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Evolution of the Dark Matter Distribution at the Galactic Center

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Neutralinos in supersymmetry are likely candidates for the non-baryonic dark matter \[1, 2\]. If neutralinos make up a large fraction of the dark matter in the galactic halo, pair annihilations will produce an excess of photons which may be observed in gamma ray detectors \[3\].

The galactic center (GC) is a promising target for such searches since the dark matter density is predicted to exhibit a central minimum \[12\]. However in fainter elliptical galaxies \[17\], the most likely sites for steeply-rising dark matter densities and hence for the steeply-rising stellar densities are the most plausible sites for SBH to remain high. Since stars and dark matter re-solve. Scattering decreases the density of a dark matter spike by kinetic heating, and by driving particles into the SBH. The result, after 10 Gyr, is the virtual dissolution of the dark matter spike in a galaxy like the Milky Way. This result suggests that enhancements in the dark matter density around the GC SBH would be modest whether or not the Milky Way bulge has experienced the scouring effects of a binary SBH.

Evidence for the scouring effect of binary SBHs is seen at the centers of the brightest galaxies \[7, 17\], where the stellar density profiles are nearly flat and sometimes even exhibit a central minimum \[12\]. However in fainter elliptical galaxies and in the bulges of spiral galaxies like the Milky Way, steeply-rising stellar densities are observed: \(\rho_\star \sim r^{-\gamma}, 1.5 \lesssim \gamma \lesssim 2.5\). In these galaxies, the most recent mergers may have taken place before the era at which SBHs formed, allowing the stellar density near the SBH to remain high. Since stars and dark matter respond similarly to the presence of a SBH, galaxies with steeply-rising stellar densities are the most plausible sites for steeply-rising dark matter densities and hence for the detection of annihilation radiation \[13\].

Stars near the SBH would also interact with the dark matter via gravitational scattering \[14, 13\]. Here, the time-dependent equations describing the scattering of dark matter particles off of stars in the presence of a SBH are solved. Scattering decreases the density of a dark matter spike by kinetic heating, and by driving particles into the SBH. The result, after 10 Gyr, is the virtual dissolution of the dark matter spike in a galaxy like the Milky Way. This result suggests that enhancements in the dark matter density around the GC SBH would be modest whether or not the Milky Way bulge has experienced the scouring effects of a binary SBH.
$M_\bullet = 3 \times 10^6 M_\odot$. The Coulomb logarithm was set to $\ln \Lambda = \ln (0.4 N)$ with $N \approx 6 \times 10^6$ the number of stars within $r_h$ \cite{19}. The time to heat the dark matter is nearly independent of radius and shorter by a factor $\sim 5$ than the age ($\sim 10$ Gyr; \cite{20}) of the stellar bulge.

The change with time of the dark matter density can be computed from the Fokker-Planck equation describing the evolution of $f(r, v, t)$, the mass density of dark matter particles in phase space, due to gravitational interactions with stars \cite{21}. We assume that $f$ is isotropic in velocity space, $f = f(E, t)$, with $E \equiv -v^2/2 + \phi(r)$ the binding energy per unit mass of a dark matter particle, and $\phi(r) = -\Phi(r)$, with $\Phi(r)$ the gravitational potential due to the SBH and the stars. The kinetic equation describing the evolution of $f(E, t)$ due to scattering off of stars with masses $\gg m$ is

\begin{equation}
4\pi^2 p(E) \frac{\partial f(E, t)}{\partial t} = -\frac{\partial F_E}{\partial E} - F_{lc}(E, t),
\end{equation}

\begin{equation}
F_E(E, t) = -D_{EE}(E) \frac{\partial f}{\partial E},
\end{equation}

\begin{equation}
D_{EE}(E) = 64\pi^4 G^2 \ln \Lambda \times \left[ q(E) \int_{-\infty}^{E} dE' h_s(E') + \int_{E}^{\infty} dE' q(E') h_s(E') \right],
\end{equation}

