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The Infusion Length of Material Filled Inside Hollow Core Fiber

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The Infusion Length of Material Filled Inside Hollow Core Fiber

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December 6, 2017

MS Thesis

Telecommunications Engineering Technology

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The Infusion Length of Material
Filled Inside Hollow Core Fiber

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Thank you, everyone!
Abstract

We have designed a MATLAB code to perform the simulations on the infusion length of fluid inside the capillary. Previous research work has shown few results but the equation used for plotting the graph of infusion length v/s time has a numerical error. The work we have done has shown the step-by-step procedure to derive the list of forces acting on the capillary getting filled with the material and has also plotted the graph of infusion length v/s time from the equation we get after proof of derivation. The significant changes can be seen in the overall infusion length from the previous work and the work we have presented with mathematical derivations. The impact of overhead pressure, as well as capillary pressure on the infusion length, has been specifically discussed and the impact of the absence of one or the other could be clearly seen from the plots. The MATLAB simulation environment has been designed to go with any material to calculate the infusion length of the material over the time. List of factors affecting the filling length and calculation for same are presented in this report. The capillaries of hollow core fiber have been considered for the design looking at the applications of the fiber and future work.
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Chapter 1

1. Introduction

1.1 Background

The fiber optic communication has become a popular topic in the world of telecommunication as it seems to be the fastest way of communication between two nodes with low losses. There has been an increase in demand for higher data rates and in the world of cloud technologies, the fiber optic communication is providing support to data center fulfilling the needs of higher data rate transfer [1]. Light is a way of carrying an information using optical fibers. Though there are many other materials being used for manufacturing of these waveguides, the cheapest and easily available material is silica [2]. Information is carried using light within these waveguides. There are different generations of fibers that evolved after its discovery which focused on data speed, transmission distance, losses and remedies to overcome these losses and improve transmission.
The single mode fibers are the physical medium for carrying the light over long distance and there have been many different designs and application being developed that could reduce the loss and improve the quality of data getting transferred from end to end over different generations of fiber [3]. The evolving requirements and applications lead to the generation of the new structure of fiber name hollow core fiber. The hollow core fibers were introduced by Dr. Peter Russell and that leads to ultra-high-power transmission [4] as well as biomedical applications[5]. The fiber is made up of a hollow core surrounded by glass capillaries just like a honeycomb structure instead of the solid core like standard single mode fiber [4].

The hollow core of the fiber gave a lot of flexibility for researchers to fill the hole with different materials and develop the applications that best support their needs. The standard fiber like SMF have been so far doped with material like Erbium for amplification purposes [6] but that needs a lot of money as one must follow the fiber manufacturing process with the introduction of doping concentration. The hollow core fibers once manufactured provided researcher’s solution to doped the inner core of the hollow core fiber and test the fiber without going through the manufacturing process.
1.2 Motivation for Simulation

The advancing applications of hollow core fiber and doping flexibility provided by it has fascinated me to learn more about its application in the field of photonics. The previous research has shown the amplification and sensing applications being developed by the researchers with great results by filling the core with ytterbium and erbium respectively [7][8]. The single mode fibers have the core diameters of around 8-10 um which supports wavelength of operation around 1310-1550nm. The procedures defined in [9] includes filling and cleaving the fiber to make sure the only center of the core is getting filled. The procedure may sound easy just to fill the core of the hollow core fiber but it needs advanced tooling and technique to make sure we are only filling the core of the fiber and not the surrounding capillaries. This has motivated me to perform the simulation of the filling process to make sure within a given environment the center hole of different sizes can get filled up to the expected length of a fiber. The goal of my thesis is to outline and discuss the mathematical solution for the filling of capillary tubes and provide simulated results for different radii of capillary tubes getting filled. The thesis will also focus on the design of the generalized model that can simulate the filling of capillary tubes for any type of material and will also consider different inclination of capillary tubes to incorporate the length of material getting filled over the time. This thesis will also point out the correction in the equation mentioned in the
research paper presented by author Kristen Nielsen in his paper on “Selective filling of photonic crystal fibers” (Nielsen, Noordegraaf, Sørensen, Bjarklev, & Hansen, 2005). The mathematical calculations presented in this thesis will correct the equation used for plotting the graph of length of capillary getting filled v/s time.
Chapter 2

2. Structural details of Hollow core fiber

2.1 Fiber types and Transmission Mechanism

There are two types of fibers that we know as single mode fiber (SMF) and multimode fiber (MMF). These types have been used for different applications in fiber optic systems. MMF is limited by the distance it can carry the signal to and SMF has been going through many advancements as each generation needs higher data rates and transmission distances.

The phenomenon called as total internal reflection is used by light to travel down the optical fiber. The two important conditions for light to travel inside these silica core optical fibers are:

1. The angle of incidence needs to be greater than the critical angle
2. The light should travel through denser medium within the less dense medium
The fiber is made of a core surrounded by cladding where most of light travels inside a core. When light enters the core of optical fiber it interacts with the core and cladding boundary. Because the density of the core is higher than the cladding the light will be reflected and trapped inside the core [10].

The hollow core fiber (HCF) is a member of the family of photonic crystal fiber. The hollow core fiber is manufactured with hollow core stacked with capillaries [11-18]. The hollow core fiber can transmit the light in two different ways. It can be used the same phenomenon used by standard single mode fiber or multimode fiber i.e. total internal reflection (TIR) based on the doping concentration used inside the hollow core or combination of the hollow core as well as cladding capillaries. The other type of transmission mechanism used by hollow core fiber is known as photonic bandgap effect. The photonic bandgap is created inside the crystalline structure as the hollow core is completely hollow and can only consist of air without doping while the cladding has different capillaries made of silicon. The refractive index of silica is higher than the refractive index of air those total internal reflection phenomenon does not apply. The photonic bandgap effect causes
the beam of light to travel through the capillaries and create a bandgap effect that allowed light to travel through the fiber. In this scenario, light does not confine inside the core rather it travels through the cladding structure design in such a fashion that light travels back and forth the cladding structure through the core [19-24]. The popular Bragg diffraction grating is an analogy to this concept.

The advantages of hollow core fiber are listed below:

a. Tight confinement of light
b. Non-linear optics application
c. Low transmission loss
d. Controllable chromatic dispersion

The Bragg theory plays a significant role in understanding the transmission of light under a defined structure. The light travels through the structure and it goes through constructive or destructive interference pattern to generate a beam of light reflecting off the surface [25]. The Bragg’s theory tells the significance of pitch between the structure as it only allows certain Bragg’s wavelength to travel through the structure considering the change in effective refractive index remains constant. This dependency of Bragg’s wavelength to the pitch (\(\Lambda\)) of the structure is given as [26].

\[
\lambda_B = 2\ \text{n_eff} \ \Lambda
\]  

\[\text{(2.1)}\]
The pitch is nothing but the distance between two adjacent gratings or capillaries. So, once we decide the wavelength of operation we need to think of arranging the structure in such a way that calculated wavelength shall travel through the design structure. The other factor that plays a significant role is the change in the refractive index at the boundary condition. Once the pitch is decided the light wave travel through the structure and scatters a spherical wave pattern. These spherical waves then interfere and create constructive interference to generate a beam of light. The Bragg’s condition for constructive interference as given in [27]

while light scatters at an angle $\theta$, the condition defined for constructive interference of waves of the same frequency as mentioned in [27]
The difference of path travel by two ways is an integral multiple of wavelength where ‘n’ is an integer.

\[ 2dsin\theta = n\lambda \]  

(2.2)

The expressions are shown in equations (2.1) and (2.2) clearly shows the dependencies of wavelength to the pitch of the structure and angle of diffraction to the variation in \( \lambda \). The book by Lukas Chrostowski has given great details about the diffraction gratings. The relation between a pitch and the wavelength of operation as mentioned in [28]

\[ \Lambda = \frac{\lambda}{n_{\text{eff}} - n_{\text{clad}} sin\theta} \]  

(2.3)

These operational details are important for understanding the working of hollow core fiber and the way it has been designed. The geometry plays an important role and those will help us to understand the level of accuracy required to fill those capillaries. The geometrical analysis will be done in the next section of this chapter, we will be focusing on the minute details about the geometry of hollow core fiber as it will help us in the designing a mathematical model and understand the importance of these factors while designing a system to calculate the filling length of the capillaries.
2.2 Geometry of hollow core fiber

The geometry is key to the performance of hollow core fiber as we have seen in the previous section. The geometry of hollow core fiber consists of core, cladding and coating diameters. All the capillaries and the hollow core is circular and have been designed with certain dimensions.

