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A vortex lens is a useful optical device having applications ranging from astronomy to microscopy. Current vortex masks operate across a narrow bandwidth. Two design schemes are proposed for creating a vortex across a bandwidth exceeding 100 nm in the visible region of the spectrum. © 2006 Optical Society of America

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Taking advantage of the unique properties of optical vortices or so-called “twisted light,” scientists from diverse areas have developed a wide range of applications. Techniques for creating an optical vortex, which usually require quasi-monochromatic light, restrict these uses. Such applications include optical spatial filtering, optical tweezers, high-resolution fluorescence microscopy, lithography, quantum cryptography, quantum entanglement, infrared vortex polarizers, and stellar coronagraphy. Additional uses may be possible once the broadband properties of vortices are fully explored. Such broadband vortices may also be expected to provide propagation dynamics that differ from naturally occurring vortices such as those in caustics and those from chromatic vortex masks.

An optical vortex of topological charge \( m \) may be induced on a noncirculating wave front (a plane wave for example) by an optical element having the following spatial transmission distribution for a field oscillating at angular frequency \( \omega \):

\[
t(\theta, \omega) = \exp[im(\omega)\theta] = \sum_{l=-\infty}^{\infty} C_l(\omega) \exp(il\theta) , \tag{1a}
\]

\[
C_l(\omega) = (2\pi)^{-1} \int_{-\pi}^{\pi} t(\theta, \omega) \exp(-il\theta) d\theta , \tag{1b}
\]

where \( C_l(\omega) = \text{sinc}[\pi m(\omega) - \pi l] \) shall be called the \( l \)-th order vortex spectrum and \( \theta \) is the angular coordinate in the transverse plane of the propagating beam. The function \( C_l(\omega) \) represents the fraction of light that is transmitted into the quantized mode, \( \exp(il\theta) \), where \( l \) is an integer. For example, when \( m \) has an integer value all the light is transmitted into the \( l=\pm m \) mode, i.e., \( C_m^l(\omega) = \delta_{m,l} \), where \( \delta_{m,l} \) is the Kronecker delta function.

A vortex lens, such as those shown in Fig. 1, may be fabricated to satisfy Eq. (1a). This element is a transparent plate with a single (or multiple) spiral etched into the surface. The thickness of the substrate in Fig. 1 may be expressed as

\[
d(\theta) = d_{\text{base}} + \theta' m_0 \lambda_0 / 2\pi(n_s - n_0)|_{\lambda_0} , \tag{2}
\]

where \( d_{\text{base}} \) is the base thickness, \( m_0 \) is the topological charge produced at the design vacuum wavelength, \( \lambda_0 = 2\pi c / \omega_0 \) (where \( c \) is the speed of light in vacuum and \( \omega_0 \) is the design angular frequency), and the refractive indices of the substrate and superstrate at the design wavelength are \( n_s(\lambda_0) \) and \( n_0(\lambda_0) \), respectively. Further, \( \theta' = \text{mod}(\theta, 2\pi/\mu) \), where \( \theta \) is the angle circumscribing the axis of the spiral and \( \mu \) is the pitch multiplicity, where \( m_0/\mu \) is an integer. The multiplicity values in Fig. 1(a) and 1(b) are \( \mu = 1 \) and \( \mu = 2 \), respectively. The pitch of the substrate surface is given by

\[
\Delta d = m_0 \lambda_0 / \mu(n_s - n_0)|_{\lambda_0} . \tag{3}
\]

For a conventional air–glass system, \( n_s(\lambda_0) - n_0(\lambda_0) \approx 0.5 \), and thus the pitch for an \( m_0 = 1 \) vortex lens is roughly twice the design vacuum wavelength. The superstrate thickness is given by \( D - d(\theta) \), where \( D \) is the net thickness of the lens.