Here $p(E) = 4\sqrt{2} \int r^{r_{\text{max}}(E)} dr r^2 \sqrt{\phi(r) - E} = -\partial q/\partial E$ is the phase space volume accessible per unit of energy, with $\phi(r_{\text{max}}) = E$. The mass density of dark matter particles in $E$-space is $N(E)dE = 4\pi^2 p(E)f(E)dE$. The dark matter heating rate is determined by $h_s(E) = \int m_s f_s(E, m_s) dm_s$ with $f_s(E, m_s)dm_s$ the mass density of stars in phase space in the interval $m_s$ to $m_s + dm_s$. If the distribution $n(m_s)dm_s$ of stellar masses is assumed independent of energy (i.e. distance from the black hole), then $h_s(E) = \bar{m}_s f_s(E)$, with $f_s(E)$ the total mass density of stars in phase space and

\begin{equation}
\bar{m}_s = \frac{\int n(m_s)m_s^2 dm_s}{\int n(m_s)m_s dm_s}.
\end{equation}

$F_{lc}(E)$ is the flux of stars that are scattered from low angular momentum orbits into the SBH and is discussed in more detail below. Eqs. (1) assume that small-angle scatterings dominate the evolution of $f$ and that the gravitational potential changes on a time scale long compared with $T_{\text{local}}$; the latter assumption is valid as long as the gravitational acceleration is not produced dominantly by the dark matter particles themselves.

Neglecting $F_{lc}$, the total energy $E = \int_{E_1}^{E_2} N(E)EdE$ of dark matter particles in the energy range $E_1 < E < E_2$ changes with time according to

\begin{equation}
\frac{dE}{dt} = \int_{E_1}^{E_2} dE E \frac{\partial F_E}{\partial E},
\end{equation}

\begin{equation}
= \left[ EF_E \right]_{E_1}^{E_2} - \left[ D_{EE} \frac{E}{E} \right]_{E_1}^{E_2} \int_{E_1}^{E_2} dEN(E)Q(E),
\end{equation}

\begin{equation}
Q(E) = 16\pi^2 G^2 \bar{m}_s \ln \Lambda \int_{E}^{\infty} dE' f_s(E').
\end{equation}

The third term in eq. (6), which is always negative, represents heating of the dark matter. We accordingly define the (non-local) time scale for heating of the dark matter particles to be

\begin{equation}
T^{-1}_{\text{heat}} = \frac{\int_{E_1}^{E_2} dEN(E)Q(E)}{\int_{E_1}^{E_2} dEEN(E)}.
\end{equation}

This expression may be used to estimate the dissolution time of a dark matter spike. Assume that both stars and dark matter particles initially have power-law density profiles near the SBH: $\rho(r, t = 0) \propto r^{-\gamma} \rho_s(r) \propto r^{-\gamma}$, $r \lesssim r_h$, and that $\phi(r) = GM_\bullet/r$. The isotropic distribution function corresponding to an $r^{-\gamma}$ density profile in an $r^{-1}$ potential is $f(E) \propto E^{\gamma-3/2}$. Setting $E_1 = \phi(r_h) = GM_\bullet/r_h$ and $E_2 \rightarrow \infty$, $T_{\text{heat}}$ becomes

\begin{equation}
T_{\text{heat}} = A(\gamma, \gamma_{sp}) \frac{M_\bullet}{\bar{m}_s} \left( \frac{GM_\bullet}{r_h^3} \right)^{-1/2} \frac{1}{\ln \Lambda},
\end{equation}

\begin{equation}
A(\gamma, \gamma_{sp}) = \frac{1}{2} \sqrt{\frac{\pi}{3}} \left( 3 - \gamma \right)^{(2 - \gamma_{sp})} \left( \frac{\Gamma(\gamma + 1)}{\Gamma(\gamma + 1/2)} \right)^2.
\end{equation}

When $\gamma = 3/2$, equal within the uncertainties with the slope of the stellar cusp around the Milky Way SBH \cite{19}, the coefficient $A(\gamma, \gamma_{sp})$ in eq. (8) is independent of $\gamma_{sp}$ and

\begin{equation}
T_{\text{heat}} = 4\sqrt{3} \frac{M_\bullet}{27 \bar{m}_s} \left( \frac{GM_\bullet}{r_h^3} \right)^{-1/2} \frac{1}{\ln \Lambda}
\end{equation}

\begin{equation}
= 1.25 \times 10^9 \text{yr} \times \left( \frac{M_\bullet}{3 \times 10^6 M_\odot} \right)^{1/2} \left( \frac{r_h}{2 \text{ pc}} \right)^{3/2} \left( \frac{\bar{m}_s}{M_\odot} \right)^{-1} \left( \frac{\ln \Lambda}{15} \right)^{-1}.
\end{equation}