The figure below is an example of a hollow core fiber by NKT photonics:

![Hollow core fiber from NKT Photonics](image)

*Figure 3. Hollow core fiber from NKT Photonics*

We can see that the coating is made of solid silica glass, while the core is hollow and surrounded by microstructure cladding. The pitch (Λ) is also clearly mentioned in Figure 3 which shows the distance between the center of the capillaries and the diameter of the capillaries mentioned as (d). The cladding consists of only small glass capillaries while the fiber also has a protective coating to keep it safe from any contaminants. The hollow core is an air channel and the cladding area is filled
with silica. The refractive index of air is close to unity and the cladding has a refractive index that of silica which approximately equals to 1.45.

2.3 Specification of Hollow core fiber

In this section, we will focus on the technical details of hollow core fiber. The hollow core fiber that we have considered for performing the simulation at a different inclination of capillary has following specifications.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Core diameter</td>
<td>10 ± 1 µm</td>
</tr>
<tr>
<td>Cladding pitch</td>
<td>3.8 ± 0.1µm</td>
</tr>
<tr>
<td>Diameter of PCF region</td>
<td>70 ± 5 µm</td>
</tr>
<tr>
<td>Cladding diameter</td>
<td>120 ± 2 µm</td>
</tr>
<tr>
<td>Coating diameter</td>
<td>220 ± 30 µm</td>
</tr>
<tr>
<td>Coating material</td>
<td>Single layer acrylate</td>
</tr>
</tbody>
</table>

*Table 1. Physical Properties of hollow core fiber*

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Design Wavelength</td>
<td>1550 nm</td>
</tr>
<tr>
<td>Operating wavelength</td>
<td>1490 – 1680 nm</td>
</tr>
</tbody>
</table>

*Table 2. Optical Properties of hollow core fiber*

The above details are from the hollow core fiber purchased from Thorlabs Inc. The hollow core fiber comes in various dimensions depending on the range of operating wavelength, wavelength and loss factors. The numerical aperture of this fiber is around ~0.2 and can be filled with gas or any other material.
2.4 Importance of understanding the Geometry of capillary tube

It is important to understand the geometry of hollow core fiber because there are many factors that depend on the geometrical shape of the fiber whether it is a circular waveguide or a rectangular one. The one that we have considered is a circular waveguide structure and those simulating the environment for circular capillaries equivalent to the core of the fiber. The radius of the circular waveguide/capillaries has significant importance and it calculated the surface area getting filled. The area of the circular waveguide is given as:

\[
\text{Area} = \pi r^2
\]  
(2.4)

Let us refer to the Figure 4. We have circular capillary in the picture.

![Figure 4. Volume inserted inside a capillary with area A and fluid traveling with velocity V. The d is the distance is the length of filled for the capillary over the period t.](image)

From equation (2.4), we know the equation for the area of circular capillaries and it’s clearly mentioned in . The length over which the capillary got filled is shown by letter’. The filled capillary is colored with blue and the capillary
is yet to be filled is shown with white empty space. The velocity of fluid inside the capillary is noted by ‘V’. The r in equation (2.4) stands for the radius of the circular tube.

From equation (2.4) and Figure 4, and referring to (“Dynamics of liquid rise in a vertical capillary tube,” 2013) the rate of volume getting filled then given as

\[
\text{Volume inserted inside tube} = \text{Area} \times \text{Velocity} \times \text{time} \quad (2.5)
\]

The equation (2.5) clearly shows the dependency of the rate of volume getting filled to the area of the capillary tube. After modifying the equation (2.5) the time require filling the capillary tube with certain volume keeping velocity of liquid constant can be given as

\[
\text{time} = \frac{\text{Volume inserted inside tube}}{\text{Area} \times \text{Velocity}} \quad (2.6)
\]

The equation (2.6) shows the relation between the time it will take to fill the capillary with calculated velocity and volume from (2.5), assuming values remain constant used in equation (2.6). The time is inversely proportional to the area of the capillaries. The smaller the capillaries more time it will take for them to get filled.
The above discussion shows that the geometry discussed in section 2.3 is significant. The rate of volume getting filled will depend on the surface area of the structure getting filled as well as it will also depend on the velocity of the liquid inside the structure. The velocity of the liquid is a function of the change in distance over the time and the rate at which it will flow will depend on the mechanical and natural forces applied to the capillary. As this thesis will calculate and plot length of capillary getting filled over the period, the dependency of time over the radius has significant importance in considering different radii for designing a generalized model for calculations. The mathematical calculations made in the upcoming chapters will put more light on the importance of understanding the geometry of the structures getting filled.
Chapter 3

3. Filling Processes for Hollow core fiber

3.1 Introduction

There have been a lot of research on the development of hollow core fiber as we have seen in the previous chapters as well as listed in [30-31]. A considerable amount of research has also been done on the filling of these hollow core fiber to generate innovative solution from it. The [32-33] has shown the filling of hollow core fiber with gases and liquids to produce plasmonic and sensing applications. In [34] they have discussed the Fresnel reflection and water core filling of hollow core fiber. The water core filling fiber is something we are more interested in as we have simulated our environment for water core filled fiber. The filling of hollow core is a very important process to discussed as it will decide the list of equipment and procedures under consideration for developing the model. The list of processes we will be discussing in the next few sections of this chapter will help us to determine the important factors that might affect the filling of hollow core fiber with liquid or
any type of material as well as the impact of those on infusion length of fiber over the period.

3.2 List of hollow core filling processes and applications

There have been a lot of research on the filling of hollow core fiber with the selective core as well as filling selective capillaries with the gases, liquid or manufacturing the hollow core with specified dimensions with only air hole for different operational wavelength. In this section, we will list few popular methods and procedure to fill the hollow core of the fiber and evaluate a list of parameters we need to consider for filling.

3.2.1 Fusion splicing methodology

As one of the popular method discussed in [35] has detailed about the fusion splicing technique to splice to single-mode fiber with hollow core fiber and technique of collapsed cladding holes with the used of SUMITOMO fusion splicer. The paper has detailed the images of collapsed cladding holes as well as the arc current required to achieve the collapsed cladding. The arc current is nothing but the current required to be provided to the electrode of the fusion splicer to generate the heat to melt the silica glass. This paper has detailed the use of a range of arc current to the produced certain amount of heat that could collapse the cladding
capillaries but keeps the core in good shape. There are two types of fusion splicer typically used for carrying out the experiment. The arc fusion splicer and filament fusion splicer. The [36] mentioned the fusion splicing within air environment with nothing inside the hollow core fiber while [37] discusses the manufacturing and fusion splicing the hollow core fiber. The filament splicers are controllable and one can control the amount of current supplied to the electrode while the arc fusion splicers mostly have fixed heat-generating electrode. The [38] has discussed some more details of the fusion splicing with the variable electrode current and impact of heat on the collapsing the cladding holes. The important of performing the experiment using fusion splicer is to make sure not to collapse the core hole. The only way to do that is to use an arc of short duration with low discharge current [39].

In standard splicing, the fibers need to be exactly at the center of the discharge electrode which generates heat to melt and splice the silica glass together.
Our purpose to collapse the cladding, the [40-41] has described the procedure for the same. The hollow core fiber will be treated with the electrode heat away from the center of the electrode to reduce the impact of the discharge on the center hole of the fiber. The hollow core fiber will be offset during the discharge from the electrode, the rate of collapse than will be given as

\[
\text{Rate of collapse} = \frac{\gamma}{2\eta}
\]  

(3.1)

The rate of collapse is a function of surface tension and the viscosity of silica. The viscosity of the silica is temperature dependent and will greatly reduce the increase in temperature that leads to collapsing the cladding holes.