The topological charge produced on a beam having an arbitrary vacuum wavelength, \( \lambda \), may be determined from the expression

\[
m(\lambda) = m_0(\lambda/\lambda_0)(n_s - n_0)\lambda(n_s - n_0)|_{\lambda_0} . \tag{4}
\]

Equation (4) is a dispersive relation, showing that the topological charge varies continuously with the wavelength of the beam. If material dispersion can be ignored, \( m(\omega) \approx m_0 \omega / \omega_0 \) and thus the vortex spectrum is given by \( C_l(\omega) = \text{sinc}(m_0 \pi\omega / \omega_0 - l\pi) \). If \( |\omega - \omega_0| / \omega_0 < 1 \), or equivalently \( |\lambda - \lambda_0| / \lambda_0 < 1 \), the zero-order topological transmission (corresponding to a noncirculating transmitted field) may be written \( C_0^l(\omega) = (\delta\lambda / \lambda_0)^2 \), where \( \delta\lambda = \lambda - \lambda_0 \). Thus a fraction of the transmitted beam is unaffected by the vortex lens. Another undesirable consequence of a chromatic vortex lens is that strong nearest neighbor modes \( (l = m_0 \pm 1) \) may be produced.

![Fig. 1. Lower substrate and upper superstrate layers of vortex lenses (phase masks) of topological charge \( m \) and pitch multiplicity \( \mu \), where (A) \( \mu = 1 \) and (B) \( \mu = 2 \).](image_url)
Achromatic optical components may be created by combining elements that have different refractive properties. An achromatic vortex lens requires the condition $m(\lambda) = m_0$ to be satisfied across a wide bandwidth of wavelengths. Using Eq. (4), one may express the achromatic condition as $m(\lambda)/m_0 = 1$. Successful fabrication of an achromatic vortex lens for use in the vicinity of $\lambda_0$, therefore depends on finding suitably matched materials.

Toward this end the refractive index may be expressed as a truncated Taylor series expansion:

$$n(\lambda) = n(\lambda_0) + n'(\lambda_0)(\lambda - \lambda_0) + (1/2)n''(\lambda_0)(\lambda - \lambda_0)^2,$$

(5)

where $n'(\lambda_0) = (\partial n/\partial \lambda)|_{\lambda_0}$ and $n''(\lambda_0) = (\partial^2 n/\partial \lambda^2)|_{\lambda_0}$. For many optical glasses the first two terms in Eq. (5) are sufficient to describe the refractive index in the vicinity of $\lambda_0$ across a bandwidth of roughly 100 nm. Combining Eqs. (4) and (5), the achromatic condition may be written as an error parameter:

$$n(\lambda) = n(\lambda_0) + n'(\lambda_0)\partial\lambda + (1/2)n''(\lambda_0)\partial\lambda^2,$$

(6)

where $\varepsilon = 0$ satisfies the condition $m(\lambda)/m_0 = 1$. The first term in Eq. (6) cannot be made equal to zero; otherwise $\Delta d = \infty$ [see Eq. (3)]. In practice, the value of $\Delta d$ should be made smaller than the characteristic diffraction length of the beam. Setting $\partial\lambda = 0$, a pair of materials that satisfies $\varepsilon = 0$ at a single wavelength, $\lambda_0$, may be explored. For optical glasses the second term in Eq. (6) is of the order of $10^{-2}$, and thus the index difference between the glasses must also be small. This difference will result in small Fresnel reflections at the interface but will also require a value of $\Delta d$ of the order of 100$\lambda_0$. To obtain smaller values of $\Delta d$ other materials must be explored. Dispersion equations such as the Sellmeier formula may be used to determine the refractive index and its derivatives for a given material. An analysis of more than 100 Schott glasses shows the trend, $|\varepsilon| \approx 1.2 \times |n_0(\lambda_0) - n_0(\lambda_0)|$. Thus large index differences (which are desirable to achieve small values of $\Delta d$) require large achromatic errors for these glasses. Table 1 shows pairs of Schott glasses having large index differences and small errors. Note that the labeling of “substrate” and “superstrate” is arbitrary, and thus Glass$_1$ and Glass$_2$ are listed.

