Rtio	=	0.4
			Mean Square Successive Difference Tests			
Normal z	=	15.1		MSSD SD	=	0.2
			Spec Info			
LSL	=	8.4		% Under LSL	=	13.9
USL	=	10.4		% Over USL	=	0.0
Nominal	=	9.4		% In Specs.	=	86.1

As one can see, the process average is slightly below the nominal (9.1 vs. 9.4). The regular standard deviation is much bigger than the MSSD estimate of the standard deviation (0.6 vs. 0.2); and the significance test has a z value of 15.1 indicating that the two variance estimators are significantly different. This in turn implies that the data may have trend and/or long-term cycles. In other words, the process does not seem to be stable during the period the data is gathered. It also indicates that the process variation could potentially be reduced to 0.2 and the process capability be improved. The skewness test for normality indicates that the data is not quite normal. Specification analysis shows that the process will not be able to meet the specifications 100%; roughly 14% of the data will fall below LSL.

The second data set is on the diameter of transmission covers. This data shows the sign of first order autocorrelation (Figure 1), which violates the assumption of the independence of the control charts. Thus, the autocorrelation should be modeled and removed from the data before the control chart analysis is applied.

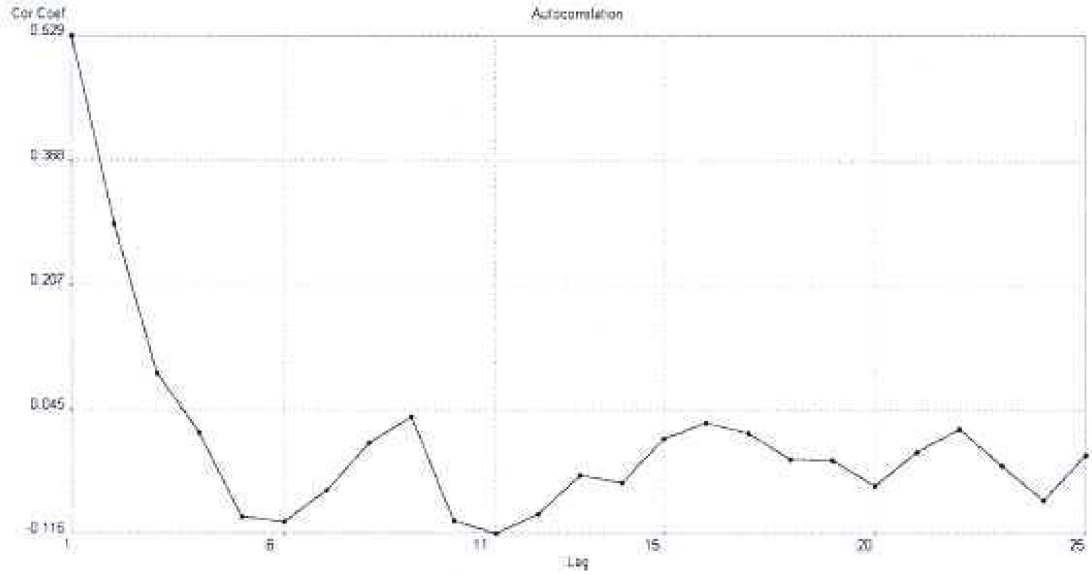

Figure 1. Autocorrelation chart.

The third data set is on the distribution of particle weight percentages for five screen sizes. The correlation matrix below shows strong correlations between some of the five variables. Thus, these correlated variables should be analyzed using multivariate SPC techniques to minimize the type I and/or type II error.

	S1	S2	S3	S4	S5
S1	1.00				
S2	0.59	1.00			
S3	0.96	0.39	1.00		
S4	0.87	0.74	0.77	1.00	
S5	0.34	-0.56	0.54	-0.01	1.00

Figure 2. Correlation matrix

CONCLUSION

In this paper we proposed more and better use of descriptive statistics in SPC. Better use of descriptive statistical analysis would make the other phases of the process control more effective by helping choose the right process control and process capability techniques, as well as helping managers set the proper targets for process improvement projects.

REFERENCES

[1] Alwan, L.C. and Roberts, H.V. "Time series modeling for statistical process control," *Journal of Bus. Econ. Stat.*, 1988, 6(1), 87-95.

[2] Alwan, L.C. "Autocorrelation: Fixed versus variable control limits," *Quality Engineering*, 1991-92, 4(2), 167-188.

[3] Dixon, W.J. and Massey, F.J., *Introduction to Statistical Analysis*, New York, NY: McGraw-Hill, 1969, 353-354.

[4] Duncan, A.J., *Quality Control and Industrial Statistics*, 5th edition, Homewood, IL: Irwin, 1986.

[5] Hald, A. *Statistical Theory with Engineering Applications*, NewYork, NY: Wiley, 1952, pp.357-360.

[6] Holmes, D.S., *Introduction to Statistical Process Control*, Schenectady, NY: Stochos, Inc., 2003.

[7] Holmes, D.S. and Mergen, A.E. "Problems with Non-Normal Distributions and Process Performance Measures," *The 1997 Northeast Decision Sciences Institute Meeting Proceedings*, Annapolis, Maryland, April 2-4, 1997, pp.299-301.

[8] Holmes, D.S. and Mergen, A.E., "An Alternative Method to test for Randomness of a Process," *Quality and Reliability Engineering International*, Vol.11, 1995, pp.171-174.

[9] Holmes, D.S. and Mergen, A.E. "Estimating Potential capability of an Unstable Process," *The 2004 Northeast Decision Sciences Institute Meeting Proceedings*, Atlantic City, NJ, March 24-26, 2004, pp.290-292.

[10] Holmes, D.S. and Gordon, W. "Automation can make your SPC charts send wrong messages," *Quality*, June 1992, 20-21.

[11] Montgomery, D.C., *Introduction to Statistical Quality Control*, 4th ed., New York, NY: Wiley, 2001.

[12] Montgomery, D.C. and Mastrangelo, C.M. "Some statistical process control methods for autocorrelated data," *Journal of Quality Technology*, 1991, 23(3), 179-193.

[13] Neumann, J.V., Kent, R.H., Bellinson, H.R. and Hart, B.I., "The Mean Square Successive Difference," *Annals of Mathematical Statistics*, Vol.12, 1941, pp.153-162.

[14] Roes, K.C.B., Does, R.M.M. and Schuring, Y., "Shewhart-Type Control Charts for Individual Observations," *Journal of Quality Technology*, Vol.25, No.3, 1993, pp.188-198.