The applications of this method are clearly seen in [42]. They have filled the hollow core of the fiber with the ethanol. The ethanol was filled in liquid form. These are a list of steps being followed for filling the hollow core of the fiber with the fusion splicing filling process.

a. Fusion splice the hollow core fiber to close the cladding holes
b. Suck the material into the core using a vacuum pump.
3.2.2 Cleaving methodology

The capillary filling using cut and cleaving methodology is widely used to avoid the complexity of the fusion splicing method. The [43] has shown the methodology to fill the hollow core with vacuum on one end of the fiber while the other end put into the liquid material. The similar methodology was accepted in [44] and the core was filled with silica aerogel. There have been selective filling techniques invented by the researchers. The selective filling includes filling the core and selective capillaries around the core. In [45-46] the core and six outer capillaries were only filled to create tunable birefringence optical fiber. The [47] has shown the applications of filling the core with water and ethanol as well as it has also a detailed number of samples taken with different material filled inside the hollow core of the fiber. The filling of different material on the loss of fiber as well as the impact on the operational bandwidth was also discussed. Some of these papers have explained the methodology they have used to fill the fiber, the mostly used methodology was by applying the external pressure to the capillaries using a vacuum pump or by using capillary action.

In [48] the researchers have discussed details of cut and cleave method. The paper defined the steps carried out in the process. The material used was NOA73 polymer material which can be UV cured and hardened after its get filled inside hollow capillaries. They have used syringe pump to fill the capillaries. They have
neglected the capillary pressure because it is negligible in front of external pressure applied by the syringe pump. The flow defined under the capillary with the applied external pressure was laminar flow, that means the meniscus of the liquid had concave shape toward the syringe pump end. The middle portion had a greater velocity than the subsequent layers of the liquid and it was minimum at the solid-liquid interface. The steps followed for filling is explained as below and data has been acquired form [48].

```markdown
Stage 1: ![HCF](image1) Filling of HCF with liquid
Stage 2: ![HCF](image2) Cleaved from the place core get filled
Stage 3: ![HCF](image3) Hollow core filled with liquid

*Figure 6.* Cut and cleave method of hollow core filling

- Cladding capillaries getting filled with the liquid
- Core getting filled with the liquid

In the first stage, the liquid was sucked into the hollow core fiber with the one end of the fiber inside the liquid while the other end connected to the syringe. As the
laminar flow established inside the hollow core fiber the velocity of infiltration of liquid inside the hollow core fiber given in [48] is

\[ V = \frac{\Delta P r^2}{8l\mu} \]  \hspace{1cm} (3.2)

The \( \Delta P \) is the differential pressure at the two ends of the fiber while the \( r \) is the radius of the fiber. The \( \mu \) is the viscosity of the liquid material getting filled while the \( l \) stands for the infusion length of the liquid inside the hollow core fiber. The stage of the filling includes filling the hollow core fiber with the liquid. As per the equation (3.2) and Figure 5, we can see that the cladding holes get filled over longer length than the core hole. The filled adhesive then UV cured to hardened inside the capillaries after cleaving from the end where the core gets filled in stage 1. In stage 2 the liquid filled again with syringe pump and applied external pressure. The hardened outside capillaries do not allow any further infusion while the core gets filled. The hollow core fiber then again cleaved after filling the core to greater extent than the cladding holes. The final sample includes only the filled hollow core of the fiber. The similar procedures with fusion splicing with single mode fiber has also been explained.
3.3 Importance of understanding filling process and simulation

It is important to understand different filling process and mechanism for designing the simulation model. The two main processes discussed in section 3.2.1 and 3.2.2 has so many factors that might need to consider for someone to design the generalized model. The section 3.2.2 is more interesting because it is less complex and has less destructive technique than one discussed in 3.2.1. The factors that affect the simulation model fusion splice method are the tapered core of the fiber, the radius under consideration for calculating the velocity of liquid as discussed in (3.2) is important. So, if one needs to design a model, the variation over the certain length of the tapered radius needs to be considered then followed by the constant radius over longer length for calculating the infusion length of liquid over the period. The common properties of the material have been highlighted in this section such as the viscosity of the material, surface tension. These properties of material decide the flow of liquid inside the capillaries. Some parameters are temperature dependent so one needs to design these parameters as a function of temperature. The importance of pressure difference at two ends of the capillary has been explained in equation (3.2). The pressure difference results in a flow of liquid those directly proportional to the velocity of liquid flow. So, if we need to increase the filling speed or should reduce the filling time for the constant length of capillary we need to increase the pressure difference at two ends of capillaries. The one
equipment mostly used is syringe and vacuum pump. The vacuum pump can be set to any desired pressure to suck the liquid into the hollow core fiber. The graph of pressure versus time has been plotted in [49]. It clearly shows the required time for different pressure values under consideration. The papers also used the term laminar flow of liquid inside the capillary and we will see more details about it in the upcoming chapter. Those understanding the filling process was important from this thesis point of view as it helped to note down the factors need to be paid attention to while designing a simulation model.
Chapter 4

4. Dynamics of capillary flow

4.1 Introduction

In this section, we will be doing a mathematical analysis of capillary action and difference of liquid flow inside a capillary for different radii and applied pressure. The equation will be generated that can be used to plot the infusion length of liquid inside a capillary v/s the time required to fill the capillary. The above sections will be recalled during the explanation of this chapter as most of the concept relates to the details we have studied in the previous chapter. The dynamics of capillary flow include different properties of the material, the impact of the capillary flow of liquid, analysis of liquid column inside a capillary, list of forces acting on the capillary, the law of conservation of energy, thermodynamics and Newton’s law will be discussed.

The dynamics of capillary flow inside a circular capillary is the base to understand the dynamics of mathematical modeling. The [50] has given a detail
about the capillary flow and according to it, different factors that impact the flow of liquid inside the capillary are below.

a. Pressure
b. Flow rate
c. Shear stress
d. Viscosity
e. Surface tension
f. Contact angle between solid and liquid surfaces
g. List of forces
   1. Capillary force
   2. Gravitational force
   3. Driving force
   4. Friction force
h. Infusion time.