The effective stellar mass $\bar{m}_s$ that appears in equation (9a) depends via equation (7) on the mass function $n(m_s)$ of stars in the GC stellar cusp. When $n(m_s)$ is not strongly constrained by observations, either at the high or low mass ends, it is sometimes assumed (e.g. \cite{10}) to be a power law with Salpeter \cite{21} index, $n(m_s) \propto m^{-(1+\alpha)}$, $\alpha \approx 1.35$. Setting $m_{s, \min} = 0.08 M_\odot$ and $m_{s, \max} = (5)10(20)M_\odot$ then yields $\bar{m}_s \approx (0.8)(1.2)(1.8)M_\odot$. 
$T \text{ is the time in units of } t < 20$ Gyr, $r < 67 \text{ pc}$ and $\gamma_c = 1$. Times shown are $\tau = 0$ (heavy curves) and $\tau = 2, 4, \ldots, 20$ where $\tau$ is the time in units of $T_{\text{heat}}$ (eq. 9); $10 \leq \tau \leq 20$ corresponds roughly to the age of the galactic bulge.

We take $T_{\text{heat}}$ as defined in eq. 9 as our unit of time in what follows, with $\tau \equiv t/T_{\text{heat}}$. The age of the majority of the stars near the GC is $\gtrsim 10$ Gyr [20], although some much younger ($t \lesssim 10^7$ yr) stars are present in the cusp at distances $\lesssim 0.1$ pc from the SBH [23,24]. Setting $t = 10$ Gyr, $r_h = 1.67$ pc and $1 \lesssim \dot{m}_*/M_{\odot} \lesssim 2$ gives $10 \leq \tau \lesssim 20$. If the young stellar population is continually replenished, $m_*$ and $\tau$ could be larger, implying a higher mean rate of dark matter heating.

Diffusion in energy will cause a modest loss of stars to the SBH, $\dot{M} = -F_{\text{LC}}(E_2)$, $E_2 \approx c^2$. A much greater capture rate is implied by scattering of dark matter particles on low angular momentum (eccentric) orbits into the SBH [25]. The loss rate is given approximately by

\begin{equation}
F_{\text{LC}}(E) \approx 4\pi^2 P(E)J_c^2(E)\Pi(E)R \frac{\partial f}{\partial R} \tag{10a}
\end{equation}

\begin{equation}
\approx S(E)f(E), \tag{10b}
\end{equation}

\begin{equation}
S(E) = 4\pi^2 P(E)J_c^2(E)\Pi(E) \left[ \ln R_0(E)^{-1} \right]^{-1}, \tag{10c}
\end{equation}

Here $R \equiv J^2/J_c^2(E)$ is a scaled angular momentum variable with $J_c(E)$ the angular momentum of a circular orbit of energy $E$; $P$ is the period of a radial orbit; $\Pi$ is the orbit-averaged angular momentum diffusion coefficient $\langle (\Delta R)^2/2R \rangle$; and $R_0$ is the value of the angular momentum variable at which $f$ drops to zero due to the competing effects of capture and diffusion. The final line of eq. 10a assumes $f \sim \ln[R/R_0(E)]$ near the loss cone [26,27]. Cohn & Kulsrud [27] give expressions for $R_0$ as a function of $E$ based on solutions to the $R$–dependent Fokker-Planck equation; we adopt their expressions here. The angular momentum diffusion coefficients used by these authors, for modelling systems containing a single stellar mass, may be shown to remain unchanged when the scattered objects (here dark matter) have masses much less that those of the scatterers (stars). The rate of loss of stars predicted by eq. 10a depends only weakly on the radius of the capturing sphere, which we set to $2GM_*/c^2$. The diffusion coefficient $\Pi$ is of order $T_{\text{heat}}^{-1}$, hence $F_{\text{LC}}(E) \approx N(E)/T_{\text{heat}}(E) \ln R_0^{-1}$.