The relative topological charge, $m(\lambda)/m_0$, and the topological transmission spectrum, $C^2$, may be numerically computed across the spectrum for a given pair of glasses. For example, Fig. 2 shows the values for Schott glasses N-LASF44 and N-SF14 and for the parameters $m_0 = 2$ and $\lambda_0 = 0.55 \mu m$. The bandwidth of the element depends on the fidelity of the transmission into the desired mode, $m_0$. For example, the

| Glass$_1$ | $n_1$ | Glass$_2$ | $n_2$ | $|n_1-n_2|/10^{-2}$ | (550 nm) | (500,600 nm) |
|-----------|-------|-----------|-------|-----------------|----------|-------------|
| N-LASF44  | 1.80789 | N-SF14    | 1.76786 | 4.00            | 4.91     | 1.79        |
| N-LAF36   | 1.80352 | SF14      | 1.76786 | 3.57            | 9.58     |
| N-LAK33A  | 1.75702 | N-SF1     | 1.72249 | 3.45            | 5.85     | 0.16        |
| F4        | 1.62017 | N-LAK21   | 1.64278 | 2.26            | 8.04     |
| N-LAF34   | 1.77583 | N-LAF7    | 1.75406 | 2.18            | 7.83     |
| F5        | 1.60677 | N-SK15    | 1.62526 | 1.85            | 6.95     | 4.14        |
| N-BASF64  | 1.70778 | N-SF8     | 1.69360 | 1.42            | 1.02     | 3.44        |
| N-BAP4    | 1.60864 | N-PSK53   | 1.62223 | 1.36            | 5.20     | 7.38        |
| N-KZFS11  | 1.64094 | N-LAK7    | 1.65398 | 1.30            | 4.45     | 6.34        |
| N-KZFS11  | 1.64094 | N-LAK22   | 1.65362 | 1.27            | 10.0     | 8.34        |
| N-BAF52   | 1.61140 | N-PSK53   | 1.62223 | 1.08            | 5.40     | 7.14        |
99.8% bandwidth, $\lambda_{\text{max}} - \lambda_{\text{min}}$ may be established by use of the integral

$$\frac{1}{(\lambda_{\text{max}} - \lambda_{\text{min}})} \int_{\lambda_{\text{min}}}^{\lambda_{\text{max}}} C_{m_0}^2 \, d\lambda = 0.998. \quad (7)$$

The data in Fig. 2 has a 99.8% bandwidth of 130 nm. In comparison, the 99.8% bandwidth of a chromatic fused silica mask in air is 20 nm. As expected, Fig. 2(b) shows no transmission into neighboring modes ($l \neq 2$) at $\lambda_0$. The next-nearest modes $C_0^2$ and $C_4^2$ have transmission values roughly five times smaller than $C_1^2$ and $C_3^2$ near $\lambda_0$.

Whereas the foregoing analysis describes a means of satisfying the achromatic condition in the vicinity of a single wavelength, it is also possible to achieve the same value of topological charge at two wavelengths, $\lambda_0 \pm \delta \lambda$. The achromatic error in this case is also described by Eq. (6), with the last term retained. Pairs of Schott glasses having $|\varepsilon| < 10^{-3}$ are listed in the final column of Table 1 for $\lambda_0 = 0.55 \, \mu\text{m}$ and $\delta \lambda = 0.05 \, \mu\text{m}$. The relative topological charge and the topological transmission spectrum is shown in Fig. 3 for Schott glasses F5 and N-SK15. In this case $m_0(0.5 \, \mu\text{m}) = m_0(0.6 \, \mu\text{m}) = 2.0$. Applying Eq. (7) the 99.8% bandwidth is found to be 140 nm. This value is 7.7% larger than the case in Fig. 2 and 700% larger than the case in Fig. 2(b) shows no transmission into neighboring modes ($l \neq 2$) at $\lambda_0 \pm \delta \lambda$.

The method of fabrication will depend on the properties of the substrate and superstrate. Glass, plastic, semiconductor, crystalline, or liquid materials may be used. For example, the substrate may be etched using photolithographic techniques, and the superstrate may then be deposited using vaporization, molding or spin-coating methods. To achieve small values of $\Delta l$, material pairs having a large value of $|\Delta l - n_0| \lambda_0$ are desirable.

In conclusion, an achromatic vortex lens comprises two adjacent materials whose interface resembles a single or multiple helicoid. The outer surfaces may be polished flat or curved to provide optical power. Two methods of designing an achromatic vortex lens have been discussed. One design seeks to achieve small achromatic errors in the vicinity of a single wavelength, whereas the other does this at two wavelengths. A bandwidth as large as 140 nm has been calculated for a pair of optical glasses. Other materials may be explored to achieve larger bandwidths.

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