In [51] researchers from Denmark have discussed the selective filling of photonic crystal fiber. They have also calculated the list of forces acting on the capillaries as fluid flow through the capillary. The meniscus variation and contact angle between solid and liquid have been presented in the paper. This paper has also listed the model of capillary filling. The paper though has listed all the forces and presented the equation that can be used to plot the infusion length of liquid v/s
time, the calculated final equation used for plotting the graphs has some modification as we will see in the upcoming section. The paper has claimed the model they have designed also being verified with the experimental results but with the calculations, we will make in the upcoming sections, it will be clearly seen that factor of two can make a lot of difference in the overall infusion length over the same time. This thesis will make the corrections in the equation and will plot the same graph and differentiate between two results w.r.t to the equation they have used to plot the graph and the equation we will derive in this chapter.

4.2 Density of fluid

The density of fluid [52] is proportionality of mass to the given volume. The mass of the fluid can be calculated considering the mass of empty vessel and mass of vessel filled with the fluid. The relation to calculate the mass of the fluid then will be given as

\[ \text{Total mass} = \text{mass of vessel} + \text{mass of fluid} \]  \hspace{1cm} (4.1)

The mass of the fluid then can be calculated re-arranging the equation (4.1) as,

\[ \text{Mass of fluid} = \text{Total mass} - \text{mass of vessel} \]  \hspace{1cm} (4.2)

Once, we calculate the mass of fluid the density of fluid can be given as,
4.3 Viscosity of fluid

The fluid that we will be considered for making the calculation is water because the correction we are making in the equation used to plot the infusion length v/s time plot in [51] have used the water and its material property to plot it. According to [52] the viscosity of any fluid can be measured as below.

\[ \text{Density of fluid} = \frac{\text{Mass of fluid}}{\text{volume}} \]  

\[ \rho_1: \text{density of sphere} \]
\[ \rho_2: \text{density of fluid} \]
\[ V_T: \text{velocity of sphere} \]
\[ g: \text{acceleration due to gravity} \]
\[ r^2: \text{radius of the sphere} \]

*Figure 7. Experimental diagram for calculating viscosity of fluid*
The equation to calculate the viscosity of the fluid then will be given as

\[ \mu = \frac{\rho_1 - \rho_2}{\sqrt{T}} \frac{2}{9} g r^2 \]  (4.4)

The viscosity of the material is given by \( \mu \). It is nothing but the resistance of the fluid and can even be calculated using viscometer. The dynamic viscosity of the fluid is force per unit area acting parallel to surface elements (fluid plate) of fluid which causes due to friction between plates. According to [52] the dynamic viscosity of fluid can be calculated as

\[ \tau(y) = \mu \frac{du}{dy} \]  (4.5)

The \( \tau \) is tensile stress while the \( \frac{du}{dy} \) is change in velocity with respect to diameter of the tube. Then \( \mu \) viscosity of fluid can be calculated by re-arranging the equation (4.5) as,

\[ \mu = \frac{\tau(y)}{\frac{du}{dy}} \]  (4.6)

The equations (4.4) and (4.6) can be used to calculate the viscosity of the fluid. These values then will be inserted in the final equation derived at the end of this chapter.
4.4 Contact angle between solid and liquid surface

The contact angle between solid and liquid boundary is important to understand as it will decide under no external pressure the liquid will be pulled inside the capillary of pushed outside the capillary. In [51], they have given a great explanation about the rise of capillary-based on the angle of contact between solid and liquid surfaces. Consider a vertical capillary inserted inside a vessel of water. If the contact angle between the capillary and the liquid surface is less than 90 degrees then the liquid will be pulled inside the capillary while for contact angle $\theta$ greater than 90, the liquid will be forced out of the capillary. This capillary force has named as capillary action the capillary filled with the liquid column.

According to [53] there are two types of flow possibly could flow through the circular capillaries and those are laminar and turbulent. The laminar flow is the flow of liquid with all its layer traveling parallel to each other and there is no disruption in the layers. The other type is turbulent flow in which there is a disruption in the flow of liquid inside the capillaries. The [54] has detailed about Reynold’s number that can be used to differentiate between these two types of flow. The Reynold’s number is a ration of inertial forces to viscous forces and is given as
The Reynolds number $\gg 2300$ is turbulent while Reynolds number $\ll 2300$ is laminar flow.

The [54] has provided great details on the capillary physics. The Bernoulli’s principle states that “the increase in fluid’s speed is based on the decrease in the potential energy of the fluid.”

4.5 Bernoulli’s principle

\[ E_i = E_o \quad \text{(4.8)} \]
Where $E_i$ is the input energy while $E_0$ is output energy at another end of the capillary. The input energy is the sum of work done by the fluid and the addition of potential as well as kinetic energy generated. Following up with the equation (4.8)

$$Work\ done_i + PE_i + KE_i = Work\ done_o + PE_o + KE_o$$

(4.9)

The work done is the distance over which force is applied to the material. Therefore, the work done is given as

$$Work\ done = force \times displacement$$

(4.10)

Applying equation (4.10) to equation (4.9) we get,

$$force \times displacement + PE_i + KE_i = force \times displacement + PE_o + KE_o$$

(4.11)

From [55], we get the definition of the potential and kinetic energy and accordingly, potential and kinetic energy will be given as,

$$Potential\ Energy = mass \times acceleration\ due\ to\ gravity \times height$$

$$P.E = m \times g \times h$$

(4.12)

$$Kinetic\ Energy = 0.5 \times mass \times (velocity)^2$$

$$K.E = 0.5 \times m \times (v)^2$$

(4.13)
Therefore, applying the results of equation (4.12) and (4.13) into (4.11) we get,

\[
\text{force} \times \text{displacement} + m \times g \times h + 0.5 \times m \times (v1)^2 = \text{force} \times \text{displacement} + m \times g \times h + 0.5 \times m \times (v2)^2
\]

The force acting on the capillary of circular dimension is nothing but the pressure per unit area. The pressure applied to the unit area will then make an object move or stay based on the way it’s been applied. Therefore, the force can be defined as,

\[
\text{force} = \text{Pressure} \times \text{area}
\]

(4.15)

The pressure then can be measured in newton per square meter. After applying equation (4.15) into equation (4.14) we get,

\[
\text{Pressure} \times \text{area} \times \text{displacement} + m \times g \times h + 0.5 \times m \times (v1)^2 = \text{Pressure} \times \text{area} \times \text{displacement} + m \times g \times h + 0.5 \times m \times (v2)^2
\]

(4.16)

Following with the law of conservation of energy, the equation (4.16) can be re-written as,

\[
Pi \times A \times \text{displacement} + m \times g \times h + 0.5 \times m \times (v1)^2 = Po \times A \times \text{displacement}
\]

(4.17)

\[+ m \times g \times h + 0.5 \times m \times (v2)^2\]

The \(Pi\) and \(Po\) can be better understood by looking at Figure 7.
Now, the velocity of fluid inside the capillary is the displacement of the liquid with the given time. It is same as the law of physics for calculating the velocity of a particle in the given environment.

The speed is the scalar quantity while velocity is a vector quantity and those direction of flow liquid is important to know in this case. The flow as we can see from Figure. 7 is from left to right and the equation (4.17) has been balanced such that the quantity remains positive.

Therefore, the equation (4.17) can be re-modified after replacing the displacement by velocity as