The detailed evolution of the dark matter density around the GC SBH was computed by integrating eq. 9h forward in time. The stellar density was modelled via Dehnen’s [28] density law, $\rho_*(r) \propto (r/r_0)^{-\gamma} (1 + r/r_0)^{\gamma-1}$ with $\gamma = 1.4$ and $r_0$ chosen to match the observed stellar density at $r \lesssim r_h$ [16]. Fig. 1 shows the evolution of the dark matter density assuming $\gamma_{sp} = 7/3$ and $\gamma_c = 1$, the values corresponding to a spike that developed in response to adiabatic growth of a SBH in a dark matter halo with $\gamma_c = 1.0$. Fig. 1 shows that scattering of dark matter particles by stars causes the dark matter density to drop and the spike to flatten, within a radius $\sim r_h$ where the heating time is shorter than the age of the bulge. Fig. 2 shows the dark matter density at $\tau = 10$ for a range of initial spike profiles, $1.0 \leq \gamma_{sp} \leq 2.75$. The mean density within $r_h$ drops by several orders of magnitude when $\gamma_{sp} \approx 2$. When $\gamma_{sp} = \gamma_c = 1$, i.e. no spike, the net effect of the heating is to increase the dark matter density slightly.

In the absence of scattering into the SBH, eqs. 9h have the time-independent solution $f(E) = \text{constant}$, $\rho \sim r^{-3/2}$ [15]. As Figs. 1 and 2 show, there is a tendency to evolve toward this characteristic profile, although a number of factors keep it from being precisely reached, including the finite evolution time; the presence of the loss term $F_{\text{LC}}$; and the fact that $f = \text{constant}$ can only hold true over a finite range of energies given the boundary conditions on $f$. Nevertheless, at late times ($\tau \gtrsim 20$), the solutions found here are generally well described by
$\rho \sim r^{-3/2}$ at radii $10^{-5} \lesssim r/r_h \lesssim 10^{-2}$.

The flux of dark matter annihilation photons along a direction that makes an angle $\psi$ with respect to the GC is proportional to the line-of-sight integral $\int_{\psi}^{\psi} \rho^2(\ell) d\ell$. Following earlier authors [3], we define the dimensionless form factor $J(\psi) \equiv K \int_{\psi}^{\psi} \rho^2(\ell) d\ell$, $K^{-1} = (8.5 \text{ kpc})/(0.3 \text{ GeV/cm}^3)$. Given a photon detector with angular acceptance $\Delta \Omega$ directed toward the GC, the signal is proportional to

$$\langle J \rangle \equiv \frac{1}{\Delta \Omega} \int_{-\Delta \Omega}^{\Delta \Omega} J(\psi) d\Omega.$$  \hspace{1cm} (11)

Figure 3 shows the evolution of $\langle J \rangle$ for $\Delta \Omega = 10^{-5}(10^{-3})$ sr; the first value is the approximate solid angle of the detectors in GLAST [29] and in atmospheric Cerenkov telescopes like VERITAS [30], while the larger angle corresponds approximately to EGRET [31]. The dark matter density was normalized to a fiducial value of $100 M_\odot pc^{-3}$ at $r = r_h$; $\langle J \rangle$ scales as $\rho^2(r_h)$.

We note that $\rho(r_h)$ is uncertain and could be much lower [4, 32, 33]. Figure 3 shows that the very large initial values of $\langle J \rangle$ are rapidly diminished as the spike is dissolved; by $\tau = 10$, $\langle J \rangle$ has dropped below $\sim (10^4, 10^5)$, $\Delta \Omega = (10^{-5}, 10^{-3})$ for all $\gamma_{sp} \gtrsim 2.5$. These values are similar to what would be predicted for the central regions of a dark matter halo in the absence of a SBH [3].

The dark mass captured by the SBH after 10 Gyr is less than $10^4 M_\odot$ ($\rho(r_h) = 100 M_\odot pc^{-3}$) for all the integrations presented here. Various schemes have been discussed for increasing the captured mass in stars or dark matter to much greater values, perhaps of order $M_\star$. These include making the dark matter collisional [34]; assuming instantaneous replenishment of the loss cone [35, 36]; or allowing the stellar potential to be non-axisymmetric [37]. The first two mechanisms are ad hoc; the third, if it applies to the GC, might allow the persistence of a dark matter spike in the face of scattering and capture by increasing the mass in dark matter particles that can interact with the SBH.

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