\[ \pi x A x \text{velocity} x \text{time} + m x g x h + 0.5 x m x (v1)^2 = P_0 x A x \text{velocity} x \text{time} + m x g x h + 0.5 x m x (v2)^2 \]  

(4.18)

After inputting the acronym for velocity (v) and for time as (t), the equation can be written as

\[ \pi x A x v x t + m x g x h + 0.5 x m x (v1)^2 = P_0 x A x v x t + m x g x h + 0.5 x m x (v2)^2 \]  

(4.19)
After following with the equation (2.5) and Figure 4. Equation (4.19) can be written as

\[ Pi \times \text{volume of fluid inserted} + mxgh + 0.5mx(v_1)^2 = Po \times \text{volume of fluid inserted} + mxgh + 0.5mx(v_2)^2 \] (4.20)

After considering the equation (4.3) the volume of fluid can be replaced by mass and density. The modified equation then can be represented as

\[ Pi \times \text{mass/density} + mxgh + 0.5mx(v_1)^2 = Po \times \text{mass/density} + mxgh + 0.5mx(v_2)^2 \] (4.21)

As the mass of the fluid remains constant, we can neglect the mass by cancelling out from above equation. The equation then will be minimized into

\[ Pi/density + gh + 0.5x(v_1)^2 = Po/density + gh + 0.5x(v_2)^2 \] (4.22)

Also, the length of the capillary will remain the same while calculating the pressure difference at two ends of the capillaries those we can neglect the gravitational constant and height for this calculation. After canceling out the term the equation will be
After taking the terms containing pressure from RHS to LHS and shifting the rest of the terms not containing pressure term from LHS to RHS we get,

\[
\frac{P_i}{\text{density}} + 0.5 \times (v_1)^2 = \frac{P_o}{\text{density}} + 0.5 \times (v_2)^2
\]  

(4.23)

As the density of the fluid considered to be same, the relation between the pressure difference at two ends of the capillary and the velocity of the fluid at respective ends of the capillary will be given as,

\[
\frac{P_i}{\text{density}} - \frac{P_o}{\text{density}} = 0.5 \times (v_2)^2 - 0.5 \times (v_1)^2
\]  

(4.24)

For varying density at two ends,

\[
P_i \times \text{density}_2 - P_o \times \text{density}_1/\text{density}_1 \times \text{density}_2 = 0.5 \times [(v_2)^2 - (v_1)^2]
\]  

(4.26)
From equation (4.25) and (4.26) we can easily calculate the pressure difference at two ends of the capillaries.

After carefully reading the [56] and following up with the physics of capillary flow. There is an important law that needs to be analyzed for finding the mathematical expression to fluid flow inside the capillary. The law is named as Poiseuille’s law. According to Poiseuille’s law, the fluid flow relates to pressure (P), viscosity (μ), length (d) and radius of the capillary (r). It also calculates the pressure drop under the laminar flow of fluid. We will derive the equation for flow rate and see the dependency of the flow rate on the pressure drop in the following section.
4.6 Poiseuille’s law

As we are considering water as our material to be flowing inside the capillary for the deriving our mathematical equation to calculate infusion length, the flow type for that will be laminar. Then we can calculate the flow rate of fluid using Poiseuille’s law as follows

![Diagram for calculating flow rate based on Poiseuille's law](image)

The $\Delta P$ pressure difference causes more volume to flow inside a tube. The driving force [54] can be given as

$$Driving\ Force\ (F_D) = \Delta P \times Area$$

$$F_D = \Delta P \times \pi \times R^2 \quad (4.27)$$
The driving pressure is analogous to the overhead pressure applied to the capillaries. The $\Delta P$ is the pressure difference at two ends of the capillaries and referring to Figure. 8 is $(P_2) - (P_1)$.

Just like a driving force drives the fluid, the resistive force will oppose the flow of fluid inside the capillaries. The resistive force that oppose the flow of fluid then will be given as

$$F_R = -\mu \times 2\pi R L \frac{dV}{dR}$$

(4.28)

For a steady state conditions, we will consider $F_D = F_R$. Those equating equations (4.27) and (4.28), we will get

$$\Delta P \times \pi R^2 = -\mu \times 2\pi R L \frac{dV}{dR}$$

(4.29)

After canceling out the $\pi$ and $R$ terms on both side of the equation. We will be left with

$$-\frac{dV}{dR} = \Delta P \times R / 2 \times L \times \mu$$

(4.30)

Then multiplying by the $dR$ term on both side we get,

$$-dV = \Delta P \times R / 2 \times L \times \mu \times dR$$

(4.31)
Then we will perform the integration of both sides of equation (4.31)

\[ \int_{0}^{\nu} -dV = \left( \Delta P / 2 \times L \times \mu \right) \times \int_{0}^{R} R \, dR \]  

(4.32)

After solving the integration term in the equation (4.32),

\[-[V]_{0}^{\nu} = \Delta P \times R / 2 \times L \times \mu \times \left[ \frac{R^2}{2} \right]_{0}^{R} \]

(4.33)

Solving the equation over the integral limit we get,

\[ -V = \frac{p_2 - p_1}{2 \times \mu \times L} \times \frac{R^2}{2} + C \]  

(4.34)

For C, consider \( R = r \) and \( V = 0 \). The equation (4.34) will get modified into,

\[ 0 = \frac{p_2 - p_1}{2 \times \mu \times L} \times \frac{r^2}{2} + C \]

(4.35)

Therefore, after re-arranging the terms in equation (4.35), we get

\[ C = \frac{-(p_2 - p_1)}{2 \times \mu \times L} \times \frac{r^2}{2} \]

(4.36)

After Inserting the value of equation (4.36) into the equation (4.34) gave us,
After re-arranging the terms of equation (4.37), we get the equation of the parabola.

This shows the flow profile of the fluid inside the capillary of radius R.

\[
V = \frac{P_2 - P_1}{2 \mu L} x \frac{R^2}{2} + \frac{-(P_2 - P_1)}{2 \mu L} x \frac{r^2}{2} \tag{4.37}
\]

The discharge rate is nothing but the volume of fluid traveling through the cross-sectional area A. In our case its circular capillary so the cross-sectional area will be the area of the circle. So, discharge rate can be derived as follows

\[
\frac{dV}{dt} = \text{Area} \times \text{Velocity} \tag{4.39}
\]

Substitute \(\frac{dV}{dt} = dQ,\)

\[
dQ = \text{Area} \times \text{Velocity} \tag{4.40}
\]

Dividing both sides by cross-sectional area will give,

\[
\frac{dQ}{dA} = \text{Velocity} \tag{4.41}
\]

Now substituting equation (4.38) for the value of velocity we get,
Re-arranging the terms in the equation (4.42) will give us,

\[
\frac{dQ}{dA} = \frac{P_2 - P_1}{2 \times \mu \times L} \times \left( \frac{R^2}{2} - \frac{r^2}{2} \right) \quad (4.42)
\]

Then integrating the equation (4.43) over the limit going from 0 to R, turn the equation into the following form,

\[
dQ = \frac{P_2 - P_1}{2 \times \mu \times L} \times \left( \frac{R^2}{2} - \frac{r^2}{2} \right) dA \quad (4.43)
\]

Substituting \(dA = 2 \times \Pi \times r\) we get,

\[
Q = \int_{r=0}^{r=R} \frac{P_2 - P_1}{2 \times \mu \times L} \times \left( \frac{R^2}{2} - \frac{r^2}{2} \right) \times 2 \times \Pi \times r \, dr \quad (4.44)
\]

The integration performs on equation (4.45) turns into,

\[
Q = \frac{\Delta P \times 2 \times \Pi}{2 \times \mu \times L} \times \int_{r=0}^{r=R} \left( \frac{R^2}{2} - \frac{r^3}{2} \right) dr \quad (4.46)
\]
After performing the integration, the equation will be presented as,

\[ Q = \frac{\Delta P \times 2 \times \Pi}{4 \times \mu \times L} \times x \left[ \frac{R^2r^2}{2} - \frac{r^4}{4}\right]_0^R \]  \hspace{1cm} (4.47)

Solving the equation (4.47) over the given limit give us,

\[ Q = \frac{\Delta P \times 2 \times \Pi}{4 \times \mu \times L} \times x \left[ \frac{R^4}{2} - \frac{R^4}{4}\right] \] \hspace{1cm} (4.48)

Finally, solving the equation and re-arranging the terms of equation (4.48) will get us,

\[ Q = \frac{\Delta P \times 2 \times \Pi}{4 \times \mu \times L} \times \left[ \frac{R^4}{4}\right] \] \hspace{1cm} (4.49)

Cancelling the factor of 2 on the RHS,

\[ Q = \frac{\Delta P \times \Pi}{2 \times \mu \times L} \times \left[ \frac{R^4}{4}\right] \] \hspace{1cm} (4.50)

The popular equation for calculating the flow rate under overhead pressure or shear stress then be represented as,

\[ Q = \frac{\Delta P \times \Pi \times R^4}{8 \times \mu \times L} \] \hspace{1cm} (4.51)
As we have seen in the previous chapters there is a capillary pressure and the flow rate will be calculated based on the capillary pressure applied only onto the fluid flowing inside the capillaries and there is also an overhead pressure that could be applied to suck the fluid inside the capillaries. We have already seen the flow rate w.r.t. overhead pressure or shear stress and in the next section we will be studying the impact of overhead pressure on the flow rate of the fluid. Once the flow rate is derived it will get easy to depict the infusion length based on the list of other forces applied onto the capillary.

4.7 Velocity of Fluid under capillary pressure

Again, considering the physics of capillaries and following up with the [54] and its references, we will study the equation for the velocity of fluid under capillary pressure. According to Poiseuille’s law in capillary,

\[
dV = \frac{n \times \sum P}{8 \times \mu \times L} \times (r^4 + 4 \times \varepsilon \times r^3) dt \tag{4.52}
\]

The \( r \) is nothing but the radius of the capillary, \( \varepsilon \) is coefficient of slip, \( \sum P \) is summation three different pressure acts on the capillary namely hydrostatic pressure, capillary pressure and atmospheric pressure. In this scenario, we will be only considering the capillary pressure for simplicity, \( \mu \) is the viscosity of fluid and
we have seen earlier calculation for the viscosity of fluid. The L in the equation stands for the length of the capillary and it can be view in Figure.8

According to equation (2.5) and Figure. 4 , the volume that inserted inside the capillary is given as

\[ dV = \Pi x r^2 dl \]  

(4.53)

Inserting the value of \( dV \) from equation (4.53) into equation (4.52), we get

\[ \Pi x r^2 dl = \frac{\Pi x \Sigma P}{8 \mu x L} x (r^4 + 4 x e x r^3) dt \]  

(4.54)

Re-arranging the terms and dividing both RHS and LHS by \( dt \). The equation (4.54) turns into

\[ dl/dt = \frac{\Pi x \Sigma P}{8 \mu x L x r^2} x (r^4 + 4 x e x r^3) / \Pi x r^2 \]  

(4.55)

The velocity of fluid inside capillary is,

\[ dl/dt = \frac{\Pi x \Sigma P}{8 \mu x L x r^2} x (r^4 + 4 x e x r^3) \]  

(4.56)
As we discussed the $\sum P$ is the summation of three different pressures, the hydrostatic pressure term can be represented as,

$$P_h = h \times g \times d - l_s \times g \times d \sin \varphi$$  \hspace{1cm} (4.57)

Similarly, the capillary pressure will be represented as,

$$P_c = \frac{2 \times \sigma}{r} \cos \theta$$  \hspace{1cm} (4.58)

The $\sigma$ stands for the surface tension and $\theta$ is the contact angle between solid and liquid interface. As we know the pressure is defined as the force per unit area. We can calculate the capillary force referring to equation (4.58)

$$Pressure = \frac{Force}{Area}$$

$$Force = Pressure \times Area$$

$$Force = \frac{2 \times \sigma}{r} \cos \theta \times Area$$

$$Force = \frac{2 \times \sigma}{r} \cos \theta \times \pi r^2$$

$$Force = 2 \times \sigma \times \pi r \cos \theta$$  \hspace{1cm} (4.59)

The $\sum P$ then can be represented as, refer to equation (4.57) and (4.58),

$$\sum P = P_A + h \times g \times d - l_s \times g \times d \sin \varphi + \frac{2 \times \sigma}{r} \cos \theta$$  \hspace{1cm} (4.60)
Substituting the value, we get in equation (4.60) into equation (4.56), we get

\[
dl/dt = \frac{\pi x (PA + h x g x d - ls x g x d \sin \phi + \frac{2 x \sigma}{r} \cos \theta)}{8 x \mu L x r^2} x (r^4 + 4 x e x r^3) \tag{4.61}
\]

Now, we will consider specific condition for calculating the volume rate under capillary pressure.

For horizontal capillary, \( \phi = 0, \epsilon = 0, PA = 0 \). Then, integrating both sides of equation (4.61), and re-arranging the term we see,

\[
\int dl = \int \frac{\pi x (PA + h x g x d - ls x g x d \sin \phi + \frac{2 x \sigma}{r} \cos \theta)}{8 x \mu L x r^2} x (r^4 + 4 x e x r^3) dt \tag{4.62}
\]

Integrating both RHS and LHS over the time from 0 to \( t \) gave us,

\[
l^2 = \frac{(PA + h x g x d + \frac{2 x \sigma}{r} \cos \theta)}{4 x \mu} x (r^2 + 4 x e x r) x t \tag{4.63}
\]

Now, for \( PA = 0 \),

\[
l^2 = \frac{(h x g x d + \frac{2 x \sigma}{r} \cos \theta)}{4 x \mu} x (r^2 + 4 x e x r) x t \tag{4.64}
\]
For $h = 0$ and $\varepsilon = 0$,

$$l^2 = \frac{\sigma \times \cos \theta}{2 \times \mu} \times r \times t \quad (4.65)$$

Dividing both sides by $t$ and differentiating LHS as function of $t$ give us,

$$\frac{dl}{dt} = \frac{\sigma \times \cos \theta \times r}{4 \times \mu \times l} \quad (4.66)$$

So, we conclude that for higher radius the velocity of the fluid is higher under capillary pressure.

Just like driving force and resistive forces, according to physics of capillary flow, there are forces like frictional forces and gravitation forces those applied to the capillary flow of fluids. The frictional force will be given as

$$F_f = - 8 \times \Pi \times \mu \times L \times U \quad (4.67)$$

The $\mu$ is the viscosity of the fluid and the $L$ stands for the length of the capillary. The $U$ is the velocity of the liquid inside the capillary. Similarly, the gravitational force acts on the capillary is given as,

$$F_g = - \Pi \times \rho \times g \times a^2 \times L \quad (4.68)$$
4.8 Derivation of the equation of infusion length of fluid

After carefully studying the equations for flow rate under capillary and overhead pressure as well as deriving equations to calculate different factors that might affect the infusion length. We will be using the law of conservation of momentum to finally calculate the infusion length of fluid inside a capillary [54].

According to the law of conservation of momentum, the change in momentum is equal to the summation of all different forces acting on the capillary.

\[ \frac{d}{dt} (\text{momentum}) = \sum F \]  

(4.69)

The momentum is defined as the product of mass into velocity. Replacing the momentum by this term then give us,

\[ \frac{d}{dt} (\text{mass} \times \text{velocity}) = \sum F \]  

(4.70)

Now, referring to the section on calculating the density of the material. We know, the density of the material is a ratio of mass to volume. Replacing the mass by the terms of density will give us,

\[ \frac{d}{dt} (\text{density} \times \text{volume} \times \text{velocity}) = \sum F \]  

(4.71)
The velocity of the fluid inside the capillary changes in length of capillary getting fluid to the time it takes to fill the capillary. Referring to equation (2.5) and . The equation (4.71) can be represented as

\[ \frac{d}{dt} (\text{density} \times \text{volume} \times \frac{dL}{dt}) = \Sigma F \]  

(4.72)

Taking the \( \frac{dL}{dt} \) term outside the bracket and solving for the equation we get,

\[ \frac{d^2L}{dt^2} (\text{density} \times \text{volume} ) = \Sigma F \]  

(4.73)

From equation (2.5) we know the volume inserted inside the capillary is given as

\[ dV = \pi r^2 x dL \]  

(4.74)

Referring to equation (4.74), the volume is the product of the area of the capillary and infusion length of fluid inside the capillary. Substituting for volume will give us,

\[ \frac{d^2L}{dt^2} (\text{density} \times \text{area} \times L ) = \Sigma F \]  

(4.75)

Taking the L outside the bracket will get,

\[ \frac{d^2L^2}{dt^2} (\text{density} \times \text{area}) = \Sigma F \]  

(4.76)
The area here is an area of the circular capillary which is equal to the area of a circle. Substituting for the area of circle in the equation (4.76),

\[
\frac{d^2 L^2}{dt^2} (\text{density} \times \pi r^2) = \sum F
\]  \hspace{1cm} (4.77)

Referring to list of forces mentioned in (4.59), (4.67), (4.68) and substituting in the RHS will elaborate the RHS into

\[
\sum F = 2\pi r \sigma \cos \theta + \Delta P \pi r^2 - 8 \pi \mu x L x U - \pi \rho x g x a^2 x L
\]  \hspace{1cm} (4.78)

Combining equations (4.77) and (4.78) we get,

\[
\frac{d^2 L^2}{dt^2} (\text{density} \times \pi r^2) = 2\pi r \sigma \cos \theta + \Delta P \pi r^2 - 8 \pi \mu x L x U - \pi \rho x g x r^2 x L
\]  \hspace{1cm} (4.79)

Dividing the RHS by \text{density} \times \pi r^2 in equation (4.79), we get

\[
\frac{d^2 L^2}{dt^2} = 2 \sigma \cos \theta / \pi \rho + \Delta P / \rho - 8\mu L U / r^2 \rho - g L
\]  \hspace{1cm} (4.80)
Replacing the U by \( \frac{dL}{dt} \), in the equation (4.80),

\[
\frac{d^2L}{dt^2} + \frac{8\mu}{\rho r^2} \frac{dL}{dt} L + gL = \frac{2\sigma \cos \theta + r \Delta P}{r \rho} \tag{4.81}
\]

Substituting for the constant terms in the equation (4.81),

\[
A = \frac{2\sigma \cos \theta + r \Delta P}{r \rho} \quad \text{and} \quad B = \frac{8\mu}{\rho r^2}
\]

\[
\frac{d^2L}{dt^2} + B \frac{dL}{dt} L + gL = A \tag{4.82}
\]

The equation (4.82) have been used in the MATLAB simulation to calculate the infusion length as a function of \( t \). The MATLAB uses an ode45 function to solve the non-linear differential equation. The aim of this thesis is not just to derive the equation for infusion length but also to correct the factor of 2 which was there in the equation presented by [51]. The upcoming section will differentiate the results we get by plotting the infusion length v/s time graph using the equation mentioned in [51] and derived in this thesis.

The equation presented in [51] is

\[
\frac{d^2L^2}{dt^2} + \frac{8\mu}{\rho r^2} \frac{dL}{dt} L + 2gL = \frac{4\sigma \cos \theta + 2r \Delta P}{r \rho} \tag{4.83}
\]
And the simplifies version looks like

\[
\frac{d^2 L^2}{dt^2} + B \frac{dL}{dt} L + 2gL = A
\]

(4.84)

\[
A = \frac{4 \sigma \cos \theta + 2 r \Delta P}{rp} \quad \text{and} \quad B = \frac{8\mu}{\rho r^2}
\]

The impact of a factor of extra 2 in the pressure term will double the pressure and those it will show that capillaries get filled over a longer length in given time than it should be. The details discussion is presented in the next chapter.
Chapter 5

5. MATLAB Simulation and results

5.1 Introduction

In this chapter, I have presented the MATLAB simulation results from the .m scripts I have designed using the equations from chapter 4. The reliability of code is verified by plotting the same graph in [51]. Then the calculated equation (4.81) in chapter 4 was used to plot the infusion length v/s time graph and differentiated data is presented. The data clearly shows the impact of a factor of 2 on the overall infusion length of capillary over the same time. The graphs are plotted for different radii of hollow core fiber. The specific radii considered by [51] are 1 um, 5 um and 10 um. The material used to fill the hollow core of the fiber is a water. The MATLAB function ODE45 has been used to solve the non-linear differential equation. The second order non-linear non-homogenous differential equation is
first converted into first order equation and then solved by the function to calculate the infusion length as a function of time.

5.2 MATLAB simulation and Results

In this section, we will explore the MATLAB simulation results and will see the difference between the plots we get by using equation [4.84] and equation [4.82].

5.2.1 Reliability check of MATLAB code

At first, we have plotted the graph using

\[
\frac{d^2 L^2}{dt^2} + \frac{8\mu}{\rho r^2} \frac{dL}{dt} L + 2gL = \frac{4 \sigma \cos \theta + 2r \Delta P}{\rho} \tag{5.1}
\]

In [51] they have considered the contact angle between the liquid and solid interface as 0 as well as the gravitational term has been neglected as the impact of gravitation on the horizontal capillary is negligible. The paper has provided us with the values of surface tension = 72 dyne, the viscosity of water as \( \mu = 1 \) cp.

![Figure 10](image.png)

*Figure 10. The plot of infusion length versus time for \( \theta=0, \sigma=72 \) dyne and \( \mu=1 \) cP.*
The graph we get by substituting these value in equation (5.1) and solving with the MATLAB code I have designed I got the above graph. The graph that they [51] had for the provided value after solving the equation analytically after neglecting the gravitational term was

![Graph](image_url)

*Figure 11. Plot for infusion length from reference 51*

There was a lot of difference between the plot we get for solving the same equation using MATLAB and graph presented in [51]. Then I realized even though we consider a horizontal capillary there should be some pressure difference at two ends of the capillary either due to the capillary force of hydrostatic pressure or externally applied pressure. The term missing in the calculation or has not been provided in
the paper was the values of overhead pressure. Those calculating the pressure value and inserted them into the MATLAB code for different radii we get,

![Plot for infusion length after considering pressure values for radii of 1 um, 5um and 10um](image)

*Figure 12. Plot for infusion length after considering pressure values for radii of 1 um, 5um and 10um*

The Figure 11 has shown two graphs, the graph with the solid line is when the capillaries are vertical while the graph with a dotted line is when the capillaries are horizontal. For the horizontal capillaries, 2g term was neglected. As we compared the Figure 12 graph with the Figure 11 graphs with the dotted line we got a match. Those the reliability of our MATLAB code has been verified.
The results of the graph from Figure 12 can be summarized in the below table.

<table>
<thead>
<tr>
<th>Infusion Length</th>
<th>Radius of the capillary</th>
<th>$\Delta P$ ((\text{dyne/cm}^2))</th>
<th>Time duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>29.3 cm</td>
<td>1 $\mu$m</td>
<td>56000000</td>
<td>60 minutes</td>
</tr>
<tr>
<td>66 cm</td>
<td>5 $\mu$m</td>
<td>11000000</td>
<td>60 minutes</td>
</tr>
<tr>
<td>81.8 cm</td>
<td>10 $\mu$m</td>
<td>4200000</td>
<td>60 minutes</td>
</tr>
</tbody>
</table>

*Table 3. Details of the infusion length v/s time plot for the assumed overhead pressure*

5.2.2 MATLAB simulation with corrected equation

As we found the MATLAB code was generating the exact similar plot for the equation provided in [51], we take it to next level by plotting with the corrected equation as presented by equation (4.82). The difference in the plots for the corrected values without any applied pressure and similar values of surface tension and viscosity for water we get,

*Figure 13. MATLAB simulation with corrected equation using $\sigma=\mu$, $\sigma=1$ dyne and $\mu=1$ cP.*
We can clearly see the difference between the Figure. 13 and Figure 10. There is a shift in the graph in Figure 13, as the equation (4.84) is different by the factor of 2 than equation (4.82).

The equation that we have used for plotting this graph is

\[ \frac{d^2 L^2}{dt^2} + \frac{8\mu}{\rho r^2} \frac{dL}{dt} L + gL = \frac{2\sigma \cos \theta + r \Delta P}{r \rho} \]  

(5.2)

Now, that we have verified the shift in the graph. We put more light into the equation and applying the overhead pressure values we got for three different radii under consideration of the capillaries. Using these pressure values and equation (5.2) we have plotted the graph for infusion length of the capillaries.

*Figure 14.* Plot of infusion length v/s time with corrected equation
From above figure, we can see a clear difference between results of Figure 12 and Figure 14. The infusion length is reduced by the factor 0.707. This is because the exact solution of the differential equation contains the square root term.

The result of the graphs from Figure 14 can be summarized in the table below.

<table>
<thead>
<tr>
<th>Infusion Length</th>
<th>Radius of the capillary</th>
<th>( \Delta P ) (dyne/cm²)</th>
<th>Time duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>20.75 cm</td>
<td>1 µm</td>
<td>56000000</td>
<td>60 minutes</td>
</tr>
<tr>
<td>46.66 cm</td>
<td>5 µm</td>
<td>11000000</td>
<td>60 minutes</td>
</tr>
<tr>
<td>57.8 cm</td>
<td>10 µ</td>
<td>4200000</td>
<td>60 minutes</td>
</tr>
</tbody>
</table>

*Table 4. Details of the infusion length v/s time plot with the corrected equation*

5.3 Impact of \( \theta \) variation on infusion length.

The contact angle between the solid and liquid interface has been discussed in the previous sections. In this section, we will see the impact of the contact angle on the infusion length.

As we have seen in equation (4.58), the capillary pressure can be given as

\[
P_c = \frac{2 \times \sigma}{r} \cos \theta
\]

And we also know the overall pressure acting on the capillary sum of capillary pressure and overhead pressure. The graphs we get initially as in Figure 10.
was different from the one Figure 11 or Figure 12 because it does not include the overhead pressure and force associated with it. That means the only pressure acting on the capillaries is due to capillary force.

In section 4.8, the derivation has defined the summation of force as summation of all the listed forces in the previous section considered to be acting on the capillary. The equation was given as

\[ \sum F = 2\pi r \sigma \cos \theta + \Delta P \pi r^2 - 8 \pi \mu \rho x L x U - \pi \rho x g x a^2 x L \]

If neglect or canceled out the overhead pressure term \( \Delta P \) from the equation then the final equation we get

\[ \sum F = 2\pi r \sigma \cos \theta - 8 \pi \mu \rho x L x U - \pi \rho x g x a^2 x L \]

The final equation referring to 4.79 and following equations of it will reduce it to,

\[
\frac{d^2 L^2}{dt^2} + \frac{8\mu}{\rho r^2} \frac{dL}{dt} + gL = \frac{2\sigma \cos \theta}{r \rho} \quad (5.3)
\]

The equation presented in [51] can then be represented as,

\[
\frac{d^2 L^2}{dt^2} + \frac{8\mu}{\rho r^2} \frac{dL}{dt} + 2gL = \frac{4\sigma \cos \theta}{r \rho} \quad (5.4)
\]
The equations (5.3) and (5.4) represents the differential equation to solve for calculating the infusion length as a function of time. So, we can see that in the absence of overhead pressure the only pressure that acts on the fluid is due to capillary. Those varying the contact angle between the solid and liquid will vary the infusion length as capillary pressure is directly proportional to the angle of contact and inversely proportional to a radius as per equation (4.58).

After considering the above conditions we have re-arrange the equations and plotted the graph for the different values of contact angle. We specifically considered θ= 45 and 90-degree cases. As cos is an even function the -45 and -90 cases will return the same values. The factor of 2g needs to be considered here as the gravitational field will also act on the capillary carrying the fluid.

*Figure 15. Infusion length v/s time under capillary pressure for ΔP = 0, and θ =45 degree*
Therefore, for $\Delta P = 0$, and $\theta =45$ degree and considering equation (5.4) the infusion length v/s time plot looks like above graph.

Similarly, to see the impact of the contact angle on the capillary pressure and those on the infusion length of the capillary we choose to repeat the similar process for the higher value of $\theta$ and those for $\Delta P = 0$, and $\theta =90$ degree we get the below plot.

*Figure 16. Infusion length v/s time plot for $\Delta P = 0$, and $\theta =90$ degree*
Looking at Figure 16, we can see that if the contact angle equals to 90 degrees there is not impact from the capillary pressure on the infusion length as the vertical capillary does not pull the liquid inside the capillary but try to push the liquid out of the capillaries. The radii of the capillaries under consideration has shown the values equal to 0 for the entire period of filling. The result of the graphs from Figure 15 and Figure 16 can be summarized in the table below.

<table>
<thead>
<tr>
<th>Infusion Length</th>
<th>Radius of the capillary</th>
<th>( \theta )</th>
<th>( \Delta P ) (dyne/cm(^2))</th>
<th>Time duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.4 cm</td>
<td>1 ( \mu ) m</td>
<td>45</td>
<td>0</td>
<td>60 minutes</td>
</tr>
<tr>
<td>12.2 cm</td>
<td>5 ( \mu ) m</td>
<td>45</td>
<td>0</td>
<td>60 minutes</td>
</tr>
<tr>
<td>17.8 cm</td>
<td>10 ( \mu ) m</td>
<td>45</td>
<td>0</td>
<td>60 minutes</td>
</tr>
</tbody>
</table>

*Table 5. Details of the infusion length v/s time plotted for \( \Delta P = 0, \) and \( \theta =45 \) degree*

<table>
<thead>
<tr>
<th>Infusion Length</th>
<th>Radius of the capillary</th>
<th>( \theta )</th>
<th>( \Delta P ) (dyne/cm(^2))</th>
<th>Time duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 cm</td>
<td>1 ( \mu ) m</td>
<td>90</td>
<td>0</td>
<td>60 minutes</td>
</tr>
<tr>
<td>0 cm</td>
<td>5 ( \mu ) m</td>
<td>90</td>
<td>0</td>
<td>60 minutes</td>
</tr>
<tr>
<td>0 cm</td>
<td>10 ( \mu ) m</td>
<td>90</td>
<td>0</td>
<td>60 minutes</td>
</tr>
</tbody>
</table>

*Table 6. Details of the infusion length v/s time plotted for \( \Delta P = 0, \) and \( \theta =90 \) degree*
Similarly, we have performed the calculations for the equation we got in (4.82) which has corrected the equation presented in [51].

The graph for $\Delta P = 0$, and $\theta=45$

![Diagram showing infusion length vs time plot for $\Delta P = 0$, and $\theta=45$ degree for corrected equation.](image)

*Figure 17. Infusion length v/s time plot for $\Delta P = 0$, and $\theta=45$ degree for corrected equation*
Similarly, the graph for $\Delta P = 0$, and $\theta = 90$

![Graph](image)

*Figure 18. Infusion length v/s time plot for $\Delta P = 0$, and $\theta = 90$ degree for corrected equation*

The result of the graphs from Figure 17 and Figure 18 can be summarized in the table below.

<table>
<thead>
<tr>
<th>Infusion Length</th>
<th>Radius of the capillary</th>
<th>$\theta$</th>
<th>$\Delta P$ (dyne/cm²)</th>
<th>Time duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.7 cm</td>
<td>1 µm</td>
<td>45</td>
<td>0</td>
<td>60 minutes</td>
</tr>
<tr>
<td>8.9 cm</td>
<td>5 µm</td>
<td>45</td>
<td>0</td>
<td>60 minutes</td>
</tr>
<tr>
<td>12.2 cm</td>
<td>10 µm</td>
<td>45</td>
<td>0</td>
<td>60 minutes</td>
</tr>
</tbody>
</table>

*Table 7. Details of the infusion length v/s time plotted for $\Delta P = 0$, and $\theta = 45$ degree for corrected equation*
Sanity check for θ= 90 degree and ΔP = 0

Referring to equations (4.82) and (4.84) as well as equations (5.3) and (5.4). For the equation derived in (4.82), we have,

\[ A = \frac{2 \sigma \cos \theta + r \Delta P}{\rho r} \quad \text{and} \quad B = \frac{8 \mu}{\rho r^2} \cdot \frac{d^2L^2}{dt^2} + \frac{8 \mu}{\rho r^2} \frac{dL}{dt} \quad L + gL = \frac{2 \sigma \cos \theta}{\rho r} \]  

(5.5)

And for the equation derived in (4.84), we have,

\[ A = \frac{4 \sigma \cos \theta + 2 r \Delta P}{\rho r} \quad \text{and} \quad B = \frac{8 \mu}{\rho r^2} \quad \text{and} \]

\[ \frac{d^2L^2}{dt^2} + \frac{8 \mu}{\rho r^2} \frac{dL}{dt} \quad L + 2gL = \frac{4 \sigma \cos \theta}{\rho r} \]  

(5.6)
For $\theta=90$ degree, $A$ becomes 0 as the cosine of 90 is 0. After solving the remaining equation, we only left with a constant value of $B$. so our graph is a straight line.
Chapter 6

Conclusion

The goal of this thesis is to correct the equations presented in [51] and provide the detailed mathematical calculations for the proof of the corrected equation. The differences in infusion length were verified using the equations and graphs in chapter 5. The importance of understanding the mechanism of operation of hollow core fiber was explained and different factors affecting the infusion length has been studied in this thesis. The factors then studied in detailed in chapter 4 and calculation methods were presented. The impact of overhead pressure on the infusion length has been seen from the difference of Figure 10 and Figure 12. Figure 14 has shown the significant change in the infusion length as we plot the graph with the corrected equation. The thesis has also verified and presented the data on the impact of the contact angle on the infusion length while neglecting any overhead pressure. The result significantly proved the concept discussed in section 4.4. The infusion length for $\theta=45$ and $\theta=90$ has been plotted and verified at the end of chapter 5.
Bibliography


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